## PEP 2017

## Assignment 13

27.4 - A particle with mass $1.81 \times 10^{-3} \mathrm{~kg}$ and a charge of $1.22 \times 10^{-8} \mathrm{C}$ has, at a given instant, a velocity $\overrightarrow{\boldsymbol{v}}=(3.00 \times$ $\left.10^{4} \mathrm{~m} / \mathrm{s}\right) \hat{\boldsymbol{J}}$. What are the magnitude and direction of the particle's acceleration produced by a uniform magnetic field $\overrightarrow{\boldsymbol{B}}=(1.63 \mathrm{~T}) \hat{\boldsymbol{\imath}}+$ (0.980 T) $\hat{\boldsymbol{J}}$ ?
27.14 •. The magnetic field ${ }^{-} \overrightarrow{\boldsymbol{B}}$ in a certain region is 0.128 T , and its direction is that of the $+z$-axis in Fig. E27.14. (a) What is the magnetic flux across the surface abcd in the figure? (b) What is the magnetic flux across the surface befc? (c) What is the magnetic flux across the surface aefd? (d) What is the net flux through all five surfaces that enclose the shaded volume?
27.22 - In an experiment with cosmic rays, a vertical beam of particles that have charge of magnitude $3 e$ and mass 12 times the proton mass enters a uniform horizontal magnetic field of 0.250 T and is bent in a semicircle of diameter 95.0 cm , as shown in Fig. E27.22. (a) Find the speed of the particles Figure E27.22
 and the sign of their charge. (b) Is it reasonable to ignore the gravity force on the particles? (c) How does the speed of the particles as they enter the field compare to their speed as they exit the field?
27.39 •. A long wire carrying 4.50 A of current makes two $90^{\circ}$ bends, as shown in Fig. E27.39. The bent part of the wire passes through a uniform $0.240-\mathrm{T}$ magnetic field directed as shown in the figure and confined to a limited region of space. Find the magnitude and direction of the force that the magnetic field exerts on the wire.
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27.45 - The $20.0 \mathrm{~cm} \times \bar{x} 35.0 \mathrm{~cm}$ rectangular circuit shown in Fig. E27.45 is hinged along side $a b$. It carries a clockwise $5.00-\mathrm{A}$ current and is located in a uniform 1.20-T magnetic field oriented perpendicular to two of its sides, as shown. (a) Draw a clear diagram showing the direction of the force that the magnetic field exerts on each segment of the circuit $(a b, b c$, etc.). (b) Of the four forces you

Figure E27.45
 drew in part (a), decide which ones exert a torque about the hinge $a b$. Then calculate only those forces that exert this torque. (c) Use your results from part (b) to calculate the torque that the magnetic field exerts on the circuit about the hinge axis $a b$.

Figure E27.39

27.78 • The rectangular loop shown in Fig. P27.78 is pivoted about the $y$-axis and carries a current of 15.0 A in the direction indicated. (a) If the loop is in a uniform magnetic field with magnitude 0.48 T in the $+x$ direction, find the magnitude and direction of the torque required to hold the loop in the position shown. (b) Repeat part (a) for the case in which the field is in the $-z$-direction. (c) For each of the above magnetic fields, what torque would be required if the loop were pivoted about an axis through its center, parallel to the $y$-axis?

Figure P27.78

27.4. Identify: Apply Newton's second law, with the force being the magnetic force.

SET UP: $\hat{\boldsymbol{j}} \times \hat{\boldsymbol{i}}=-\hat{\boldsymbol{k}}$
EXECUTE: $\overrightarrow{\boldsymbol{F}}=m \overrightarrow{\boldsymbol{a}}=q \overrightarrow{\boldsymbol{v}} \times \overrightarrow{\boldsymbol{B}}$ gives $\overrightarrow{\boldsymbol{a}}=\frac{q \overrightarrow{\boldsymbol{v}} \times \overrightarrow{\boldsymbol{B}}}{m}$ and

$$
\overrightarrow{\boldsymbol{a}}=\frac{\left(1.22 \times 10^{-8} \mathrm{C}\right)\left(3.0 \times 10^{4} \mathrm{~m} / \mathrm{s}\right)(1.63 \mathrm{~T})(\hat{\boldsymbol{j}} \times \hat{\boldsymbol{i}})}{1.81 \times 10^{-3} \mathrm{~kg}}=-\left(0.330 \mathrm{~m} / \mathrm{s}^{2}\right) \hat{\boldsymbol{k}}
$$

27.14. Identify: When $\overrightarrow{\boldsymbol{B}}$ is uniform across the surface, $\Phi_{B}=\overrightarrow{\boldsymbol{B}} \cdot \overrightarrow{\boldsymbol{A}}=B A \cos \phi$.

