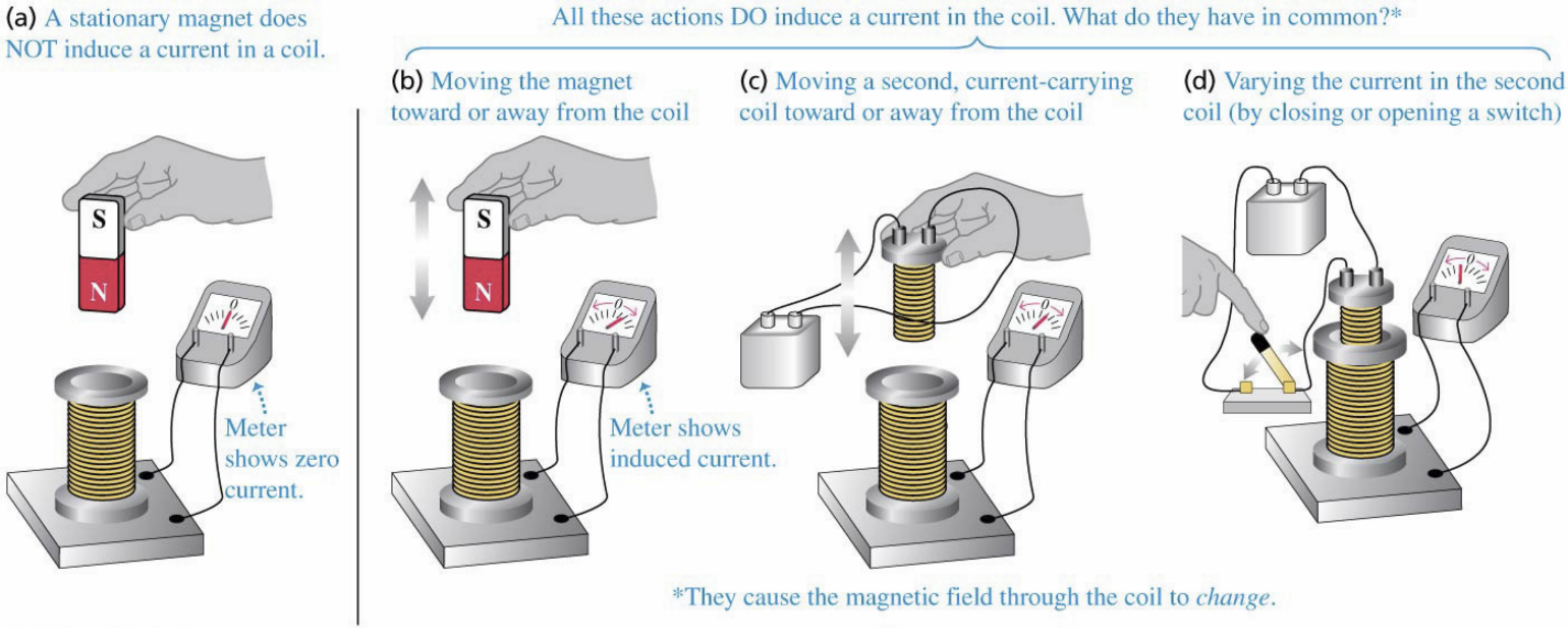
Electromagnetic Induction

Demonstration:



1. [Electromagnetic induction](#)

Faraday's Law of Electromagnetic induction

A time-varying magnetic flux induces an emf

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

⚠ Magnetic flux $\Phi_B = \int \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{A}}$ in Tm^2 or Wb. Do not confuse with line integral $\oint \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{l}}$ in Ampere's law

Lenz's law – The direction of any magnetic induction effect is such as to oppose the cause of the effect.

⚠ Strategy: ignore the –ve sign and find the magnitude of the induced emf. Then determine direction by Lenz's law (you have learned it in high school). We will use the –ve sign later.

