Longitudinal waves

Physics Enhancement Programme for Gifted Students

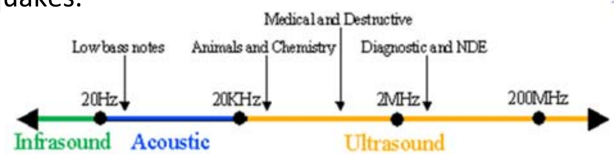
The Hong Kong Academy for Gifted Education
and
Department of Physics, HKBU

Waves

1. **Mechanical waves**
   - e.g. water waves, sound waves, seismic waves, strings in musical instruments
2. **Electromagnetic waves**
   - light (ultraviolet, visible, infrared), mcrowaves, radio waves, television waves, X-rays
3. **Matter (=quantum) waves**
   - electrons, protons, other fundamental particles, atoms and molecules
4. **Gravity waves ← never observed!**

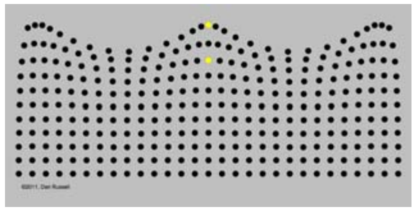
Sound waves in fluid

- Sound in a fluid (gas or liquid) is a longitudinal mechanical vibration with frequencies from about 20 Hz to about 20000 Hz which is the typical range of human hearing.
- Longitudinal waves of higher frequency, which are called ultrasonic waves ("ultrasound"), are used in locating underwater objects and in medical imaging.
- Longitudinal mechanical waves of lower frequency, called infrasonic waves ("infrasound"), occur as seismic waves in earthquakes.



Sound waves in fluid

- The branch of physics and engineering deals with the study of mechanical waves of all frequencies, with both transverse and longitudinal vibrations in the case of solids.
- In this lecture, we consider mainly sound waves in air, which are strictly longitudinal.



Water waves
(both transverse and longitudinal vibrations)

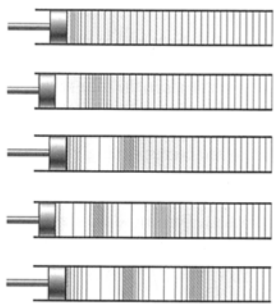


FIGURE 19-1. Sound waves generated in a tube by a moving piston, which might represent the moving cone of a loudspeaker. The vertical lines divide the compressible medium in the tube into layers of equal mass.

Travelling sound waves

- As the piston oscillates, it causes variations in the density of the air in the tube.
  - ❖ The regions of high density are called compressions, the regions of low density are called rarefactions.
  - ❖ As the sound wave travels, the compressions and rarefactions travel along the tube.
- The air density in the tube can be expressed as

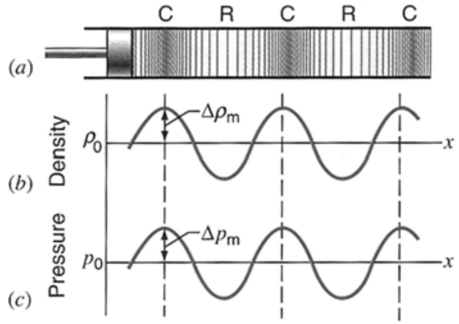
$$\rho(x,t) = \rho_o + \Delta\rho(x,t)$$

, where $|\Delta\rho(x,t)| \ll \rho_o$ is the small fluctuation in the density caused by the sound wave

Travelling sound waves

- If the piston is driven so that its position can be described by a sine function, then the density and pressure of air in the tube will also vary sinusoidally.

$$\Delta p(x,t) = \Delta p_m \sin(kx - \omega t)$$



Relationship between Δp_m and $\Delta\rho_m$

- The relationship between the pressure amplitude Δp_m and the density amplitude $\Delta\rho_m$ depends on the mechanical properties of the medium.

We introduce the bulk modulus B of the medium → $B = -\Delta p/(\Delta V/V)$ ······ (1)
which describes the relative change in volume of an element of fluid in response to a change in pressure.

With density $\rho = m/V$, then $d\rho = -\left(m/V^2\right)dV = -\left(\rho/V\right)dV$

$$\Delta\rho = -\left(\rho/V\right)\Delta V \quad \cdots\cdots (2)$$

Combine (1) and (2), we have $\Delta\rho = -\rho\dfrac{\Delta V}{V} = -\rho_o\dfrac{-\Delta p}{B} = \Delta p\dfrac{\rho_o}{B}$

In terms of the density and pressure amplitudes, $\boxed{\Delta\rho_m = \Delta p_m \dfrac{\rho_o}{B}}$

The speed (velocity) of sound

- Similar to the case of the transverse mechanical wave, the speed of a sound wave depends on the ratio of an elastic property of the medium and an inertial property.

$$\boxed{v = \sqrt{\frac{B}{\rho_o}}}$$

- The speed of sound in fluids (gases and liquids) is in terms of the bulk modulus B and the density ρ_m. Note that the speed of sound in a fluid depends only on the properties of the medium and NOT on the frequency or wavelength of the wave.

