PEP 2017
Assignment 19

37.6 •• As you pilot your space utility vehicle at a constant speed toward the moon, a race pilot flies past you in her spaceracer at a constant speed of $0.800c$ relative to you. At the instant the spaceracer passes you, both of you start timers at zero. (a) At the instant when you measure that the spaceracer has traveled 1.20×10^8 m past you, what does the race pilot read on her timer? (b) When the race pilot reads the value calculated in part (a) on her timer, what does she measure to be your distance from her? (c) At the instant when the race pilot reads the value calculated in part (a) on her timer, what do you read on yours?

37.7 •• A spacecraft flies away from the earth with a speed of 4.80×10^6 m/s relative to the earth and then returns at the same speed. The spacecraft carries an atomic clock that has been carefully synchronized with an identical clock that remains at rest on earth. The spacecraft returns to its starting point 365 days (1 year) later, as measured by the clock that remained on earth. What is the difference in the elapsed times on the two clocks, measured in hours? Which clock, the one in the spacecraft or the one on earth, shows the shorter elapsed time?

37.21 •• Two particles in a high-energy accelerator experiment approach each other head-on with a relative speed of $0.890c$. Both particles travel at the same speed as measured in the laboratory. What is the speed of each particle, as measured in the laboratory?

37.25 • **Tell It to the Judge.** (a) How fast must you be approaching a red traffic light ($\lambda = 675$ nm) for it to appear yellow ($\lambda = 575$ nm)? Express your answer in terms of the speed of light. (b) If you used this as a reason not to get a ticket for running a red light, how much of a fine would you get for speeding? Assume that the fine is \$1.00 for each kilometer per hour that your speed exceeds the posted limit of 90 km/h.

37.32 • **Relativistic Baseball.** Calculate the magnitude of the force required to give a 0.145-kg baseball an acceleration $a = 1.00\ \text{m/s}^2$ in the direction of the baseball's initial velocity when this velocity has a magnitude of (a) 10.0 m/s; (b) $0.900c$; (c) $0.990c$. (d) Repeat parts (a), (b), and (c) if the force and acceleration are perpendicular to the velocity.

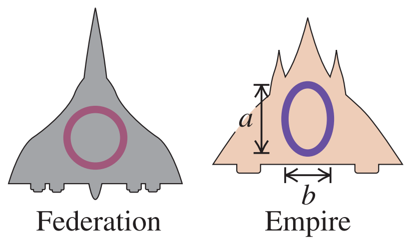
37.33 •• What is the speed of a particle whose kinetic energy is equal to (a) its rest energy and (b) five times its rest energy?

37.46 • **Creating a Particle.** Two protons (each with rest mass $M = 1.67 \times 10^{-27}$ kg) are initially moving with equal speeds in opposite directions. The protons continue to exist after a collision that also produces an η^0 particle (see Chapter 44). The rest mass of the η^0 is $m = 9.75 \times 10^{-28}$ kg. (a) If the two protons and the η^0 are all at rest after the collision, find the initial speed of the protons, expressed as a fraction of the speed of light. (b) What is the kinetic energy of each proton? Express your answer in MeV. (c) What is the rest energy of the η^0, expressed in MeV? (d) Discuss the relationship between the answers to parts (b) and (c).

37.47 • The sun produces energy by nuclear fusion reactions, in which matter is converted into energy. By measuring the amount of energy we receive from the sun, we know that it is producing energy at a rate of 3.8×10^{26} W. (a) How many kilograms of matter does the sun lose each second? Approximately how many tons of matter is this (1 ton = 2000 lbs)? (b) At this rate, how long would it take the sun to use up all its mass?

37.51 ••• The starships of the Solar Federation are marked with the symbol of the federation, a circle, while starships of the Denebian Empire are marked with the empire's symbol, an ellipse whose major axis is 1.40 times longer than its minor axis ($a = 1.40b$ in Fig. P37.51). How fast, relative to an observer, does an empire ship have to travel for its marking to be confused with the marking of a federation ship?

Figure **P37.51**



37.60 •• Two protons are moving away from each other. In the frame of each proton, the other proton has a speed of $0.600c$. What does an observer in the rest frame of the earth measure for the speed of each proton?