⚠ Unit on the left-hand-side is $\mathrm{Tm^2/s} = [\mathrm{N/Cms^{-1}}][\mathrm{m^2/s}] = \mathrm{Nm/C} = \mathrm{J/C} = \mathrm{V}$

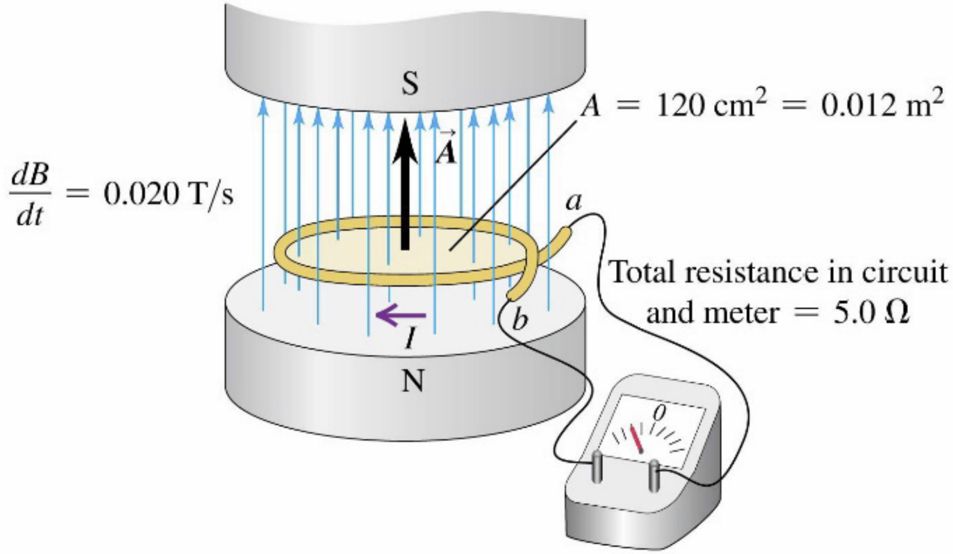
Example 29.1

A circular loop of wire inside a uniform B field which is increasing at $0.020\ \mathrm{T/s}$. Magnitude of induced emf in the loop

$$\mathcal{E} = \frac{d\Phi_B}{dt} = \frac{d(BA)}{dt} = A\frac{dB}{dt}$$
$$= (0.012\ \mathrm{m}^2)(0.020\ \mathrm{T/s})$$
$$= 2.4 \times 10^{-4}\ \mathrm{V}$$

Induced current in the loop is

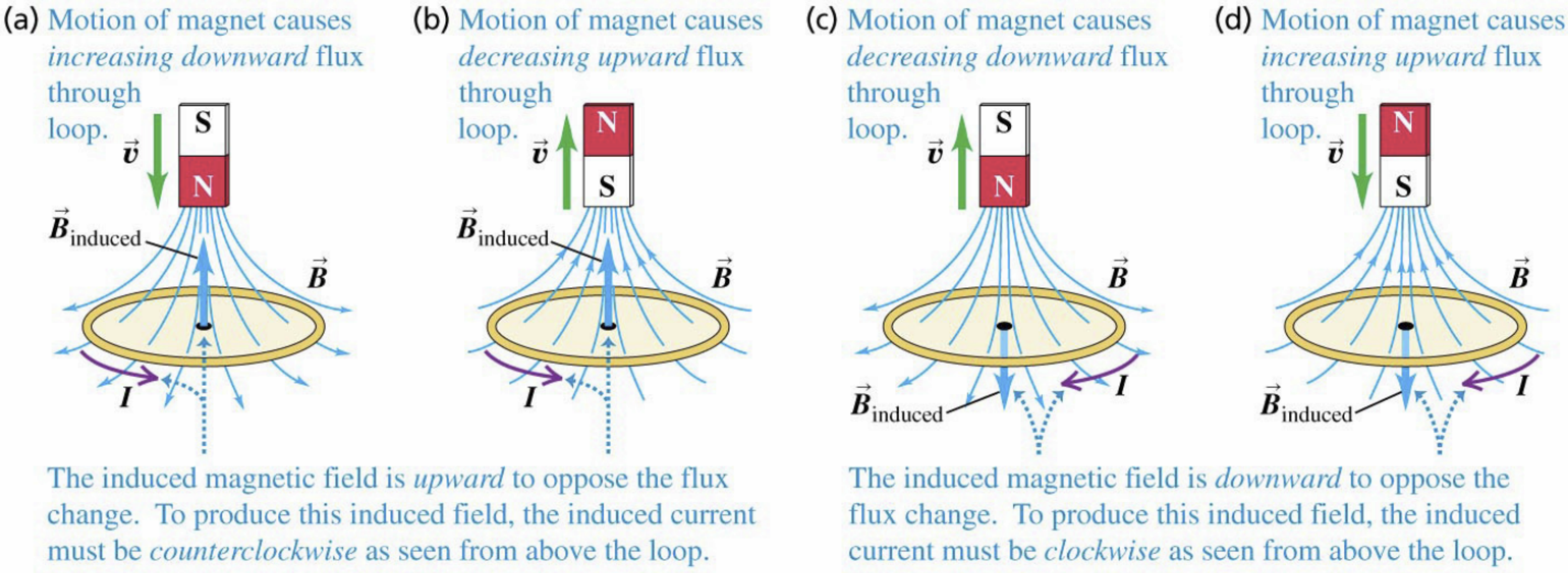
$$I = \frac{\mathcal{E}}{R} = \frac{2.4 \times 10^{-4}\ \mathrm{V}}{5.0\ \Omega} = 4.8 \times 10^{-5}\ \mathrm{A}$$



