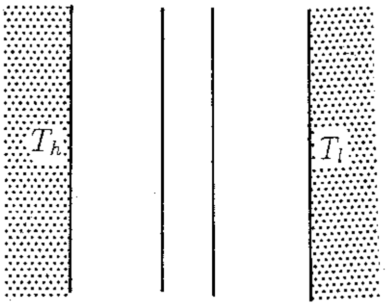
Contemporary, atomic, nuclear, and particle physics

1 Blackbody radiation as a thermal equilibrium condition (in vacuum this is the only heat loss)

Example-1

A black plane surface at a constant high temperature T_h is parallel to another black plane surface at a constant lower temperature T_l. Between the plates is vacuum.

In order to reduce the heat flow due to radiation, a heat shield consisting of two thin black plates, thermally isolated from each other, is placed between the warm and the cold surfaces and parallel to these. After some time stationary conditions are obtained.



By what factor ξ is the stationary heat flow reduced due to the presence of the heat shield? Neglect end effects due to the finite size of the surfaces. *(1.5 points)*

Solution:

Under stationary conditions the net heat flow is the same everywhere:

$$J = \sigma(T_h^4 - T_1^4)$$
$$J = \sigma(T_1^4 - T_2^4)$$
$$J = \sigma(T_2^4 - T_L^4)$$

Adding these three equations we get

$$3J = \sigma(T_h^4 - T_L^4) = J_0,$$

where J_0 is the heat flow in the absence of the heat shield. Thus $\xi = J/J_0$ takes the value

$$\underline{\underline{\xi = \tfrac{1}{3}}}. \qquad \text{Ans.}$$

Example 2

Take Earth as a blackbody, estimate its surface temperature (given Sun's surface temperature T_s, radius R_s, sun-earth distance d, Earth radius R_e).

Solution:

Earth takes all its energy from the sun, so first we need to know the amount of energy, in terms of radiation from the sun, that reaches Earth.
The total amount of energy emitted by the sun is $4\pi\sigma R_s^2 T_s^4$.
When this amount of energy reaches a distance d from the sun its light intensity I is

$$4\pi\sigma R_s^2 T_s^4 = 4\pi d^2 I .$$

The amount of energy Earth receives is then, considering only half the Earth is facing the sun and Earth's projection,

$$E = \pi R_e^2 I .$$

In thermal equilibrium, this must equal to the amount of energy emitted by Earth, again in the form of blackbody radiation.

$$E = 4\pi R_e^2 \sigma T_e^4 .$$

Combining the above equations, we have

$$T_e = T_s (R_s^2 / 4d^2)^{1/4} . \qquad \text{Ans.}$$

We see that the result is independent of the Earth diameter. In fact, objects of any size at the same distance to the sun would have the same temperature, if all regarded as blackbody. If the object has a wavelength independent reflectivity r, then $T = T_s((1-r)R_s^2 / 4d^2)^{1/4}$. If r is wavelength dependent, which is usually the case, then we need to integrate the blackbody radiation to find the total amount of energy received per unit area $I = \int_0^\infty I(\lambda)\left(1-r(\lambda)\right) d\lambda$,

where $I(\lambda)$ is given by the blackbody radiation formula $I(\lambda) = \frac{2\pi hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda k_B T} - 1}$.

2 Nuclear physics

A Introduction

A nucleus is made of particles called protons and neutrons. A proton carries positive charge $+e$ while neutrons carry no net charge. The protons and neutrons in a nucleus are bond together by *strong interaction* force which is stronger than the Coulomb force in the short distance but drops exponentially over large distance. Protons and neutrons together are called nucleons.
A nucleus can have a total number of A nucleons, of which Z are protons and $N=A-Z$ are neutrons. The number Z also determines the number of electrons the atom has, and its chemical element. Elements with same Z but different A's (and therefore N's) are called isotopes.