The speed of sound

TABLE 19-1 The Speed of Sound[a]

Medium	Speed (m/s)
Gases	
Air (0°C)	331
Air (20°C)	343
Helium	965
Hydrogen	1284
Liquids	
Water (0°C)	1402
Water (20°C)	1482
Seawater[b]	1522
Solids[c]	
Aluminum	6420
Steel	5941
Granite	6000

[a] At 0°C and 1 atm pressure, unless indicated otherwise.
[b] At 20°C and 3.5% salinity.
[c] Longitudinal waves; speeds of transverse waves are about half as large as those for longitudinal waves.

Section summary

- ❖ Human audio spectrum: 20Hz – 20kHz
  Ultrasound: > 20kHz
  Infrasound: < 20Hz
- ❖ Longitudinal oscillations of air → density and pressure
  (1) compressions: regions of high density
  (2) rarefractions: regions of low density

$\rho = m/V$

- ❖ Bulk modulus B of medium, relationship btw. Δp_m and $\Delta\rho_m$

$$B = -\Delta p/(\Delta V/V) \qquad \Delta\rho_m = \Delta p_m \frac{\rho_o}{B}$$

- ❖ Speed of sound ← depends on the properties of the medium

$$v = \sqrt{\frac{B}{\rho_o}}$$

Stiffer medium = faster sound wave
Denser medium = slower sound wave
Speed of sound in air (20°C) = 343 ms^{-1}

Intensity of sound waves

- When we compare different sounds, it is useful to use the intensity (average power per unit area) of the wave.
- The response of the ear to sound of increasing intensity is approximately logarithmic, thus it is convenient to introduce a logarithmic scale of intensity called the sound level SL:

$$SL = 10\log\frac{I}{I_o}$$

Unit: dB or decibel

- The SL is defined with respect to a reference intensity I_o, which is chosen to be 10^{-12} W/m^2 (a typical value for the threshold of human hearing).

Sound Level and Decibel (dB)

- Sound level is measured in decibels (dB).
- A sound of intensity I_o (10^{-12} W/m^2) has a SL of 0 dB, whereas sound at the upper range of human hearing, called the threshold of pain, has an intensity of 1 W/m^2 and a SL of 120 dB (12 orders of magnitude stronger!).
- Multiplication of the intensity I by a factor 10 corresponds to adding 10 dB to the SL.

Sound Level and Decibel (dB)

- We can use dB as a relative measure to compare different sounds with one another.

  Suppose we wish to compare two sounds of intensities I_1 and I_2:

$$SL_1 - SL_2 = 10\log\frac{I_1}{I_o} - 10\log\frac{I_2}{I_o}$$

$$= 10\log\frac{I_1}{I_2}$$ ← use log (a) – log (b) = log (a/b)

For example, two sounds whose intensity ratio is 2 differ in SL by 10 log 2 = 3 dB.

Sound Level and Decibel (dB)

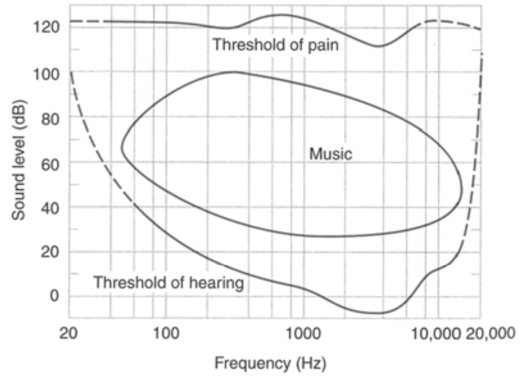


FIGURE 19-5. The average range of sound levels for human hearing. Note the dependence of the threshold levels on frequency. A sound that we can just hear at 100 Hz must have 1000 times the acoustic power (30 dB greater sound level) than one we can just hear at 1000 Hz, because our ear is that much less sensitive at 100 Hz.

