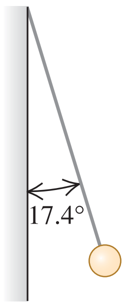
PEP 2017
Assignment 10

21.67 •• **CP** Two positive point charges Q are held fixed on the x-axis at $x = a$ and $x = -a$. A third positive point charge q, with mass m, is placed on the x-axis away from the origin at a coordinate x such that $|x| \ll a$. The charge q, which is free to move along the x-axis, is then released. (a) Find the frequency of oscillation of the charge q. (*Hint:* Review the definition of simple harmonic motion in Section 14.2. Use the binomial expansion $(1 + z)^n = 1 + nz + n(n-1)z^2/2 + \cdots$, valid for the case $|z| < 1$.) (b) Suppose instead that the charge q were placed on the y-axis at a coordinate y such that $|y| \ll a$, and then released. If this charge is free to move anywhere in the xy-plane, what will happen to it? Explain your answer.

21.73 •• **CP** A small 12.3-g plastic ball is tied to a very light 28.6-cm string that is attached to the vertical wall of a room (Fig. P21.73). A uniform horizontal electric field exists in this room. When the ball has been given an excess charge of $-1.11\ \mu C$, you observe that it remains suspended, with the string making an angle of 17.4° with the wall. Find the magnitude and direction of the electric field in the room.

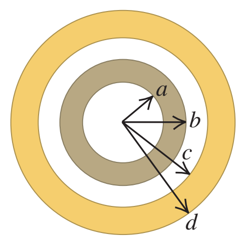
Figure **P21.73**



21.95 • **CALC** Positive charge $+Q$ is distributed uniformly along the $+x$-axis from $x = 0$ to $x = a$. Negative charge $-Q$ is distributed uniformly along the $-x$-axis from $x = 0$ to $x = -a$. (a) A positive point charge q lies on the positive y-axis, a distance y from the origin. Find the force (magnitude and direction) that the positive and negative charge distributions together exert on q. Show that this force is proportional to y^{-3} for $y \gg a$. (b) Suppose instead that the positive point charge q lies on the positive x-axis, a distance $x > a$ from the origin. Find the force (magnitude and direction) that the charge distribution exerts on q. Show that this force is proportional to x^{-3} for $x \gg a$.

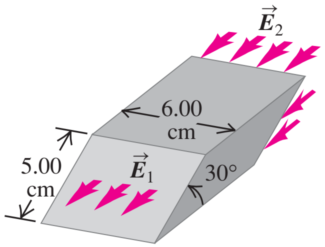
22.47 • **Concentric Spherical Shells.** A small conducting spherical shell with inner radius a and outer radius b is concentric with a larger conducting spherical shell with inner radius c and outer radius d (Fig. P22.47). The inner shell has total charge $+2q$, and the outer shell has charge $+4q$. (a) Calculate the electric field (magnitude and direction) in terms of q and the distance r from the common center of the two shells for (i) $r < a$; (ii) $a < r < b$; (iii) $b < r < c$; (iv) $c < r < d$; (v) $r > d$. Show your results in a graph of the radial component of $\vec{E}$ as a function of r. (b) What is the total charge on the (i) inner surface of the small shell; (ii) outer surface of the small shell; (iii) inner surface of the large shell; (iv) outer surface of the large shell?

Figure **P22.47**



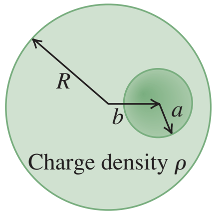
22.37 •• The electric field $\vec{E}_1$ at one face of a parallelepiped is uniform over the entire face and is directed out of the face. At the opposite face, the electric field $\vec{E}_2$ is also uniform over the entire face and is directed into that face (Fig. P22.37). The two faces in question are inclined at 30.0° from the horizontal, while $\vec{E}_1$ and $\vec{E}_2$ are both horizontal; $\vec{E}_1$ has a magnitude of 2.50×10^4 N/C, and $\vec{E}_2$ has a magnitude of 7.00×10^4 N/C. (a) Assuming that no other electric field lines cross the surfaces of the parallelepiped, determine the net charge contained within. (b) Is the electric field produced only by the charges within the parallelepiped, or is the field also due to charges outside the parallelepiped? How can you tell?

