PEP 2017
Assignment 6

15.81 ••• **CP** A large rock that weighs 164.0 N is suspended from the lower end of a thin wire that is 3.00 m long. The density of the rock is 3200 kg/m^3. The mass of the wire is small enough that its effect on the tension in the wire can be neglected. The upper end of the wire is held fixed. When the rock is in air, the fundamental frequency for transverse standing waves on the wire is 42.0 Hz. When the rock is totally submerged in a liquid, with the top of the rock just below the surface, the fundamental frequency for the wire is 28.0 Hz. What is the density of the liquid?

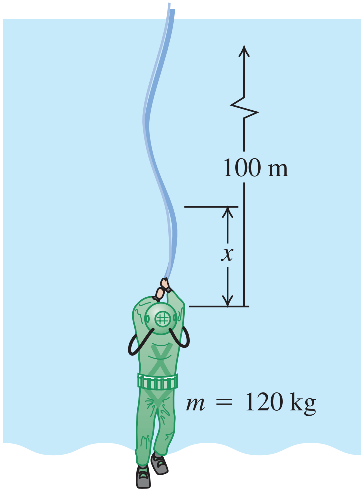
15.70 ••• **CALC** **Energy in a Triangular Pulse.** A triangular wave pulse on a taut string travels in the positive x-direction with speed v. The tension in the string is F, and the linear mass density of the string is μ. At $t = 0$, the shape of the pulse is given by

$$y(x, 0) = \begin{cases} 0 & \text{if } x < -L \\ h(L + x)/L & \text{for } -L < x < 0 \\ h(L - x)/L & \text{for } 0 < x < L \\ 0 & \text{for } x > L \end{cases}$$

(a) Draw the pulse at $t = 0$. (b) Determine the wave function $y(x, t)$ at all times t. (c) Find the instantaneous power in the wave. Show that the power is zero except for $-L < (x - vt) < L$ and that in this interval the power is constant. Find the value of this constant power.

15.84 ••• **CP** **CALC** A deep-sea diver is suspended beneath the surface of Loch Ness by a 100-m-long cable that is attached to a boat on the surface (Fig. P15.84). The diver and his suit have a total mass of 120 kg and a volume of 0.0800 m^3. The cable has a diameter of 2.00 cm and a linear mass density of $\mu = 1.10$ kg/m. The diver thinks he sees something moving in the murky depths and jerks the end of the cable back and forth to send transverse waves up the cable as a signal to his companions in the boat. (a) What is the tension in the cable at its lower end, where it is attached to the diver? Do not forget to include the buoyant force that the water (density 1000 kg/m^3) exerts on him. (b) Calculate the tension in the cable a distance x above the diver. The buoyant force on the cable must be included in your calculation. (c) The speed of transverse waves on the cable is given by $v = \sqrt{F/\mu}$ (Eq. 15.13). The speed therefore varies along the cable, since the tension is not constant. (This expression neglects the damping force that the water exerts on the moving cable.) Integrate to find the time required for the first signal to reach the surface.