Sound Level

TABLE 19-2 Some Intensities and Sound Levels

Sound	*Intensity I* (W/m^2)	*Sound Level* (dB)
Threshold of hearing	1×10^{-12}	0
Rustle of leaves	1×10^{-11}	10
Whisper (at 1 m)	1×10^{-10}	20
City street, no traffic	1×10^{-9}	30
Office, classroom	1×10^{-7}	50
Normal conversation (at 1 m)	1×10^{-6}	60
Jackhammer (at 1 m)	1×10^{-3}	90
Rock group	1×10^{-1}	110
Threshold of pain	1	120
Jet engine (at 50 m)	10	130
Space shuttle engine (at 50 m)	1×10^{8}	200

Example 1

Spherical sound waves are emitted uniformly in all directions from a point source, the radiated power P being 25 W. What are the intensity and the sound level of the sound wave a distance r = 2.5 m from the source?

P = 25 W and r = 2.5 m are given.

$$SL = 10\log\frac{I}{I_o}$$

I is unknown. Calculate I using given P and r.

I_o is known and $I_o = 10^{-12}$ W/m^2

Answer to Example 1

All the radiated power P must pass through a sphere of radius r centered on the source. Thus

$$I = \frac{P}{4\pi r^2}$$

We see that the intensity of the sound drops off as the inverse square of the distance from the source. Numerically, we have

$$I = \frac{25\ \text{W}}{(4\pi)(2.5\ \text{m})^2} = 0.32\ \text{W/m}^2$$

$$SL = 10\log\frac{I}{I_o} = 10\log\frac{0.32\ \text{W/m}^2}{10^{-12}\ \text{W/m}^2} = 115\ \text{dB}$$

A comparison of this result with the list in Table 19-2 shows this sound level to be dangerous to a person's hearing.

Section summary

- ❖ Sound level: $SL = 10\log\frac{I}{I_o}$ Unit: dB or decibel
- ❖ Intensity = power / area → $I \propto \frac{1}{r^2}$
- ❖ Threshold of hearing: $I_o = 10^{-12}$ W/m^2 (i.e. 0 dB)
- ❖ Threshold of pain: $I = 1$ W/m^2 (i.e. 120 dB)
- ❖ Threshold of hearing or pain → frequency dependent
- ❖ Comparison between two sounds: $SL_1 - SL_2 = 10\log\frac{I_1}{I_2}$

Interference of sound waves

- The principle of superposition also applies to sound waves.
- The figure below shows two loudspeakers driven from a common source. At point P, the pressure variation due only to speaker S_1 is Δp_1, and that due to S_2 alone is Δp_2. The total pressure disturbance at point P is $\Delta p = \Delta p_1 + \Delta p_2$.

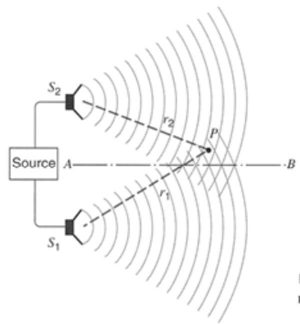
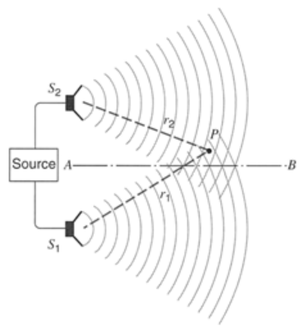


FIGURE 19-6. Two loudspeakers S_1 and S_2, driven by a common source, send signals to point P, where the signals interfere.

Interference of sound waves

- The type of interference that occurs at point P depends on the phase difference $\Delta\phi$ between the waves. The phase difference $\Delta\phi$ between the two waves arriving at P depends on the path difference $\Delta L = |r_1 - r_2|$



- $\Delta\phi$ and ΔL are related by

$$\frac{\Delta\phi}{2\pi} = \frac{\Delta L}{\lambda}$$

(since $\Delta L = \lambda$ gives $\Delta\varphi = 2\pi$)

Interference of sound waves

- For some locations of point P, the pressure variations arrive in phase ($\Delta\phi = 0, 2\pi, 4\pi, \ldots$) and interfere constructively. That is

$$\Delta\phi = m(2\pi) \qquad m = 0,1,2,\ldots \qquad \because \frac{\Delta\phi}{2\pi} = \frac{\Delta L}{\lambda}$$

❖ Path difference for constructive interference $\Delta L = m\lambda \qquad (m = 0,1,2,\ldots)$