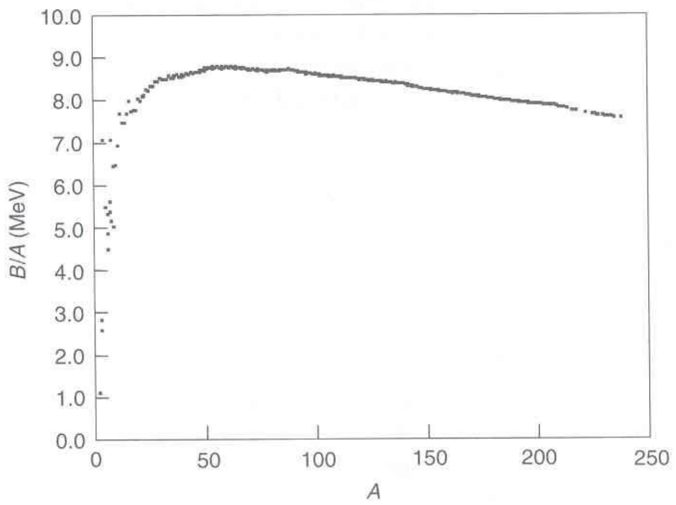
B *Binding energy*

A stable nucleus should have a total mass *m* that is less than the combined mass of the total number of protons and neutrons it contains. The difference is called the binding energy *B*, which is simply defined as

$$B/c^2 = (A-Z)m_n + Zm_p - m. \qquad (1)$$

m_n, m_p are the masses of *free* neutron and proton, respectively.

Figure 4.1 | Binding Energy per Nucleon as a Function of Mass Number, *A*



There are many other particles like electrons, neutrinos (like photons, they have nearly zero mass so always moves at speed of light), pion (π), muon (μ), W-particles, Z^0-particle, K (kappa), ∑, Λ, etc. But quarks are never in free particle form. They are always glued together by *gluons*.

The liquid drop model

The interaction between nucleons is short-range, similar to interaction between molecules in a liquid. A nucleus is then modeled as a drop of liquid, and below are the energies associated with the liquid drop.
Volume term $B_V = a_V A$

Surface term $B_S = -a_S A^{2/3}$

Coulomb term $B_C = -a_c \dfrac{Z(Z-1)}{A^{1/3}}$

Symmetry term $B_{Sym} = -a_{Sym} \dfrac{(A-2Z)^2}{A}$

Pairing term $B_P = \dfrac{-a_P}{A^{3/4}}$

$$\frac{B(A,Z)}{A} = a_V - \frac{a_S}{A^{1/3}} - \frac{a_c Z(Z-1)}{A^{4/3}} - \frac{a_{sym}(A-2Z)^2}{A^2} - \frac{a_p}{A^{7/4}} \qquad (2)$$

Total mass of a nucleus with (A, Z) is

$$m = (A-Z)m_n + Zm_p - B/c^2 \qquad (3).$$

m_n, m_p are the masses of neutron and proton, respectively.

For atomic mass, add the mass of Z electrons Zm_e.

a_V=15.5 MeV, a_S=16.8 MeV, a_C=0.72 MeV, a_{Sym}=23.2 MeV,

$$a_P = \begin{cases} 34 MeV, \mathbf{(N = odd,\ \ Z = odd)} \\ 0, \mathbf{(N + Z = odd)} \\ \mathbf{-}\ 34\ MeV, \mathbf{(N = even, Z = even)} \end{cases} \qquad (4)$$

C $\quad$ *β-decay*

A nucleus (mother) with (A,Z) emits an electron and a neutrino and becomes a daughter (A,Z+1) nucleus. The total kinetic energy shared by the three particles is

$$K = (m_m - m_d - m_e)c^2 = B(A,Z+1) - B(A,Z) + (m_n - m_p)c^2 - m_e c^2 \qquad (5)$$

Likewise, for β^+-decay, a positron (anti-electron) is created so

$$K = (m_m - m_d - m_e)c^2 = B(A,Z-1) - B(A,Z) - (m_n - m_p)c^2 - m_e c^2 \qquad (6)$$

Obviously, if *K* is negative then the process will not occur. Other nuclear processes (fission of U^{235} into two fragments, α-decay, etc.) can be calculated in the same way.

