PEP 2017
Assignment 16

32.11 • An electromagnetic wave has an electric field given by $\vec{E}(y, t) = (3.10 \times 10^5 \text{ V/m})\hat{k} \cos[ky - (12.65 \times 10^{12} \text{ rad/s})t]$. (a) In which direction is the wave traveling? (b) What is the wavelength of the wave? (c) Write the vector equation for $\vec{B}(y, t)$.

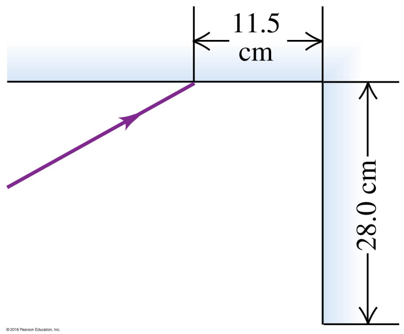
32.17 •• **Fields from a Light Bulb.** We can reasonably model a 75-W incandescent light bulb as a sphere 6.0 cm in diameter. Typically, only about 5% of the energy goes to visible light; the res goes largely to nonvisible infrared radiation. (a) What is the visible-light intensity (in W/m^2) at the surface of the bulb? (b) Wha are the amplitudes of the electric and magnetic fields at this surface, for a sinusoidal wave with this intensity?

32.18 •• A sinusoidal electromagnetic wave from a radio statio passes perpendicularly through an open window that has are 0.500 m^2. At the window, the electric field of the wave has rms value 0.0200 V/m. How much energy does this wave carr through the window during a 30.0-s commercial?

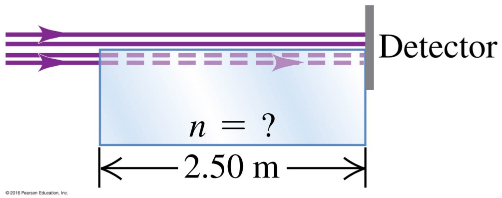
32.31 • **Microwave Oven.** The microwaves in a certain microwave oven have a wavelength of 12.2 cm. (a) How wide must this oven be so that it will contain five antinodal planes of the electric field along its width in the standing-wave pattern? (b) What is the frequency of these microwaves? (c) Suppose a manufacturing error occurred and the oven was made 5.0 cm longer than specified in part (a). In this case, what would have to be the frequency of the microwaves for there still to be five antinodal planes of the electric field along the width of the oven?

32.54 •• **CP** **Solar Sail 2.** NASA is giving serious consideration to the concept of *solar sailing*. A solar sailcraft uses a large, low-mass sail and the energy and momentum of sunlight for propulsion. (a) Should the sail be absorbing or reflective? Why? (b) The total power output of the sun is 3.9×10^{26} W. How large a sail is necessary to propel a 10,000-kg spacecraft against the gravitational force of the sun? Express your result in square kilometers. (c) Explain why your answer to part (b) is independent of the distance from the sun.

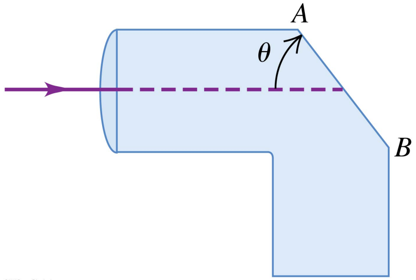
(33.1-A) Two plane mirrors intersect at right angles. A laser beam strikes the first of them at a point 11.5cm from their point of intersection. For what angle of incidence at the first mirror will this ray strike the midpoint of the second mirror (which is 28cm long) after reflecting from the first mirror?



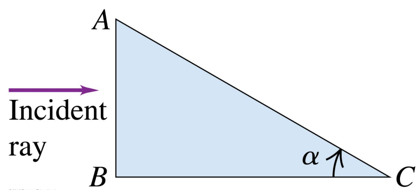
(33.8-A) A laser beam shines along the surface of a block of transparent material. Half of the beam goes straight to travels through the block an then hits the detector. The time delay between the arrival of the two light beams at the detector is 6.15ns. What is the index of refraction of this material?