- For other locations of P, the waves arrives out of phase ($\Delta\phi = \pi, 3\pi, 5\pi, \ldots$) and interfere destructively. That is

$$\Delta\phi = \left(m + \frac{1}{2}\right)2\pi \quad m = 0,1,2,\ldots$$

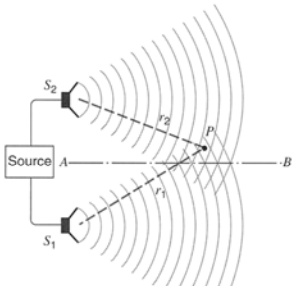
❖ Path difference for destructive interference $\Delta L = \left(m + \frac{1}{2}\right)\lambda \quad (m = 0,1,2,\ldots)$

Interference of sound waves

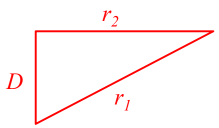
- At the points with destructive interference, that is, at locations where $|r_1 - r_2| = \lambda/2, 3\lambda/2, 5\lambda/2, \ldots$, the intensity has a minimum value (not necessarily zero, because in general the two waves arrive at point P with difference amplitudes).
- Locations of destructive interference correspond to "dead spots" in the listening environment of the speakers.

Example 2

In the geometry of Fig. 19-6, a listener is seated at a point a distance of 1.2 m directly in front of one speaker. Two speakers, which are separated by a distance D of 2.3 m, emit pure tones of wavelength λ. The waves are in phase when they leave the speakers. For what wavelengths will the listener hear a minimum in the sound intensity?



Minimum intensity → destructive interference



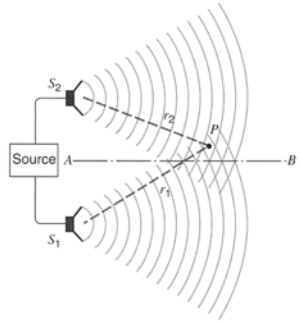
D and r_2 are given.

$\Delta L = |r_1 - r_2| = \lambda/2, 3\lambda/2, 5\lambda/2, \ldots$ → Find λ.

FIGURE 19-6. Two loudspeakers S_1 and S_2, driven by a common source, send signals to point P, where the signals interfere.

Answer to Example 2

The minimum sound intensity occurs when the waves from the two speakers interference destructively. If the listener is seated in front of speaker 2, then r_2 = 1.2 m, and r_1 can be found from the Pythagorean formula,



$$r_1 = \sqrt{r_2^2 + D^2} = \sqrt{(1.2\text{ m})^2 + (2.3\text{ m})^2} = 2.6\text{ m}$$

Then $r_1 - r_2$ = 2.6 m – 1.2 m = 1.4 m.

For destructive interference, $\Delta L = |r_1 - r_2| = \lambda/2, 3\lambda/2, 5\lambda/2, \ldots$

corresponding to λ = 2.8 m, 0.93 m, 0.56 m, ...

Note: Complete destructive interference will not occur at this location, because the two waves arriving at the observation point have different amplitudes, if they leaves the speakers with equal amplitudes.

Section summary

❖ Phase difference $\Delta\phi$ and path difference ΔL: $\dfrac{\Delta\phi}{2\pi} = \dfrac{\Delta L}{\lambda}$

$$\Delta L = |r_1 - r_2|$$

❖ Constructive interference
→ in-phase ($\Delta\phi = 0, 2\pi, 4\pi, \ldots$) → $\Delta L = m\lambda \qquad (m = 0, 1, 2, \ldots)$

❖ Destructive interference
→ out of phase ($\Delta\phi = \pi, 3\pi, 5\pi, \ldots$) → $\Delta L = \left(m + \frac{1}{2}\right)\lambda$

$(m = 0, 1, 2, \ldots)$

→ minimum intensity, i.e. dead spot
(not zero, because of different amplitudes)

Source of Musical Sound

- Closed end → node
  Open end → antinode
  Standing wave patterns

- Consider pipes with both ends open:

Fundamental or first harmonic
(2 antinodes at the ends, 1 node in the middle)

$$L = \frac{\lambda}{2}, \qquad \lambda = 2L$$

Resonant frequency: $f = \dfrac{v}{\lambda} = \dfrac{v}{2L}$

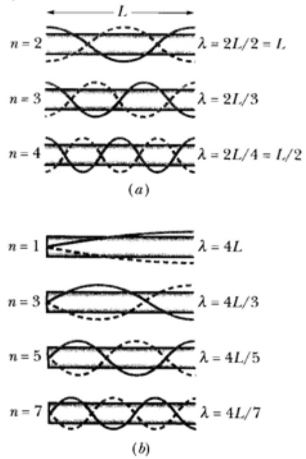


FIG. 17-15 Standing wave patterns for string waves superimposed on pipes to represent standing sound wave patterns in the pipes. (*a*) With *both* ends of the pipe open, any harmonic can be set up in the pipe. (*b*) With only *one* end open, only odd harmonics can be set up.