Figure **P22.37**



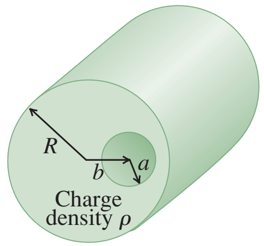
22.61 • (a) An insulating sphere with radius a has a uniform charge density ρ. The sphere is not centered at the origin but at $\vec{r} = \vec{b}$. Show that the electric field inside the sphere is given by $\vec{E} = \rho(\vec{r} - \vec{b})/3\epsilon_0$. (b) An insulating sphere of radius R has a spherical hole of radius a located within its volume and centered a distance b from the center of the sphere, where $a < b < R$ (a cross section of the sphere is shown in Fig. P22.61). The solid part of the sphere has a uniform volume charge density ρ. Find the magnitude and direction of the electric field $\vec{E}$ inside the hole, and show that $\vec{E}$ is uniform over the entire hole. [*Hint:* Use the principle of superposition and the result of part (a).]

Figure **P22.61**



22.62 • A very long, solid insulating cylinder with radius R has a cylindrical hole with radius a bored along its entire length. The axis of the hole is a distance b from the axis of the cylinder, where $a < b < R$ (Fig. P22.62). The solid material of the cylinder has a uniform volume charge density ρ. Find the magnitude and direction of the electric field $\vec{E}$ inside the hole, and show that $\vec{E}$ is uniform over the entire hole. (*Hint:* See Problem 22.61.)

Figure **P22.62**



21.67. **(a) IDENTIFY:** Use Coulomb's law to calculate the force exerted by each Q on q and add these forces as vectors to find the resultant force. Make the approximation $x \gg a$ and compare the net force to $F = -kx$ to deduce k and then $f = (1/2\pi)\sqrt{k/m}$.

SET UP: The placement of the charges is shown in Figure 21.67a.

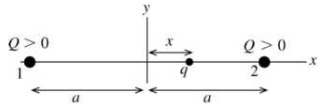


Figure 21. 67a

EXECUTE: Find the net force on q.

$F_x = F_{1x} + F_{2x}$ and $F_{1x} = +F_1, F_{2x} = -F_2$

Figure 21. 67b

$$F_1 = \frac{1}{4\pi\epsilon_0}\frac{qQ}{(a+x)^2}, F_2 = \frac{1}{4\pi\epsilon_0}\frac{qQ}{(a-x)^2}$$

$$F_x = F_1 - F_2 = \frac{qQ}{4\pi\epsilon_0}\left[\frac{1}{(a+x)^2} - \frac{1}{(a-x)^2}\right]$$

$$F_x = \frac{qQ}{4\pi\epsilon_0 a^2}\left[+\left(1+\frac{x}{a}\right)^{-2} - \left(1-\frac{x}{a}\right)^{-2}\right]$$

Since $x \ll a$ we can use the binomial expansion for $(1-x/a)^{-2}$ and $(1+x/a)^{-2}$ and keep only the first two terms: $(1+z)^n \approx 1+nz$. For $(1-x/a)^{-2}$, $z=-x/a$ and $n=-2$ so $(1-x/a)^{-2} \approx 1+2x/a$. For $(1+x/a)^{-2}$, $z=+x/a$ and $n=-2$ so $(1+x/a)^{-2} \approx 1-2x/a$. Then $F \approx \frac{qQ}{4\pi\epsilon_0 a^2}\left[\left(1-\frac{2x}{a}\right)-\left(1+\frac{2x}{a}\right)\right] = -\left(\frac{qQ}{\pi\epsilon_0 a^3}\right)x.$

For simple harmonic motion $F=-kx$ and the frequency of oscillation is $f=(1/2\pi)\sqrt{k/m}$. The net force here is of this form, with $k = qQ/\pi\epsilon_0 a^3$. Thus $f = \frac{1}{2\pi}\sqrt{\frac{qQ}{\pi\epsilon_0 m a^3}}$.

(b) The forces and their components are shown in Figure 21.67c.

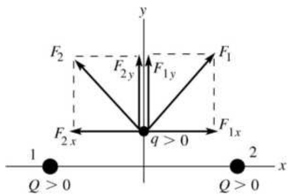


Figure 21.67c

The x-components of the forces exerted by the two charges cancel, the y-components add, and the net force is in the $+y$-direction when $y > 0$ and in the $-y$-direction when $y < 0$. The charge moves away from the origin on the y-axis and never returns.

EVALUATE: The directions of the forces and of the net force depend on where q is located relative to the other two charges. In part (a), $F = 0$ at $x = 0$ and when the charge q is displaced in the $+x$- or $-x$-direction the net force is a restoring force, directed to return q to $x = 0$. The charge oscillates back and forth, similar to a mass on a spring.

