Longitudinal Waves

Reading: Chapter 17, Sections 17-7 to 17-10

Sources of Musical Sound

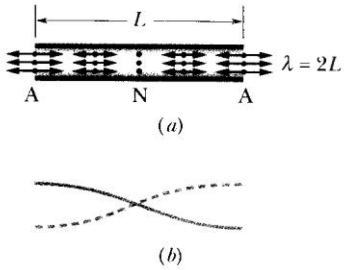


FIG. 17-14 (*a*) The simplest standing wave pattern of displacement for (longitudinal) sound waves in a pipe with both ends open has an antinode (A) across each end and a node (N) across the middle. (The longitudinal displacements represented by the double arrows are greatly exaggerated.) (*b*) The corresponding standing wave pattern for (transverse) string waves.

Pipe

Closed end: node
Open end: antinode

Standing wave pattern:

Fundamental or first harmonic: 2 nodes at the ends, 1 antinode in the middle

$$L = \frac{\lambda}{2}, \qquad \lambda = 2L.$$

Resonant frequency: $f = \dfrac{v}{\lambda} = \dfrac{v}{2L}.$

In general, for harmonic number *n*,

$$L = \frac{n\lambda}{2}, \quad \lambda = \frac{2L}{n}, \quad \text{for } n = 1, 2, 3,$$

Resonant frequency:

$$f = \frac{v}{\lambda} = n\frac{v}{2L}, \qquad \text{for } n = 1, 2, 3,$$

For pipes with only one open end,

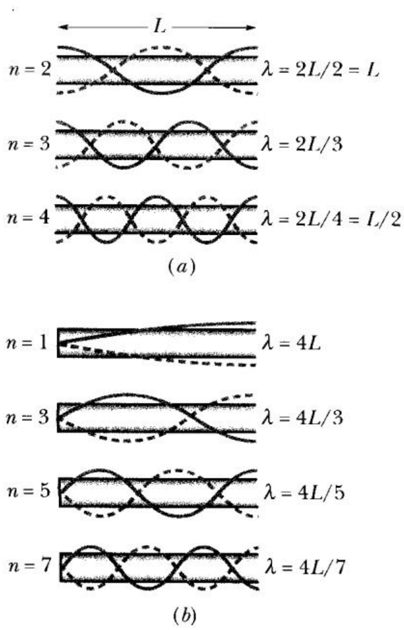


FIG. 17-15 Standing wave patterns for string waves superimposed on pipes to represent standing sound wave patterns in the pipes. (*a*) With *both* ends of the pipe open, any harmonic can be set up in the pipe. (*b*) With only *one* end open, only odd harmonics can be set up.

$$L = \frac{\lambda}{4}, \; \frac{3\lambda}{4}, \; \frac{5\lambda}{4}, \; \frac{7\lambda}{4} \cdots$$

$$\lambda = 4L, \; \frac{4L}{3}, \; \frac{4L}{5}, \; \frac{4L}{7} \cdots$$

In general,

$$\lambda = \frac{4L}{n}, \qquad \text{for } n = 1, 3, 5, \ldots$$

Resonant frequencies:

$$f = \frac{v}{\lambda} = \frac{nv}{4L}, \qquad \text{for } n = 1, 3, 5, \ldots$$

In general, when a musical instrument produces a tone, the fundamental as well as higher harmonics are generated simultaneously. This gives rise to the different waveforms generated by different instruments.
Hence different instruments have different sounds.

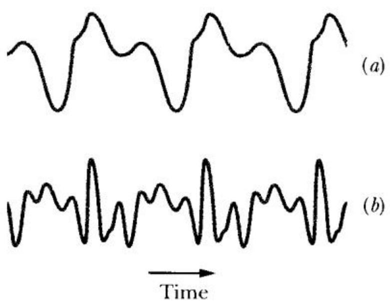
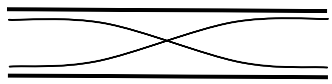


FIG. 17-17 The wave forms produced by (a) a flute and (b) an oboe when played at the same note, with the same first harmonic frequency.

Example

17-6 Weak background noises from a room set up the fundamental standing wave in a cardboard tube of length L = 67.0 cm with two open ends. Assume that the speed of sound in the air within the tube is 343 ms^{-1}.
(a) What frequency do you hear from the tube?
(b) If you jam your ear against one end of the tube, what fundamental frequency do you hear from the tube?

