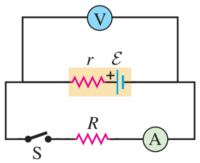
PEP 2017
Assignment 12

25.16 •• A ductile metal wire has resistance R. What will be the resistance of this wire in terms of R if it is stretched to three times its original length, assuming that the density and resistivity of the material do not change when the wire is stretched? (*Hint:* The amount of metal does not change, so stretching out the wire will affect its cross-sectional area.)

25.33 • When switch S in Fig. E25.33 is open, the voltmeter V of the battery reads 3.08 V. When the switch is closed, the voltmeter reading drops to 2.97 V, and the ammeter A reads 1.65 A. Find the emf, the internal resistance of the battery, and the circuit resistance R. Assume that the two meters are ideal, so they don't affect the circuit.

Figure **E25.33**

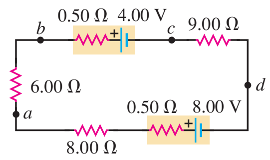


25.34 • In the circuit of Fig. E25.32,

25.52 •• A typical small flashlight contains two batteries, each having an emf of 1.5 V, connected in series with a bulb having resistance 17 Ω. (a) If the internal resistance of the batteries is negligible, what power is delivered to the bulb? (b) If the batteries last for 5.0 h, what is the total energy delivered to the bulb? (c) The resistance of real batteries increases as they run down. If the initial internal resistance is negligible, what is the combined internal resistance of both batteries when the power to the bulb has decreased to half its initial value? (Assume that the resistance of the bulb is constant. Actually, it will change somewhat when the current through the filament changes, because this changes the temperature of the filament and hence the resistivity of the filament wire.)

25.68 • (a) What is the potential difference V_{ad} in the circuit of Fig. P25.68? (b) What is the terminal voltage of the 4.00-V battery? (c) A battery with emf 10.30 V and internal resistance 0.50 Ω is inserted in the circuit at d, with its negative terminal connected to the negative terminal of the 8.00-V battery. What is the difference of potential V_{bc} between the terminals of the 4.00-V battery now?

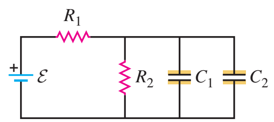
Figure **P25.68**



25.74 •• A cylindrical copper cable 1.50 km long is connected across a 220.0-V potential difference. (a) What should be its diameter so that it produces heat at a rate of 75.0 W? (b) What is the electric field inside the cable under these conditions?

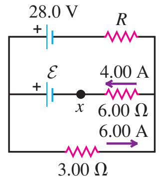
25.84 •• **CP** Consider the circuit shown in Fig. P25.84. The battery has emf 60.0 V and negligible internal resistance. $R_2 = 2.00\ \Omega$, $C_1 = 3.00\ \mu\text{F}$, and $C_2 = 6.00\ \mu\text{F}$. After the capacitors have attained their final charges, the charge on C_1 is $Q_1 = 18.0\ \mu\text{C}$. (a) What is the final charge on C_2? (b) What is the resistance R_1?

Figure **P25.84**



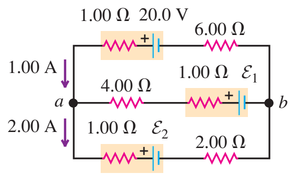
26.25 • In the circuit shown in Fig. E26.25 find (a) the current in resistor R; (b) the resistance R; (c) the unknown emf $\mathcal{E}$. (d) If the circuit is broken at point x, what is the current in resistor R?

Figure **E26.25**



26.26 • Find the emfs $\mathcal{E}_1$ and $\mathcal{E}_2$ in the circuit of Fig. E26.26, and find the potential difference of point b relative to point a.

Figure **E26.26**



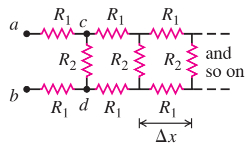
26.42 • A 12.4-μF capacitor is connected through a 0.895-MΩ resistor to a constant potential difference of 60.0 V. (a) Compute the charge on the capacitor at the following times after the connections are made: 0, 5.0 s, 10.0 s, 20.0 s, and 100.0 s. (b) Compute the charging currents at the same instants. (c) Graph the results of parts (a) and (b) for t between 0 and 20 s.