21.73. **IDENTIFY:** The electric field exerts a horizontal force away from the wall on the ball. When the ball hangs at rest, the forces on it (gravity, the tension in the string, and the electric force due to the field) add to zero.
SET UP: The ball is in equilibrium, so for it $\sum F_x = 0$ and $\sum F_y = 0$. The force diagram for the ball is given in Figure 21.73. F_E is the force exerted by the electric field. $\vec{F} = q\vec{E}$. Since the electric field is horizontal, $\vec{F}_E$ is horizontal. Use the coordinates shown in the figure. The tension in the string has been replaced by its x- and y-components.

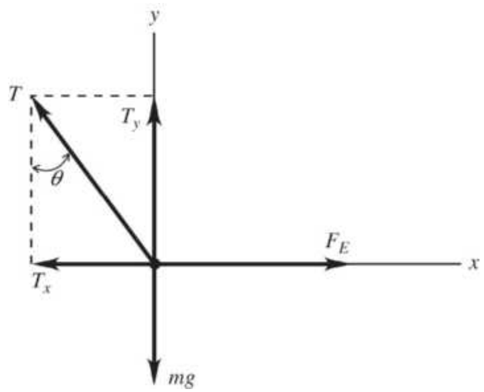


Figure 21.73

EXECUTE: $\sum F_y = 0$ gives $T_y - mg = 0$. $T\cos\theta - mg = 0$ and $T = \dfrac{mg}{\cos\theta}$. $\sum F_x = 0$ gives $F_E - T_x = 0$.
$F_E - T\sin\theta = 0$. Combing the equations and solving for F_E gives

$$F_E = \left(\frac{mg}{\cos\theta}\right)\sin\theta = mg\tan\theta = (12.3\times10^{-3}\ \text{kg})(9.80\ \text{m/s}^2)(\tan 17.4°) = 3.78\times10^{-2}\ \text{N}.$$ $F_E = |q|E$ so

$$E = \frac{F_E}{|q|} = \frac{3.78\times10^{-2}\ \text{N}}{1.11\times10^{-6}\ \text{C}} = 3.41\times10^4\ \text{N/C}.$$ Since q is negative and $\vec{F}_E$ is to the right, $\vec{E}$ is to the left in the figure.

EVALUATE: The larger the electric field E the greater the angle the string makes with the wall.

21.95. **IDENTIFY:** Find the resultant electric field due to the two point charges. Then use $\vec{F} = q\vec{E}$ to calculate the force on the point charge.

SET UP: Use the results of Problems 21.90 and 21.89.

EXECUTE: **(a)** The y-components of the electric field cancel, and the x-component from both charges, as given in Problem 21.90, is $E_x = \frac{1}{4\pi\epsilon_0}\frac{-2Q}{a}\left(\frac{1}{y} - \frac{1}{(y^2+a^2)^{1/2}}\right)$. Therefore,

$\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{-2Qq}{a}\left(\frac{1}{y} - \frac{1}{(y^2+a^2)^{1/2}}\right)\hat{\boldsymbol{i}}$. If $y \gg a$, $\vec{F} \approx \frac{1}{4\pi\epsilon_0}\frac{-2Qq}{ay}(1-(1-a^2/2y^2+\cdots))\hat{\boldsymbol{i}} = -\frac{1}{4\pi\epsilon_0}\frac{Qqa}{y^3}\hat{\boldsymbol{i}}$.

(b) If the point charge is now on the x-axis the two halves of the charge distribution provide different forces, though still along the x-axis, as given in Problem 21.89: $\vec{F}_+ = q\vec{E}_+ = \frac{1}{4\pi\epsilon_0}\frac{Qq}{a}\left(\frac{1}{x-a} - \frac{1}{x}\right)\hat{\boldsymbol{i}}$

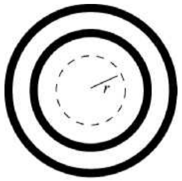
and $\vec{F}_- = q\vec{E}_- = -\frac{1}{4\pi\epsilon_0}\frac{Qq}{a}\left(\frac{1}{x} - \frac{1}{x+a}\right)\hat{\boldsymbol{i}}$. Therefore, $\vec{F} = \vec{F}_+ + \vec{F}_- = \frac{1}{4\pi\epsilon_0}\frac{Qq}{a}\left(\frac{1}{x-a} - \frac{2}{x} + \frac{1}{x+a}\right)\hat{\boldsymbol{i}}$. For

$x \gg a$, $\vec{F} \approx \frac{1}{4\pi\epsilon_0}\frac{Qq}{ax}\left(\left(1+\frac{a}{x}+\frac{a^2}{x^2}+\ldots\right) - 2 + \left(1-\frac{a}{x}+\frac{a^2}{x^2}-\ldots\right)\right)\hat{\boldsymbol{i}} = \frac{1}{4\pi\epsilon_0}\frac{2Qqa}{x^3}\hat{\boldsymbol{i}}$.