Source of Musical Sound

- Case 1: pipes with both open ends:

In general for harmonic number n, $\quad L = n\dfrac{\lambda}{2}, \qquad \lambda = \dfrac{2L}{n} \qquad$ for $n = 1, 2, 3, \ldots$

Resonant frequency: $\boxed{f = \dfrac{v}{\lambda} = n\dfrac{v}{2L}}$ for $n = 1, 2, 3, \ldots$

- Case 2: pipes with only one open end (another end closed):

$$L = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \frac{7\lambda}{4}, \ldots \qquad \lambda = 4L, \frac{4L}{3}, \frac{4L}{5}, \frac{4L}{7}, \ldots$$

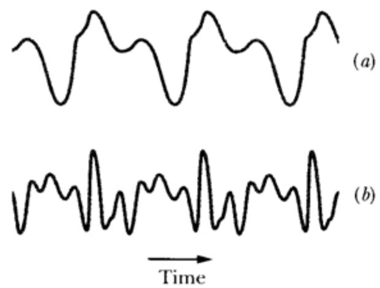
In general, $\lambda = \dfrac{4L}{n}$ for $n = 1, 3, 5, \ldots$

Resonant frequency: $\boxed{f = \dfrac{v}{\lambda} = n\dfrac{v}{4L}}$ for $n = 1, 3, 5, \ldots$

Source of Musical Sound

- In general, when a musical instrument produces a tone, the fundamental as well as higher harmonics are generated simultaneously.
  → This gives rise to the different waveforms generated by different instruments.
- Hence different instruments have different sounds: different "timbres".

FIG. 17-17 The wave forms produced by (*a*) a flute and (*b*) an oboe when played at the same note, with the same first harmonic frequency.



Example 3

Weak background noises from a room set up the fundamental standing wave in a cardboard tube of length L = 67.0 cm with two open ends (due to resonance). Assume that the speed of sound in the air within the tube is 343 ms^{-1}.
(a) What frequency do you hear from the tube?
(b) If you jam your ear against one end of the tube, what fundamental frequency do you hear from the tube?

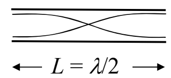
*Fundamental standing wave
*Given L = 67.0 cm and v = 343 ms^{-1}

(a) → Case: two open ends

(b) → Case: one open end, one closed end

Answers to Example 3

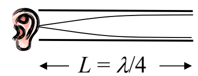
(a) Two open ends



$$L=\frac{\lambda}{2} \qquad \lambda = 2L$$

$$f=\frac{v}{\lambda}=\frac{v}{2L}=\frac{343}{2(0.67)}=256\text{ Hz}$$

(b) Ear closes one end: One fixed end and one open end,



$$L=\frac{\lambda}{4} \qquad \lambda = 4L$$

$$f=\frac{v}{\lambda}=\frac{v}{4L}=\frac{343}{4(0.67)}=128\text{ Hz}$$

Beats

- Consider two sound waves with slightly different frequencies:

  $s_1 = s_m \cos\omega_1 t$ and $s_2 = s_m \cos\omega_2 t$

- Resultant displacement:

  $s = s_1 + s_2 = s_m(\cos\omega_1 t + \cos\omega_2 t).$

- Using trigonometric identity

  $\cos\alpha+\cos\beta=2\cos\frac{1}{2}(\alpha-\beta)\cos\frac{1}{2}(\alpha+\beta)$

  we obtain

  $s=2s_m\cos\frac{1}{2}(\omega_1-\omega_2)t\,\cos\frac{1}{2}(\omega_1+\omega_2)t$

  low "beat" frequency vs. high frequency

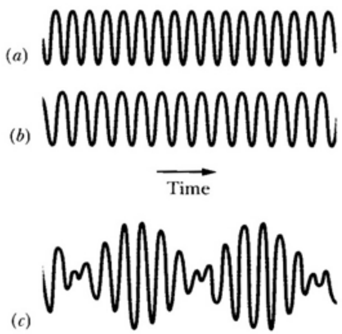
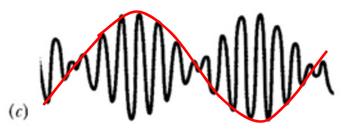


FIG. 17-18 (*a, b*) The pressure variations Δp of two sound waves as they would be detected separately. The frequencies of the waves are nearly equal. (*c*) The resultant pressure variation if the two waves are detected simultaneously.

