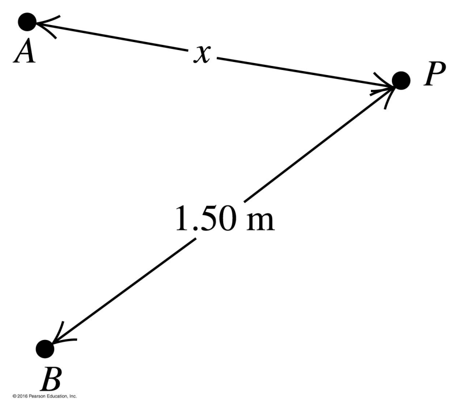
# PEP 2017
# Assignment 18

**(32.12-A)** An electromagnetic wave has a magnetic field given by $\mathbf{B}(x,t) = -(8.25 \times 10^{-9}T)\hat{\jmath}\cos[(1.38 \times 10^4\text{rad/x})x + \omega t]$. (a) In which direction is the wave traveling? (b) What is the frequency f of the wave? (c) Write the vector equation for $\mathbf{E}(x,t)$

**(35.1-A)** Two small stereo speakers A and B that are 1.5m apart are sending out sound of wavelength 36.6cm in all directions and all in phase. A person at point P starts out equidistant from both speakers and walks so that he is always 1.5m from speaker B. For what values of x will the sound this person hears be (a) maximally reinforced, (b) is cancelled? Limit your solution to the case where $x \leq 1.5$m.




**(35.3-T)** A radio transmitting station operating at a frequency of 115 Mhz has two identical antennas that radiate in phase. Anteea B is 9.05m to the right of antenna A. Consider point P between the antennas and along the line connecting them, a horizontal distance x to the right of antenna A. For what values of x will constructive interference occur at point P.

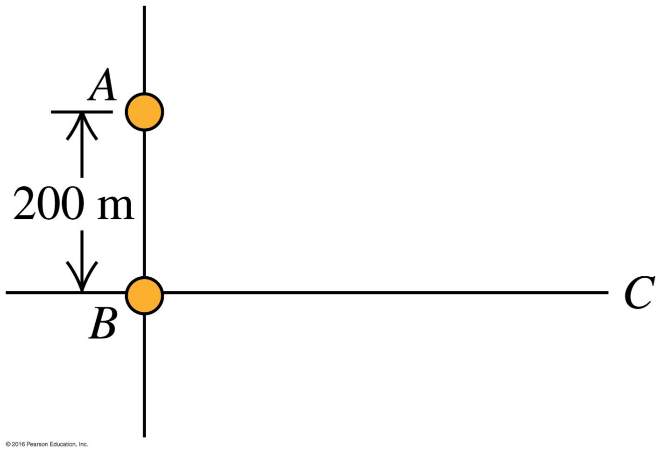
**(35.14-A)** Coherent light that contains two wavelengths: 660nm (red) and 470nm (blue), passes through two narrow slits that are separated by 0280mm. Their interference pattern is observed on a screen 4.2m from the slits. What is the distance on the screen between the first-order bright fringes for the two wavelengths?

**(35.22-A)** Two slits spaced 0.0720mm apart are 0.8m from a screen. Coherent light of wavelength $\lambda$ passes through the two slits. In their interference pattern on the screen, the distance from the centre of the central maximum to the first minimum is 3.00mm. If the intensity at the peak of the central maximum is $0.0600W/m^2$, what is the intensity at points on the screen that are (a) 2.00mm and (b) 1.50mm from the centre of the central maximum?

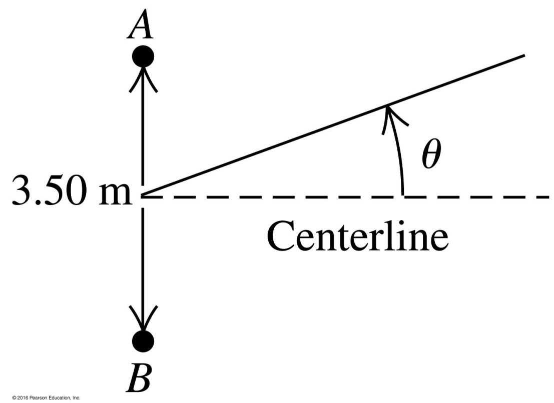
**(35.24-A)** When viewing a piece of art that is behind glass, one often is affected by the light that is reflected off the front of the glass (called glare), which can make it difficult to see the art clearly. One solution is to coat the outer surface of the glass with a film to cancel part of the glare. (a) If the glass has a refractive index of 1.55 and you use $TiO_2$, which has an index of refraction of 2.62, as the coating, what is the minimum film thickness that will cancel light of wavelength 550nm? (b) If this coating is too thin to stand up to wear, what other thickness would also work? Find the three thinnest ones.