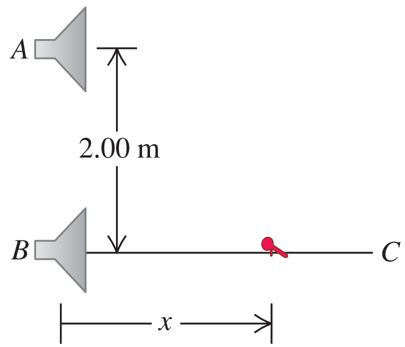
Figure **P15.84**



16.79 •• **Supernova!** The gas cloud known as the Crab Nebula can be seen with even a small telescope. It is the remnant of a *supernova,* a cataclysmic explosion of a star. The explosion was seen on the earth on July 4, 1054 C.E. The streamers glow with the characteristic red color of heated hydrogen gas. In a laboratory on the earth, heated hydrogen produces red light with frequency 4.568×10^{14} Hz; the red light received from streamers in the Crab Nebula pointed toward the earth has frequency 4.586×10^{14} Hz. (a) Estimate the speed with which the outer edges of the Crab Nebula are expanding. Assume that the speed of the center of the nebula relative to the earth is negligible. (You may use the formulas derived in Problem 16.78. The speed of light is 3.00×10^{8} m/s.) (b) Assuming that the expansion speed has been constant since the supernova explosion, estimate the diameter of the Crab Nebula. Give your answer in meters and in light-years. (c) The angular diameter of the Crab Nebula as seen from earth is about 5 arc minutes ($1 \text{ arc minute} = \frac{1}{60}$ degree). Estimate the distance (in light-years) to the Crab Nebula, and estimate the year in which the supernova explosion actually took place.

16.70 •• Two identical loudspeakers are located at points *A* and *B*, 2.00 m apart. The loudspeakers are driven by the same amplifier and produce sound waves with a frequency of 784 Hz. Take the speed of sound in air to be 344 m/s. A small microphone is moved out from point *B* along a line perpendicular to the line connecting *A* and *B* (line *BC* in Fig. P16.70). (a) At what distances from *B* will there be *destructive* interference? (b) At what distances from *B* will there be *constructive* interference? (c) If the frequency is made low enough, there will be no positions along the line *BC* at which destructive interference occurs. How low must the frequency be for this to be the case?

Figure **P16.70**



15.70. **IDENTIFY:** The wave moves in the +*x* direction with speed v, so to obtain $y(x,t)$ replace *x* with $x - vt$ in the expression for $y(x,0)$.

SET UP: $P(x,t)$ is given by Eq. (15.21).

EXECUTE: **(a)** The wave pulse is sketched in Figure 15.70.

(b)

$$y(x,t) = \begin{cases} 0 & \text{for} \quad (x - vt) < -L \\ h(L + x - vt)/L & \text{for} \quad -L < (x - vt) < 0 \\ h(L - x + vt)/L & \text{for} \quad 0 < (x - vt) < L \\ 0 & \text{for} \quad (x - vt) > L \end{cases}$$

(c) From Eq. (15.21):

$$P(x,t) = -F\frac{\partial y(x,t)}{\partial x}\frac{\partial y(x,t)}{\partial t} = \begin{cases} -F(0)(0) = 0 & \text{for} \quad (x - vt) < -L \\ -F(h/L)(-hv/L) = Fv(h/L)^2 & \text{for} \quad -L < (x - vt) < 0 \\ -F(-h/L)(hv/L) = Fv(h/L)^2 & \text{for} \quad 0 < (x - vt) < L \\ -F(0)(0) = 0 & \text{for} \quad (x - vt) > L \end{cases}$$

Thus the instantaneous power is zero except for $-L < (x - vt) < L$, where it has the constant value $Fv(h/L)^2$.

EVALUATE: For this pulse the transverse velocity v_y is constant in magnitude and has opposite sign on either side of the peak of the pulse.

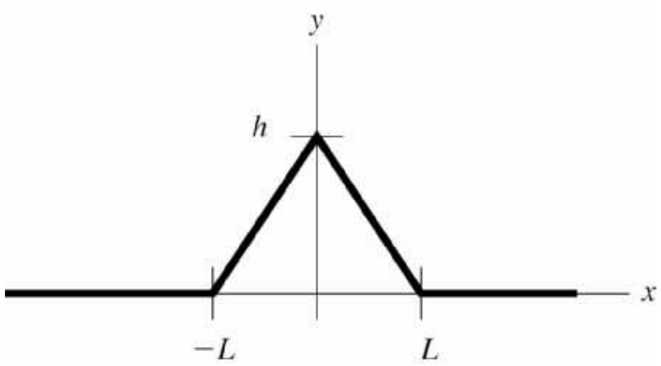


Figure 15.70

15.81. **IDENTIFY:** When the rock is submerged in the liquid, the buoyant force on it reduces the tension in the wire supporting it. This in turn changes the frequency of the fundamental frequency of the vibrations of the wire. The buoyant force depends on the density of the liquid (the target variable). The vertical forces on the rock balance in both cases, and the buoyant force is equal to the weight of the liquid displaced by the rock (Archimedes's principle).