Atomic unit mass (*u*) = 931.494 MeV/c^2.
Neutron mass m_n = 1.008664904 *u*.
Proton mass m_p = 1.007825032 *u*.

Exercise: Verify the results shown in the figures below.

Figure 4.3 | Mass Parabola for $A = 135$ Showing One Stable Nuclide (as Expected for Odd A) with $Z = 56$

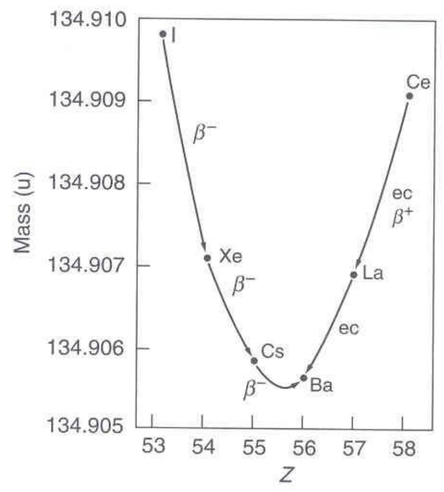
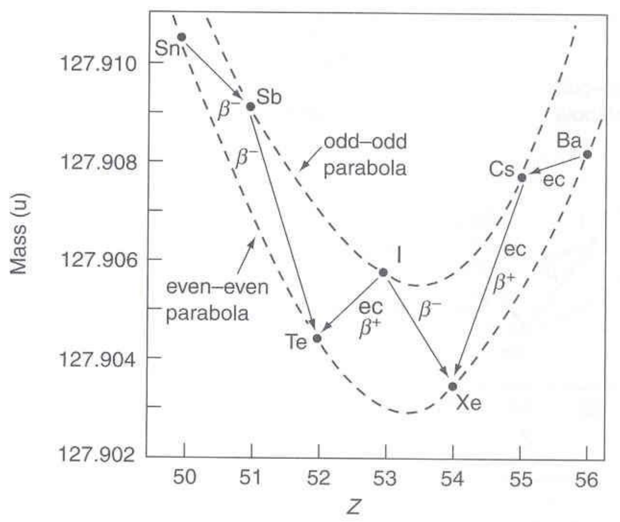


Figure 4.5 | Mass Parabola for $A = 128$ Showing Two Stable Nuclides with $Z = 52$ and 54



D General decay process

Within a unit time interval the probability of a nucleus decay is λ. If there are *N(t)* identical nuclei at time *t* then after time interval *dt* there are $dN=N\lambda dt$ nuclei that have decayed within the time interval *dt*. The differential equation is then

$$-dN = N\lambda dt \qquad (7)$$

The solution is

$$N(t) = N(0)e^{-\lambda t} \qquad (8).$$

If there are several simultaneous decay processes with probability $\lambda_1, \lambda_2, \lambda_3$..., then

$$-dN = N(\lambda_1 + \lambda_2 + \lambda_3..)dt \qquad (9)$$

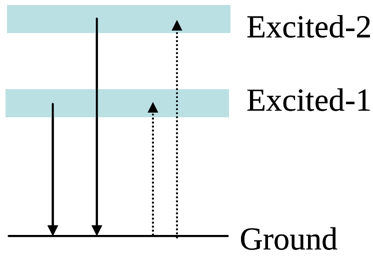
The solution is then

$$N(t) = N(0)e^{-(\lambda_1+\lambda_2+..)t} \qquad (10).$$

Since nucleus is much heavier than electron (and neutrino) it shares very little of the total kinetic energy (recall $E_k = p^2/2m$), the excessive energy K is shared by the electron and the neutrino. K is typically about 0.5 MeV, which is comparable to the electron rest mass (0.511 MeV), so the electron is quite *relativistic*. Note that the momentum of the electron plus that of the neutrino is not zero because the daughter nucleus can have comparable momentum. So the electron and the neutrino can just fly in any uncorrelated directions.

