PEP 2017 Assignment 5 Solutions

13.89. **IDENTIFY:** As suggested in the problem, divide the disk into rings of radius r and thickness dr.

SET UP: Each ring has an area $dA = 2\pi r\,dr$ and mass $dM = \frac{M}{\pi a^2}dA = \frac{2M}{a^2}r\,dr$.

EXECUTE: The magnitude of the force that this small ring exerts on the mass m is then $(Gm\,dM)(x/(r^2+x^2)^{3/2})$. The contribution dF to the force is $dF = \frac{2GMmx}{a^2}\frac{rdr}{(x^2+r^2)^{3/2}}$.

The total force F is then the integral over the range of r;

$$F = \int dF = \frac{2GMmx}{a^2}\int_0^a \frac{r}{(x^2+r^2)^{3/2}}dr.$$

The integral (either by looking in a table or making the substitution $u = r^2 + a^2$) is

$$\int_0^a \frac{r}{(x^2+r^2)^{3/2}}dr = \left[\frac{1}{x} - \frac{1}{\sqrt{a^2+x^2}}\right] = \frac{1}{x}\left[1 - \frac{x}{\sqrt{a^2+x^2}}\right].$$

Substitution yields the result $F = \frac{2GMm}{a^2}\left[1 - \frac{x}{\sqrt{a^2+x^2}}\right]$. The force on m is directed toward the center of the ring. The second term in brackets can be written as

$$\frac{1}{\sqrt{1+(a/x)^2}} = (1+(a/x)^2)^{-1/2} \approx 1 - \frac{1}{2}\left(\frac{a}{x}\right)^2$$

if $x \gg a$, where the binomial expansion has been used. Substitution of this into the above form gives

$F \approx \frac{GMm}{x^2}$, as it should.

EVALUATE: As $x \to 0$, the force approaches a constant.

13.77. **(a) IDENTIFY and SET UP:** Use Eq. (13.17), applied to the satellites orbiting the earth rather than the sun.

EXECUTE: Find the value of a for the elliptical orbit:

$2a = r_a + r_p = R_E + h_a + R_E + h_p$, where h_a and h_p are the heights at apogee and perigee, respectively.

$$a = R_E + (h_a + h_p)/2$$

$$a = 6.38\times10^6\text{ m} + (400\times10^3\text{ m} + 4000\times10^3\text{ m})/2 = 8.58\times10^6\text{ m}$$

$$T = \frac{2\pi a^{3/2}}{\sqrt{GM_E}} = \frac{2\pi(8.58\times10^6\text{ m})^{3/2}}{\sqrt{(6.673\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\text{ kg})}} = 7.91\times10^3\text{ s}$$

(b) Conservation of angular momentum gives $r_a v_a = r_p v_p$

$$\frac{v_p}{v_a} = \frac{r_a}{r_p} = \frac{6.38\times10^6\text{ m} + 4.00\times10^6\text{ m}}{6.38\times10^6\text{ m} + 4.00\times10^5\text{ m}} = 1.53$$

(c) Conservation of energy applied to apogee and perigee gives $K_a + U_a = K_p + U_p$

$$\tfrac{1}{2}mv_a^2 - Gm_E m/r_a = \tfrac{1}{2}mv_p^2 - Gm_E m/r_p$$

$$v_p^2 - v_a^2 = 2Gm_E(1/r_p - 1/r_a) = 2Gm_E(r_a - r_p)/r_a r_p$$

But $v_p = 1.532 v_a$, so $1.347 v_a^2 = 2Gm_E(r_a - r_p)/r_a r_p$

$v_a = 5.51\times 10^3$ m/s, $v_p = 8.43\times 10^3$ m/s

(d) Need v so that $E = 0$, where $E = K + U$.

at perigee: $\frac{1}{2}mv_p^2 - Gm_E m/r_p = 0$

$$v_p = \sqrt{2Gm_E/r_p} = \sqrt{2(6.673\times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.97\times 10^{24}\ \text{kg})/6.78\times 10^6\ \text{m}} = 1.084\times 10^4\ \text{m/s}$$

This means an increase of $1.084\times 10^4\ \text{m/s} - 8.43\times 10^3\ \text{m/s} = 2.41\times 10^3\ \text{m/s}$.

at apogee:

$$v_a = \sqrt{2Gm_E/r_a} = \sqrt{2(6.673\times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.97\times 10^{24}\ \text{kg})/1.038\times 10^7\ \text{m}} = 8.761\times 10^3\ \text{m/s}$$

This means an increase of $8.761\times 10^3\ \text{m/s} - 5.51\times 10^3\ \text{m/s} = 3.25\times 10^3\ \text{m/s}$.

EVALUATE: Perigee is more efficient. At this point r is smaller so v is larger and the satellite has more kinetic energy and more total energy.