37.63 •• **CALC** A particle with mass m accelerated from rest by a constant force F will, according to Newtonian mechanics, continue to accelerate without bound; that is, as $t \to \infty$, $v \to \infty$. Show that according to relativistic mechanics, the particle's speed approaches c as $t \to \infty$. [*Note:* A useful integral is $\int (1 - x^2)^{-3/2} dx = x/\sqrt{1 - x^2}$.]

37.6. **IDENTIFY:** Apply Eq. (37.8).

SET UP: For part (a) the proper time is measured by the race pilot. $\gamma = 1.667$.

EXECUTE: **(a)** $\Delta t = \dfrac{1.20\times10^8\ \text{m}}{(0.800)(3.00\times10^8\ \text{m/s})} = 0.500\ \text{s}$. $\Delta t_0 = \dfrac{\Delta t}{\gamma} = \dfrac{0.500\ \text{s}}{1.667} = 0.300\ \text{s}$.

(b) $(0.300\ \text{s})(0.800c) = 7.20\times10^7\ \text{m}$.

(c) You read $\dfrac{1.20\times10^8\ \text{m}}{(0.800)(3\times10^8\ \text{m/s})} = 0.500\ \text{s}$.

EVALUATE: The two events are the spaceracer passing you and the spaceracer reaching a point 1.20×10^8 m from you. The timer traveling with the spaceracer measures the proper time between these two events.

37.7. **IDENTIFY and SET UP:** A clock moving with respect to an observer appears to run more slowly than a clock at rest in the observer's frame. The clock in the spacecraft measurers the proper time Δt_0. $\Delta t = 365 \text{ days} = 8760 \text{ hours}$.

EXECUTE: The clock on the moving spacecraft runs slow and shows the smaller elapsed time. $\Delta t_0 = \Delta t\sqrt{1-u^2/c^2} = (8760 \text{ h})\sqrt{1-(4.80\times10^6/3.00\times10^8)^2} = 8758.88 \text{ h}$. The difference in elapsed times is $8760 \text{ h} - 8758.88 \text{ h} = 1.12 \text{ h}$.

37.21. IDENTIFY: The relativistic velocity addition formulas apply since the speeds are close to that of light.

SET UP: The relativistic velocity addition formula is $v_x' = \dfrac{v_x - u}{1 - \dfrac{uv_x}{c^2}}$.

EXECUTE: In the relativistic velocity addition formula for this case, v_x' is the relative speed of particle 1 with respect to particle 2, v is the speed of particle 2 measured in the laboratory, and u is the speed of particle 1 measured in the laboratory, $u = -v$.

$v_x' = \dfrac{v-(-v)}{1-(-v)v/c^2} = \dfrac{2v}{1+v^2/c^2}$. $\dfrac{v_x'}{c^2}v^2 - 2v + v_x' = 0$ and $(0.890c)v^2 - 2c^2v + (0.890c^3) = 0$.

This is a quadratic equation with solution $v = 0.611c$ (v must be less than c).

EVALUATE: The nonrelativistic result would be $0.445c$, which is considerably different from this result.

37.25. **IDENTIFY** and **SET UP:** Source and observer are approaching, so use Eq. (37.25): $f = \sqrt{\frac{c+u}{c-u}} f_0$. Solve for u, the speed of the light source relative to the observer.

(a) EXECUTE: $f^2 = \left(\frac{c+u}{c-u}\right) f_0^2$

$(c-u)f^2 = (c+u)f_0^2$ and $u = \frac{c(f^2 - f_0^2)}{f^2 + f_0^2} = c\left(\frac{(f/f_0)^2 - 1}{(f/f_0)^2 + 1}\right)$

$\lambda_0 = 675\ \text{nm}, \quad \lambda = 575\ \text{nm}$

$u = \left(\frac{(675\ \text{nm}/575\ \text{nm})^2 - 1}{(675\ \text{nm}/575\ \text{nm})^2 + 1}\right) c = 0.159c = (0.159)(2.998\times10^8\ \text{m/s}) = 4.77\times10^7\ \text{m/s}$; definitely speeding

(b) $4.77\times10^7\ \text{m/s} = (4.77\times10^7\ \text{m/s})(1\ \text{km}/1000\ \text{m})(3600\ \text{s}/1\ \text{h}) = 1.72\times10^8\ \text{km/h}$. Your fine would be $\$1.72\times10^8$ (172 million dollars).