26.91 ••• **An Infinite Network.** As shown in Fig. P26.91, a network of resistors of resistances R_1 and R_2 extends to infinity toward the right. Prove that the total resistance R_T of the infinite network is equal to

$$R_T = R_1 + \sqrt{R_1^2 + 2R_1R_2}$$

(*Hint:* Since the network is infinite, the resistance of the network to the right of points c and d is also equal to R_T.)

Figure **P26.91**



25.16. **IDENTIFY:** The geometry of the wire is changed, so its resistance will also change.

SET UP: $R = \dfrac{\rho L}{A}$. $L_{\text{new}} = 3L$. The volume of the wire remains the same when it is stretched.

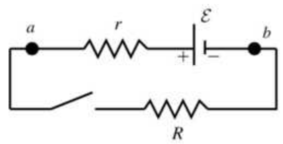
EXECUTE: Volume $= LA$ so $LA = L_{\text{new}} A_{\text{new}}$. $A_{\text{new}} = \dfrac{L}{L_{\text{new}}} A = \dfrac{A}{3}$.

$$R_{\text{new}} = \frac{\rho L_{\text{new}}}{A_{\text{new}}} = \frac{\rho(3L)}{A/3} = 9\frac{\rho L}{A} = 9R.$$

EVALUATE: When the length increases the resistance increases and when the area decreases the resistance increases.

25.33. **IDENTIFY:** The voltmeter reads the potential difference V_{ab} between the terminals of the battery.

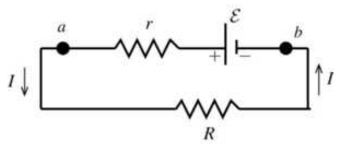
SET UP: open circuit $I = 0$. The circuit is sketched in Figure 25.33a.



EXECUTE: $V_{ab} = \mathcal{E} = 3.08$ V

Figure 25.33a

SET UP: switch closed The circuit is sketched in Figure 25.33b.



EXECUTE: $V_{ab} = \mathcal{E} - Ir = 2.97$ V

$$r = \frac{\mathcal{E} - 2.97 \text{ V}}{I}$$

$$r = \frac{3.08 \text{ V} - 2.97 \text{ V}}{1.65 \text{ A}} = 0.067\ \Omega$$

Figure 25.33b

And $V_{ab} = IR$ so $R = \frac{V_{ab}}{I} = \frac{2.97 \text{ V}}{1.65 \text{ A}} = 1.80\ \Omega.$

EVALUATE: When current flows through the battery there is a voltage drop across its internal resistance and its terminal voltage V is less than its emf.

25.52. **IDENTIFY:** The power delivered to the bulb is I^2R. Energy $= Pt$.

SET UP: The circuit is sketched in Figure 25.52. r_{total} is the combined internal resistance of both batteries.

EXECUTE: **(a)** $r_{\text{total}} = 0$. The sum of the potential changes around the circuit is zero, so

$1.5\text{ V} + 1.5\text{ V} - I(17\ \Omega) = 0$. $I = 0.1765\text{ A}$. $P = I^2R = (0.1765\text{ A})^2(17\ \Omega) = 0.530\text{ W}$. This is also $(3.0\text{ V})(0.1765\text{ A})$.

(b) Energy $= (0.530\text{ W})(5.0\text{ h})(3600\text{ s/h}) = 9540\text{ J}$

(c) $P = \dfrac{0.530\text{ W}}{2} = 0.265\text{ W}$. $P = I^2R$ so $I = \sqrt{\dfrac{P}{R}} = \sqrt{\dfrac{0.265\text{ W}}{17\ \Omega}} = 0.125\text{ A}$.

The sum of the potential changes around the circuit is zero, so $1.5\text{ V} + 1.5\text{ V} - IR - Ir_{\text{total}} = 0$.

$$r_{\text{total}} = \frac{3.0\text{ V} - (0.125\text{ A})(17\ \Omega)}{0.125\text{ A}} = 7.0\ \Omega.$$

EVALUATE: When the power to the bulb has decreased to half its initial value, the total internal resistance of the two batteries is nearly half the resistance of the bulb. Compared to a single battery, using two identical batteries in series doubles the emf but also doubles the total internal resistance.