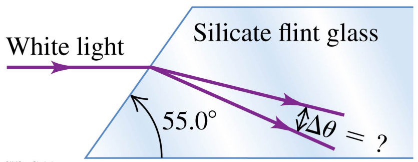
(33.15-A) Light enters a solid pipe made of plastic having an index of refraction of 1.69. The light travels parallel to the upper part of the pipe. You want to cut the face AB so that all the light will reflect back into the pipe after it first strike that face. (a) What is the largest θ can b if the pipe is in air? (b) if th pipe is immured in water of refractive index 1.33, what is the largest θ can be?



(33.21-A) Light is incident along the normal on face AB of a loads prism of refractive index 1.52. Find the largest value the angle α can have without any light refracted out of the prism at face AC if (a) the prism is immersed in air and (b) the prism is immersed in water.



(33.23-A) A narrow beam of white light strikes one face of a slab of silicate flint glass. The light is traveling parallel to the two adjoining faces. For the transmitted light inside the glass, through what angle $\Delta\theta$ is the portion of the visible spectrum between 400nm ($n_{\text{purple}} = 1.66$) and 700nm ($n_{\text{red}} = 1.61$) dispersed?



(33.28-A) Light of original intensity I_0 passes through two ideal polarising filters having their polarising axes oriented as shown. You want to adjust the angle ϕ so that the intensity at point P is equal to $I_0/18$. (a) If the original light is unpolarised, what should ϕ be? (b) If the original light is linearly polarized in the same direction as the polarising axis of the first polariser the light reaches, what should ϕ be?

32.11. **IDENTIFY** and **SET UP:** Compare the $\vec{E}(y,t)$ given in the problem to the general form given by Eq. (32.17). Use the direction of propagation and of $\vec{E}$ to find the direction of $\vec{B}$.

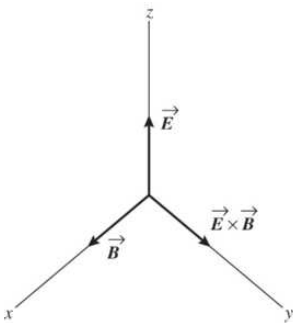
(a) EXECUTE: The equation for the electric field contains the factor $\cos(ky-\omega t)$ so the wave is traveling in the $+y$-direction.

(b) $\vec{E}(y,t)=(3.10\times10^5\ \text{V/m})\hat{k}\cos[ky-(12.65\times10^{12}\ \text{rad/s})t]$

Comparing to Eq. (32.17) gives $\omega=12.65\times10^{12}$ rad/s

$$\omega=2\pi f=\frac{2\pi c}{\lambda}\text{ so }\lambda=\frac{2\pi c}{\omega}=\frac{2\pi(2.998\times10^8\ \text{m/s})}{(12.65\times10^{12}\ \text{rad/s})}=1.49\times10^{-4}\ \text{m}$$

(c)



$\vec{E}\times\vec{B}$ must be in the $+y$-direction (the direction in which the wave is traveling). When $\vec{E}$ is in the $+z$-direction then $\vec{B}$ must be in the $+x$-direction, as shown in Figure 32.11.

Figure 32.11

$$k=\frac{2\pi}{\lambda}=\frac{\omega}{c}=\frac{12.65\times10^{12}\ \text{rad/s}}{2.998\times10^8\ \text{m/s}}=4.22\times10^4\ \text{rad/m}$$

$E_{\text{max}}=3.10\times10^5\ \text{V/m}$

Then $B_{\text{max}}=\dfrac{E_{\text{max}}}{c}=\dfrac{3.10\times10^5\ \text{V/m}}{2.998\times10^8\ \text{m/s}}=1.03\times10^{-3}\ \text{T}$

Using Eq. (32.17) and the fact that $\vec{B}$ is in the $+\hat{i}$ direction when $\vec{E}$ is in the $+\hat{k}$ direction,

$\vec{B}=+(1.03\times10^{-3}\ \text{T})\hat{i}\cos[(4.22\times10^4\ \text{rad/m})y-(12.65\times10^{12}\ \text{rad/s})t]$

EVALUATE: $\vec{E}$ and $\vec{B}$ are perpendicular and oscillate in phase.