Beats

The slowly varying amplitude $2s_m\cos\omega' t$ (where $\omega' = ½(\omega_1 - \omega_2)$) is maximum when $\cos\omega' t = \pm 1$.
→ i.e. 2 times in each repetition of the cosine function.
(There are 2 beats in each period T, so we appear to hear double the frequency compared to ω'!)

Hence the beat frequency is:



$$\omega_{\text{beat}} = 2|\omega'| = |\omega_1 - \omega_2|$$

$$f_{\text{beat}} = |f_1 - f_2| \longleftarrow \because \omega = 2\pi f$$

See "Demonstration of Beats"
https://www.youtube.com/watch?v=IQ1q8XvOW6g

Beats

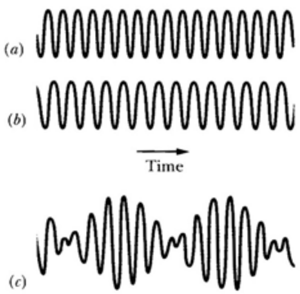
Conclusion:

$$s=[2s_m \cos\omega' t]\cos\omega t,$$

where $\omega' = \frac{1}{2}(\omega_1-\omega_2)$ and $\omega=\frac{1}{2}(\omega_1+\omega_2).$

Since ω_1 *and* ω_2 are nearly equal, $\omega >> \omega'$.

Hence the resultant displacement consists of an oscillation with angular frequency ω and a slowly changing amplitude with angular frequency ω'.



Beats

Example:

A tuning fork with a frequency of 440Hz along with the middle A string of a violin, and 2 beats per second are heard.
What are the possible frequencies of the A string?

Difference between the string and tuning fork frequencies = 2 Hz

$f_{beat} = |f_1 - f_2|$

Possible frequencies = 438 Hz, 442 Hz

Section summary

- Pipes with both open ends

→ $f=\frac{v}{\lambda}=n\frac{v}{2L}$ for $n=1,2,3,...$ Any harmonic can be set up in the pipe.

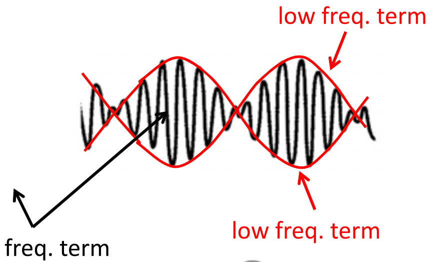
- Pipes with only one open end (another end closed)

→ $f=\frac{v}{\lambda}=n\frac{v}{4L}$ for $n=1,3,5,...$ Only odd harmonic can be set up in the pipe.

- Beat frequency: $f_{\text{beat}}=f_1-f_2$

$s=[2s_m \cos\omega' t]\cos\omega t$, where

$\omega'=\frac{1}{2}(\omega_1-\omega_2)$ and $\omega=\frac{1}{2}(\omega_1+\omega_2)$

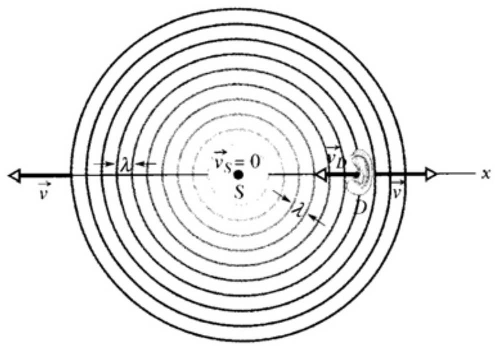


Doppler Effect

See Youtube "Doppler effect, shock wave , and sonic boom"
https://www.youtube.com/watch?v=h4OnBYrbCjY
https://www.youtube.com/watch?v=XmDVvGNtgMg

Moving Detector; Stationary Source

FIG. 17-19 A stationary source of sound S emits spherical wavefronts, shown one wavelength apart, that expand outward at speed v. A sound detector D, represented by an ear, moves with velocity $\vec{v}_D$ toward the source. The detector senses a higher frequency because of its motion.



Doppler Effect

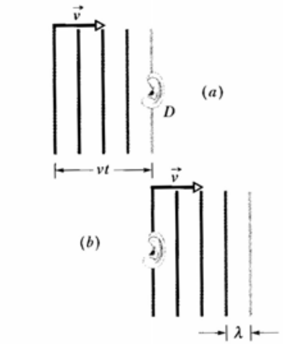


FIG. 17-20 The wavefronts of Fig. 17-19, assumed planar, (a) reach and (b) pass a stationary detector D; they move a distance vt to the right in time t.