Set Up: $\overrightarrow{\boldsymbol{A}}$ is normal to the surface and is directed outward from the enclosed volume. For surface $a b c d$, $\overrightarrow{\boldsymbol{A}}=-A \hat{\boldsymbol{i}}$. For surface befc, $\overrightarrow{\boldsymbol{A}}=-A \hat{\boldsymbol{k}}$. For surface $a e f d, \cos \phi=3 / 5$ and the flux is positive.
Execute: (a) $\Phi_{B}(a b c d)=\overrightarrow{\boldsymbol{B}} \cdot \overrightarrow{\boldsymbol{A}}=0$.
(b) $\Phi_{B}($ befc $)=\overrightarrow{\boldsymbol{B}} \cdot \overrightarrow{\boldsymbol{A}}=-(0.128 \mathrm{~T})(0.300 \mathrm{~m})(0.300 \mathrm{~m})=-0.0115 \mathrm{~Wb}$.
(c) $\Phi_{B}($ aefd $)=\overrightarrow{\boldsymbol{B}} \cdot \overrightarrow{\boldsymbol{A}}=B A \cos \phi=\frac{3}{5}(0.128 \mathrm{~T})(0.500 \mathrm{~m})(0.300 \mathrm{~m})=+0.0115 \mathrm{~Wb}$.
(d) The net flux through the rest of the surfaces is zero since they are parallel to the $x$-axis. The total flux is the sum of all parts above, which is zero.
Evaluate: The total flux through any closed surface, that encloses a volume, is zero.
27.22. IDENTIFY: For motion in an arc of a circle, $a=\frac{v^{2}}{R}$ and the net force is radially inward, toward the center of the circle.
SET UP: The direction of the force is shown in Figure 27.22. The mass of a proton is $1.67 \times 10^{-27} \mathrm{~kg}$. EXECUTE: (a) $\overrightarrow{\boldsymbol{F}}$ is opposite to the right-hand rule direction, so the charge is negative. $\overrightarrow{\boldsymbol{F}}=m \overrightarrow{\boldsymbol{a}}$ gives $|q| v B \sin \phi=m \frac{v^{2}}{R} . \phi=90^{\circ}$ and $v=\frac{|q| B R}{m}=\frac{3\left(1.60 \times 10^{-19} \mathrm{C}\right)(0.250 \mathrm{~T})(0.475 \mathrm{~m})}{12\left(1.67 \times 10^{-27} \mathrm{~kg}\right)}=2.84 \times 10^{6} \mathrm{~m} / \mathrm{s}$.
(b) $F_{B}=|q| v B \sin \phi=3\left(1.60 \times 10^{-19} \mathrm{C}\right)\left(2.84 \times 10^{6} \mathrm{~m} / \mathrm{s}\right)(0.250 \mathrm{~T}) \sin 90^{\circ}=3.41 \times 10^{-13} \mathrm{~N}$. $w=m g=12\left(1.67 \times 10^{-27} \mathrm{~kg}\right)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)=1.96 \times 10^{-25} \mathrm{~N}$. The magnetic force is much larger than the weight of the particle, so it is a very good approximation to neglect gravity.
Evaluate: (c) The magnetic force is always perpendicular to the path and does no work. The particles move with constant speed.


Figure 27.22
27.39. Identify: Apply $F=I l B \sin \phi$.