**(35.41-A)** Suppose you illuminate two thin slits by monochromatic coherent light in air and find that they produce their first interference minima at $\pm 35.09°$ on either side of the central bright spot. You then immerse these slits in a transparent liquid and illuminate them with the same light. Now you find that the first minima occur at $\pm 19.36°$ instead. What is the index of refraction of this liquid?

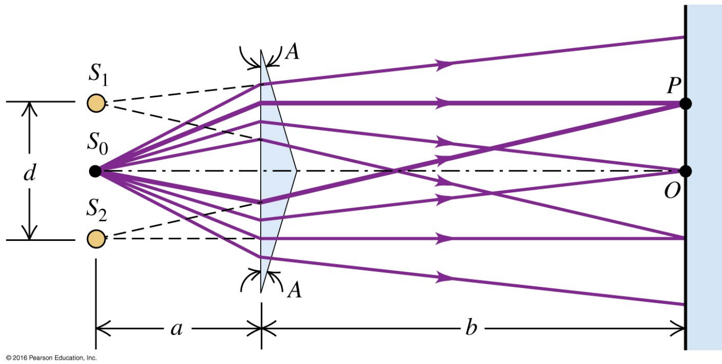
**(35.43-A)** Two radio antennas radiating in phase are located at points A and B, 200m apart. The radio waves have a frequency of 5.80 MHz. A radio receiver is moved out from point B along a line perpendicular to the line connecting A and B (line BC shown in the figure). At what distances from B will there be destructive interference?




**(35.44-T)** Two speakers A and B are 3.50m apart, and each one is emitting a frequency of 444Hz. However, because of signal delays in the cables, speaker A is one-fourth of a period ahead of speaker B. For points far from the speakers, find all the angles relative to the centerline at which the sound from these speakers cancels. Include angles on both sides of the centerline. The speed of sound is 340m/s.



**(35.56-T)** Figure shows an interferometer known as Fresnel's biprism. The magnitude of the prism angle A is extremely small. (a) If $S_0$ is a very narrow source slit, show that the separation of the two virtual coherent sources $S_1$ and $S_2$ is given by $d = 2aA(n-1)$, where n is the index of refraction of the material of the prism. (b) Calculate the spacing of the fringes of green light with wavelength 500nm on a screen 2.00m from the biprism. Take $a = 0.200m$, $A = 0.0035$rad, and $n = 1.50$.




**(36.10-A)** Light waves, for which the electric field is given by $E_y(x,t) = E_{\max}\sin\left((1.4\times10^7 m^{-1})x - \omega t\right)$, pass throughout a slit and produce the first dark bands at $\pm 28.6°$ from the centre of the diffraction pattern. (a) What is the frequency of this light? (b) How wide is the slit? (c) At which angles will other dark bands occur?

**(36.12-T)** Public radio station broadcasts at 88.9 MHz. The radio waves pass between two tall skyscrapers that are 15.0m apart along their closest walls. (a) At what horizontal angles, relative to the original direction of the waves, will a distant antenna not receive any signal from this station? (b) If the maximum intensity is $3.5W/m^2$ at the antenna, what is the intensity at $\pm 5.00°$ from the centre of the central maximum at the distant antenna?

**(36.17-A)** A single-slit diffraction pattern is formed by monochromatic electromagnetic radiation from a distant source passing through a slit 0.102mm wide. At the point in the patter 3.11° from the centre of the central maximum, the total phase difference between wavelets from the top and bottom of the slit is 57.4 rad. (a) What is the wavelength of the radiation? (b) What is the intensity at this point, if the intensity at the centre of the central maximum is $I_0$?

**(36.18-A)** Parallel rays of monochromatic light with wavelength 579nm illuminate two identical slits and produce an interference pattern on a screen that is 75.0cm from the slits. The centres of the slits are 0640mm apart and the width of each slit is 0.434 mm. If the intensity at the centre of the central maximum is $3.5 \times 10^{-4} W/m^2$, what is the intensity at a point on the screen that is 0.800mm from the central of the central maximum?

**(36.20-T)** Consider the interference pattern produced by two parallel slits of width a and separation d, in which $d = 3a$. The slits are illuminated by normally incident light of wavelength $\lambda$. (a) First we ignore diffraction effect due the the slit width. At what angles $\theta$ from the central maximum will the next four maxima in the two-slit interference pattern occur? (b) Now we include the effects of diffraction. If the intensity at $\theta = 0°$ is $I_0$, what is the intensity at each of the angles in part (a)? (c) Which double-slit interference maxima are missing in the pattern?

**(36.22-A)** Laser light of wavelength 506nm illuminates two identical slits, producing an interference pattern on a screen 88cm from the slits. The bright bands are 1.20cm apart, and the third bright bands on either side of the central maximum are missing in the pattern. Find the width and the separation of two slits.

**(36.24-A)** Monochromatic light is at normal incidence on a plane transmission grating. The first-order maximum in the interference pattern is at an angle of 8.01°. What is the angular position of the forth-order maximum?