(a) Two open ends,



$\longleftarrow L = \lambda/2 \longrightarrow$

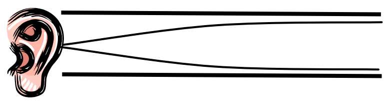
$$L = \frac{\lambda}{2}$$

$$\lambda = 2L$$

$$f = \frac{v}{\lambda} = \frac{v}{2L}$$

$$= \frac{343}{2(0.67)} = 256\,\text{Hz} \quad \text{(ans)}$$

(b) One fixed end and one open end,



$\longleftarrow L = \lambda/4 \longrightarrow$

$$L = \frac{\lambda}{4}$$

$$\lambda = 4L$$

$$f = \frac{v}{\lambda} = \frac{v}{4L}$$

$$= \frac{343}{4(0.67)} = 128\,\text{Hz} \quad \text{(ans)}$$

Beats

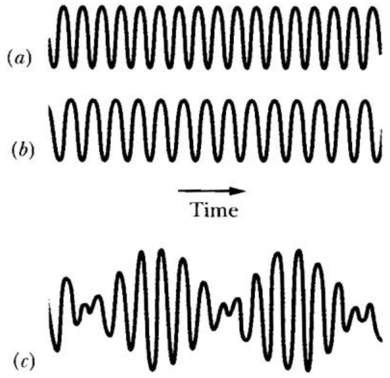


FIG. 17-18 (a, b) The pressure variations Δp of two sound waves as they would be detected separately. The frequencies of the waves are nearly equal. (c) The resultant pressure variation if the two waves are detected simultaneously.

Consider two sound waves with slightly different frequencies:

$$s_1 = s_m \cos \omega_1 t \text{ and } \quad s_2 = s_m \cos \omega_2 t$$

Resultant displacement:

$$s = s_1 + s_2 = s_m (\cos \omega_1 t + \cos \omega_2 t).$$

Using the trigonometric identity

$$\cos\alpha + \cos\beta = 2\cos\tfrac{1}{2}(\alpha - \beta)\cos\tfrac{1}{2}(\alpha + \beta),$$

we obtain

$$s = 2s_m \cos\tfrac{1}{2}(\omega_1 - \omega_2)t \cos\tfrac{1}{2}(\omega_1 + \omega_2)t.$$

Conclusion:

$$\boxed{s = [2s_m \cos\omega' t]\cos\omega t,}$$

where

$$\omega' = \tfrac{1}{2}(\omega_1 - \omega_2) \qquad \text{and} \qquad \omega = \tfrac{1}{2}(\omega_1 + \omega_2).$$

Since ω_1 and ω_2 are nearly equal, $\omega >> \omega'$.
Hence the resultant displacement consists of an oscillation with angular frequency ω and a slowly changing amplitude with angular frequency ω'.

The amplitude $2s_m\cos\omega' t$ is maximum when $\cos\omega' t = \pm 1$, i.e. 2 times in each repetition of the cosine function.
Hence the beat frequency is:

$$\omega_{\text{beat}} = 2\omega' = \omega_1 - \omega_2.$$

$$\boxed{f_{\text{beat}} = f_1 - f_2.}$$

Musicians use the beat phenomenon in tuning their instruments.
See Animation "Beats"

The Doppler Effect

See Youtube "Fire Engine siren demonstrates the Doppler Effect" and "Example of Doppler Shift using car horn"

Detector Moving; Source Stationary

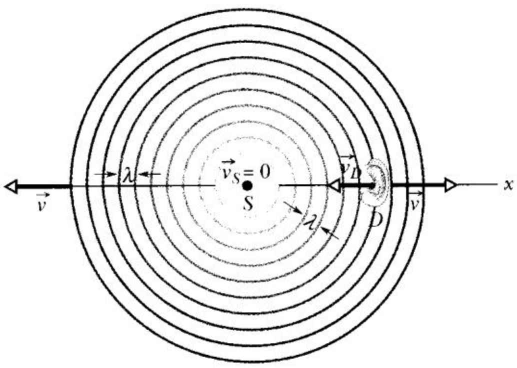


FIG. 17-19 A stationary source of sound S emits spherical wavefronts, shown one wavelength apart, that expand outward at speed v. A sound detector D, represented by an ear, moves with velocity $\vec{v}_D$ toward the source. The detector senses a higher frequency because of its motion.