EVALUATE: If the charge distributed along the x-axis were all positive or all negative, the force would be proportional to $1/y^2$ in part (a) and to $1/x^2$ in part (b), when y or x is very large.

22.47. **IDENTIFY:** Apply Gauss's law to a spherical Gaussian surface with radius r. Calculate the electric field at the surface of the Gaussian sphere.

(a) SET UP: (i) $r < a$: The Gaussian surface is sketched in Figure 22.47a.



EXECUTE: $\Phi_E = EA = E(4\pi r^2)$

$Q_{\text{encl}} = 0$; no charge is enclosed

$\Phi_E = \dfrac{Q_{\text{encl}}}{\epsilon_0}$ says

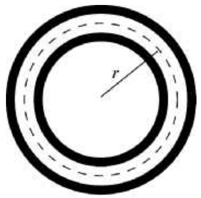
$E(4\pi r^2) = 0$ and $E = 0$.

Figure 22.47a

(ii) $a < r < b$: Points in this region are in the conductor of the small shell, so $E = 0$.

(iii) **SET UP:** $b < r < c$: The Gaussian surface is sketched in Figure 22.47b.

Apply Gauss's law to a spherical Gaussian surface with radius $b < r < c$.



EXECUTE: $\Phi_E = EA = E(4\pi r^2)$

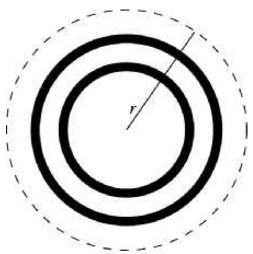
The Gaussian surface encloses all of the small shell and none of the large shell, so $Q_{\text{encl}} = +2q$.

Figure 22.47b

$\Phi_E = \dfrac{Q_{\text{encl}}}{\epsilon_0}$ gives $E(4\pi r^2) = \dfrac{2q}{\epsilon_0}$ so $E = \dfrac{2q}{4\pi\epsilon_0 r^2}$. Since the enclosed charge is positive the electric field is radially outward.

(iv) $c < r < d$: Points in this region are in the conductor of the large shell, so $E = 0$.

(v) **SET UP:** $r > d$: Apply Gauss's law to a spherical Gaussian surface with radius $r > d$, as shown in Figure 22.47c.



EXECUTE: $\Phi_E = EA = E(4\pi r^2)$

The Gaussian surface encloses all of the small shell and all of the large shell, so $Q_{\text{encl}} = +2q + 4q = 6q$.

Figure 22.47c

$\Phi_E = \dfrac{Q_{\text{encl}}}{\epsilon_0}$ gives $E(4\pi r^2) = \dfrac{6q}{\epsilon_0}$

$E = \dfrac{6q}{4\pi\epsilon_0 r^2}$. Since the enclosed charge is positive the electric field is radially outward.

The graph of E versus r is sketched in Figure 22.47d.

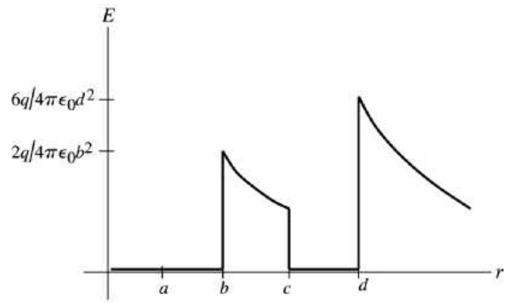


Figure 22.47d

(b) IDENTIFY and **SET UP:** Apply Gauss's law to a sphere that lies outside the surface of the shell for which we want to find the surface charge.

EXECUTE: (i) charge on inner surface of the small shell: Apply Gauss's law to a spherical Gaussian surface with radius $a < r < b$. This surface lies within the conductor of the small shell, where $E = 0$, so $\Phi_E = 0$. Thus by Gauss's law $Q_{\text{encl}} = 0$, so there is zero charge on the inner surface of the small shell.