Lenz's law: I produces a B field that oppose the *increasing* $\vec{\boldsymbol{B}}$, i.e, it produces a field pointing (upwards / downwards). Direction of I is from (a to b / b to a)

⚠ Direction of I reversed if the external field is *decreasing* in magnitude ($dB/dt < 0$) instead of increasing, even though its direction remains the same as shown in the diagram.

Lenz's Law and Response to Flux Change

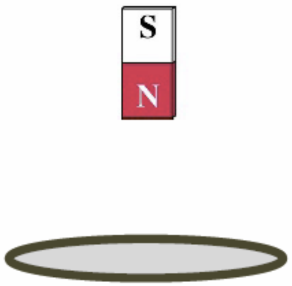


⚠ Depend on relative motion between the magnet and flux only. Doesn't matter whether the magnet of loop is/are moving

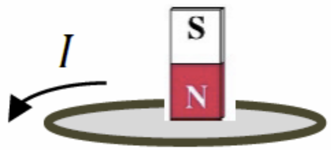
Electromagnetic induction resist flux change. How well it can do it depends on the resistance of the loop.

1. If loop is non-conducting, R is infinity, no induced current, *cannot* resist flux change

2. If loop is **superconducting**, R is zero. Even after flux stop to change, $I = \frac{\mathcal{E}}{R} = \frac{0}{0}$ still persists (a *persistent current*), making final flux equals to initial flux. *Completely* resist any flux change.



Before, flux through loop smaller

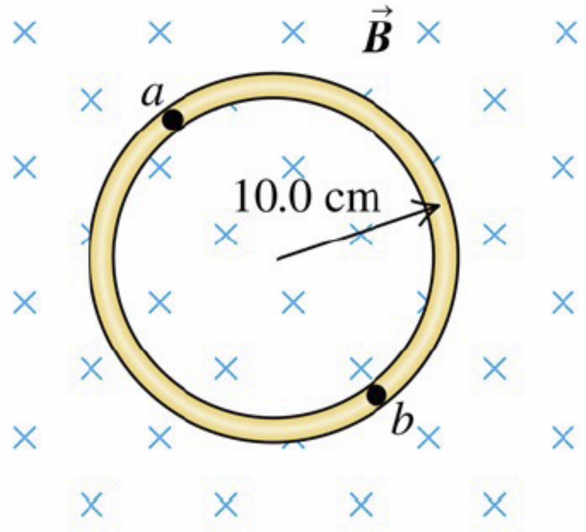


After, flux due to magnet larger, but reduced by persistent induced current

Clicker Questions

Q29.2

A circular loop of wire is in a region of spatially uniform magnetic field. The magnetic field is directed into the plane of the figure. If the magnetic field magnitude is *decreasing*,



A. the induced emf in the loop is clockwise.

B. the induced emf in the loop is counterclockwise.

C. the induced emf in the loop is zero.

D. either A or B is possible.

E. any of A, B, or C is possible.

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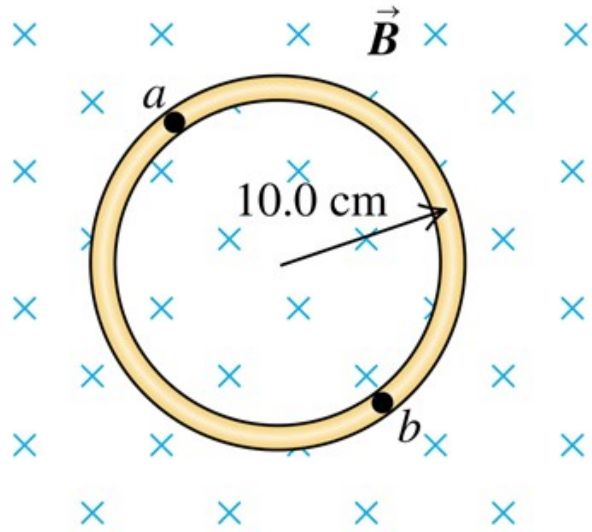

