Purpose
This experiment has two goals: (i) to observe the emission spectrum of hydrogen and (ii) to verify the relationship between the discrete emission lines in the Balmer series of hydrogen.

Equipment and components
Hydrogen spectrum lamp, spectrometer, two diffraction gratings (300 lines/mm and 600 lines/mm), convex lens with a holder and magnifying glass.

Background
The development of quantum theory was stimulated by experimental observations of several striking phenomena. One of the phenomena was that the emitted light by atoms is only at discrete wavelengths, \( \lambda \). If a prism or grating is used to disperse the emitted light, there will be a line spectrum of light called the atomic spectrum. Figure 1 below indicates the spectrum of hydrogen, which consists of a series of lines designated as \( H_\alpha, H_\beta, H_\gamma \) and so on.

Figure 1 Observation of the visible spectrum of hydrogen

An important feature of the hydrogen spectrum is that the discrete lines become increasingly closer at smaller wavelengths. In 1885 Johann Balmer, a Swiss school teacher, published an empirical equation that described the wavelengths of the known hydrogen lines within the limits of experimental error. Balmer's equation can be written in the form

\[
\frac{1}{\lambda} = R \left( \frac{1}{2^2} - \frac{1}{N^2} \right),
\]

where \( N \) is an integer greater than or equal to 3 and \( R \), now known as the Rydberg constant, was adjusted to match the data. In Eq. (1), \( N = 3 \) corresponds to the \( H_\alpha \) line, \( N = 4 \) corresponds to the \( H_\beta \) line, and so on. It was later discovered that the emission spectrum of hydrogen has several other series of lines in the ultraviolet and infrared ranges of wavelengths. These spectra can be more generally described by the equation

\[
\frac{1}{\lambda} = R \left( \frac{1}{M^2} - \frac{1}{N^2} \right),
\]

with \( M \geq 1 \) and \( N > M \). The Layman series is predicted by this equation with \( M = 1 \), the Balmer series corresponds to \( M = 2 \), the Paschen series corresponds to \( M = 3 \), the Brackett
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series corresponds to M = 4, and the Pfund series corresponds to M = 5. Equation (2) can be derived using Bohr's model, which is a quantum theory of atoms.

Procedure

I. Apparatus

1. Light source – A sealed tube containing hydrogen gas will be used as a spectrum light source. A high voltage (HV) power supply is used to provide an electric field between the two electrodes at the end of the tube. Under such a strong applied electric field, some gas molecules are ionized and the resulting ions and electrons are accelerated. These energetic ions and electrons will excite the gas molecules by collision. The excited gas molecules emit light with a characteristic line spectrum.

2. Diffraction grating – Diffraction grating is an optical component consisting of a large number of equidistant parallel slits or rules on a glass or metal surface. When a plane wave of wavelength $\lambda$ is incident normally on the grating, wavelets from each slit will interfere and give rise to an outgoing light, whose intensity varies with the angle $\theta$ relative to the incident direction. As shown in Fig. 2, when the path difference of the light from adjacent slits is equal to the wavelength, $\lambda$, the wavelets will all be in phase, giving rise to a maximum intensity. This situation occurs at multiple angles $\theta_n$ which satisfy the condition,

$$d \cdot \sin \theta_n = n\lambda,$$

where $d$ is the spacing between the slits and $n$ is an integer. The light that corresponds to the direct transmission is called the zero order maximum and is denoted $n = 0$. The other maxima occur at angles which are represented by non-zero integers $n$. Note that $n$ can be either positive or negative, resulting in a diffracted light on both sides of the zero order beam.

![Figure 2 Schematic of a diffraction grating](image)

3. Spectrometer – As shown in Fig. 3, a spectrometer consists of three basic components: a collimator, a diffracting element, and a telescope. The light to be analysed enters the collimator through a narrow slit, which is positioned at the focal point of the collimating lens. The light leaving the collimator is therefore a thin parallel beam, ensuring that the incident beam of light into the diffracting element is at the same angle of incidence. This is necessary for a better angular resolution.