SET UP: The wave speed is $v = \sqrt{\dfrac{F}{\mu}}$ and $v = f\lambda$. $B = \rho_{\text{liq}} V_{\text{rock}} g$. $\Sigma F_y = 0$.

EXECUTE: $\lambda = 2L = 6.00\text{ m}$. In air, $v = f\lambda = (42.0\text{ Hz})(6.00\text{ m}) = 252\text{ m/s}$. $v = \sqrt{\dfrac{F}{\mu}}$ so

$$\mu = \frac{F}{v^2} = \frac{164.0\text{ N}}{(252\text{ m/s})^2} = 0.002583\text{ kg/m}.$$ In the liquid, $v = f\lambda = (28.0\text{ Hz})(6.00\text{ m}) = 168\text{ m/s}$.

$F = \mu v^2 = (0.002583\text{ kg/m})(168\text{ m/s})^2 = 72.90\text{ N}$. $F + B - mg = 0$.

$B = mg - F = 164.0\text{ N} - 72.9\text{ N} = 91.10\text{ N}$. For the rock, $V = \dfrac{m}{\rho} = \dfrac{(164.0\text{ N}/9.8\text{ m/s}^2)}{3200\text{ kg/m}^3} = 5.230\times 10^{-3}\text{ m}^3$.

$B = \rho_{\text{liq}} V_{\text{rock}} g$ and $\rho_{\text{liq}} = \dfrac{B}{V_{\text{rock}} g} = \dfrac{91.10\text{ N}}{(5.230\times 10^{-3}\text{ m}^3)(9.8\text{ m/s}^2)} = 1.78\times 10^3\text{ kg/m}^3$.

EVALUATE: This liquid has a density 1.78 times that of water, which is rather dense but not impossible.

15.84. **IDENTIFY:** Apply $\Sigma F_y = 0$ to segments of the cable. The forces are the weight of the diver, the weight of the segment of the cable, the tension in the cable and the buoyant force on the segment of the cable and on the diver.

SET UP: The buoyant force on an object of volume V that is completely submerged in water is $B = \rho_{\text{water}} V g.$

EXECUTE: **(a)** The tension is the difference between the diver's weight and the buoyant force, $F = (m - \rho_{\text{water}} V)g = (120\text{ kg} - (1000\text{ kg/m}^3)(0.0800\text{ m}^3))(9.80\text{ m/s}^2) = 392\text{ N}.$

(b) The increase in tension will be the weight of the cable between the diver and the point at x, minus the buoyant force. This increase in tension is then

$(\mu x - \rho(Ax))g = (1.10\text{ kg/m} - (1000\text{ kg/m}^3)\pi(1.00\times 10^{-2}\text{ m})^2)(9.80\text{ m/s}^2)x = (7.70\text{ N/m})x.$ The tension as a function of x is then $F(x) = (392\text{ N}) + (7.70\text{ N/m})x.$

(c) Denote the tension as $F(x) = F_0 + ax,$ where $F_0 = 392\text{ N}$ and $a = 7.70\text{ N/m}.$ Then the speed of transverse waves as a function of x is $v = \dfrac{dx}{dt} = \sqrt{(F_0 + ax)/\mu}$ and the time t needed for a wave to reach the surface is found from $t = \int dt = \int \dfrac{dx}{dx/dt} = \int \dfrac{\sqrt{\mu}}{\sqrt{F_0 + ax}}\,dx.$

Let the length of the cable be L, so $t = \sqrt{\mu}\int_0^L \dfrac{dx}{\sqrt{F_0 + ax}} = \sqrt{\mu}\dfrac{2}{a}\sqrt{F_0 + ax}\Big|_0^L = \dfrac{2\sqrt{\mu}}{a}(\sqrt{F_0 + aL} - \sqrt{F_0}).$

$$t = \frac{2\sqrt{1.10\text{ kg/m}}}{7.70\text{ N/m}}(\sqrt{392\text{ N} + (7.70\text{ N/m})(100\text{ m})} - \sqrt{392\text{ N}}) = 3.89\text{ s}.$$

EVALUATE: If the weight of the cable and the buoyant force on the cable are neglected, then the tension would have the constant value calculated in part (a). Then $v = \sqrt{\dfrac{F}{\mu}} = \sqrt{\dfrac{392\text{ N}}{1.10\text{ kg/m}}} = 18.9\text{ m/s}$ and $t = \dfrac{L}{v} = 5.29\text{ s}.$

The weight of the cable increases the tension along the cable and the time is reduced from this value.

