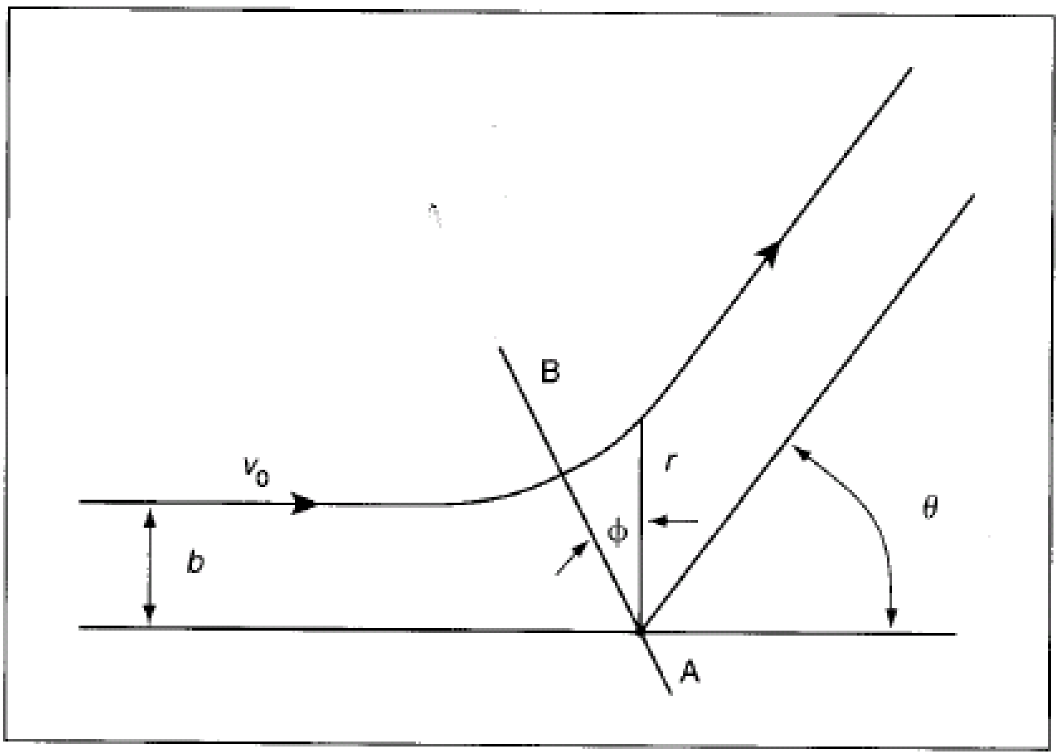
3.2 RUTHERFORD SCATTERING

Much of what has been learned about the structure of the nucleus and the distribution of matter within the nucleus has come from studies of scattering cross sections. The simplest type of scattering that can be observed is Rutherford scattering. This is the result of the nonrelativistic coulombic scattering of point charges. Deviations of experimental results from the scattering cross sections predicted by the Rutherford formula are very important in understanding the structure of the nucleus. We begin, therefore, with the basic nonrelativistic derivation of the Rutherford scattering cross section. This simple derivation assumes that the nucleus is sufficiently massive that it does not recoil during the scattering process.

The geometry of the Rutherford scattering problem is illustrated in Figure 3.2. The scattering particle (with charge ze) approaches the scatterer (with charge Ze) at an impact parameter defined in the figure as b and with an initial velocity v_0. At any time the location of the scattering particle is given relative to the scatterer by the coordinates (r, ϕ) where the angle ϕ is measured relative to the line AB. For the point $\phi = 0$ the radial velocity of the particle is zero and the distance, r, is a minimum. Conservation of angular momentum requires that

$$mv_0 b = mr^2 \frac{d\phi}{dt}. \tag{3.1}$$

Figure 3.2 | Geometry for the Rutherford Scattering Problem



The force acting on the particle due to the coulombic interaction is given in terms of the linear momentum, p, as

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{Zze^2}{4\pi\varepsilon_0 r^3}\vec{r}. \tag{3.2}$$

The change in momentum over a time interval t_1 to t_2 is found by integrating (3.2);

$$\Delta\vec{p} = \int_{t_1}^{t_2} \vec{F}\,dt. \tag{3.3}$$

The time at which the particle is at $\phi = 0$ is defined as $t = 0$. The linear momentum of the particle at $t = -\infty$ along the direction AB is

$$p = -mv_0 \sin\frac{\theta}{2} \tag{3.4}$$

where θ is the scattering angle as defined in the figure. Over the time interval $t = -\infty$ to $+\infty$, equation (3.3) can be written in terms of the Coulomb force as

$$2mv_0 \sin\frac{\theta}{2} = \int_{-\infty}^{+\infty} \frac{Zze^2}{4\pi\varepsilon_0 r^2}\cos\phi\,dt. \tag{3.5}$$

A change of variables in the integration from t to ϕ can be accomplished using equation (3.1) to give

$$\sin\frac{\theta}{2} = \frac{Zze^2}{8\pi\varepsilon_0 m v_0^2 b}\int_{-(\pi-\theta)/2}^{+(\pi-\theta)/2} \cos\phi\,d\phi. \tag{3.6}$$

Integration of equation (3.6) is straightforward and yields

$$\tan\frac{\theta}{2} = \frac{Zze^2}{4\pi\varepsilon_0 m v_0^2 b}. \qquad = \frac{GM}{v_0^2 b} \tag{3.7}$$

This expression shows that increasing the impact parameter decreases the scattering angle. Thus all particles that are scattered by an angle greater than some value of θ, must have impact parameters less than some value of b. Incident particles with b less than a particular value define an area, or cross section, given by

$$\sigma = \pi b^2. \tag{3.8}$$

Thus it can be said from equations (3.6) and (3.7) that the scattering cross section for scattering by an angle greater than θ is

$$\sigma = \pi\left[\frac{Zze^2 \cot\frac{\theta}{2}}{4\pi\varepsilon_0 m v_0^2}\right]^2. \tag{3.9}$$