Moving Detector; Stationary Source

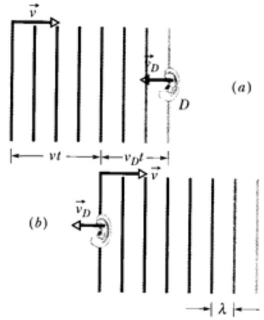


FIG. 17-21 Wavefronts traveling to the right (a) reach and (b) pass detector D, which moves in the opposite direction. In time t, the wavefronts move a distance vt to the right and D moves a distance $v_D t$ to the left.

In time t:
- the wavefronts move a distance vt,
- the detector moves a distance $v_D t$,
- the range of waves intercepted by the detector = $vt+v_D t$,
- the number of wavefronts intercepted by the detector = $(vt+v_D t)/\lambda$.

Doppler Effect

Moving Detector; Stationary Source

The frequency observed by the detector approaching the source:

$$f' = \frac{(vt+v_D t)/\lambda}{t} = \frac{v+v_D}{\lambda}$$

Since $\lambda = v/f$,

$$f' = \frac{v+v_D}{v/f} = f\frac{v+v_D}{v}$$

Similarly, if the detector moves away from the source,

$$f' = f\frac{v-v_D}{v}$$

Summarizing,

$$f' = f\frac{v \pm v_D}{v}$$

Doppler Effect

Moving Source ; Stationary Detector

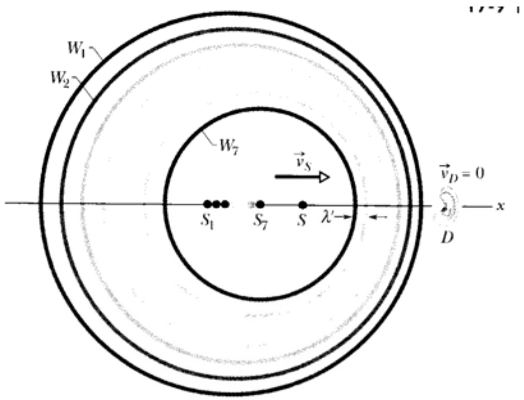


FIG. 17-22 A detector D is stationary, and a source S is moving toward it at speed v_S. Wavefront W_1 was emitted when the source was at S_1, wavefront W_7 when it was at S_7. At the moment depicted, the source is at S. The detector senses a higher frequency because the moving source, chasing its own wavefronts, emits a reduced wavelength λ' in the direction of its motion.

Doppler Effect

Moving Source ; Stationary Detector

In a period T,
the distance moved by the wavefront $W_1 = vT$,
the distance moved by the source = $v_S T$,
the distance between the wavefronts W_1 and $W_2 = vT - v_S T$.

The frequency observed by the detector (source is approaching):

$$f' = \frac{v}{\lambda'} = \frac{v}{vT - v_S T} = \frac{v}{v/f - v_S/f} = f\frac{v}{v - v_S}$$

If the source moves away from the detector,

$$f' = f\frac{v}{v + v_S}$$

Summarizing,

$$f' = f\frac{v}{v \mp v_S}$$

General Doppler Effect Equation

❖ When both the source and detector are moving (along the same line; otherwise vectorial),

$$f' = f\,\frac{v \pm v_D}{v \mp v_S}$$

$v_S = 0$ reduces to the equation for stationary source.
$v_D = 0$ reduces to the equation for stationary detector.

$$f' = f\,\frac{v \pm v_D}{v \mp v_S}$$

Numerator:
- upper +ve sign → detector moves towards source
- lower –ve sign → detector moves away from source

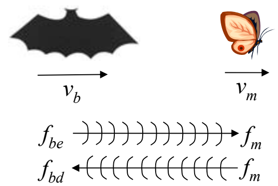
Denominator:
- upper –ve sign → source moves towards detector
- lower +ve sign → source moves away from detector

Example 4

Bats navigate and search out prey by emitting, and then detecting reflections of, ultrasonic waves, which are sound waves with frequencies greater than what can be heard by a human. Suppose a bat emits ultrasound at frequency f_{be} = 82.52 kHz while flying with velocity v_b = 9 ms^{-1}. It chases a moth that flies with velocity v_m = 8 ms^{-1}.

(a) What frequency f_{md} does the moth detect?