3 Energy levels

In the microscopic world, everything is governed by quantum mechanics, and the system (like atoms which is made of electrons surrounding the nucleus, or a nucleus which is made of a bunch of protons and neutrons) is described by *states* with associated *energy levels* (and other quantities like angular momentum, etc). For isolated atoms and nuclei the energy levels are discrete. For solids and liquids where atoms form molecules and molecules interact with one another the energy levels are continuous and are therefore called energy bands.



However, even the discrete levels in atoms (and nuclei) are not without width. And an atom at a higher energy level will not stay there forever. After a while it will transit back to the lowest level (ground state) and emit a photon (spontaneous emission). One can define a lifetime $\tau = 1/\lambda$ and the transition process is mathematically the same as the decay process. Furthermore, the energy level width Γ is related to τ by

$$\tau\Gamma = \frac{1}{2}\hbar = \frac{1}{2}\frac{h}{2\pi} \qquad (11).$$

where h is the Planck constant.

Typical energy of photons emitted/absorbed by atoms involving the change of energy levels of an electron in the outermost shell is of the order of eV and in the visible to near ultraviolet wavelength. As a convenient formula
energy (eV) = 1240/wavelength-in-nanometer.

The energy levels in the inner shells are of the order of KeV, and the photons emitted/absorbed by transitions to/from these levels to the outermost ones are X-rays.

Atoms in a molecule (like H_2O or CO) can vibrate around their equilibrium positions and the energy is typically 10 meV ($=\hbar\omega$, where $\omega = \sqrt{\frac{k}{m}}$) and the photons are in the mid-infrared (~ 10 μm).

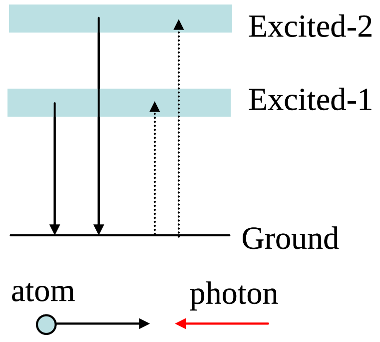
Transition energy in a nucleus (a proton or neutron dropping from a higher level to ground state) is around MeV and these are called γ-ray photons.

Pauli exclusive principle – Each state can hold only one fermion (electron, proton, neutron, etc.).

Bosons (photons, etc.) can occupy a single state.

4 Laser cooling of atoms

A photon can be absorbed by an atom and the absorbed energy is used by the atom to transit from one energy level (usually the lowest, or *ground state)* to a higher (excited) level if the photon energy equal to the energy difference. The momentum of the photon is also transferred to the atom. After spending certain time on the higher level, the atom will spontaneously drop back to the ground level but the photon emitted in the process can go in any direction. The absorption-then-emission process can take place many times, and on average, the absorbed photons all have the same momentum, but the emitted photons have momentum in all directions, therefore on average their net contribution to the momentum of the atom is zero. The velocity of the atom is therefore lowered if its initial direction is opposite of that of the incoming photons. This is the general concept of laser cooling.



5 De- Broglie wavelength

Any microscopic particles in motion can be described by a plane wave with wavelength diffraction and interference

$$\frac{2\pi}{\lambda} \equiv k = \frac{p}{\hbar} \qquad (12)$$

where p is the momentum of the particle. Electron (and other particle) beam can therefore produce interference and diffraction patterns in exactly the same way as an EM plane wave with the same wavelength. The wavelength of a beam of electrons with ~ 100 KeV kinetic energy is around 0.5 nm so it can be used, as X-ray diffraction, to explore the crystal structure of solids. This is the working principle of electron microscopes.

6 Heisenberg uncertainty principle

$$\frac{\hbar}{2} = <\Delta p><\Delta x> = <\Delta E><\Delta t> \qquad (13)$$

Estimation of ground state energy in a confined system

$<\Delta x>=a$, so $<\Delta p> = <p> = \frac{\hbar}{2a}$, and $<E> = \frac{<p^2>}{2m} = \frac{1}{2m}(\frac{\hbar}{2a})^2$