14.102. **IDENTIFY:** Calculate F_{net} and define k_{eff} by $F_{net} = -k_{eff}x$. $T = 2\pi\sqrt{m/k_{eff}}$.

SET UP: If the elongations of the springs are x_1 and x_2, they must satisfy $x_1 + x_2 = 0.200$ m.

EXECUTE: **(a)** The net force on the block at equilibrium is zero, and so $k_1x_1 = k_2x_2$ and one spring (the one with $k_1 = 2.00$ N/m) must be stretched three times as much as the one with $k_2 = 6.00$ N/m. The sum of the elongations is 0.200 m, and so one spring stretches 0.150 m and the other stretches 0.050 m, and so the equilibrium lengths are 0.350 m and 0.250 m.

(b) When the block is displaced a distance x to the right, the net force on the block is $-k_1(x_1 + x) + k_2(x_2 - x) = -[k_1x_1 - k_2x_2] - (k_1 + k_2)x$. From the result of part (a), the term in square brackets is zero, and so the net force is $-(k_1 + k_2)x$, the effective spring constant is $k_{eff} = k_1 + k_2$ and the period of vibration is $T = 2\pi\sqrt{\dfrac{0.100\ \text{kg}}{8.00\ \text{N/m}}} = 0.702$ s.

EVALUATE: The motion is the same as if the block were attached to a single spring that has force constant k_{eff}.

14.95. IDENTIFY: Apply conservation of energy to the motion before and after the collision. Apply conservation of linear momentum to the collision. After the collision the system moves as a simple pendulum. If the maximum angular displacement is small, $f = \dfrac{1}{2\pi}\sqrt{\dfrac{g}{L}}$.

SET UP: In the motion before and after the collision there is energy conversion between gravitational potential energy mgh, where h is the height above the lowest point in the motion, and kinetic energy.

EXECUTE: Energy conservation during downward swing: $m_2 g h_0 = \frac{1}{2} m_2 v^2$ and

$v = \sqrt{2gh_0} = \sqrt{2(9.8\text{ m/s}^2)(0.100\text{ m})} = 1.40\text{ m/s}$.

Momentum conservation during collision: $m_2 v = (m_2 + m_3)V$ and

$$V = \frac{m_2 v}{m_2 + m_3} = \frac{(2.00\text{ kg})(1.40\text{ m/s})}{5.00\text{ kg}} = 0.560\text{ m/s}.$$

Energy conservation during upward swing: $Mgh_f = \dfrac{1}{2}MV^2$ and

$$h_f = V^2/2g = \frac{(0.560\text{ m/s})^2}{2(9.80\text{ m/s}^2)} = 0.0160\text{ m} = 1.60\text{ cm}.$$

Figure 14.95 shows how the maximum angular displacement is calculated from h_f. $\cos\theta = \dfrac{48.4\text{ cm}}{50.0\text{ cm}}$ and

$\theta = 14.5°$. $f = \dfrac{1}{2\pi}\sqrt{\dfrac{g}{l}} = \dfrac{1}{2\pi}\sqrt{\dfrac{9.80\text{ m/s}^2}{0.500\text{ m}}} = 0.705\text{ Hz}$.

EVALUATE: $14.5° = 0.253$ rad. $\sin(0.253\text{ rad}) = 0.250$. $\sin\theta \approx \theta$ and Eq. (14.34) is accurate.

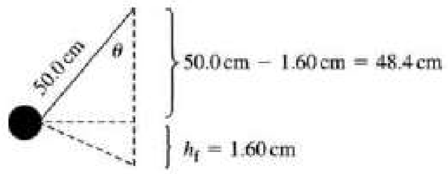

Figure 14.95

9.90. IDENTIFY: Energy conservation: Loss of U of box equals gain in K of system. Both the cylinder and pulley have kinetic energy of the form $K = \frac{1}{2}I\omega^2$.

$$m_{\text{box}}gh = \frac{1}{2}m_{\text{box}}v_{\text{box}}^2 + \frac{1}{2}I_{\text{pulley}}\omega_{\text{pulley}}^2 + \frac{1}{2}I_{\text{cylinder}}\omega_{\text{cylinder}}^2.$$

SET UP: $\omega_{\text{pulley}} = \dfrac{v_{\text{box}}}{r_p}$ and $\omega_{\text{cylinder}} = \dfrac{v_{\text{box}}}{r_{\text{cylinder}}}$.

Let B = box, P = pulley and C = cylinder.

EXECUTE: $m_B gh = \dfrac{1}{2}m_B v_B^2 + \dfrac{1}{2}\left(\dfrac{1}{2}m_P r_P^2\right)\left(\dfrac{v_B}{r_P}\right)^2 + \dfrac{1}{2}\left(\dfrac{1}{2}m_C r_C^2\right)\left(\dfrac{v_B}{r_C}\right)^2$.