**(36.58-T)** If has been proposed to use an array of infrared telescopes spread over thousands of kilometres of space to observe planets orbiting other stars. Consider such an array that has an effective diameter of 600km and observed infrared radiation at a wavelength of $10\mu m$. If it is used to observe a planet orbiting the star 70 Virginis, which is 59 light-years from our solar system, what is the size of the smallest details that the array might resolve on the planet? How does this compare to the diameter of the planet, which is assumed to be similar to that of Jupiter $(1.4 \times 10^5 km)$?

**32.12.** **IDENTIFY:** Apply Eqs. (32.17) and (32.19). $f = c/\lambda$ and $k = 2\pi/\lambda$.

**SET UP:** $B_y(x,t) = -B_{\text{max}}\cos(kx + \omega t)$.

**EXECUTE:** **(a)** The phase of the wave is given by $kx + \omega t$, so the wave is traveling in the $-x$-direction.

**(b)** $k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi f}{c}$. $f = \dfrac{kc}{2\pi} = \dfrac{(1.38\times10^4\ \text{rad/m})(3.0\times10^8\ \text{m/s})}{2\pi} = 6.59\times10^{11}\ \text{Hz}$.

**(c)** Since the magnetic field is in the $-y$-direction, and the wave is propagating in the $-x$-direction, then the electric field is in the $-z$-direction so that $\vec{E}\times\vec{B}$ will be in the $-x$-direction.

$$\vec{E}(x,t) = +cB(x,t)\hat{k} = -cB_{\text{max}}\cos(kx + \omega t)\hat{k}.$$

$$\vec{E}(x,t) = -c(8.25\times10^{-9}\ \text{T})\cos\left[(1.38\times10^4\ \text{rad/m})x + (4.14\times10^{12}\ \text{rad/s})t\right]\hat{k}.$$

$$\vec{E}(x,t) = -(2.48\ \text{V/m})\cos\left[(1.38\times10^4\ \text{rad/m})x + (4.14\times10^{12}\ \text{rad/s})t\right]\hat{k}.$$

**EVALUATE:** $\vec{E}$ and $\vec{B}$ have the same phase and are in perpendicular directions.

**35.1. IDENTIFY:** The sound will be maximally reinforced when the path difference is an integral multiple of wavelengths and cancelled when it is an odd number of half wavelengths.

**SET UP:** Constructive interference occurs for $r_2 - r_1 = m\lambda,\ m = 0, \pm 1, \pm 2, \ldots.$ Destructive interference occurs for $r_2 - r_1 = (m + \frac{1}{2})\lambda,\ m = 0, \pm 1, \pm 2\ldots.$ For this problem, $r_2 = 150\text{ cm}$ and $r_1 = x.$ The path taken by the person ensures that $x$ is in the range $0 \le x \le 150\text{ cm}.$

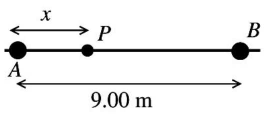
**EXECUTE:** **(a)** $150\text{ cm} - x = m(36.6\text{ cm}).$ $x = 150\text{ cm} - m(36.6\text{ cm}).$ For $m = 0, 1, 2, 3, 4$ the values of $x$ are 150 cm, 113.4 cm, 76.8 cm, 40.2 cm, 3.6 cm.

**(b)** $150\text{ cm} - x = (m + \frac{1}{2})(36.6\text{ cm}).$ $x = 150\text{ cm} - (m + \frac{1}{2})(36.6\text{ cm}).$ For $m = 0, 1, 2, 3$ the values of $x$ are 131.7 cm, 95.1 cm, 58.5 cm, 21.9 cm.

**EVALUATE:** When $x = 116\text{ cm}$ the path difference is $150\text{ cm} - 113.4\text{ cm} = 36.6\text{ cm},$ which is one wavelength. When $x = 131.7\text{ cm}$ the path difference is 18.3 cm, which is one-half wavelength.

**35.3. IDENTIFY:** Use $c = f\lambda$ to calculate the wavelength of the transmitted waves. Compare the difference in the distance from $A$ to $P$ and from $B$ to $P$. For constructive interference this path difference is an integer multiple of the wavelength.

**SET UP:** Consider Figure 35.3 (next page).




The distance of point $P$ from each coherent source is $r_A = x$ and $r_B = 9.05 \text{ m} - x$.

**Figure 35.3**

**EXECUTE:** The path difference is $r_B - r_A = 9.05 \text{ m} - 2x$.

$r_B - r_A = m\lambda, m = 0, \pm 1, \pm 2, \ldots$

$$\lambda = \frac{c}{f} = \frac{2.998 \times 10^8 \text{ m/s}}{115 \times 10^6 \text{ Hz}} = 2.61 \text{ m}.$$

Thus $9.05 \text{ m} - 2x = m(2.61 \text{ m})$ and $x = \dfrac{9.05 \text{ m} - m(2.61 \text{ m})}{2} = 4.525 \text{ m} - (1.305 \text{ m})m.$ $x$ must lie in the range 0 to 9.05 m since $P$ is said to be between the two antennas.