32.17. **IDENTIFY:** $I = P/A$. $I = \frac{1}{2}\epsilon_0 c E_{\text{max}}^2$. $E_{\text{max}} = cB_{\text{max}}$.

SET UP: The surface area of a sphere of radius r is $A = 4\pi r^2$. $\epsilon_0 = 8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2$.

EXECUTE: **(a)** $I = \dfrac{P}{A} = \dfrac{(0.05)(75\ \text{W})}{4\pi(3.0\times10^{-2}\ \text{m})^2} = 330\ \text{W/m}^2$.

(b) $E_{\text{max}} = \sqrt{\dfrac{2I}{\epsilon_0 c}} = \sqrt{\dfrac{2(330\ \text{W/m}^2)}{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(3.00\times10^8\ \text{m/s})}} = 500\ \text{V/m}$.

$B_{\text{max}} = \dfrac{E_{\text{max}}}{c} = 1.7\times10^{-6}\ \text{T} = 1.7\ \mu\text{T}$.

EVALUATE: At the surface of the bulb the power radiated by the filament is spread over the surface of the bulb. Our calculation approximates the filament as a point source that radiates uniformly in all directions.

32.18. **IDENTIFY:** The intensity of the electromagnetic wave is given by Eq. (32.29): $I = \frac{1}{2}\epsilon_0 c E_{\text{max}}^2 = \epsilon_0 c E_{\text{rms}}^2$.

The total energy passing through a window of area A during a time t is IAt.

SET UP: $\epsilon_0 = 8.85\times10^{-12}$ F/m

EXECUTE: $\text{Energy} = \epsilon_0 c E_{\text{rms}}^2 At = (8.85\times10^{-12}\ \text{F/m})(3.00\times10^8\ \text{m/s})(0.0200\ \text{V/m})^2(0.500\ \text{m}^2)(30.0\ \text{s}) = 15.9\ \mu\text{J}$

EVALUATE: The intensity is proportional to the square of the electric field amplitude.

32.31. **IDENTIFY:** The nodal and antinodal planes are each spaced one-half wavelength apart.

SET UP: $2\frac{1}{2}$ wavelengths fit in the oven, so $\left(2\frac{1}{2}\right)\lambda = L,$ and the frequency of these waves obeys the equation $f\lambda = c.$

EXECUTE: **(a)** Since $\left(2\frac{1}{2}\right)\lambda = L,$ we have $L = (5/2)(12.2 \text{ cm}) = 30.5 \text{ cm}.$

(b) Solving for the frequency gives $f = c/\lambda = (3.00\times10^8 \text{ m/s})/(0.122 \text{ m}) = 2.46\times10^9 \text{ Hz}.$

(c) $L = 35.5$ cm in this case. $\left(2\frac{1}{2}\right)\lambda = L,$ so $\lambda = 2L/5 = 2(35.5 \text{ cm})/5 = 14.2 \text{ cm}.$

$$f = c/\lambda = (3.00\times10^8 \text{ m/s})/(0.142 \text{ m}) = 2.11\times10^9 \text{ Hz}$$

EVALUATE: Since microwaves have a reasonably large wavelength, microwave ovens can have a convenient size for household kitchens. Ovens using radiowaves would need to be far too large, while ovens using visible light would have to be microscopic.