The diffracting element diffracts the incident light. If the beam of light is composed of different colors, each color is diffracted to a different angle. The telescope can be rotated to collect the diffracted light at a precisely determined angle. With the telescope focused at infinity and positioned at an angle to collect the light of a particular color, a sharp image of the collimator slit can also be seen. The sharp edge of the slit image can be used to determine the angle of diffraction. Using the working principle of the diffracting element, one can determine the wavelengths of the diffracted light from the measured diffraction angles.
II. Equipment setup

Alignment of the spectrometer

1. While looking through the telescope, as shown in Fig. 4, slide the eyepiece in and out until the cross-hairs come into sharp focus. If necessary, loosen the graticule lock ring and rotate the graticule until one of the cross-hairs is vertical. Tighten the lock ring and then refocus the cross hairs if necessary.

2. Focus the telescope at infinity. This is best accomplished by focusing on a distant object (e.g. an object outside the window) with the telescope focus knob.

3. Adjust the collimator slit width to about 1mm (by using the slit width adjustment screw).

4. Align the telescope directly opposite the collimator as shown in Fig. 5.

5. Looking through the telescope, adjust the focus of the collimator and, if necessary, the position of the telescope until the slit comes into sharp focus. Do not change the focus of the telescope.
6. Tighten the telescope rotation lock-screw, then use the fine adjustment knob to align the vertical line of the graticule with the fixed edge of the slit. If the slit is not vertical, loosen the slit lock ring, realign the slit, and retighten the lock ring. Measurements of the diffraction angle are always made with the graticule line aligned along the fixed edge of the slit, so a very narrow slit is not necessarily advantageous.

Alignment of the diffraction grating and light source

**NOTE:** The diffraction grating is a delicate optical component. Be careful not to scratch its surface and always place it in the protective foam wrapping when not in use.

7. Loosen the spectrometer table lock-screw, as shown in Fig. 6. Align the engraved line on the spectrometer table so that it is collinear with the optical axes of the telescope and the collimator. Tighten the lock-screw and **do not change this alignment.**

8. Insert a 300 lines/mm diffraction grating into the clips of the mount.

9. Place the hydrogen lamp and a convex lens in front of the collimator slit. Adjust their positions so that the lens can focus the light from the hydrogen lamp onto the slit. If necessary, adjust the height of the lens, so that the central part of the light (in red) from the hydrogen lamp is located at the slit.

**NOTE:** (a) Stray light can obscure the slit image. Use the spectrometer in a semi-darkened room.

(b) The hydrogen lamp has a finite lifetime and should be in operation only when needed. **Switch the lamp off when it is idle.**

10. To check the orientation of the grating, look at the hydrogen lamp through the grating and notice how the grating disperses the light into various color components. When placed in the grating mount, the grating should spread the colors of the incident light horizontally, so that you can see the image of the slit with different colors by rotating the telescope.
Measurement of angle of diffraction

Before making the measurement, it is important to establish a vernier reading of the undeflected beam in order to measure the angles of diffraction.

11. Rotate the telescope to align the vertical cross-hair with the fixed edge of the slit image for the undeflected beam, as shown in Fig. 7.

![Figure 7 Measurement of angle of diffraction](image)

12. Read the vernier scale and record the zero-point reading $\theta_0$ in Table 1. Refer to the Appendix, “Reading of the vernier scales,” for details about how to read the vernier scale.

13. The angle $\theta$ of diffraction can now be measured by reading the vernier scale of the deflected beam. The actual angle of diffraction is $\theta - \theta_0$. If the table base is moved for some reason during the measurements, the zero point reading may change and needs to be recalibrated.

III. Measurement of the hydrogen spectrum

Once the initial setup is finished, the spectrometer is ready to observe and measure the spectrum of the light source.

1. Visually observe through the telescope of the spectrometer the first and second orders ($n = \pm 1, 2$) of the spectral lines of the hydrogen lamp with a 300 lines/mm diffraction grating.

2. Measure the angles of diffraction, $\theta - \theta_0$, of all the spectral lines observed in step 1. Record your results in Table 1. Calculate the average value of $|\theta - \theta_0|$ for the same order of the spectral lines (on both sides).