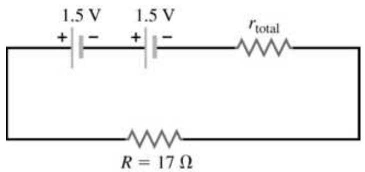


Figure 25.52

25.68. **IDENTIFY:** Consider the potential changes around the circuit. For a complete loop the sum of the potential changes is zero.

SET UP: There is a potential drop of IR when you pass through a resistor in the direction of the current.

EXECUTE: **(a)** $I = \dfrac{8.0\ \text{V} - 4.0\ \text{V}}{24.0\ \Omega} = 0.167\ \text{A}$. $V_d + 8.00\ \text{V} - I(0.50\ \Omega + 8.00\ \Omega) = V_a$, so

$V_{ad} = 8.00\ \text{V} - (0.167\ \text{A})(8.50\ \Omega) = 6.58\ \text{V}$.

(b) The terminal voltage is $V_{bc} = V_b - V_c$. $V_c + 4.00\ \text{V} + I(0.50\ \Omega) = V_b$ and

$V_{bc} = +4.00\ \text{V} + (0.167\ \text{A})(0.50\ \Omega) = +4.08\ \text{V}$.

(c) Adding another battery at point d in the opposite sense to the 8.0-V battery produces a counterclockwise current with magnitude $I = \dfrac{10.3\ \text{V} - 8.0\ \text{V} + 4.0\ \text{V}}{24.5\ \Omega} = 0.257\ \text{A}$. Then $V_c + 4.00\ \text{V} - I(0.50\ \Omega) = V_b$ and

$V_{bc} = 4.00\ \text{V} - (0.257\ \text{A})\,(0.50\ \Omega) = 3.87\ \text{V}$.

EVALUATE: When current enters the battery at its negative terminal, as in part (c), the terminal voltage is less than its emf. When current enters the battery at the positive terminal, as in part (b), the terminal voltage is greater than its emf.

25.74. **(a) IDENTIFY:** The rate of heating (power) in the cable depends on the potential difference across the cable and the resistance of the cable.

SET UP: The power is $P = V^2/R$ and the resistance is $R = \rho L/A.$ The diameter D of the cable is twice its radius. $P = \dfrac{V^2}{R} = \dfrac{V^2}{(\rho L/A)} = \dfrac{AV^2}{\rho L} = \dfrac{\pi r^2 V^2}{\rho L}.$ The electric field in the cable is equal to the potential difference across its ends divided by the length of the cable: $E = V/L.$

EXECUTE: Solving for r and using the resistivity of copper gives

$$r = \sqrt{\frac{P\rho L}{\pi V^2}} = \sqrt{\frac{(50.0\text{ W})(1.72\times10^{-8}\ \Omega\cdot\text{m})(1500\text{ m})}{\pi(220.0\text{ V})^2}} = 9.21\times10^{-5}\text{ m}.\quad D = 2r = 0.184\text{ mm}$$

(b) SET UP: $E = V/L$

EXECUTE: $E = (220\text{ V})/(1500\text{ m}) = 0.147\text{ V/m}$

EVALUATE: This would be an extremely thin (and hence fragile) cable.

25.84. **IDENTIFY:** No current flows to the capacitors when they are fully charged.

SET UP: $V_R = RI$ and $V_C = Q/C$.

EXECUTE: **(a)** $V_{C_1} = \frac{Q_1}{C_1} = \frac{18.0\ \mu\text{C}}{3.00\ \mu\text{F}} = 6.00\ \text{V}$. $V_{C_2} = V_{C_1} = 6.00\ \text{V}$.

$Q_2 = C_2 V_{C_2} = (6.00\ \mu\text{F})(6.00\ \text{V}) = 36.0\ \mu\text{C}$.

(b) No current flows to the capacitors when they are fully charged, so $\mathcal{E} = IR_1 + IR_2$.