32.54. **IDENTIFY:** For a totally reflective surface the radiation pressure is $\frac{2I}{c}$. Find the force due to this pressure and express the force in terms of the power output P of the sun. The gravitational force of the sun is $F_g = G\frac{mM_{sun}}{r^2}$.

SET UP: The mass of the sun is $M_{sun} = 1.99 \times 10^{30}$ kg. $G = 6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2$.

EXECUTE: **(a)** The sail should be reflective, to produce the maximum radiation pressure.

(b) $F_{rad} = \left(\frac{2I}{c}\right)A$, where A is the area of the sail. $I = \frac{P}{4\pi r^2}$, where r is the distance of the sail from the sun. $F_{rad} = \left(\frac{2A}{c}\right)\left(\frac{P}{4\pi r^2}\right) = \frac{PA}{2\pi r^2 c}$. $F_{rad} = F_g$ so $\frac{PA}{2\pi r^2 c} = G\frac{mM_{sun}}{r^2}$.

$$A = \frac{2\pi c G m M_{sun}}{P} = \frac{2\pi(3.00\times10^{8}\ \text{m/s})(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(10{,}000\ \text{kg})(1.99\times10^{30}\ \text{kg})}{3.9\times10^{26}\ \text{W}}.$$

$A = 6.42 \times 10^6\ \text{m}^2 = 6.42\ \text{km}^2$.

(c) Both the gravitational force and the radiation pressure are inversely proportional to the square of the distance from the sun, so this distance divides out when we set $F_{rad} = F_g$.

EVALUATE: A very large sail is needed, just to overcome the gravitational pull of the sun.

33.1. IDENTIFY: For reflection, $\theta_r = \theta_a$.

SET UP: The desired path of the ray is sketched in Figure 33.1.

EXECUTE: $\tan\phi = \dfrac{14.0\text{ cm}}{11.5\text{ cm}}$, so $\phi = 50.6°$. $\theta_r = 90° - \phi = 39.4°$ and $\theta_r = \theta_a = 39.4°$.

EVALUATE: The angle of incidence is measured from the normal to the surface.

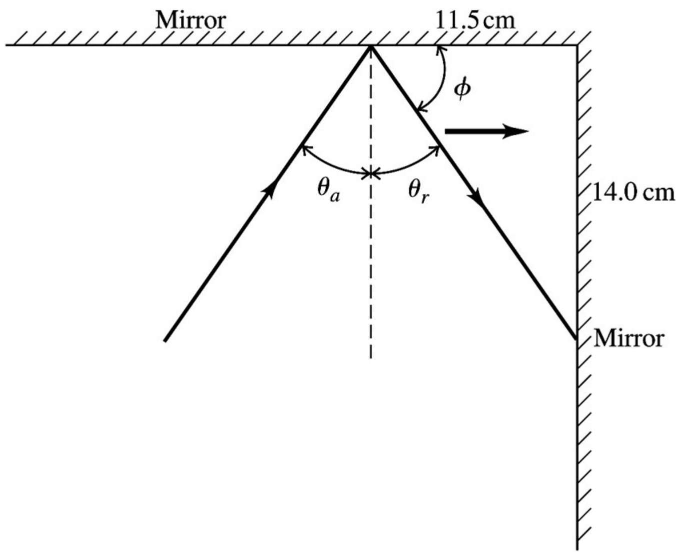


Figure 33.1

33.8. IDENTIFY: The time delay occurs because the beam going through the transparent material travels slower than the beam in air.

SET UP: $v = \dfrac{c}{n}$ in the material, but $v = c$ in air.