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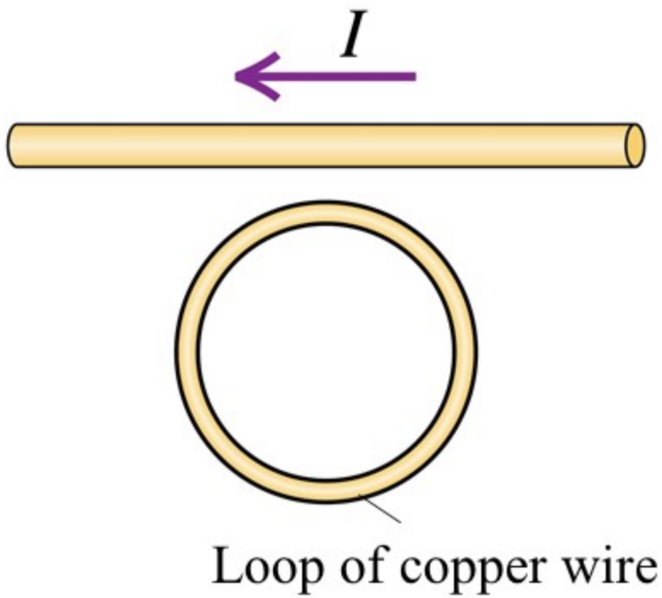
Q29.3

A circular loop of copper wire is placed next to a long, straight wire. The current I in the long, straight wire is increasing. What current does this induce in the circular loop?



A. a clockwise current

B. a counterclockwise current

C. zero current

D. either A or B

E. any of A, B, or C

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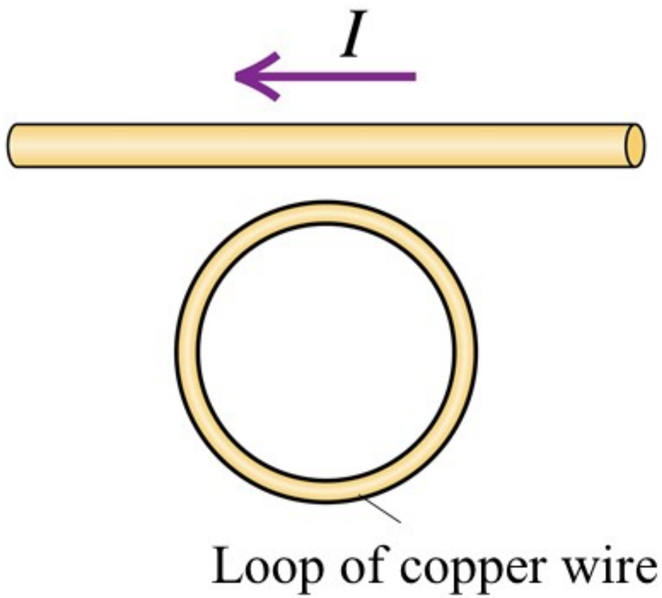

A. a clockwise current

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C. zero current

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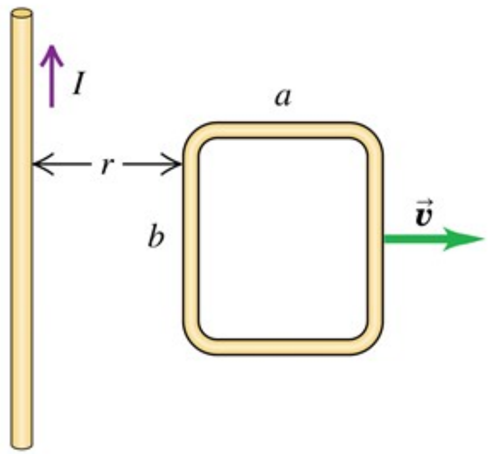
Q29.7