$V_{R_2} = V_{C_1} = 6.00\ \text{V}$. $I = \frac{V_{R_2}}{R_2} = \frac{6.00\ \text{V}}{2.00\ \Omega} = 3.00\ \text{A}$.

$R = \frac{\mathcal{E} - IR_2}{I} = \frac{60.0\ \text{V} - 6.00\ \text{V}}{3.00\ \text{A}} = 18.0\ \Omega$.

EVALUATE: When a capacitor is fully charged, it acts like an open circuit and prevents any current from flowing though it.

26.25. **IDENTIFY:** Apply Kirchhoff's point rule at point a to find the current through R. Apply Kirchhoff's loop rule to loops (1) and (2) shown in Figure 26.25a to calculate R and ε. Travel around each loop in the direction shown.

(a) SET UP:

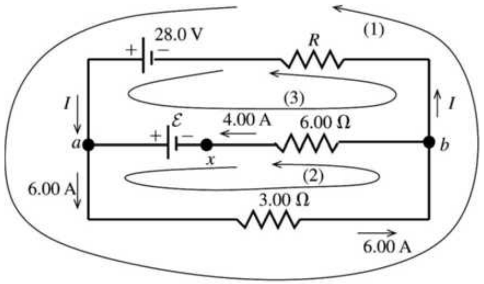


Figure 26.25a

EXECUTE: Apply Kirchhoff's point rule to point a: $\sum I = 0$ so $I + 4.00\text{ A} - 6.00\text{ A} = 0$

$I = 2.00\text{ A}$ (in the direction shown in the diagram).

(b) Apply Kirchhoff's loop rule to loop (1): $-(6.00\text{ A})(3.00\ \Omega) - (2.00\text{ A})R + 28.0\text{ V} = 0$

$-18.0\text{ V} - (2.00\ \Omega)R + 28.0\text{ V} = 0$

$$R = \frac{28.0\text{ V} - 18.0\text{ V}}{2.00\text{ A}} = 5.00\ \Omega$$

(c) Apply Kirchhoff's loop rule to loop (2): $-(6.00\text{ A})(3.00\ \Omega) - (4.00\text{ A})(6.00\ \Omega) + \varepsilon = 0$

$\varepsilon = 18.0\text{ V} + 24.0\text{ V} = 42.0\text{ V}$

EVALUATE: Can check that the loop rule is satisfied for loop (3), as a check of our work:

$28.0\text{ V} - \varepsilon + (4.00\text{ A})(6.00\ \Omega) - (2.00\text{ A})R = 0$

$28.0\text{ V} - 42.0\text{ V} + 24.0\text{ V} - (2.00\text{ A})(5.00\ \Omega) = 0$

$52.0\text{ V} = 42.0\text{ V} + 10.0\text{ V}$

$52.0\text{ V} = 52.0\text{ V}$, so the loop rule is satisfied for this loop.

(d) IDENTIFY: If the circuit is broken at point x there can be no current in the 6.00-Ω resistor. There is now only a single current path and we can apply the loop rule to this path.

SET UP: The circuit is sketched in Figure 26.25b.

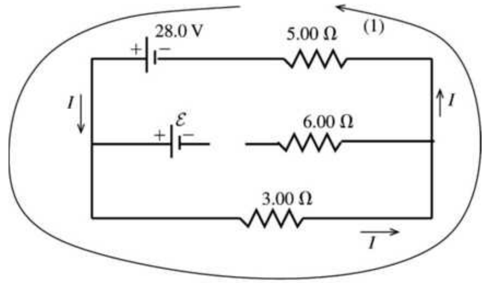


Figure 26.25b

EXECUTE: $+28.0\text{ V} - (3.00\ \Omega)I - (5.00\ \Omega)I = 0$

$$I = \frac{28.0\text{ V}}{8.00\ \Omega} = 3.50\text{ A}$$

EVALUATE: Breaking the circuit at x removes the 42.0-V emf from the circuit and the current through the 3.00-Ω resistor is reduced.

26.26. **IDENTIFY:** Apply the loop rule and junction rule.

