Modern Physics Laboratory
MP1 Direct Measurement of the Speed of Light

Purpose

In this experiment, you will measure the speed of light in air and in various materials with different refractive indices.

Equipment and components

Oscilloscope, rail with scale, speed of light apparatus (including the measuring unit, light transmitter and light receiver), converging lens (x2), synthetic resin block, water pipe with holders and measuring tape.

Background

The speed of light is an important quantity in many areas of physics. For example, it plays an important role in Einstein's special theory of relativity. In particular, the theory assumes that the speed of light in free space, $c = 2.998 \times 10^8$ m/s, is an upper bound on the speed at which energy, matter, and information can travel. In the history of physics, many efforts have been made to measure the value of *c*.

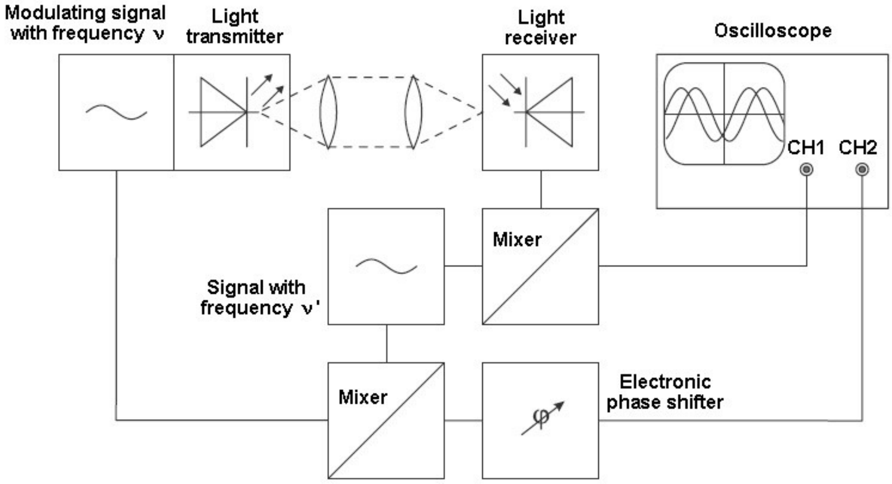


Figure 1 Block diagram showing the working principle of the experiment.

In this experiment, you will measure the speed of light in air and in various materials with different refractive indices. Figure 1 shows a block diagram illustrating the working principle of the experiment. A light signal is emitted from Light Transmitter and its intensity is modulated by Modulation Unit ν. The intensity of the modulated light can be written as

$$I(t) = \langle I \rangle + I_0 \cos(\omega t), \qquad (1)$$

where $\langle I \rangle$ is the average intensity, I_0 is the amplitude of the intensity modulation, and $\omega = 2\pi\nu = 2\pi/T$ is the angular frequency with ν being the modulation frequency in units of Hz and T the modulation period in units of second. Through two collimating lenses, the modulated light is detected by Light Receiver, which converts light into a voltage signal V(t) with the time dependence:

$$V(t) = V_0 \cos(\omega t - \varphi), \qquad (2)$$

where V_0 is the amplitude of the voltage signal and $\varphi = \omega\Delta t$ is the phase shift relative to the original signal I(t). The phase shift is caused by the fact that it takes a period of time Δt for the light signal to travel a distance *s* from Light Transmitter to Light Receiver. Therefore, we have $s = c\Delta t$ and

$$\varphi = 2\pi\nu\frac{s}{c}. \tag{3}$$

One can use Eq. (3) to determine the speed of light *c* once the phase shift φ is measured.