$m = 0$ gives $x = 4.525$ m.

$m = +1$ gives $x = 4.525 \text{ m} - 1.305 \text{ m} = 3.22 \text{ m}.$

$m = +2$ gives $x = 4.525 \text{ m} - 2.61 \text{ m} = 1.915 \text{ m}.$

$m = +3$ gives $x = 4.525 \text{ m} - 3.75 \text{ m} = 0.61 \text{ m}.$

$m = -1$ gives $x = 4.525 \text{ m} + 1.305 \text{ m} = 5.83 \text{ m}.$

$m = -2$ gives $x = 4.525 \text{ m} + 2.61 \text{ m} = 7.135 \text{ m}.$

$m = -3$ gives $x = 4.525 \text{ m} + 3.915 \text{ m} = 8.44 \text{ m}.$

All other values of $m$ give values of $x$ out of the allowed range. Constructive interference will occur for $x$ = 0.61 m, 1.915 m, 3.22 m, 4.525 m, 5.83 m, 7.135 m, and 8.44 m.

**EVALUATE:** Constructive interference occurs at the midpoint between the two sources since that point is the same distance from each source. The other points of constructive interference are symmetrically placed relative to this point.

**35.14. IDENTIFY:** For small angles: $y_m = R\dfrac{m\lambda}{d}$.

**SET UP:** First-order means $m = 1$.

**EXECUTE:** The distance between corresponding bright fringes is

$$\Delta y = \frac{Rm}{d}\Delta\lambda = \frac{(4.20\ \text{m})(1)}{(0.280\times 10^{-3}\ \text{m})}(660-470)\times(10^{-9}\ \text{m}) = 2.85\times 10^{-3}\ \text{m} = 2.85\ \text{mm}.$$

**EVALUATE:** The separation between these fringes for different wavelengths increases when the slit separation decreases.

**35.22. IDENTIFY:** Light from the two slits interferes on the screen. The bright and dark fringes are very close together compared to the distance between the screen and the slits, so we can use the small-angle approximation. $\phi = \dfrac{2\pi}{\lambda}(r_1 - r_2)$. The intensity is $I = I_0 \cos^2\left(\dfrac{\phi}{2}\right)$.

**SET UP:** The intensity is $I = I_0 \cos^2\left(\dfrac{\phi}{2}\right)$. $\dfrac{\phi}{2} = \dfrac{\pi d \sin\theta}{\lambda}$, but for small angles $\dfrac{\phi}{2} \approx \dfrac{\pi dy}{R\lambda}$.

**EXECUTE:** At the first minim, $y$ = 3.00 mm and $\phi/2 = \pi/2$. At $y$ = 2.00 mm, which is 2/3 of 3.00 mm, $\phi/2 = (2/3)(\pi/2) = \pi/3 = 60°$. Therefore the intensity at $x$ = 2.00 mm is $I = (0.0600\ \text{W/m}^2)\cos^2(60°) = 0.0150\ \text{W/m}^2$.

**(b)** Using the same reasoning as in (a), 1.50 mm is $\frac{1}{2}$ of 3.00 m, so $\phi/2 = (1/2)(\pi/2) = \pi/4 = 45°$. So $I = (0.0600\ \text{W/m}^2)\cos^2(45°) = 0.0300\ \text{W/m}^2$.

**EVALUATE:** As a check, we could first find $\lambda$ and then use it to find the intensities. At the first minimum, $\phi/2 = \pi/2 = \pi dy/R\lambda$, which gives $\lambda = 2dy/R = 5.40\times10^{-4}$ mm. Now use this to calculate the intensities using $I = I_0 \cos^2\left(\dfrac{\phi}{2}\right)$ and $\dfrac{\phi}{2} \approx \dfrac{\pi dy}{R\lambda}$.

**35.24. IDENTIFY:** Require destructive interference for light reflected at the front and rear surfaces of the film.

**SET UP:** At the front surface of the film, light in air $(n = 1.00)$ reflects from the film $(n = 2.62)$ and there is a $180°$ phase shift due to the reflection. At the back surface of the film, light in the film $(n = 2.62)$ reflects from glass $(n = 1.55)$ and there is no phase shift due to reflection. Therefore, there is a net $180°$ phase difference produced by the reflections. The path difference for these two rays is $2t$, where $t$ is the thickness of the film. The wavelength in the film is $\lambda = \dfrac{550\text{ nm}}{2.62}$.

**EXECUTE:** **(a)** Since the reflection produces a net $180°$ phase difference, destructive interference of the reflected light occurs when $2t = m\lambda$. $t = m\left(\dfrac{550\text{ nm}}{2[2.62]}\right) = (105\text{ nm})m$. The minimum thickness is 105 nm.

**(b)** The next three thicknesses are for $m = 2, 3$ and 4: 210 nm, 315 nm, and 420 nm.

**EVALUATE:** The minimum thickness is for $t = \lambda_0/2n$. Compare this to Problem 35.23, where the minimum thickness for destructive interference is $t = \lambda_0/4n$.