SET UP: The circuit diagram is given in Figure 26.26. The junction rule has been used to find the magnitude and direction of the current in the middle branch of the circuit. There are no remaining unknown currents.

EXECUTE: The loop rule applied to loop (1) gives:

$+20.0\text{V}-(1.00\text{ A})(1.00\,\Omega)+(1.00\text{ A})(4.00\,\Omega)+(1.00\text{ A})(1.00\,\Omega)-\mathcal{E}_1-(1.00\text{ A})(6.00\,\Omega)=0$

$\mathcal{E}_1=20.0\text{ V}-1.00\text{ V}+4.00\text{ V}+1.00\text{ V}-6.00\text{ V}=18.0\text{ V}.$ The loop rule applied to loop (2) gives:

$+20.0\text{ V}-(1.00\text{ A})(1.00\,\Omega)-(2.00\text{ A})(1.00\,\Omega)-\mathcal{E}_2-(2.00\text{ A})(2.00\,\Omega)-(1.00\text{ A})(6.00\,\Omega)=0$

$\mathcal{E}_2=20.0\text{ V}-1.00\text{ V}-2.00\text{ V}-4.00\text{ V}-6.00\text{ V}=7.0\text{ V}.$ Going from b to a along the lower branch, $V_b+(2.00\text{ A})(2.00\,\Omega)+7.0\text{ V}+(2.00\text{ A})(1.00\,\Omega)=V_a\cdot V_b-V_a=-13.0\text{ V};$ point b is at 13.0 V lower potential than point a.

EVALUATE: We can also calculate V_b-V_a by going from b to a along the upper branch of the circuit. $V_b-(1.00\text{ A})(6.00\,\Omega)+20.0\text{ V}-(1.00\text{ A})(1.00\,\Omega)=V_a$ and $V_b-V_a=-13.0\text{ V}.$ This agrees with V_b-V_a calculated along a different path between b and a.

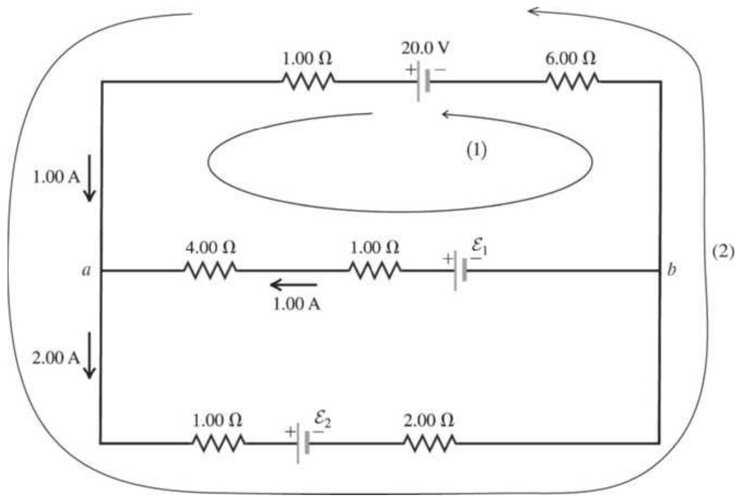


Figure 26.26

26.42. **IDENTIFY:** For a charging capacitor $q(t) = C\mathcal{E}(1 - e^{-t/\tau})$ and $i(t) = \dfrac{\mathcal{E}}{R}e^{-t/\tau}$.

SET UP: The time constant is $RC = (0.895 \times 10^6\ \Omega)\,(12.4 \times 10^{-6}\ \text{F}) = 11.1\ \text{s}$.

EXECUTE: **(a)** At $t = 0$ s: $q = C\mathcal{E}(1 - e^{-t/RC}) = 0$.

At $t = 5$ s: $q = C\mathcal{E}(1 - e^{-t/RC}) = (12.4\times10^{-6}\ \text{F})(60.0\ \text{V})(1 - e^{-(5.0\ \text{s})/(11.1\ \text{s})}) = 2.70\times10^{-4}$ C.