The loop of wire is being moved to the right at constant velocity. A constant current I flows in the long, straight wire in the direction shown. The current induced in the loop is



A. clockwise and proportional to I.

B. counterclockwise and proportional to I.

C. clockwise and proportional to I^2.

D. counterclockwise and proportional to I^2.

E. zero.

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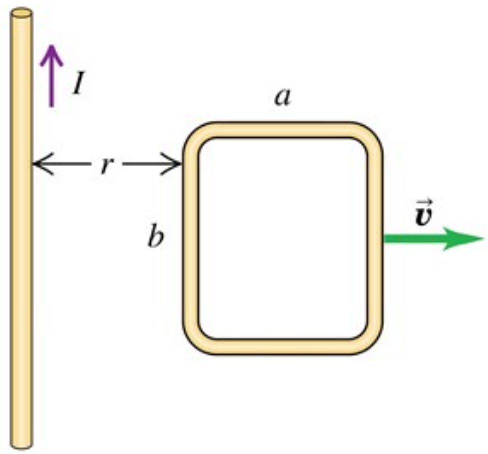


✔ A. clockwise and proportional to I.

B. counterclockwise and proportional to I.

C. clockwise and proportional to I^2.

D. counterclockwise and proportional to I^2.

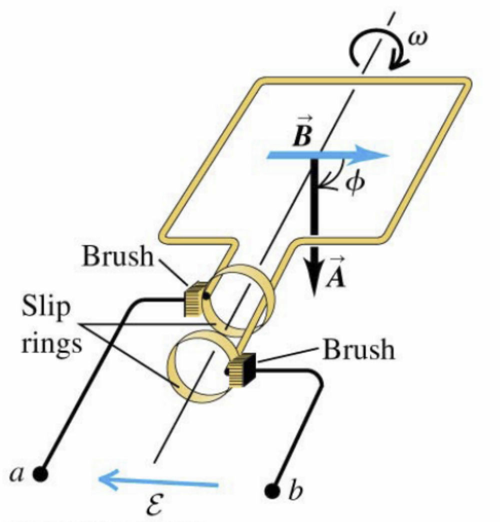
E. zero.

A Simple Alternator Example 29.2 and 29.3

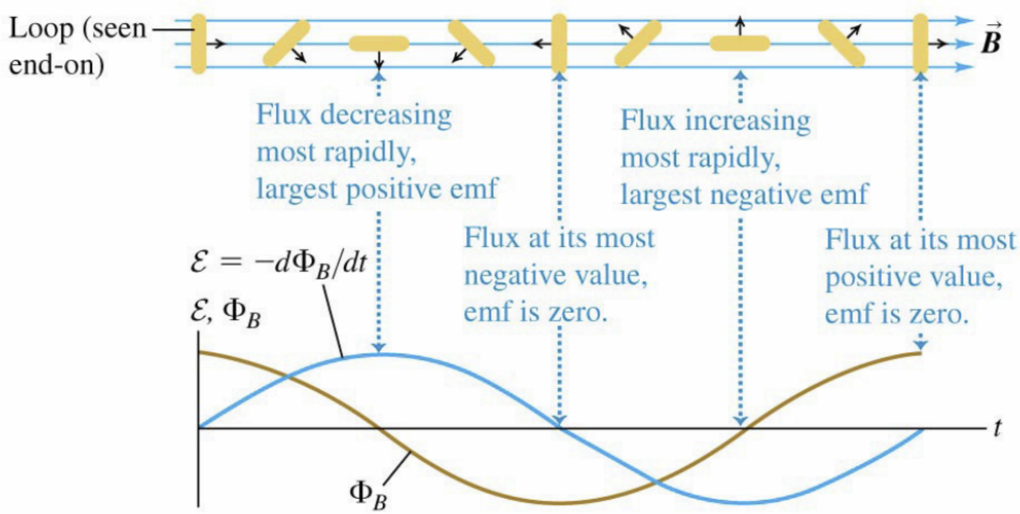
A loop being driven by an external force to rotate in a uniform constant magnetic field. $\vec{\boldsymbol{B}}$ is not changing but $\Phi_B = \vec{\boldsymbol{B}} \cdot \vec{\boldsymbol{A}}$ is.