EXECUTE: The time for the beam traveling in air to reach the detector is

$t = \dfrac{d}{c} = \dfrac{2.50 \text{ m}}{3.00\times 10^{8} \text{ m/s}} = 8.33\times 10^{-9} \text{ s}.$ The light traveling in the block takes time

$t = 8.33\times 10^{-9} \text{ s} + 6.15\times 10^{-9} \text{ s} = 1.45\times 10^{-8} \text{ s}.$ The speed of light in the block is

$v = \dfrac{d}{t} = \dfrac{2.50 \text{ m}}{1.45\times 10^{-8} \text{ s}} = 1.72\times 10^{8} \text{ m/s}.$ The refractive index of the block is $n = \dfrac{c}{v} = \dfrac{3.00\times 10^{8} \text{ m/s}}{1.72\times 10^{8} \text{ m/s}} = 1.74.$

EVALUATE: $n > 1,$ as it must be, and 1.74 is a reasonable index of refraction for a transparent material such as plastic.

33.15. **IDENTIFY:** The critical angle for total internal reflection is θ_a that gives $\theta_b = 90°$ in Snell's law.

SET UP: In Figure 33.15 the angle of incidence θ_a is related to angle θ by $\theta_a + \theta = 90°$.

EXECUTE: **(a)** Calculate θ_a that gives $\theta_b = 90°$. $n_a = 1.60$, $n_b = 1.00$ so $n_a \sin\theta_a = n_b \sin\theta_b$ gives

$(1.69)\sin\theta_a = (1.00)\sin 90°$. $\sin\theta_a = \dfrac{1.00}{1.69}$ and $\theta_a = 33.7°$. $\theta = 90° - \theta_a = 53.7°$.

(b) $n_a = 1.69$, $n_b = 1.333$. $(1.69)\sin\theta_a = (1.333)\sin 90°$. $\sin\theta_a = \dfrac{1.333}{1.69}$ and $\theta_a = 52.1°$.

$\theta = 90° - \theta_a = 37.9°$.

EVALUATE: The critical angle increases when the ratio $\dfrac{n_a}{n_b}$ decreases.

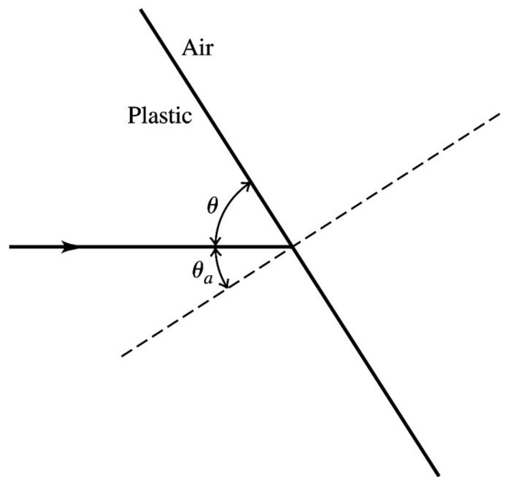


Figure 33.15

33.21. **IDENTIFY:** If no light refracts out of the glass at the glass to air interface, then the incident angle at that interface is θ_{crit}.

SET UP: The ray has an angle of incidence of $0°$ at the first surface of the glass, so enters the glass without being bent, as shown in Figure 33.21. The figure shows that $\alpha + \theta_{\text{crit}} = 90°$.

EXECUTE: **(a)** For the glass-air interface $\theta_a = \theta_{\text{crit}}$, $n_a = 1.52$, $n_b = 1.00$, and $\theta_b = 90°$.

$n_a \sin\theta_a = n_b \sin\theta_b$ gives $\sin\theta_{\text{crit}} = \dfrac{(1.00)(\sin 90°)}{1.52}$ and $\theta_{\text{crit}} = 41.1°$. $\alpha = 90° - \theta_{\text{crit}} = 48.9°$.

(b) Now the second interface is glass $\rightarrow$ water and $n_b = 1.333$. $n_a \sin\theta_a = n_b \sin\theta_b$ gives

$\sin\theta_{\text{crit}} = \dfrac{(1.333)(\sin 90°)}{1.52}$ and $\theta_{\text{crit}} = 61.3°$. $\alpha = 90° - \theta_{\text{crit}} = 28.7°$.

EVALUATE: The critical angle increases when the air is replaced by water.

