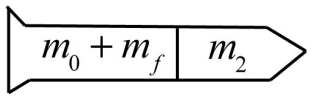
PEP 2017
Assignment 4

(1) Solve the DE $\frac{1}{2}y' + 2y = 0$ subjected to the condition that $y(0) = 3$.

(2) Solve $y'' + y = 2y'$ given that $y(0) = 1$ and $y(1) = 0$.

(3) Solve $y' = e^{(x-2y)}$ given that $y(0) = 1$.

(4) A two-stage rocket in a zero gravitational field starts from rest and burns fuel. The fuel is ejected at a speed u relative to the rocket. After all the fuel has been burned, explosive bolts release the first stage and push it backwards with a speed v_1 relative to the second stage. The mass of the first stage before ny fuel is burned is $m_1 = m_0 + m_f$, where m_f is the mass of the fuel. The mass of the second stage is m_2. The total mass of the rocket before any fuel is burnt is $m_1 + m_2$. The goal of this problem is to find the speed of the second stage after the separation.



two-stage rocket before ignition

(a) When the rocket is travelling at speed v, derive a relation between the differential of the speed of the rocket dv, and the differential of the mass of the rocket dm, and the speed u relative to the rocket of the ejected fuel.

(b) What is the speed v_f of the rocket immediately after all the fuel has been burned but before the second stage is released? Express your answer in terms of u, m_f, m_0, and m_2 as needed.

(c) What is the speed v_2 of the second stage immediately after it has been released? Express your answers in terms of v_1, v_f, m_f, m_2 and m_0 as needed.

(5) Physical pendulum A planar triangle of uniform surface density with mass M, base D and height H is hanged at the top and oscillates due to its own gravity. Assume the angle of oscillation is small, find the period of oscillation. (HINT: you need to find the position of the center of mass (CM) and the moment of inertia with respect to the rotational axis)