EVALUATE: The source and observer are approaching, so $f > f_0$ and $\lambda < \lambda_0$. Our result gives $u < c$, as it must.

37.32. **IDENTIFY and SET UP:** The force is found from Eq. (37.32) or Eq. (37.33).

EXECUTE: **(a)** Indistinguishable from $F = ma = 0.145\ \text{N}$.

(b) $\gamma^3 ma = 1.75\ \text{N}$.

(c) $\gamma^3 ma = 51.7\ \text{N}$.

(d) $\gamma ma = 0.145\ \text{N},\ 0.333\ \text{N}, 1.03\ \text{N}$.

EVALUATE: When v is large, much more force is required to produce a given magnitude of acceleration when the force is parallel to the velocity than when the force is perpendicular to the velocity.

37.33. **IDENTIFY:** Apply Eq. (37.36).

SET UP: The rest energy is mc^2.

EXECUTE: **(a)** $K = \frac{mc^2}{\sqrt{1-v^2/c^2}} - mc^2 = mc^2$

$$\Rightarrow \frac{1}{\sqrt{1-v^2/c^2}} = 2 \Rightarrow \frac{1}{4} = 1 - \frac{v^2}{c^2} \Rightarrow v = \sqrt{\frac{3}{4}}c = 0.866c.$$

(b) $K = 5mc^2 \Rightarrow \frac{1}{\sqrt{1-v^2/c^2}} = 6 \Rightarrow \frac{1}{36} = 1 - \frac{v^2}{c^2} \Rightarrow v = \sqrt{\frac{35}{36}}c = 0.986c.$

EVALUATE: If $v \ll c$, then K is much less than the rest energy of the particle.

37.46. IDENTIFY: The total energy is conserved in the collision.

SET UP: Use Eq. (37.38) for the total energy. Since all three particles are at rest after the collision, the final total energy is $2Mc^2 + mc^2$. The initial total energy of the two protons is $\gamma 2Mc^2$.

EXECUTE: **(a)** $2Mc^2 + mc^2 = \gamma 2Mc^2 \Rightarrow \gamma = 1 + \dfrac{m}{2M} = 1 + \dfrac{9.75}{2(16.7)} = 1.292.$

Note that since $\gamma = \dfrac{1}{\sqrt{1 - v^2/c^2}}$, we have that $\dfrac{v}{c} = \sqrt{1 - \dfrac{1}{\gamma^2}} = \sqrt{1 - \dfrac{1}{(1.292)^2}} = 0.6331.$

(b) According to Eq. (37.36), the kinetic energy of each proton is

$$K = (\gamma - 1)Mc^2 = (1.292 - 1)(1.67 \times 10^{-27}\ \text{kg})(3.00 \times 10^8\ \text{m/s})^2 \left(\frac{1.00\ \text{MeV}}{1.60 \times 10^{-13}\ \text{J}}\right) = 274\ \text{MeV}.$$

(c) The rest energy of η^0 is $mc^2 = (9.75 \times 10^{-28}\ \text{kg})(3.00 \times 10^8\ \text{m/s})^2 \left(\dfrac{1.00\ \text{MeV}}{1.60 \times 10^{-13}\ \text{J}}\right) = 548\ \text{MeV}.$

EVALUATE: **(d)** The kinetic energy lost by the protons is the energy that produces the η^0, $548\ \text{MeV} = 2(274\ \text{MeV})$.

37.47. **IDENTIFY:** Use $E = mc^2$ to relate the mass decrease to the energy produced.

SET UP: 1 kg is equivalent to 2.2 lbs and $1 \text{ ton} = 2000 \text{ lbs}$. $1 \text{ W} = 1 \text{ J/s}$.

EXECUTE: **(a)** $E = mc^2, m = E/c^2 = (3.8\times10^{26} \text{ J})/(2.998\times10^{8} \text{ m/s})^2 = 4.2\times10^{9} \text{ kg} = 4.6\times10^{6} \text{ tons}$.

(b) The current mass of the sun is 1.99×10^{30} kg, so it would take it

$(1.99\times10^{30} \text{ kg})/(4.2\times10^{9} \text{ kg/s}) = 4.7\times10^{20} \text{s} = 1.5\times10^{13}$ years to use up all its mass.

EVALUATE: The power output of the sun is very large, but only a small fraction of the sun's mass is converted to energy each second.