**35.41. IDENTIFY:** The liquid alters the wavelength of the light and that affects the locations of the interference minima.

**SET UP:** The interference minima are located by $d\sin\theta = (m+\tfrac{1}{2})\lambda$. For a liquid with refractive index $n$, $\lambda_{\text{liq}} = \dfrac{\lambda_{\text{air}}}{n}$.

**EXECUTE:** $\dfrac{\sin\theta}{\lambda} = \dfrac{(m+\tfrac{1}{2})}{d} =$ constant, so $\dfrac{\sin\theta_{\text{air}}}{\lambda_{\text{air}}} = \dfrac{\sin\theta_{\text{liq}}}{\lambda_{\text{liq}}}$. $\dfrac{\sin\theta_{\text{air}}}{\lambda_{\text{air}}} = \dfrac{\sin\theta_{\text{liq}}}{\lambda_{\text{air}}/n}$ and

$$n = \frac{\sin\theta_{\text{air}}}{\sin\theta_{\text{liq}}} = \frac{\sin 35.09^\circ}{\sin 19.36^\circ} = 1.734.$$

**EVALUATE:** In the liquid the wavelength is shorter and $\sin\theta = (m+\tfrac{1}{2})\dfrac{\lambda}{d}$ gives a smaller $\theta$ than in air, for the same $m$.

**35.43. IDENTIFY:** For destructive interference, $d = r_2 - r_1 = (m + \frac{1}{2})\lambda$.

**SET UP:** $r_2 - r_1 = \sqrt{(200\text{ m})^2 + x^2} - x$.

**EXECUTE:** $(200\text{ m})^2 + x^2 = x^2 + \left[(m + \frac{1}{2})\lambda\right]^2 + 2x(m + \frac{1}{2})\lambda$.

$x = \dfrac{20{,}000\text{ m}^2}{(m + \frac{1}{2})\lambda} - \frac{1}{2}(m + \frac{1}{2})\lambda$. The wavelength is calculated by $\lambda = \dfrac{c}{f} = \dfrac{3.00 \times 10^8\text{ m/s}}{5.80 \times 10^6\text{ Hz}} = 51.7\text{ m}$.

$m = 0: x = 761\text{ m}$; $m = 1: x = 219\text{ m}$; $m = 2: x = 90.1\text{ m}$; $m = 3; x = 20.0\text{ m}$.

**EVALUATE:** For $m = 3$, $d = 3.5\lambda = 181\text{ m}$. The maximum possible path difference is the separation of 200 m between the sources.

**35.44. IDENTIFY:** For destructive interference the net phase difference must be $180°$, which is one-half a period, or $\lambda/2$. Part of this phase difference is due to the fact that the speakers are 1/4 of a period out of phase, and the rest is due to the path difference between the sound from the two speakers.

**SET UP:** The phase of $A$ is $90°$ or, $\lambda/4$, ahead of $B$. At points above the centerline, points are closer to $A$ than to $B$ and the signal from $A$ gains phase relative to $B$ because of the path difference. Destructive interference will occur when $d\sin\theta = (m+\frac{1}{4})\lambda$, $m = 0, 1, 2, \ldots$. At points at an angle $\theta$ below the centerline, the signal from $B$ gains phase relative to $A$ because of the phase difference. Destructive interference will occur when $d\sin\theta = (m+\frac{3}{4})\lambda$, $m = 0, 1, 2, \ldots$ $\lambda = \dfrac{v}{f}$.

**EXECUTE:** $\lambda = \dfrac{340\text{ m/s}}{444\text{ Hz}} = 0.766\text{ m}.$

<u>Points above the centerline</u>: $\sin\theta = (m+\frac{1}{4})\dfrac{\lambda}{d} = (m+\frac{1}{4})\left(\dfrac{0.766\text{ m}}{3.50\text{ m}}\right) = 0.219(m+\frac{1}{4})$. $m = 0$: $\theta = 3.14°$;

$m = 1$: $\theta = 15.9°$; $m = 2$: $\theta = 29.5°$; $m = 3$: $\theta = 45.4°$; $m = 4$: $\theta = 68.6°$.

<u>Points below the centerline</u>: $\sin\theta = (m+\frac{3}{4})\dfrac{\lambda}{d} = (m+\frac{3}{4})\left(\dfrac{0.766\text{ m}}{3.50\text{ m}}\right) = 0.219(m+\frac{3}{4})$. $m = 0$: $\theta = 9.45°$;

$m = 1$: $\theta = 22.5°$; $m = 2$: $\theta = 37.0°$; $m = 3$: $\theta = 55.2°$.

**EVALUATE:** It is *not* always true that the path difference for destructive interference must be $(m+\frac{1}{2})\lambda$, but it *is* always true that the phase difference must be $180°$ (or odd multiples of $180°$).