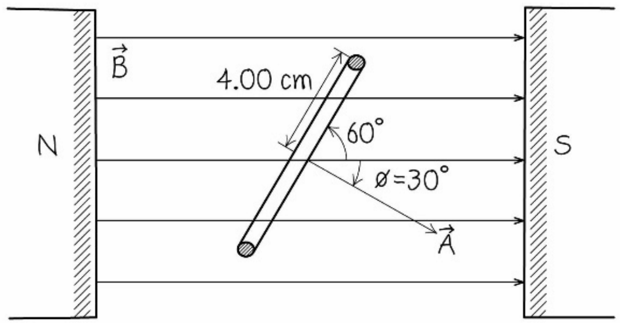
⚠ Direction of $\vec{\boldsymbol{A}}$ can be either side, but once chosen, must stick to the same choice

(a)



(b)





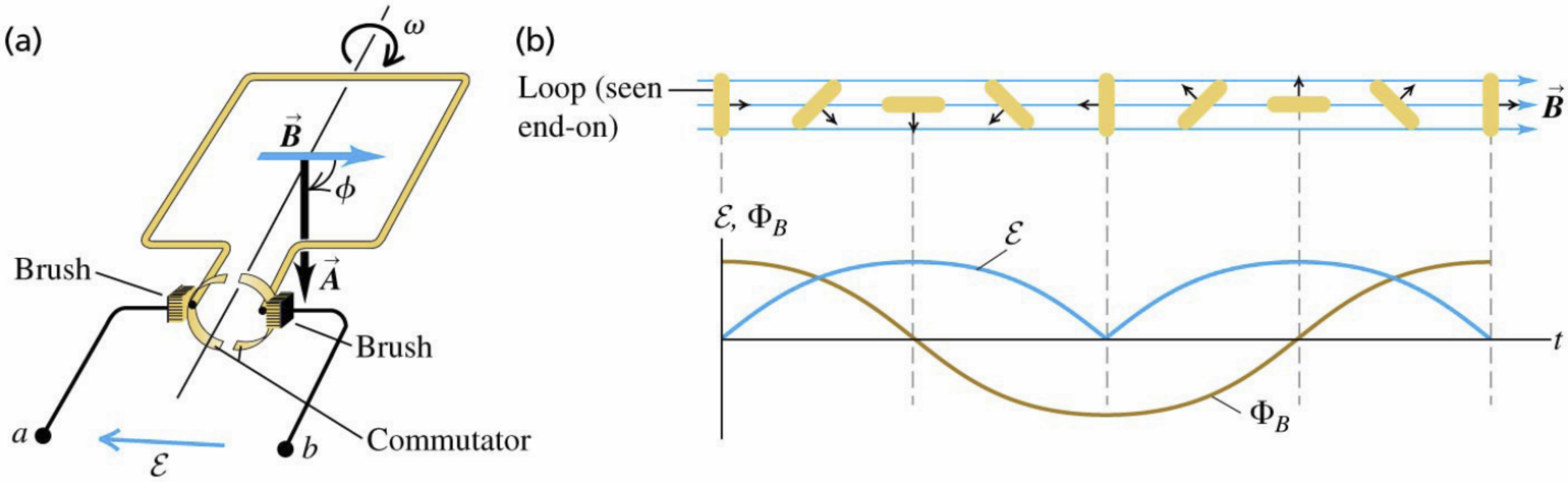
If loop is rotating with constant angular speed ω and $\phi = 0$ at $t = 0$, then $\phi = \omega t$

$$\Phi_B = BA\cos\omega t$$

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = \omega BA\sin\omega t$$

$\mathcal{E}$ across ab is sinusoidal, alternating between +ve and –ve values

A **DC generator** is similar to an alternator except that the slip rings are replaced by a single split ring called a *commutator*. I changes direction once the gaps in the commutator pass through the brushes



$$\mathcal{E} = \left|\frac{d\Phi_B}{dt}\right| = \omega BA|\sin \omega t|$$

Back emf in a motor Example 29.4

A motor is the same as a DC generator except that an external emf is applied across *ab* to drive the motor, but at the same time induce a back emf $\mathcal{E}$ just like in a DC generator

While Φ_B repeats every period $T = 2\pi/\omega$, $\mathcal{E}$ repeats every $T/2 = \pi/\omega$

To find average $\mathcal{E}$, integrate from $t = 0$ to $t = \pi/\omega$

$$\mathcal{E}_{av} = \frac{\int_0^{\pi/\omega} \mathcal{E}\, dt}{\pi/\omega} = \omega BA \frac{\int_0^{\pi/\omega} \sin \omega t\, dt}{\pi/\omega} = \frac{2\omega BA}{\pi}$$

- ⚠ Back emf oppose the applied emf, making the current smaller
- ⚠ At the moment the motor is started, ω small and so is back emf. Current is large until the motor picks up speed. This is the reason why when something like a refrigerator or an air conditioner (anything with a motor) first turns on in your house, the lights often dim momentarily.