16.70. **IDENTIFY:** Destructive interference occurs when the path difference is a half-integer number of wavelengths. Constructive interference occurs when the path difference is an integer number of wavelengths.

SET UP: $\lambda = \dfrac{v}{f} = \dfrac{344\text{ m/s}}{784\text{ Hz}} = 0.439\text{ m}$

EXECUTE: **(a)** If the separation of the speakers is denoted h, the condition for destructive interference is $\sqrt{x^2 + h^2} - x = \beta\lambda$, where β is an odd multiple of one-half. Adding x to both sides, squaring, canceling the x^2 term from both sides and solving for x gives $x = \dfrac{h^2}{2\beta\lambda} - \dfrac{\beta}{2}\lambda$. Using $\lambda = 0.439\text{ m}$ and $h = 2.00\text{ m}$ yields 9.01 m for $\beta = \frac{1}{2}$, 2.71 m for $\beta = \frac{3}{2}$, 1.27 m for $\beta = \frac{5}{2}$, 0.53 m for $\beta = \frac{7}{2}$, and 0.026 m for $\beta = \frac{9}{2}$. These are the only allowable values of β that give positive solutions for x.

(b) Repeating the above for integral values of β, constructive interference occurs at 4.34 m, 1.84 m, 0.86 m, 0.26 m. Note that these are between, but not midway between, the answers to part (a).

(c) If $h = \lambda/2$, there will be destructive interference at speaker B. If $\lambda/2 > h$, the path difference can never be as large as $\lambda/2$. (This is also obtained from the above expression for x, with $x = 0$ and $\beta = \frac{1}{2}$.) The minimum frequency is then $v/2h = (344\text{ m/s})/(4.0\text{ m}) = 86\text{ Hz}$.

EVALUATE: When f increases, λ is smaller and there are more occurrences of points of constructive and destructive interference.

16.79. **IDENTIFY:** Apply the result derived in part (b) of Problem 16.78. The radius of the nebula is $R = vt$, where t is the time since the supernova explosion.

SET UP: When the source and receiver are moving toward each other, v is negative and $f_R > f_S$. The light from the explosion reached earth 952 years ago, so that is the amount of time the nebula has expanded. $1 \text{ ly} = 9.46 \times 10^{15}$ m.

EXECUTE: **(a)** $v = c\dfrac{f_S - f_R}{f_S} = (3.00\times10^8 \text{ m/s})\dfrac{-0.018\times10^{14} \text{ Hz}}{4.568\times10^{14} \text{ Hz}} = -1.2\times10^6 \text{ m/s},$ with the minus sign indicating that the gas is approaching the earth, as is expected since $f_R > f_S$.

(b) The radius is $(952 \text{ yr})(3.156\times10^7 \text{ s/yr})(1.2\times10^6 \text{ m/s}) = 3.6\times10^{16} \text{ m} = 3.8 \text{ ly}.$

(c) The ratio of the width of the nebula to 2π times the distance from the earth is the ratio of the angular width (taken as 5 arc minutes) to an entire circle, which is 60×360 arc minutes. The distance to the nebula is then $\left(\dfrac{2}{2\pi}\right)(3.75 \text{ ly})\dfrac{(60)(360)}{5} = 5.2\times10^3 \text{ ly}.$ The time it takes light to travel this distance is 5200 yr, so the explosion actually took place 5200 yr before 1054 C.E., or about 4100 B.C.E.

EVALUATE: $\left|\dfrac{v}{c}\right| = 4.0\times10^{-3}$, so even though $|v|$ is very large the approximation required for $v = c\dfrac{\Delta f}{f}$ is accurate.