(ii) charge on outer surface of the small shell: The total charge on the small shell is $+2q$. We found in part (i) that there is zero charge on the inner surface of the shell, so all $+2q$ must reside on the outer surface.

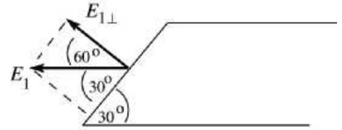
(iii) charge on inner surface of large shell: Apply Gauss's law to a spherical Gaussian surface with radius $c < r < d$. The surface lies within the conductor of the large shell, where $E = 0$, so $\Phi_E = 0$. Thus by Gauss's law $Q_{\text{encl}} = 0$. The surface encloses the $+2q$ on the small shell so there must be charge $-2q$ on the inner surface of the large shell to make the total enclosed charge zero.

(iv) charge on outer surface of large shell: The total charge on the large shell is $+4q$. We showed in part (iii) that the charge on the inner surface is $-2q$, so there must be $+6q$ on the outer surface.

EVALUATE: The electric field lines for $b < r < c$ originate from the surface charge on the outer surface of the inner shell and all terminate on the surface charge on the inner surface of the outer shell. These surface charges have equal magnitude and opposite sign. The electric field lines for $r > d$ originate from the surface charge on the outer surface of the outer sphere.

22.37. **(a) IDENTIFY:** Find the net flux through the parallelepiped surface and then use that in Gauss's law to find the net charge within. Flux out of the surface is positive and flux into the surface is negative.

SET UP: $\vec{E}_1$ gives flux out of the surface. See Figure 22.37a.



EXECUTE: $\Phi_1 = +E_{1\perp}A$

$A = (0.0600\ \text{m})(0.0500\ \text{m}) = 3.00\times10^{-3}\ \text{m}^2$

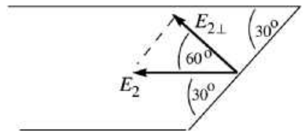
$E_{1\perp} = E_1\cos 60° = (2.50\times10^4\ \text{N/C})\cos 60°$

$E_{1\perp} = 1.25\times10^4\ \text{N/C}$

Figure 22.37a

$\Phi_{E_1} = +E_{1\perp}A = +(1.25\times10^4\ \text{N/C})(3.00\times10^{-3}\ \text{m}^2) = 37.5\ \text{N}\cdot\text{m}^2/\text{C}$

SET UP: $\vec{E}_2$ gives flux into the surface. See Figure 22.37b.



EXECUTE: $\Phi_2 = -E_{2\perp}A$

$A = (0.0600\ \text{m})(0.0500\ \text{m}) = 3.00\times10^{-3}\ \text{m}^2$

$E_{2\perp} = E_2\cos 60° = (7.00\times10^4\ \text{N/C})\cos 60°$

$E_{2\perp} = 3.50\times10^4\ \text{N/C}$

Figure 22.37b

$\Phi_{E_2} = -E_{2\perp}A = -(3.50\times10^4\ \text{N/C})(3.00\times10^{-3}\ \text{m}^2) = -105.0\ \text{N}\cdot\text{m}^2/\text{C}$

The net flux is $\Phi_E = \Phi_{E_1} + \Phi_{E_2} = +37.5\ \text{N}\cdot\text{m}^2/\text{C} - 105.0\ \text{N}\cdot\text{m}^2/\text{C} = -67.5\ \text{N}\cdot\text{m}^2/\text{C}$.

The net flux is negative (inward), so the net charge enclosed is negative.

Apply Gauss's law: $\Phi_E = \dfrac{Q_{\text{encl}}}{\epsilon_0}$

$Q_{\text{encl}} = \Phi_E\epsilon_0 = (-67.5\ \text{N}\cdot\text{m}^2/\text{C})(8.854\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2) = -5.98\times10^{-10}\ \text{C}$.

(b) EVALUATE: If there were no charge within the parallelpiped the net flux would be zero. This is not the case, so there is charge inside. The electric field lines that pass out through the surface of the parallelpiped must terminate on charges, so there also must be charges outside the parallelpiped.

22.61. **(a) IDENTIFY:** Use $\vec{E}(\vec{r})$ from Example (22.9) (inside the sphere) and relate the position vector of a point inside the sphere measured from the origin to that measured from the center of the sphere.

SET UP: For an insulating sphere of uniform charge density ρ and centered at the origin, the electric field inside the sphere is given by $E = Qr'/4\pi\epsilon_0 R^3$ (Example 22.9), where $\vec{r}'$ is the vector from the center of the sphere to the point where E is calculated.