Because the ratio s/c is of the order of 10^{-8} s for a typical travelling distance *s*, a high frequency light source is needed to generate a measureable phase shift, φ. However, it is quite difficult to directly display a high-frequency signal on a typical oscilloscope. (The bandwidth of the oscilloscope used in the experiment is 20 MHz.) To overcome this difficulty, the detected signal is mixed (multiplied) electronically by another modulation signal of a slight different frequency ν'. This is done by Mixer and Modulation Unit ν'. The mixed signal has the following form:

$$V_m(t) = V_0\cos(2\pi\nu t - \varphi)\cos(2\pi\nu' t) \tag{4}$$

Using the trigonometry addition rule, one can readily show that the mixed signal has two components of different frequencies. One component has the sum frequency, $\nu + \nu'$, and the other has the difference frequency, $\nu_1 = \nu - \nu'$. The high frequency component is removed by a low pass filter, and then the mixed signal becomes,

$$V_m(t) \approx \frac{V_0}{2}\cos(2\pi\nu_1 t - \varphi). \tag{5}$$

This signal can be displayed on an oscilloscope if the value of $\nu_1 = \nu - \nu'$ is small enough. Note that the phase shift φ is not affected by the mixing, but it is now measured in the time frame of $\varphi = 2\pi\nu_1\Delta t_1$, where ν_1 is the frequency of the mixed signal. Figure 1 also shows a reference signal with a frequency ν, which is mixed with the same ν' signal and filtered in the same way as the detected signal. This signal is connected to CH2 and can be used as the reference signal with φ = 0.

When light travels through a medium, its speed c_n is reduced and the amount of reduction depends on the refractive index *n* of that medium. The refractive index *n* measures the optical density of the medium. The value of c_n is related to the speed of light *c* in vacuum via the equation

$$c_n = c / n. \tag{6}$$

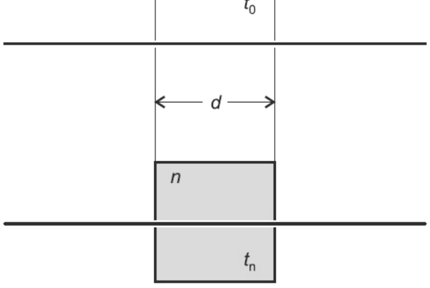


Figure 2 Propagation of light through a medium of thickness *d* with a travelling time t_n.

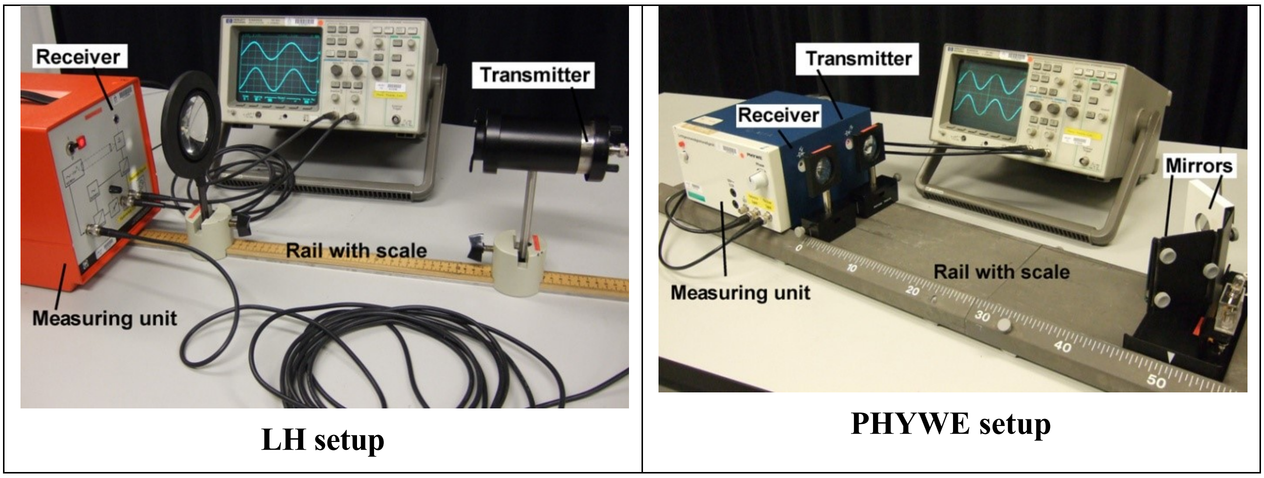
As shown in Fig. 2, the propagation time of light in a medium of thickness d is $t_n = d/c_n$, whereas in vacuum the propagation time is $t_0 = d/c$. Using the definition $\delta t = t_n - t_0$, one can readily show that