**35.56. IDENTIFY:** Apply Snell's law to the refraction at the two surfaces of the prism. $S_1$ and $S_2$ serve as coherent sources so the fringe spacing is $\Delta y = \dfrac{R\lambda}{d}$, where $d$ is the distance between $S_1$ and $S_2$.

**SET UP:** For small angles, $\sin\theta \approx \theta$, with $\theta$ expressed in radians.

**EXECUTE:** **(a)** Since we can approximate the angles of incidence on the prism as being small, Snell's law tells us that an incident angle of $\theta$ on the flat side of the prism enters the prism at an angle of $\theta/n$, where $n$ is the index of refraction of the prism. Similarly on leaving the prism, the in-going angle is $\theta/n - A$ from the normal, and the outgoing angle, relative to the prism, is $n(\theta/n - A)$. So the beam leaving the prism is at an angle of $\theta' = n(\theta/n - A) + A$ from the optical axis. So $\theta - \theta' = (n-1)A$. At the plane of the source $S_0$, we can calculate the height of one image above the source: $\dfrac{d}{2} = \tan(\theta - \theta')a \approx (\theta - \theta')a = (n-1)Aa \Rightarrow d = 2aA(n-1)$.

**(b)** To find the spacing of fringes on a screen, we use

$$\Delta y = \frac{R\lambda}{d} = \frac{R\lambda}{2aA(n-1)} = \frac{(2.00\text{ m} + 0.200\text{ m})(5.00\times10^{-7}\text{ m})}{2(0.200\text{ m})(3.50\times10^{-3}\text{ rad})(1.50-1.00)} = 1.57\times10^{-3}\text{ m}.$$

**EVALUATE:** The fringe spacing is proportional to the wavelength of the light. The biprism serves as an alternative to two closely spaced narrow slits.

**36.10. IDENTIFY:** Compare $E_y$ to the expression $E_y = E_{max}\sin(kx - \omega t)$ and determine $k$, and from that calculate $\lambda$. $f = c/\lambda$. The dark bands are located by $\sin\theta = \dfrac{m\lambda}{a}$.

**SET UP:** $c = 3.00\times10^8$ m/s. The first dark band corresponds to $m = 1$.

**EXECUTE:** **(a)** $E = E_{max}\sin(kx - \omega t)$. $k = \dfrac{2\pi}{\lambda} \Rightarrow \lambda = \dfrac{2\pi}{k} = \dfrac{2\pi}{1.40\times10^7\ \text{m}^{-1}} = 4.488\times10^{-7}$ m.

$$f\lambda = c \Rightarrow f = \frac{c}{\lambda} = \frac{3.0\times10^8\ \text{m/s}}{4.488\times10^{-7}\ \text{m}} = 6.68\times10^{14}\ \text{Hz}.$$

**(b)** $a\sin\theta = \lambda$. $a = \dfrac{\lambda}{\sin\theta} = \dfrac{4.488\times10^{-7}\ \text{m}}{\sin 28.6^\circ} = 9.38\times10^{-7}$ m.

**(c)** $a\sin\theta = m\lambda (m = 1, 2, 3, \ldots)$. $\sin\theta_2 = \pm2\dfrac{\lambda}{a} = \pm2\dfrac{4.488\times10^{-7}\ \text{m}}{9.38\times10^{-7}\ \text{m}}$ so $\theta_2 = \pm73.2^\circ$.

**EVALUATE:** For $m = 3$, $\dfrac{m\lambda}{a}$ is greater than 1 so only the first and second dark bands appear.

**36.12.** **IDENTIFY:** The space between the skyscrapers behaves like a single slit and diffracts the radio waves.

**SET UP:** Cancellation of the waves occurs when $a\ \sin\theta = m\lambda$, $m = 1, 2, 3, \ldots,$ and the intensity of the waves is given by $I_0\left(\dfrac{\sin\beta/2}{\beta/2}\right)^2$, where $\beta/2 = \dfrac{\pi a \sin\theta}{\lambda}$.

**EXECUTE:** **(a)** First find the wavelength of the waves:

$\lambda = c/f = (3.00\times10^8\ \text{m/s})/(88.9\ \text{MHz}) = 3.375\ \text{m}.$

For no signal, $a\ \sin\theta = m\lambda.$

$m = 1$: $\sin\theta_1 = (1)(3.375\ \text{m})/(15.0\ \text{m}) \Rightarrow \theta_1 = \pm13.0°.$

$m = 2$: $\sin\theta_2 = (2)(3.375\ \text{m})/(15.0\ \text{m}) \Rightarrow \theta_2 = \pm26.7°.$

$m = 3$: $\sin\theta_3 = (3)(3.375\ \text{m})/(15.0\ \text{m}) \Rightarrow \theta_3 = \pm42.4°.$

$m = 4$: $\sin\theta_4 = (4)(3.375\ \text{m})/(15.0\ \text{m}) \Rightarrow \theta_4 = \pm64.1°.$

**(b)** $I_0\left(\dfrac{\sin\beta/2}{\beta/2}\right)^2$, where $\beta/2 = \dfrac{\pi a \sin\theta}{\lambda} = \dfrac{\pi(15.0\ \text{m})\sin(5.00°)}{3.375\ \text{m}} = 1.217\ \text{rad}.$

$$I = (3.50\ \text{W/m}^2)\left[\frac{\sin(1.217\ \text{rad})}{1.217\ \text{rad}}\right]^2 = 2.08\ \text{W/m}^2.$$

**EVALUATE:** The wavelength of the radio waves is very long compared to that of visible light, but it is still considerably shorter than the distance between the buildings.