Set UP: Label the three segments in the field as $a, b$, and $c$. Let $x$ be the length of segment $a$. Segment $b$ has length 0.300 m and segment $c$ has length $0.600 \mathrm{~m}-x$. Figure 27.39a shows the direction of the force on each segment. For each segment, $\phi=90^{\circ}$. The total force on the wire is the vector sum of the forces on each segment. ExECUTE: $\quad F_{a}=I l B=(4.50 \mathrm{~A}) x(0.240 \mathrm{~T}) . \quad F_{c}=(4.50 \mathrm{~A})(0.600 \mathrm{~m}-x)(0.240 \mathrm{~T})$. Since $\overrightarrow{\boldsymbol{F}}_{a}$ and $\overrightarrow{\boldsymbol{F}}_{c}$ are in the same direction their vector sum has magnitude
$F_{a c}=F_{a}+F_{c}=(4.50 \mathrm{~A})(0.600 \mathrm{~m})(0.240 \mathrm{~T})=0.648 \mathrm{~N}$ and is directed toward the bottom of the page in Figure 27.39a. $F_{b}=(4.50 \mathrm{~A})(0.300 \mathrm{~m})(0.240 \mathrm{~T})=0.324 \mathrm{~N}$ and is directed to the right. The vector addition diagram for $\overrightarrow{\boldsymbol{F}}_{a c}$ and $\overrightarrow{\boldsymbol{F}}_{b}$ is given in Figure 27.39b.
$F=\sqrt{F_{a c}^{2}+F_{b}^{2}}=\sqrt{(0.648 \mathrm{~N})^{2}+(0.324 \mathrm{~N})^{2}}=0.724 \mathrm{~N} . \tan \theta=\frac{F_{a c}}{F_{b}}=\frac{0.648 \mathrm{~N}}{0.324 \mathrm{~N}}$ and $\theta=63.4^{\circ}$. The net
force has magnitude 0.724 N and its direction is specified by $\theta=63.4^{\circ}$ in Figure 27.39b.
Evaluate: All three current segments are perpendicular to the magnetic field, so $\phi=90^{\circ}$ for each in the force equation. The direction of the force on a segment depends on the direction of the current for that segment.


Figure 27.39
27.45. Identify: The wire segments carry a current in an external magnetic field. Only segments $a b$ and $c d$ will experience a magnetic force since the other two segments carry a current parallel (and antiparallel) to the magnetic field. Only the force on segment $c d$ will produce a torque about the hinge.
SET UP: $F=I l B \sin \phi$. The direction of the magnetic force is given by the right-hand rule applied to the
directions of $I$ and $\overrightarrow{\boldsymbol{B}}$. The torque due to a force equals the force times the moment arm, the perpendicular distance between the axis and the line of action of the force.
EXECUTE: (a) The direction of the magnetic force on each segment of the circuit is shown in Figure 27.45.
For segments $b c$ and $d a$ the current is parallel or antiparallel to the field and the force on these segments is zero


## Figure 27.45

(b) $\overrightarrow{\boldsymbol{F}}_{a b}$ acts at the hinge and therefore produces no torque. $\overrightarrow{\boldsymbol{F}}_{c d}$ tends to rotate the loop about the hinge so it does produce a torque about this axis. $F_{c d}=I l B \sin \phi=(5.00 \mathrm{~A})(0.200 \mathrm{~m})(1.20 \mathrm{~T}) \sin 90^{\circ}=1.20 \mathrm{~N}$
(c) $\tau=F l=(1.20 \mathrm{~N})(0.350 \mathrm{~m})=0.420 \mathrm{~N} \cdot \mathrm{~m}$.

Evaluate: The torque is directed so as to rotate side $c d$ out of the plane of the page in Figure 27.45.
27.78. IDentify: The torque exerted by the magnetic field is $\overrightarrow{\boldsymbol{\tau}}=\overrightarrow{\boldsymbol{\mu}} \times \overrightarrow{\boldsymbol{B}}$. The torque required to hold the loop in place is $-\vec{\tau}$.
Set Up: $\mu=I A . \vec{\mu}$ is normal to the plane of the loop, with a direction given by the right-hand rule that is illustrated in Figure 27.32 in the textbook. $\tau=I A B \sin \phi$, where $\phi$ is the angle between the normal to the loop and the direction of $\overrightarrow{\boldsymbol{B}}$.
EXECUTE: (a) $\tau=I A B \sin 60^{\circ}=(15.0 \mathrm{~A})(0.060 \mathrm{~m})(0.080 \mathrm{~m})(0.48 \mathrm{~T}) \sin 60^{\circ}=0.030 \mathrm{~N} \cdot \mathrm{~m}$, in the $-\hat{\boldsymbol{j}}$
direction. To keep the loop in place, you must provide a torque in the $+\hat{\boldsymbol{j}}$ direction.
(b) $\tau=I A B \sin 30^{\circ}=(15.0 \mathrm{~A})(0.60 \mathrm{~m})(0.080 \mathrm{~m})(0.48 \mathrm{~T}) \sin 30^{\circ}=0.017 \mathrm{~N} \cdot \mathrm{~m}$, in the $+\hat{\boldsymbol{j}}$ direction. You must provide a torque in the $-\hat{\boldsymbol{j}}$ direction to keep the loop in place.
Evaluate: (c) If the loop was pivoted through its center, then there would be a torque on both sides of the loop parallel to the rotation axis. However, the lever arm is only half as large, so the total torque in each case is identical to the values found in parts (a) and (b).