$$n = 1 + \frac{c}{d}\delta t = 1 + \frac{\lambda}{2\pi d}\varphi, \tag{7}$$

where $\varphi = 2\pi\nu\delta t$ is the phase shift and $\lambda = c/\nu$ is the wavelength of the modulated light. Equation (7) can be used to determine the refractive index n of a medium once the phase shift φ in the medium is measured. In the experiment, the phase shift φ is measured in the time frame of ν_1. In this case, we have $\varphi = 2\pi\nu_1\delta t_1$, where δt_1 is the apparent time change of δt obtained from the mixed signal.

Procedure

It will take two lab sessions to finish the present experiment. It is suggested that Part I be finished in the first lab session and Part II be finished in the second lab session.



LH setup

PHYWE setup

Figure 3 Experimental setup

Experimental apparatus:

In the experiment, we use two different brands of *speed of light apparatus*: LH and PHYWE, and they are shown in Fig. 3. The two apparatuses function based on the same working principle as described above, but there are some minor differences between the two setups:

1. Modulation frequency ν: The LH model uses 60 MHz modulation signal whereas the PHYWE model uses 50.1 MHz.
2. Light transmitter: As shown in Fig 3, the light transmitter in the LH model is separated from the measuring unit. In the PHYWE model, the transmitter and receiver are placed on the same measuring unit.
3. Equation (3) shows the relationship between the phase shift φ and the travelling distance s of the modulated light from the transmitter to receiver. In the actual experiment, however, it is quite difficult to accurately measure the absolute value of s, as there are several optical components in the light path. To overcome this difficulty, we will measure the displacement Δs and the corresponding phase shift $\Delta\varphi$ instead. The relationship between Δs and $\Delta\varphi$ remains the same as that given in Eq. (3). As shown in Fig. 4 below, the value of Δs can be directly measured in the LH setup. For the PHYWE setup, the two mirrors are set at right angles to reflect the light beam from the transmitter to the receiver. In this case, one has $\Delta s = 2\Delta x$.
4. It should be noted that the phase shift is a relative concept and one has to know the reference signal, with respective to which the phase shift is set at $\varphi = 0$. In this experiment, we adjust the knob of the electronic phase shifter so that the receiver signal

and the reference signal are in phase when the separation between the light transmitter and receiver is s. The phase change $\Delta\varphi$ is measured when the separation between the light transmitter and receiver is changed from s to $s+\Delta s$ (see Fig. 4).

5. It should also be noted that the phase shift φ (or $\Delta\varphi$) is not affected by the change of frequency of the signal, but the time delay Δt_1 in the equation, $\Delta\varphi = 2\pi\nu_1\Delta t_1$, depends on the frequency ν_1 of the signal used. To avoid confusion, it is suggested that you convert all the measured Δt_1 into $\Delta\varphi$, when you use the data to calculate other quantities.