3. Replace the 300 lines/mm diffraction grating with a 600 lines/mm one. Observe the first order ($n = \pm 1$) of the spectral lines of the hydrogen lamp.

4. Measure the angles of diffraction of the spectral lines observed in step 3. Record your results in Table 2. Calculate the average value of $|\theta - \theta_0|$ for the same order of the spectral lines (on both sides).
Appendix: Reading of the vernier scales

To read the angle shown in Figure A1, first find the location where the zero point of the vernier scale on the upper plate aligns with the degree plate (the lower plate) and record the degree value. If the zero point is in between two lines, take the smaller value. As shown in Figure A1 below, the zero point on the vernier scale is between the $155^\circ$ and $155.30'$ (or $155.5^\circ$) marks on the degree plate, so the recorded value should be $155^\circ$. Note that $1^\circ = 60'$.

Now use the magnifying glass to find the line on the vernier scale that aligns most closely with any line on the degree scale. As shown in Figure A1, this line corresponds to a reading of 15 minutes of arc. Add this value to the above reading to get the final result to within 1 minute of arc: $155^\circ + 15' = 155^\circ15'$. 

![Figure A1 Reading of the vernier scales](image)
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A. Answer the following questions BEFORE the lab session (5 pts each)

1. Derive Eq. (3) using the optical path diagram shown in Fig. 2. Briefly describe the physical meaning of Eq. (3).

2. What is the advantage of using a diffraction grating with greater lines/mm over that with a smaller lines/mm?

3. Use Eq. (1) to calculate the corresponding wavelength of the spectral lines with N = 3 and N = 4. Can a 300 lines/mm diffraction grating resolve the two wavelengths? Explain. (The Rydberg constant R = 1.097 × 10⁷ m⁻¹)
B. Results (40 pts)

Table 1  Angular reading at different diffraction orders (using the 300 lines/mm diffraction grating)

Zero point reading $\theta_0$: ______________

| Color | Angle of diffraction, $\theta$ (degree) | Average value of $|\theta - \theta_0|$ (degree) |
|-------|---------------------------------------|----------------------------------|
|       | $n = +1$ | $n = -1$ | $n = +2$ | $n = -2$ | $|n| = 1$ | $|n| = 2$ |
| violet|            |            |            |            |            |            |
| blue  |            |            |            |            |            |            |
| green |            |            |            |            |            |            |
| red   |            |            |            |            |            |            |

Table 2  Angular reading at the first diffraction order (using the 600 lines/mm diffraction grating)

| Color | Angle of diffraction, $\theta$ (degree) | Average value of $|\theta - \theta_0|$ (degree) |
|-------|----------------------------------------|----------------------------------|
|       | $n = +1$ | $n = -1$ | $|n| = 1$ |
| violet|            |            |            |
| blue  |            |            |            |
| green |            |            |            |
| red   |            |            |            |
C. Data analysis and questions (45 pts)

1. (10 pts) Using the data shown in Table 1, find the corresponding wavelength of the four spectral lines of the hydrogen spectrum. Tabulate your results for $|n| = 1$ and $|n| = 2$, separately.

2. (5 pts) Using the data shown in Table 2, find the corresponding wavelength of the four spectral lines of the hydrogen spectrum. Tabulate your results below.

3. (10 pts) In the above, you obtained three values of the wavelength for each spectral line of the hydrogen spectrum. These values are obtained either by using different diffraction gratings or different diffraction orders. In your opinion, which way gives the most accurate result? Why? Find the percentage differences between the three wavelength values and tabulate your results below.
4. (15 pts) Using the tabulated results in Question 2 and assigning the red, green, blue, and violet lines with N = 3, 4, 5, and 6, respectively, plot the $1/\lambda$ values as a function of $1/N^2$. Use the linear regression to find the slope $S = \pm$ and y-intercept $y_0 = \pm$. Show the fitting result (as a solid line) and the ‘R-square’ value on your graph. Attach your plot to the lab report.

5. (5 pts) Explain the physical meaning of the slope and y-intercept values obtained above. Compare your results (slope and y-intercept) with the theoretical expectations.