The Slideware Generator

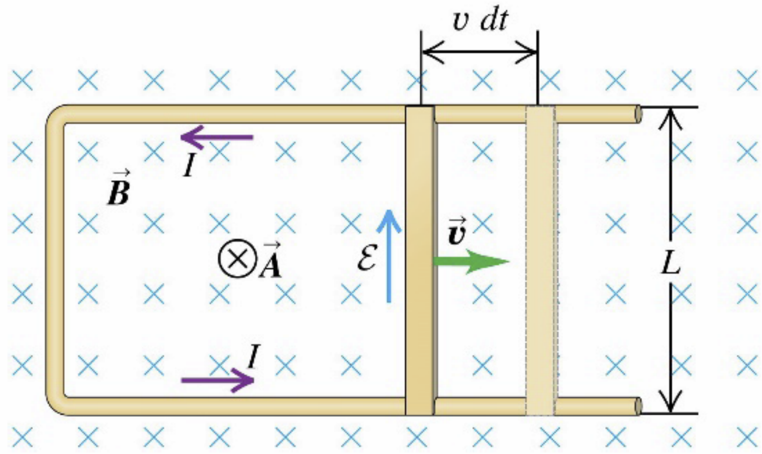
The straight slidewire driven by an external force to move with velocity $\vec{\boldsymbol{v}}$

In time interval dt, area increase by $dA = L(vdt)$

$$\Phi_B = BA \ \Rightarrow \frac{d\Phi_B}{dt} = B\frac{dA}{dt} = BLv$$

Magnitude of induced emf is $\mathcal{E} = BLv$

Lenz's law: induced current produces a B field to oppose the increasing flux, ∴ counter-clockwise

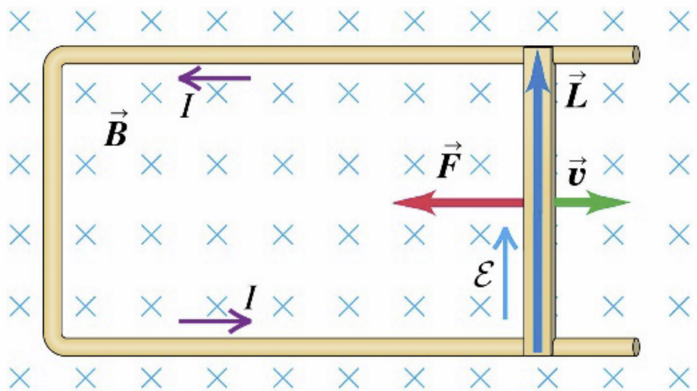


⚠ This generates a DC, but not a practical DC motor because the slidewire cannot move in one direction indefinitely

⚠ Induced current in this direction causes a magnetic force $\vec{\boldsymbol{F}} = I\vec{\boldsymbol{L}} \times \vec{\boldsymbol{B}}$ on the slidewire, opposing the motion $\vec{\boldsymbol{v}}$. Must provide and external force $-\vec{\boldsymbol{F}}$ to keep it going. Power supplied by external force

$$-\vec{\boldsymbol{F}} \cdot \vec{\boldsymbol{v}} = ILBv = I\mathcal{E}$$

Same as power dissipated by induced current

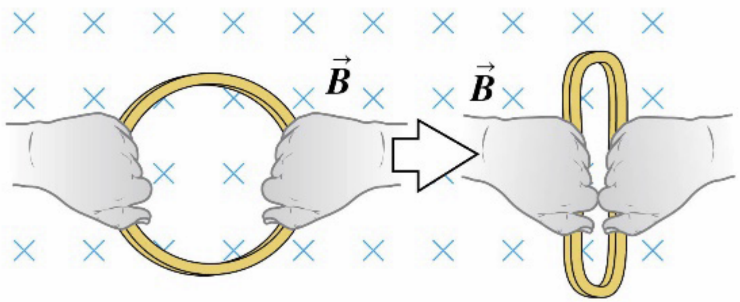


Question: what if Lenz's law doesn't hold, i.e., I in the slidewire is in the opposite direction?