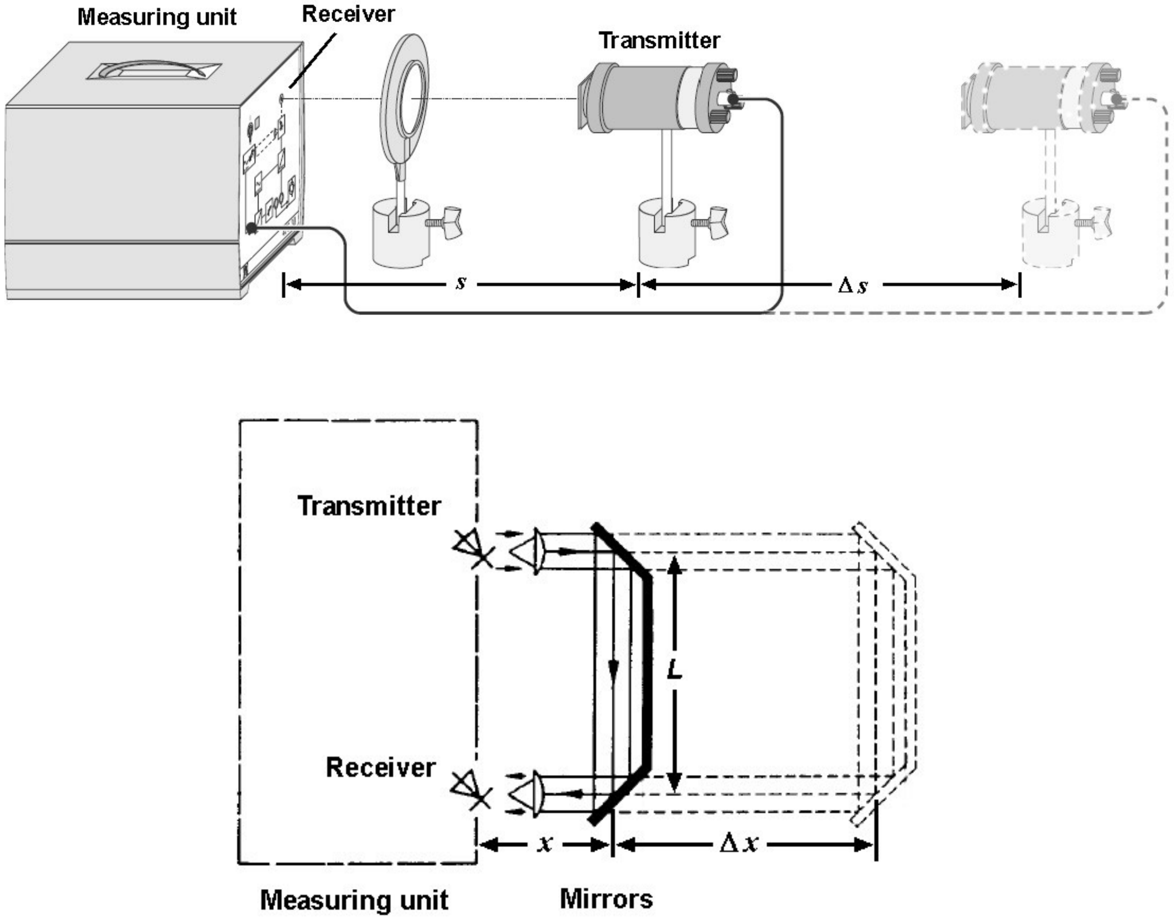


Figure 4 Optical setups of the LH model (upper) and PHYWE model (lower).

Part I Measurement of the speed of light in air

1. Identify the brand name of the apparatus you use and record it in Table 1.
2. Turn on the measuring unit.
3. Connect the output signal from the light receiver to the oscilloscope CH1 and the reference signal on the measuring unit to the oscilloscope CH2.
4. Use a 6-m-long coaxial cable to connect the measuring unit to the light transmitter as shown in Fig. 4 (for the LH setup).
5. Adjust the distance s between the light transmitter and light receiver to about 1 m. For the LH setup, move the transmitter along the rail. For the PHYWE setup, move the mirrors along the rail such that $x \sim 0.5$m. Note that, in this case, $s = 2x + \mathrm{L}$ and hence $x \sim s/2$.
6. Adjust the converging lens in front of the transmitter (LED), such that the divergent light becomes a parallel beam.