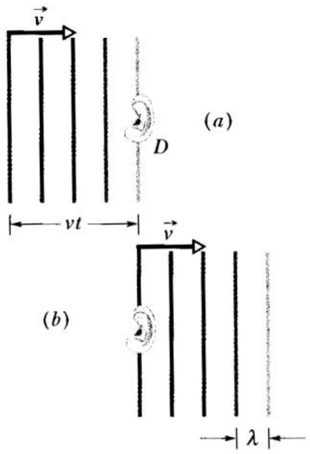


FIG. 17-20 The wavefronts of Fig. 17-19, assumed planar, (a) reach and (b) pass a stationary detector D; they move a distance vt to the right in time t.

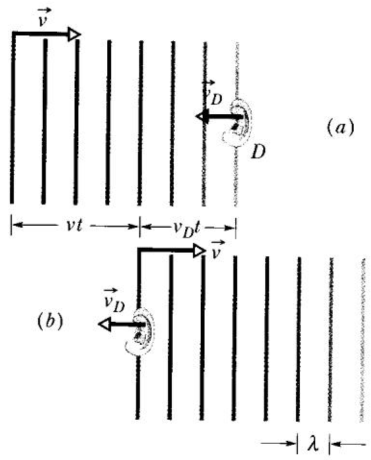


FIG. 17-21 Wavefronts traveling to the right (a) reach and (b) pass detector D, which moves in the opposite direction. In time t, the wavefronts move a distance vt to the right and D moves a distance $v_D t$ to the left.

In time t,
the wavefronts move a distance vt,
the detector moves a distance $v_D t$,
the range of waves intercepted by the detector = $vt+v_D t$,
the number of wavefronts intercepted by the detector = $(vt+v_D t)/\lambda$.

The frequency observed by the detector:

$$f' = \frac{(vt + v_D t)/\lambda}{t} = \frac{v + v_D}{\lambda}.$$

Since $\lambda = v/f$,

$$f' = \frac{v + v_D}{v/f} = f\,\frac{v + v_D}{v}.$$

Similarly, if the detector moves away from the source,

$$f' = f\,\frac{v - v_D}{v}.$$

Summarizing,

$$f' = f\,\frac{v \pm v_D}{v}.$$

Source Moving; Detector Stationary

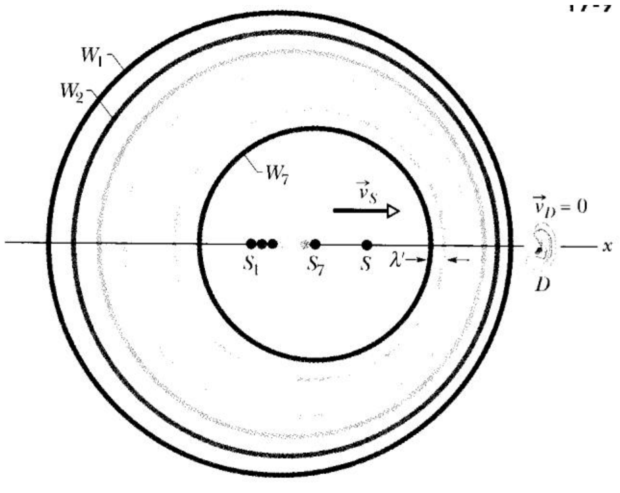


FIG. 17-22 A detector D is stationary, and a source S is moving toward it at speed v_S. Wavefront W_1 was emitted when the source was at S_1, wavefront W_7 when it was at S_7. At the moment depicted, the source is at S. The detector senses a higher frequency because the moving source, chasing its own wavefronts, emits a reduced wavelength λ' in the direction of its motion.

In a period T,
the distance moved by the wavefront $W_1 = vT$,
the distance moved by the source = $v_S T$,
the distance between the wavefronts W_1 and W_2
= $vT - v_S T$.