37.51. **IDENTIFY and SET UP:** There must be a length contraction such that the length a becomes the same as b; $l_0 = a,\ l = b.\ l_0$ is the distance measured by an observer at rest relative to the spacecraft. Use Eq. (37.16) and solve for u.

EXECUTE: $\dfrac{l}{l_0} = \sqrt{1-u^2/c^2}$ so $\dfrac{b}{a} = \sqrt{1-u^2/c^2}$;

$a = 1.40b$ gives $b/1.40b = \sqrt{1-u^2/c^2}$ and thus $1-u^2/c^2 = 1/(1.40)^2$

$u = \sqrt{1-1/(1.40)^2}\,c = 0.700c = 2.10\times10^8$ m/s

EVALUATE: A length on the spacecraft in the direction of the motion is shortened. A length perpendicular to the motion is unchanged.

37.60. **IDENTIFY:** The protons are moving at speeds that are comparable to the speed of light, so we must use the relativistic velocity addition formula.

SET UP: S is lab frame and S' is frame of proton moving in $+x$-direction. $v_x = -0.600c$. In lab frame each proton has speed αc. $u = +\alpha c$. $v_x = -\alpha c$. $v_x = \dfrac{v'_x + u}{1 + uv'_x/c^2} = \dfrac{-0.600c + \alpha c}{1 - 0.600\alpha} = -\alpha c$.

EXECUTE: $(1 - 0.600\alpha)(-\alpha) = -0.600 + \alpha$. $0.600\alpha^2 - 2\alpha + 0.600 = 0$. Quadratic formula gives $\alpha = 3.00$ or $\alpha = 0.333$. Can't have $v > c$ so $\alpha = 0.333$. Each proton has speed $0.333c$ in the earth frame.

EVALUATE: To the earth observer, the protons are separating at $2(0.333c) = 0.666c$, but to the protons they are separating at $0.600c$.

37.63. IDENTIFY and SET UP: Use Eq. (37.30), with $a = dv/dt,$ to obtain an expression for dv/dt. Separate the variables v and t and integrate to obtain an expression for $v(t)$. In this expression, let $t \to \infty$.

EXECUTE: $a = \dfrac{dv}{dt} = \dfrac{F}{m}(1 - v^2/c^2)^{3/2}$. (One-dimensional motion is assumed, and all the F, v and a refer to x-components.)

$$\frac{dv}{(1-v^2/c^2)^{3/2}} = \left(\frac{F}{m}\right)dt$$

Integrate from $t = 0,$ when $v = 0,$ to time t, when the velocity is v.

$$\int_0^v \frac{dv}{(1-v^2/c^2)^{3/2}} = \int_0^t \left(\frac{F}{m}\right)dt$$

Since F is constant, $\int_0^t \left(\frac{F}{m}\right)dt = \frac{Ft}{m}$. In the velocity integral make the change of variable $y = v/c;$ then $dy = dv/c.$

$$\int_0^v \frac{dv}{(1-v^2/c^2)^{3/2}} = c\int_0^{v/c} \frac{dy}{(1-y^2)^{3/2}} = c\left[\frac{y}{(1-y^2)^{1/2}}\right]_0^{v/c} = \frac{v}{\sqrt{1-v^2/c^2}}$$

Thus $\dfrac{v}{\sqrt{1-v^2/c^2}} = \dfrac{Ft}{m}$.

Solve this equation for v:

$$\frac{v^2}{1-v^2/c^2} = \left(\frac{Ft}{m}\right)^2 \text{ and } v^2 = \left(\frac{Ft}{m}\right)^2 (1-v^2/c^2)$$

$$v^2\left(1+\left(\frac{Ft}{mc}\right)^2\right) = \left(\frac{Ft}{m}\right)^2 \text{ so } v = \frac{(Ft/m)}{\sqrt{1+(Ft/mc)^2}} = c\frac{Ft}{\sqrt{m^2c^2+F^2t^2}}$$

As $t \to \infty$, $\dfrac{Ft}{\sqrt{m^2c^2+F^2t^2}} \to \dfrac{Ft}{\sqrt{F^2t^2}} \to 1$, so $v \to c.$

EVALUATE: Note that $\dfrac{Ft}{\sqrt{m^2c^2+F^2t^2}}$ is always less than 1, so $v < c$ always and v approaches c only when $t \to \infty$.