But $\rho = 3Q/4\pi R^3$ so this may be written as $E = \rho r/3\epsilon_0$. And $\vec{E}$ is radially outward, in the direction of $\vec{r}'$, so $\vec{E} = \rho\vec{r}'/3\epsilon_0$.

For a sphere whose center is located by vector $\vec{b}$, a point inside the sphere and located by $\vec{r}$ is located by the vector $\vec{r}' = \vec{r} - \vec{b}$ relative to the center of the sphere, as shown in Figure 22.61.

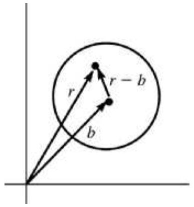


Figure 22.61

EXECUTE: Thus $\vec{E} = \dfrac{\rho(\vec{r} - \vec{b})}{3\epsilon_0}$

EVALUATE: When $b = 0$ this reduces to the result of Example 22.9. When $\vec{r} = \vec{b}$, this gives $E = 0$, which is correct since we know that $E = 0$ at the center of the sphere.

(b) IDENTIFY: The charge distribution can be represented as a uniform sphere with charge density ρ and centered at the origin added to a uniform sphere with charge density $-\rho$ and centered at $\vec{r} = \vec{b}$.

SET UP: $\vec{E} = \vec{E}_{\text{uniform}} + \vec{E}_{\text{hole}}$, where $\vec{E}_{\text{uniform}}$ is the field of a uniformly charged sphere with charge density ρ and $\vec{E}_{\text{hole}}$ is the field of a sphere located at the hole and with charge density $-\rho$. (Within the spherical hole the net charge density is $+\rho - \rho = 0$.)

EXECUTE: $\vec{E}_{\text{uniform}} = \dfrac{\rho\vec{r}}{3\epsilon_0}$, where $\vec{r}$ is a vector from the center of the sphere.

$\vec{E}_{\text{hole}} = \dfrac{-\rho(\vec{r} - \vec{b})}{3\epsilon_0}$, at points inside the hole.

Then $\vec{E} = \dfrac{\rho\vec{r}}{3\epsilon_0} + \left(\dfrac{-\rho(\vec{r} - \vec{b})}{3\epsilon_0}\right) = \dfrac{\rho\vec{b}}{3\epsilon_0}$.

EVALUATE: $\vec{E}$ is independent of $\vec{r}$ so is uniform inside the hole. The direction of $\vec{E}$ inside the hole is in the direction of the vector $\vec{b}$, the direction from the center of the insulating sphere to the center of the hole.

22.62. **IDENTIFY:** We first find the field of a cylinder off-axis, then the electric field in a hole in a cylinder is the difference between two electric fields: that of a solid cylinder on-axis, and one off-axis, at the location of the hole.
SET UP: Let $\vec{r}$ locate a point within the hole, relative to the axis of the cylinder and let $\vec{r}'$ locate this point relative to the axis of the hole. Let $\vec{b}$ locate the axis of the hole relative to the axis of the cylinder. As shown in Figure 22.62, $\vec{r}' = \vec{r} - \vec{b}$. Problem 22.42 shows that at points within a long insulating cylinder,

$$\vec{E} = \frac{\rho\vec{r}}{2\epsilon_0}.$$

EXECUTE: $\vec{E}_{\text{off-axis}} = \frac{\rho\vec{r}'}{2\epsilon_0} = \frac{\rho(\vec{r} - \vec{b})}{2\epsilon_0}$. $\vec{E}_{\text{hole}} = \vec{E}_{\text{cylinder}} - \vec{E}_{\text{off-axis}} = \frac{\rho\vec{r}}{2\epsilon_0} - \frac{\rho(\vec{r} - \vec{b})}{2\epsilon_0} = \frac{\rho\vec{b}}{2\epsilon_0}$.

Note that $\vec{E}$ is uniform.
EVALUATE: If the hole is coaxial with the cylinder, $b = 0$ and $E_{\text{hole}} = 0$.

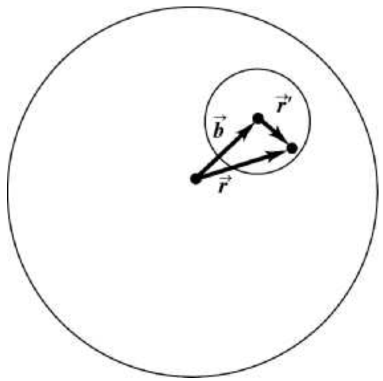


Figure 22.62