The frequency observed by the detector:

$$f' = \frac{v}{\lambda'} = \frac{v}{vT - v_S T} = \frac{v}{v / f - v_S / f} = f \frac{v}{v - v_S}.$$

If the source moves away from the detector,

$$f' = f \frac{v}{v + v_S}.$$

Summarizing,

$$f' = f \frac{v}{v \mp v_S}.$$

General Doppler Effect Equation

When both the source and detector are moving,

$$\boxed{f' = f \frac{v \pm v_D}{v \mp v_S}.}$$

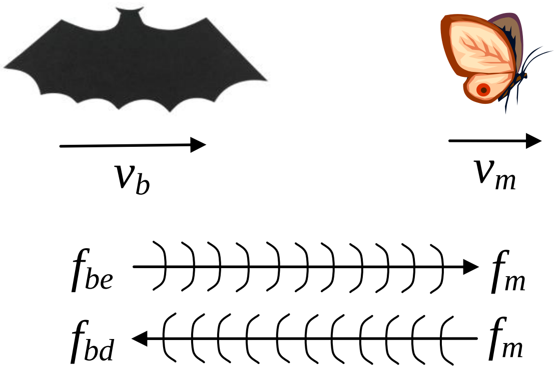
$v_S = 0$ reduces to the equation for stationary source.
$v_D = 0$ reduces to the equation for stationary detector.

Example

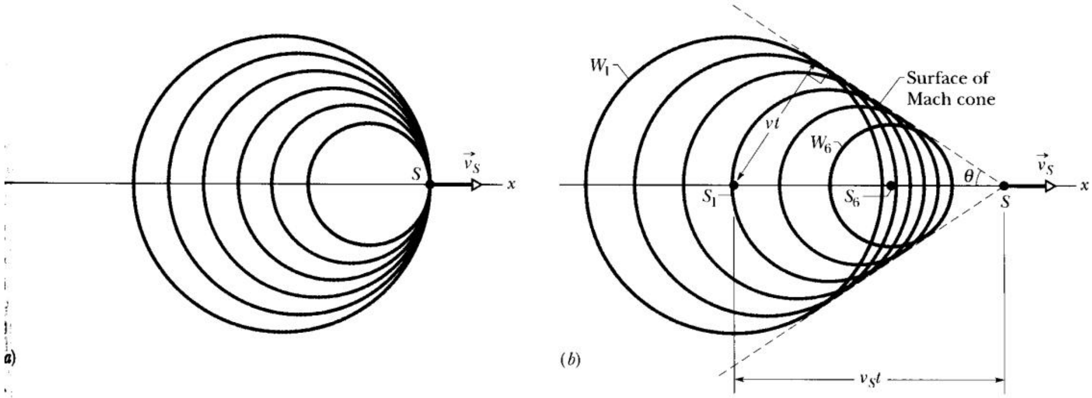
17-8 Bats navigate and search out prey by emitting, and then detecting reflections of, ultrasonic waves, which are sound waves with frequencies greater than what can be heard by a human. Suppose a bat emits ultrasound at frequency f_{be} = 82.52 kHz while flying with velocity v_b = 9.00 ms^{-1}. It chases a moth that flies with velocity v_m = 8.00 ms^{-1}.

(a) What frequency f_{md} does the moth detect?

(b) What frequency f_{bd} does the bat detect in the returning echo from the moth?



Supersonic Speeds; Shock Waves



17-23 (*a*) A source of sound S moves at speed v_S equal to the speed of sound and thus as fast as the wavefronts it gen- s. (*b*) A source S moves at speed v_S faster than the speed of sound and thus faster than the wavefronts. When the source t position S_1 it generated wavefront W_1, and at position S_6 it generated W_6. All the spherical wavefronts expand at the of sound v and bunch along the surface of a cone called the Mach cone, forming a shock wave. The surface of the cone alf-angle θ and is tangent to all the wavefronts.

FIG. 17-24 Shock waves produced by the wings of a Navy FA 18 jet. The shock waves are visible because the sudden decrease in air pressure in them caused water molecules in the air to condense, forming a fog. *(U.S. Navy photo by Ensign John Gay)*

See Youtube "Sonic Boom"

When v approaches v_S, f' becomes infinity since $f' = f\dfrac{v}{v - v_S}$.

When v exceeds v_S, the Doppler effect equation does not apply. All wavefronts bunch along a V-shaped envelope. This is called the *Mach cone*. A **shock wave** is produced. Note that the envelope touches the circular wavefronts. Therefore the radius ending at the tangent point is normal to the Mach cone.

Mach cone angle:

$$\sin\theta = \frac{vt}{v_S t} = \frac{v}{v_S}.$$

v_S/v is called the Mach number.