**36.17.** **IDENTIFY** and **SET UP:** Use $\beta = \left(\frac{2\pi}{\lambda}\right) a \sin\theta$ to calculate $\lambda$ and use $I = I_0\left(\frac{\sin\beta/2}{\beta/2}\right)^2$ to calculate $I$.

$\theta = 3.11°$, $\beta = 57.4$ rad, $a = 0.102\times10^{-3}$ m.

**EXECUTE:** **(a)** $\beta = \left(\frac{2\pi}{\lambda}\right) a \sin\theta$ so

$$\lambda = \frac{2\pi a \sin\theta}{\beta} = \frac{2\pi(0.102\times10^{-3}\text{ m})\sin 3.11°}{57.4\text{ rad}} = 606\text{ nm.}$$

**(b)** $I = I_0\left(\frac{\sin\beta/2}{\beta/2}\right)^2 = I_0\left(\frac{4}{\beta^2}\right)(\sin(\beta/2))^2 = I_0\frac{4}{(57.4\text{ rad})^2}[\sin(28.7\text{ rad})]^2 = 2.07\times10^{-4}\ I_0.$

**EVALUATE:** At the first minimum $\beta = 2\pi$ rad and at the point considered in the problem $\beta = 18.3\pi$ rad, so the point is well outside the central maximum. Since $\beta$ is close to $m\pi$ with $m = 18$, this point is near one of the minima. The intensity here is much less than $I_0$.

**36.18. IDENTIFY:** The intensity at the screen is due to a combination of single-slit diffraction and double-slit interference.

**SET UP:** $I = I_0\left(\cos^2\frac{\phi}{2}\right)\left[\frac{\sin(\beta/2)}{\beta/2}\right]^2$, where $\phi = \frac{2\pi d}{\lambda}\sin\theta$ and $\beta = \frac{2\pi}{\lambda}a\sin\theta$.

**EXECUTE:** $\tan\theta = \frac{8.00\times 10^{-4}\text{ m}}{0.750\text{ m}} = 1.067\times 10^{-3}$. $\theta$ is small, so $\sin\theta \approx \tan\theta$.

$$\phi = \frac{2\pi d}{\lambda}\sin\theta = \frac{2\pi(0.640\times 10^{-3}\text{ m})}{579\times 10^{-9}\text{ m}}(1.067\times 10^{-3}) = 7.4105\text{ rad}.$$

$$\beta = \frac{2\pi a}{\lambda}\sin\theta = \frac{2\pi(0.434\times 10^{-3}\text{ m})}{579\times 10^{-9}\text{ m}}(1.067\times 10^{-3}) = 5.0252\text{ rad}.$$

$$I = (3.50\times 10^{-4}\text{ W/m}^2)(\cos 3.7052\text{ rad})^2\left[\frac{\sin 2.5126\text{ rad}}{2.5126}\right]^2 = 1.37\times 10^{-5}\text{ W/m}^2.$$

**EVALUATE:** The intensity as decreased by a factor of almost a thousand, so it would be difficult to see the light at the screen.

**36.20. IDENTIFY:** The net intensity is the *product* of the factor due to single-slit diffraction and the factor due to double slit interference.

**SET UP:** The double-slit factor is $I_{DS} = I_0\left(\cos^2\frac{\phi}{2}\right)$ and the single-slit factor is $I_{SS} = \left(\frac{\sin\beta/2}{\beta/2}\right)^2$.

**EXECUTE: (a)** $d\sin\theta = m\lambda \Rightarrow \sin\theta = m\lambda/d.$

$\sin\theta_1 = \lambda/d, \sin\theta_2 = 2\lambda/d, \sin\theta_3 = 3\lambda/d, \sin\theta_4 = 4\lambda/d.$

**(b)** At the interference bright fringes, $\cos^2\phi/2 = 1$ and $\beta/2 = \frac{\pi a\sin\theta}{\lambda} = \frac{\pi(d/3)\sin\theta}{\lambda}.$

At $\theta_1$, $\sin\theta_1 = \lambda/d$, so $\beta/2 = \frac{\pi(d/3)(\lambda/d)}{\lambda} = \pi/3.$ The intensity is therefore

$$I_1 = I_0\left(\cos^2\frac{\phi}{2}\right)\left(\frac{\sin\beta/2}{\beta/2}\right)^2 = I_0(1)\left(\frac{\sin\pi/3}{\pi/3}\right)^2 = 0.684\, I_0.$$