$m_B gh = \dfrac{1}{2}m_B v_B^2 + \dfrac{1}{4}m_P v_B^2 + \dfrac{1}{4}m_C v_B^2$ and

$$v_B = \sqrt{\frac{m_B gh}{\frac{1}{2}m_B + \frac{1}{4}m_P + \frac{1}{4}m_C}} = \sqrt{\frac{(3.00\text{ kg})(9.80\text{ m/s}^2)(2.50\text{ m})}{1.50\text{ kg} + \frac{1}{4}(7.00\text{ kg})}} = 4.76\text{ m/s}.$$

EVALUATE: If the box was disconnected from the rope and dropped from rest, after falling 2.50 m its speed would be $v = \sqrt{2g(2.50\text{ m})} = 7.00$ m/s. Since in the problem some of the energy of the system goes into kinetic energy of the cylinder and of the pulley, the final speed of the box is less than this.

9.72. **IDENTIFY:** Use the constant angular acceleration equations, applied to the first revolution and to the first two revolutions.

SET UP: Let the direction the disk is rotating be positive. $1\text{ rev} = 2\pi$ rad. Let t be the time for the first revolution. The time for the first two revolutions is $t + 0.750$ s.

EXECUTE: **(a)** $\theta - \theta_0 = \omega_{0z}t + \frac{1}{2}\alpha_z t^2$ applied to the first revolution and then to the first two revolutions gives $2\pi \text{ rad} = \frac{1}{2}\alpha_z t^2$ and $4\pi \text{ rad} = \frac{1}{2}\alpha_z (t + 0.750 \text{ s})^2$. Eliminating α_z between these equations gives $4\pi \text{ rad} = \frac{2\pi \text{ rad}}{t^2}(t + 0.750 \text{ s})^2$. $2t^2 = (t + 0.750 \text{ s})^2$. $\sqrt{2}t = \pm(t + 0.750 \text{ s})$. The positive root is $t = \frac{0.750 \text{ s}}{\sqrt{2} - 1} = 1.81 \text{ s}$.

(b) $2\pi \text{ rad} = \frac{1}{2}\alpha_z t^2$ and $t = 1.81$ s gives $\alpha_z = 3.84 \text{ rad/s}^2$

EVALUATE: At the start of the second revolution, $\omega_{0z} = (3.84 \text{ rad/s}^2)(1.81 \text{ s}) = 6.95 \text{ rad/s}$. The distance the disk rotates in the next 0.750 s is $\theta - \theta_0 = \omega_{0z}t + \frac{1}{2}\alpha_z t^2 = (6.95 \text{ rad/s})(0.750 \text{ s}) + \frac{1}{2}(3.84 \text{ rad/s}^2)(0.750 \text{ s})^2 =$ 6.29 rad, which is two revolutions.

10.102. **IDENTIFY:** The vertical forces must sum to zero. Apply Eq. (10.33).

SET UP: Denote the upward forces that the hands exert as F_L and F_R. $\tau = (F_L - F_R)r$, where $r = 0.200$ m.

EXECUTE: The conditions that F_L and F_R must satisfy are $F_L + F_R = w$ and $F_L - F_R = \Omega \frac{I\omega}{r}$, where the second equation is $\tau = \Omega L$, divided by r. These two equations can be solved for the forces by first adding and then subtracting, yielding $F_L = \frac{1}{2}\left(w + \Omega\frac{I\omega}{r}\right)$ and $F_R = \frac{1}{2}\left(w - \Omega\frac{I\omega}{r}\right)$. Using the values

$w = mg = (8.00 \text{ kg})(9.80 \text{ m/s}^2) = 78.4 \text{ N}$ and

$\frac{I\omega}{r} = \frac{(8.00 \text{ kg})(0.325 \text{ m})^2(5.00 \text{ rev/s} \times 2\pi \text{ rad/rev})}{(0.200 \text{ m})} = 132.7 \text{ kg}\cdot\text{m/s}$ gives

$F_L = 39.2 \text{ N} + \Omega(66.4 \text{ N}\cdot\text{s})$, $F_R = 39.2 \text{ N} - \Omega(66.4 \text{ N}\cdot\text{s})$.

(a) $\Omega = 0, F_L = F_R = 39.2 \text{ N}$.

(b) $\Omega = 0.05 \text{ rev/s} = 0.314 \text{ rad/s}, F_L = 60.0 \text{ N}, F_R = 18.4 \text{ N}$.

(c) $\Omega = 0.3 \text{ rev/s} = 1.89 \text{ rad/s}, F_L = 165 \text{ N}, F_R = -86.2 \text{ N}$, with the minus sign indicating a downward force.

(d) $F_R = 0$ gives $\Omega = \frac{39.2 \text{ N}}{66.4 \text{ N}\cdot\text{s}} = 0.590 \text{ rad/s}$, which is 0.0940 rev/s.

EVALUATE: The larger the precession rate Ω, the greater the torque on the wheel and the greater the difference between the forces exerted by the two hands.