7. Adjust the converging lens in front of the receiver, such that the parallel beam is focused on the receiver.
8. Watch the receiver signal on the oscilloscope and optimize the signal by fine-tuning the positions of the light transmitter (in the LH setup) or mirrors (in the PHYWE setup) and the focusing lenses.
9. Reposition the light transmitter (in the LH setup) or mirrors (in the PHYWE setup), such that the distance s between the transmitter and receiver is about 1.8 m.
10. To start the measurement, adjust the knob of the electronic phase shifter so that the receiver signal and the reference signal are in phase. Record the initial position (read from the scaled rail) of the transmitter (for the LH setup)/ mirrors (for the PHYWE setup) in Table 1.
11. Measure the frequency ν_1 of the receiver signal and record it in Table 1.
12. Reduce the distance s by 20 cm, i.e. $\Delta s = 20\text{ cm}$. Measure the *time delay* Δt_1 using the Cursor function on the oscilloscope and record your result in Table 1. For the PHYWE setup, please be reminded that $\Delta s = 20\text{ cm}$ can be achieved by moving the mirrors with $\Delta x = 10$ cm (see Fig. 4).
13. Repeat the measurements with multiple steps of $\Delta s = 20\text{ cm}$ until $\Delta s = 160\text{ cm}$.
14. Calculate the corresponding phase shift $\Delta\varphi$ and record your results in Table 1.

Part II Measurement of the speed of light in other media

1. Set up the experiment as in Part I, steps 1 to 8.
2. Position the light transmitter (in the LH setup) or mirrors (in the PHYWE setup) such that the distance between the transmitter/mirrors and receiver is large enough to put the medium samples into the light path. Two medium samples will be used in the experiment; one is a pipe filled with water of length ~1 m (for LH setup) or 0.5 m (for PHYWE setup) and the other sample is a long block made of synthetic resin of length ~0.3 m.
3. *Method 1: Measurement of the apparent time change* δt_1
   A) Put one of the medium samples into the light path.
   B) Adjust the knob of the electronic phase shifter so that the receiver signal and the reference signal are in phase.
   C) Remove the sample from the light path and measure the *apparent time change* δt_1 using the Cursor function of the oscilloscope and record the result in Table 2.
   D) Repeat the measurement for two more trials.
   E) Repeat steps A to D for the other sample.
   F) Measure the frequency ν_1 of the receiver signal and record the value in Table 2.
   G) Measure the length of the samples and record the results in Table 2.
   H) Use the equations provided in Introduction and calculate the speed of light c_n in the media used and their refractive index n. Record your results in Table 2.
4. *Method 2: Measurement of the apparent path change* δs

   Another method to measure the speed of light in a medium is shown in Fig. 5. Since light propagates at a lower speed in most media than that in air, an apparent path change δs can be introduced to account for the decrease in the travelling time when the medium is replaced by air. Here we define δs in such way that the time needed to travel the two different light paths remains the same. In this case, one can show that

$$n = \frac{c_a}{c_n} = 1 + \frac{\delta s}{d}, \tag{9}$$

where c_a is the speed of light in air.

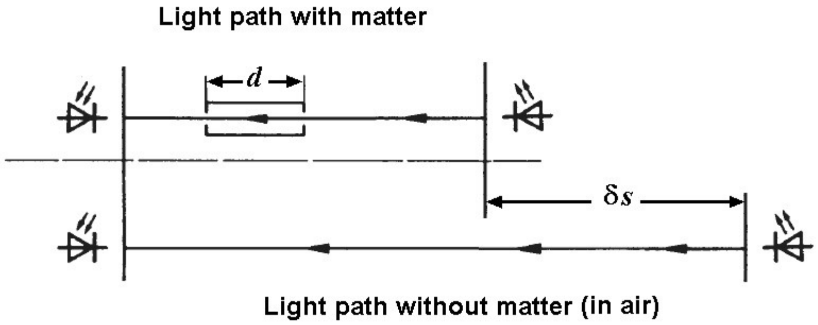


Figure 5 Equivalent light paths used in Method 2.