At $t = 10$ s: $q = C\mathcal{E}(1 - e^{-t/RC}) = (12.4\times10^{-6}\ \text{F})(60.0\ \text{V})(1 - e^{-(10.0\ \text{s})/(11.1\ \text{s})}) = 4.42\times10^{-4}$ C.

At $t = 20$ s: $q = C\mathcal{E}(1 - e^{-t/RC}) = (12.4\times10^{-6}\ \text{F})(60.0\ \text{V})(1 - e^{-(20.0\ \text{s})/(11.1\ \text{s})}) = 6.21\times10^{-4}$ C.

At $t = 100$ s: $q = C\mathcal{E}(1 - e^{-t/RC}) = (12.4\times10^{-6}\ \text{F})(60.0\ \text{V})(1 - e^{-(100\ \text{s})/(11.1\ \text{s})}) = 7.44\times10^{-4}$ C.

(b) The current at time t is given by: $i = \dfrac{\mathcal{E}}{R}e^{-t/RC}$.

At $t = 0$ s: $i = \dfrac{60.0\ \text{V}}{8.95\times10^5\ \Omega}e^{-0/11.1} = 6.70\times10^{-5}$ A.

At $t = 5$ s: $i = \dfrac{60.0\ \text{V}}{8.95\times10^5\ \Omega}e^{-5/11.1} = 4.27\times10^{-5}$ A.

At $t = 10$ s: $i = \dfrac{60.0\ \text{V}}{8.95\times10^5\ \Omega}e^{-10/11.1} = 2.72\times10^{-5}$ A.

At $t = 20$ s: $i = \dfrac{60.0\ \text{V}}{8.95\times10^5\ \Omega}e^{-20/11.1} = 1.11\times10^{-5}$ A.

At $t = 100$ s: $i = \dfrac{60.0\ \text{V}}{8.95\times10^5\ \Omega}e^{-100/11.1} = 8.20\times10^{-9}$ A.

(c) The graphs of $q(t)$ and $i(t)$ are given in Figure 26.42a and b.

EVALUATE: The charge on the capacitor increases in time as the current decreases.

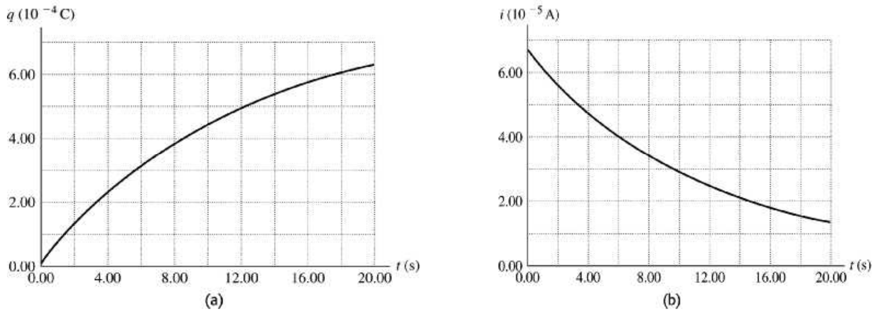


Figure 26.42

26.91. **IDENTIFY:** Consider one segment of the network attached to the rest of the network.

SET UP: We can re-draw the circuit as shown in Figure 26.91.

EXECUTE: $R_T = 2R_1 + \left(\dfrac{1}{R_2} + \dfrac{1}{R_T}\right)^{-1} = 2R_1 + \dfrac{R_2 R_T}{R_2 + R_T}$. $R_T{}^2 - 2R_1 R_T - 2R_1 R_2 = 0$.

$R_T = R_1 \pm \sqrt{R_1{}^2 + 2R_1 R_2}$. $R_T > 0$, so $R_T = R_1 + \sqrt{R_1{}^2 + 2R_1 R_2}$.

EVALUATE: Even though there are an infinite number of resistors, the equivalent resistance of the network is finite.

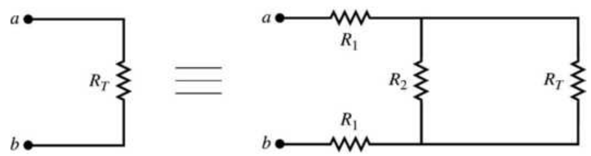


Figure 26.91