(b) What frequency f_{bd} does the bat detect in the returning echo from the moth?



speed of sound in the air = 343 ms^{-1}

Answers to Example 4

$$f' = f\,\frac{v \pm v_D}{v \mp v_S}$$

(a) Detection by moth (moth is the detector, bat is the source):

detector away from source → -ve

$$f_{md} = f_{be}\,\frac{v - v_m}{v - v_b} = 82.52\left(\frac{343-8}{343-9}\right) = 82.767 \approx 82.8 \text{ kHz}$$

source towards detector → -ve

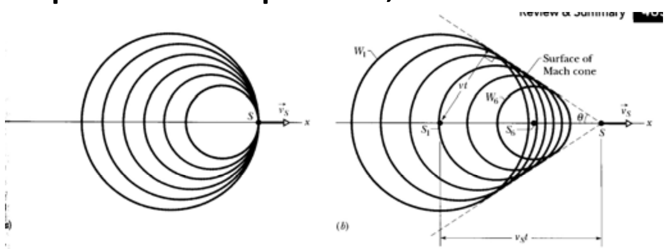
(b) Detection of echo by bat (bat is detector, moth is source of the reflected wave with Doppler-shifted frequency):

detector towards source → +ve

$$f_{bd} = f_{md}\,\frac{v + v_b}{v + v_m} = 82.767\left(\frac{343+9}{343+8}\right) = 83.00 \approx 83.0 \text{ kHz}$$

source away from detector → +ve

Supersonic speeds; Shock waves



17-23 (a) A source of sound S moves at speed v_S equal to the speed of sound and thus as fast as the wavefronts it gen- (b) A source S moves at speed v_S faster than the speed of sound and thus faster than the wavefronts. When the source position S_1 it generated wavefront W_1, and at position S_6 it generated W_6. All the spherical wavefronts expand at the of sound v and bunch along the surface of a cone called the Mach cone, forming a shock wave. The surface of the cone lf-angle θ and is tangent to all the wavefronts.

FIG. 17-24 Shock waves produced by the wings of a Navy FA 18 jet. The shock waves are visible because the sudden decrease in air pressure in them caused water molecules in the air to condense, forming a fog. *(U.S. Navy photo by Ensign John Gay)*

See Youtube "Sonic Boom Physics"
https://www.youtube.com/watch?v=GstwLblPpIM

Supersonic speeds; Shock waves

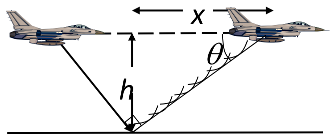
- When v approaches v_S, the Doppler frequency f' becomes infinite since

$$f' = f\frac{v}{v - v_S}$$

- When the sound speed v exceeds the source speed v_S, the Doppler effect equation does not apply. All wavefronts bunch along a V-shaped envelope.
- This is called the *Mach cone*. A **shock wave** is produced.
- Note that the envelope touches the circular wavefronts. Therefore the radius ending at the tangent point is normal to the Mach cone.

Mach cone angle: $\sin\theta = \frac{vt}{v_S t} = \frac{v}{v_S}$ ← v_S / v is called the Mach number.

When is the boom heard?

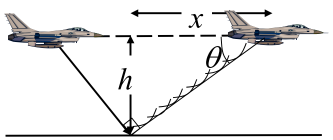


- The sonic boom detected by a ground observer was generated by a supersonic jet before it flew overhead.
- However, when the sonic boom arrives at the ground observer, the supersonic jet has flown ahead (located at the tip of the Mach cone).

Example 5

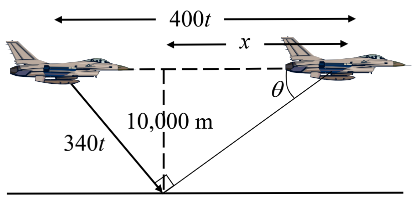
The speed of sound is 340 ms^{-1}. A plane flies horizontally at an altitude of 10,000 m and a speed of 400 ms^{-1}. When an observer on the ground hears the sonic boom, what is the horizontal distance x from the point on its path directly above the observer to the plane?

Given:
(1) h = 10000 m
(2) speed of sound = 340 ms^{-1}
(3) speed of plane = 400 ms^{-1}



* trigonometry problem → about length (or distance) and angle
* distance = speed x time

Answer to Example 5



From the figure, $\sin\theta = \frac{340t}{400t} = 0.85$

$$\theta = 58.2^o$$

$$x = 10000\cot\theta = 6200\text{ m}$$

Section summary

- ❖ Doppler effect
  - the apparent change in f of a wave caused by relative motion between the source of the wave and the observer.

- ❖ General Doppler effect equation: $f' = f\dfrac{v \pm v_D}{v \mp v_S}$

- ❖ Sonic boom, supersonic waves, shock waves
  (1) $v_S > v$ → Doppler effect does not apply
  (2) V-shaped envelope → Mach cone
  (3) Mach cone angle: $\sin\theta = \dfrac{vt}{v_S t} = \dfrac{v}{v_S}$
  (4) Mach number = $\dfrac{v_S}{v}$