A) Put one of the medium samples into the light path.

B) Adjust the knob of the electronic phase shifter so that the receiver signal and the reference signal are in phase.

C) Record the initial position (read from the scaled rail) of the transmitter (for the LH setup) or mirrors (for the PHYWE setup) in Table 3.

D) Remove the sample from the light path. Move the transmitter (for the LH setup) or mirrors (for the PHYWE setup) away from the receiver, so that the receiver signal and the reference signal are in phase again.

E) Record the final position of the transmitter (for the LH setup) or mirrors (for the PHYWE setup) in Table 3.

F) Calculate the *apparent path change* δs and record your result in Table 3. Recall that for the PHYWE setup, $\delta s = 2\delta x$, where δx is the difference between the final position of the mirrors and their initial position.

G) Repeat the measurement for two more trials.

H) Repeat steps A to G with the other sample.

I) Use Eq. (9) to calculate the speed of light c_n in the media used and their refractive index n. Record your results in Table 3.

Name	
Date	
Lab session (Day & time)	
Lab partner	

MP1 Direct Measurement of the Speed of Light Lab Report

A. Answer the following question BEFORE the first lab session (8 pts each)

1. Calculate the phase shift φ introduced to a modulated light of frequency 50 MHz, when it travels across a distance of 5 m in air.

2. Show that the mixed signal in Eq. (4) has two frequency components; one has the sum frequency, $\nu + \nu'$, and the other has the difference frequency, $\nu_1 = \nu - \nu'$.

3. Derive Eq. (7) using the sample geometry given in Fig. 2.

Name		**LA ( )**

B. Experimental results (96 pts)

Table 1 Measurement of the speed of light in air (36 pts)

Name of the setup: LH or PHYWE

Initial position of the transmitter (for the LH setup) or mirrors (for the PHYWE setup):

____

Frequency ν1 of the receiver signal = ____________________

Δs ( )	Δt_1 ( )	$\Delta\varphi$ (in radian)
0	0	0

Table 2 Measurement of the speed of light in other media [Method 1] (30 pts)

Frequency ν1 of the receiver signal = ____________________

Length of the water pipe, d_w = ____________________

Length of the synthetic resin block, d_r = ____________________

Sample	**Trial**	***Apparent time change*** δt_1 **( )**	**Speed of light** c_n **( )**	**Refractive index**
Water	1			
	2			
	3			
Synthetic resin	1			
	2			
	3			

Table 3 Measurement of the speed of light in other media [Method 2] (30 pts)

Sample	Water			Synthetic resin		
Trial	1	2	3	1	2	3
Initial position ( )						
Final position ( )						
Apparent path change δs **( )**						
Speed of light c_n **( )**						
Refractive index						

C. Data analysis and questions (16 pts each)

4. Use the data in Table 1 and plot the phase change $\Delta\varphi$ as a function of the displacement Δs. Fit the data to a linear function and record the fitting results in your lab report. Attach your plot to the lab report.

5. From your fitting results and the equations given in the "Background", determine the speed of light in air c_a. How does your fitted value of c_a compare with the published value? What are the potential sources of error in this experiment?

6. Use the data in Table 2 and calculate the mean value of the speed of light c_n of the two samples and their refractive index n. Compare the values of n for water and synthetic resin (an organic material) with the published values.

7. Use the data in Table 3 and calculate the mean value of the speed of light c_n of the two samples and their refractive index n. Compare the values of n for water and synthetic resin (an organic material) with the published values.

8. Which method is more accurate? Why? Discuss briefly how to increase the accuracy of the experiment.

D. Summary of the experiment (20 pts)

Summarize what you have done and your interpretation of the results, especially in respect to how they match the goal of the experiment and what you have learnt from this experiment (2-3 paragraphs, less than 450 words).