At $\theta_2$, $\sin\theta_2 = 2\lambda/d$, so $\beta/2 = \frac{\pi(d/3)(2\lambda/d)}{\lambda} = 2\pi/3.$ Using the same procedure as for $\theta_1$, we have

$I_2 = I_0(1)\left(\frac{\sin 2\pi/3}{2\pi/3}\right)^2 = 0.171\, I_0.$

At $\theta_3$, we get $\beta/2 = \pi$, which gives $I_3 = 0$ since $\sin\,\pi = 0.$

At $\theta_4$, $\sin\,\theta_4 = 4\lambda/d$, so $\beta/2 = 4\pi/3$, which gives $I_4 = I_0\left(\frac{\sin 4\pi/3}{4\pi/3}\right)^2 = 0.0427\, I_0.$

**(c)** Since $d = 3a$, every third interference maximum is missing.

**(d)** In Figure 36.12c in the textbook, every fourth interference maximum at the sides is missing because $d = 4a.$

**EVALUATE:** The result in this problem is different from that in Figure 36.12c in the textbook because in this case $d = 3a$, so every third interference maximum at the sides is missing. Also the "envelope" of the intensity function decreases more rapidly here than in Figure 36.12c in the text because the first diffraction minimum is reached sooner, and the decrease in intensity from one interference maximum to the next is faster for $a = d/3$ than for $a = d/4.$

**36.22. IDENTIFY:** The diffraction minima are located by $\sin\theta = \dfrac{m_\text{d}\lambda}{a}$ and the two-slit interference maxima are located by $\sin\theta = \dfrac{m_\text{i}\lambda}{d}$. The third bright band is missing because the first order single-slit minimum occurs at the same angle as the third order double-slit maximum.

**SET UP:** The pattern is sketched in Figure 36.22. $\tan\theta = \dfrac{3.6\text{ cm}}{88\text{ cm}}$, so $\theta = 2.34°$.

**EXECUTE:** Single-slit dark spot: $a\sin\theta = \lambda$ and

$$a = \frac{\lambda}{\sin\theta} = \frac{506\text{ nm}}{\sin 2.34°} = 1.24\times 10^4\text{ nm} = 12.4\ \mu\text{m (width)}.$$

Double-slit bright fringe: $d\sin\theta = 3\lambda$ and $d = \dfrac{3\lambda}{\sin\theta} = \dfrac{3(506\text{ nm})}{\sin 2.34°} = 3.72\times 10^4\text{ nm} = 37.2\ \mu\text{m}$ (separation).

**EVALUATE:** Note that $d/a = 3.0$.

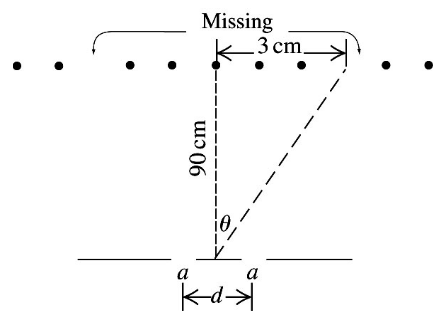



**Figure 36.22**

**36.24. IDENTIFY:** The maxima are located by $d\sin\theta = m\lambda$.

**SET UP:** The order corresponds to the values of $m$.

**EXECUTE:** First-order: $d\sin\theta_1 = \lambda$. Fourth-order: $d\sin\theta_4 = 4\lambda$.

$\frac{d\sin\theta_4}{d\sin\theta_1} = \frac{4\lambda}{\lambda}$, $\sin\theta_4 = 4\sin\theta_1 = 4\sin 8.01°$ and $\theta_4 = 33.9°$.

**EVALUATE:** We did not have to solve for $d$.

**36.58. IDENTIFY:** Apply $\sin\theta = 1.22\dfrac{\lambda}{D}$.

**SET UP:** $\theta$ is small, so $\sin\theta \approx \dfrac{\Delta x}{R}$, where $\Delta x$ is the size of the details and $R$ is the distance to the earth.

So $\dfrac{\Delta x}{R} \approx 1.22\dfrac{\lambda}{D}$. $1\text{ ly} = 9.41\times10^{15}\text{ m}$ and $10\ \mu\text{m} = 1.0\times10^{-5}\text{ m}$.

**EXECUTE:** $\Delta x = 1.22\dfrac{\lambda R}{D} = \dfrac{(1.22)(1.0\times10^{-5}\text{ m})(59\text{ ly})(9.41\times10^{15}\text{ m/ly})}{6.0\times10^{6}\text{ m}} = 1.1\times10^{6}\text{ m} = 1100\text{ km}.$

$\dfrac{\Delta x}{D_{\text{planet}}} = \dfrac{1100\text{ km}}{1.4\times10^{5}\text{ km}} = 7.9\times10^{-3}$. So $\Delta x$ is less than 1% of the diameter of the planet.

**EVALUATE:** The very large diameter of the array allows it to resolve planet-sized detail at great distances.