Longitudinal Waves

Sources of Musical Sound

Pipe

Closed end: node
Open end: antinode

Standing wave pattern:

Fundamental or first harmonic: 2 nodes at the ends, 1 antinode in the middle

\[ L = \frac{\lambda}{2}, \quad \lambda = 2L. \]

Resonant frequency: \( f = \frac{v}{\lambda} = \frac{v}{2L} \).

In general, for harmonic number \( n \),

\[ L = \frac{n\lambda}{2}, \quad \lambda = \frac{2L}{n}, \quad \text{for } n = 1, 2, 3, \]

Resonant frequency:

\[ f = \frac{v}{\lambda} = n \frac{v}{2L}, \quad \text{for } n = 1, 2, 3, \]

For pipes with only one open end,
\[ L = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \frac{7\lambda}{4} \ldots \]

\[ \lambda = 4L, \frac{4L}{3}, \frac{4L}{5}, \frac{4L}{7} \ldots \]

In general,

\[ \lambda = \frac{4L}{n}, \quad \text{for } n = 1, 3, 5, \ldots \]

Resonant frequencies:

\[ f = \frac{v}{\lambda} = \frac{nv}{4L}, \quad \text{for } n = 1, 3, 5, \ldots \]

In general, when a musical instrument produces a tone, the fundamental as well as higher harmonics are generated simultaneously. This gives rise to the different waveforms generated by different instruments. Hence different instruments have different sounds.

![Waveforms](image.png)

**FIG. 17-17** The wave forms produced by \((a)\) a flute and \((b)\) an oboe when played at the same note, with the same first harmonic frequency.
Example

17-6 Weak background noises from a room set up the fundamental standing wave in a cardboard tube of length \( L = 67.0 \) cm with two open ends. Assume that the speed of sound in the air within the tube is 343 ms\(^{-1}\).

(a) What frequency do you hear from the tube?
(b) If you jam your ear against one end of the tube, what fundamental frequency do you hear from the tube?

(a) Two open ends,

\[
L = \frac{\lambda}{2}
\]

\[
\lambda = 2L
\]

\[
f = \frac{v}{\lambda} = \frac{v}{2L}
\]

\[
= \frac{343}{2(0.67)} = 256 \text{ Hz} \quad \text{(ans)}
\]

(b) One fixed end and one open end,

\[
L = \frac{\lambda}{4}
\]

\[
\lambda = 4L
\]

\[
f = \frac{v}{\lambda} = \frac{v}{4L}
\]

\[
= \frac{343}{4(0.67)} = 128 \text{ Hz} \quad \text{(ans)}
\]
Beats

Consider two sound waves with slightly different frequencies:

\[ s_1 = s_m \cos \omega_1 t \quad \text{and} \quad s_2 = s_m \cos \omega_2 t \]

Resultant displacement:

\[ s = s_1 + s_2 = s_m (\cos \omega_1 t + \cos \omega_2 t). \]

Using the trigonometric identity

\[ \cos \alpha + \cos \beta = 2 \cos \frac{\alpha - \beta}{2} \cos \frac{\alpha + \beta}{2}, \]

we obtain

\[ s = 2s_m \cos \frac{1}{2} (\omega_1 - \omega_2) t \cos \frac{1}{2} (\omega_1 + \omega_2) t. \]
Conclusion:

\[ s = [2s_m \cos \omega' t] \cos \omega t, \]

where

\[ \omega' = \frac{1}{2}(\omega_1 - \omega_2) \quad \text{and} \quad \omega = \frac{1}{2}(\omega_1 + \omega_2). \]

Since \( \omega_1 \) and \( \omega_2 \) are nearly equal, \( \omega >> \omega' \).
Hence the resultant displacement consists of an oscillation with angular frequency \( \omega \) and a slowly changing amplitude with angular frequency \( \omega' \).

The amplitude \( 2s_m \cos \omega' t \) is maximum when \( \cos \omega' t = \pm 1 \), i.e. 2 times in each repetition of the cosine function.
Hence the beat frequency is:

\[ \omega_{\text{beat}} = 2\omega' = \omega_1 - \omega_2. \]

\[ f_{\text{beat}} = f_1 - f_2. \]

Musicians use the beat phenomenon in tuning their instruments.
See Animation “Beats”
The Doppler Effect
See Youtube “Fire Engine siren demonstrates the Doppler Effect” and “Example of Doppler Shift using car horn”

Detector Moving; Source Stationary

In time $t$, the wavefronts move a distance $vt$, the detector moves a distance $v_D t$, the range of waves intercepted by the detector = $vt+v_D t$, the number of wavefronts intercepted by the detector = $(vt+v_D t)/\lambda$. 
The frequency observed by the detector:

\[ f' = \frac{(vt + v_D t)}{t} = \frac{v + v_D}{\lambda}. \]

Since \( \lambda = \frac{v}{f} \),

\[ f' = \frac{v + v_D}{v / f} = f \cdot \frac{v + v_D}{v}. \]

Similarly, if the detector moves away from the source,

\[ f' = f \cdot \frac{v - v_D}{v}. \]

Summarizing,

\[ f' = f \cdot \frac{v \pm v_D}{v}. \]

**Source Moving; Detector Stationary**

![Diagram](image)

**FIG. 17-22** A detector \( D \) is stationary, and a source \( S \) is moving toward it at speed \( v \). Wavefront \( W_1 \) was emitted when the source was at \( S_1 \), wavefront \( W_2 \) when it was at \( S_2 \). At the moment depicted, the source is at \( S \). The detector senses a higher frequency because the moving source, chasing its own wavefronts, emits a reduced wavelength \( \lambda' \) in the direction of its motion.
In a period $T$, the distance moved by the wavefront $W_1 = vT$, the distance moved by the source $= v_S T$, the distance between the wavefronts $W_1$ and $W_2 = vT - v_S T$.

The frequency observed by the detector:

$$f' = \frac{v}{\lambda'} = \frac{v}{vT - v_S T} = \frac{v}{v/f - v_S / f} = f \frac{v}{v - v_S}.$$

If the source moves away from the detector,

$$f' = f \frac{v}{v + v_S}.$$

Summarizing,

$$f' = f \frac{v}{v \mp v_S}.$$

**General Doppler Effect Equation**

When both the source and detector are moving,

$$f' = f \frac{v \pm v_D}{v \mp v_S}.$$

$v_S = 0$ reduces to the equation for stationary source. $v_D = 0$ reduces to the equation for stationary detector.
Example

17-8 Bats navigate and search out prey by emitting, and then detecting reflections of, ultrasonic waves, which are sound waves with frequencies greater than what can be heard by a human. Suppose a bat emits ultrasound at frequency $f_{be} = 82.52$ kHz while flying with velocity $v_b = 9.00$ ms$^{-1}$. It chases a moth that flies with velocity $v_m = 8.00$ ms$^{-1}$.

(a) What frequency $f_{md}$ does the moth detect?
(b) What frequency $f_{bd}$ does the bat detect in the returning echo from the moth?
Supersonic Speeds; Shock Waves

When \( v \) approaches \( v_S \), \( f' \) becomes infinity since
\[
f' = f \frac{v}{v - v_S}.
\]

When \( v \) exceeds \( v_S \), the Doppler effect equation does not apply. All wavefronts bunch along a V-shaped envelope. This is called the Mach cone. A shock wave is produced. Note that the envelope touches the circular wavefronts. Therefore the radius ending at the tangent point is normal to the Mach cone.

Mach cone angle:
\[
\sin \theta = \frac{vt}{v_S t} = \frac{v}{v_S}.
\]

\( v_S/v \) is called the Mach number.