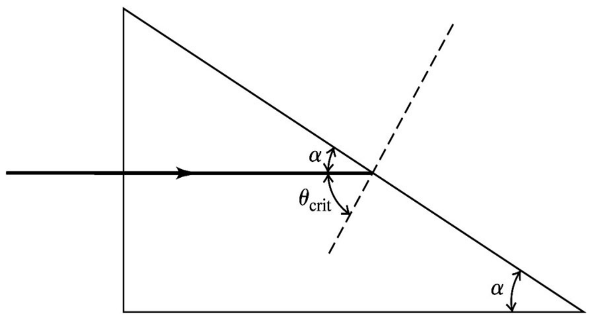


Figure 33.21

33.23. IDENTIFY: The index of refraction depends on the wavelength of light, so the light from the red and violet ends of the spectrum will be bent through different angles as it passes into the glass. Snell's law applies at the surface.

SET UP: $n_a \sin\theta_a = n_b \sin\theta_b$. From the graph in Figure 33.17 in the textbook, for $\lambda = 400$ nm (the violet end of the visible spectrum), $n = 1.67$ and for $\lambda = 700$ nm (the red end of the visible spectrum), $n = 1.62$. The path of a ray with a single wavelength is sketched in Figure 33.23.

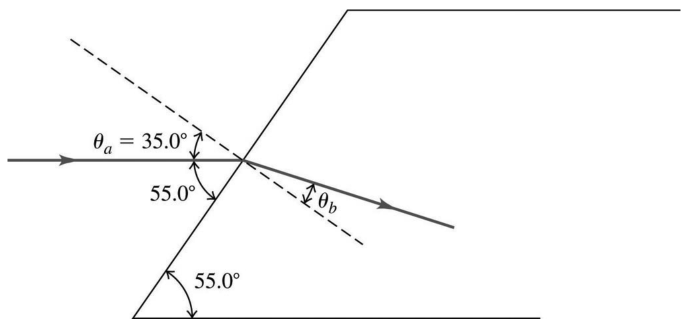


Figure 33.23

EXECUTE: For $\lambda = 400$ nm, $\sin\theta_b = \dfrac{n_a}{n_b}\sin\theta_a = \dfrac{1.00}{1.67}\sin 35.0°$, so $\theta_b = 20.1°$. For $\lambda = 700$ nm,

$\sin\theta_b = \dfrac{1.00}{1.62}\sin 35.0°$, so $\theta_b = 20.7°$. $\Delta\theta$ is about $0.6°$.

EVALUATE: This angle is small, but the separation of the beams could be fairly large if the light travels through a fairly large slab.

33.28. IDENTIFY: Set $I = I_0/18$, where I is the intensity of light passed by the second polarizer.

SET UP: When unpolarized light passes through a polarizer the intensity is reduced by a factor of $\frac{1}{2}$ and the transmitted light is polarized along the axis of the polarizer. When polarized light of intensity I_{max} is incident on a polarizer, the transmitted intensity is $I = I_{\text{max}} \cos^2 \phi$, where ϕ is the angle between the polarization direction of the incident light and the axis of the filter.

EXECUTE: **(a)** After the first filter $I = \dfrac{I_0}{2}$ and the light is polarized along the vertical direction. After the second filter we want $I = \dfrac{I_0}{18}$, so $\dfrac{I_0}{18} = \left(\dfrac{I_0}{2}\right)(\cos\phi)^2$. $\cos\phi = \sqrt{2/18}$ and $\phi = 70.5°$.

(b) Now the first filter passes the full intensity I_0 of the incident light. For the second filter $\dfrac{I_0}{10} = I_0(\cos\phi)^2$. $\cos\phi = \sqrt{1/18}$ and $\phi = 76.4°$.

EVALUATE: When the incident light is polarized along the axis of the first filter, ϕ must be larger to achieve the same overall reduction in intensity than when the incident light is unpolarized.