Conclusion: **Lenz's law is consistent with conservation of energy**

Question:

A coil is being squeezed in a uniform constant magnetic field perpendicular to it.
While the coil is being squeezed, its induced emf is (clockwise / counterclockwise / zero)
Once the coil has reached its final squeezed shape, its induced emf is (clockwise / counterclockwise / zero)



Circular wire coil

Coil squeezed into oval

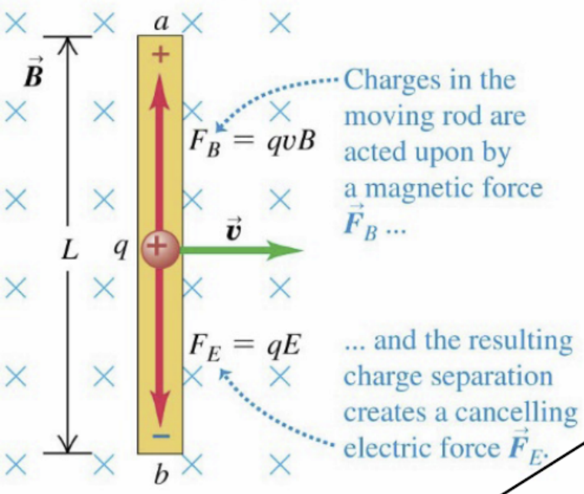
How to understand the Faraday's law, $\mathcal{E} = -d\Phi_B/dt$, i.e., where comes the emf?
Can be divided into two cases

1. A conductor is moving in a magnetic field
2. A conductor is not moving in a magnetic field

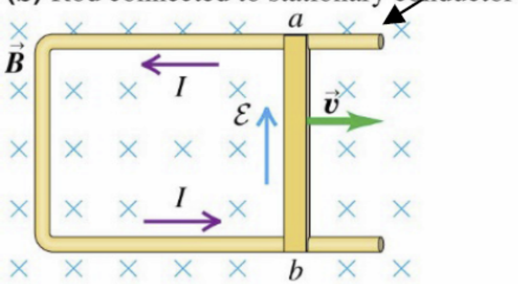
Case 1: **Motional Electromotive Force** where there is relative motion

A moving conductor acquires a potential difference due to magnetic force on charge carries (assume to be $q > 0$)

(a) Isolated moving rod



(b) Rod connected to stationary conductor



The motional emf $\mathcal{E}$ in the moving rod creates an electric field in the stationary conductor.

In equilibrium, charge under no net force

$$qE = qvB \quad \Rightarrow \quad E = vB$$

Creates potential difference

$$\mathcal{E} = V_{ab} = V_a - V_b = EL = vBL > 0$$

Rod acts like a battery, produces an emf $\mathcal{E} = vBL$

⚠ Same as result by Faraday's law obtained previously, and direction same as Lenz's law when the rod drives a current in a loop

In vector notation,

$$q\vec{E} + q\vec{v}\times\vec{B} = 0 \quad \rightarrow. \quad \vec{E} = -\vec{v}\times\vec{B}$$

$$\mathcal{E} = V_{ab} = -\vec{E}\cdot\vec{L} = \left(\vec{v}\times\vec{B}\right)\cdot\vec{L}$$

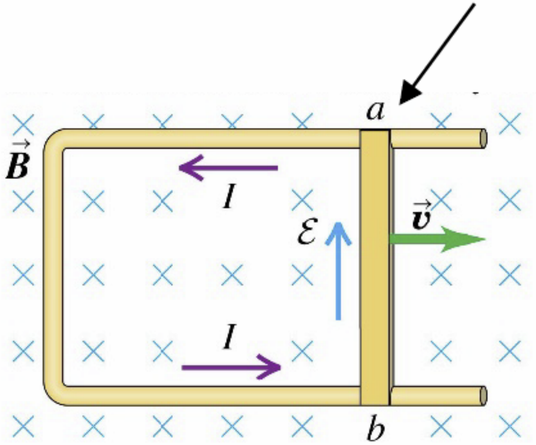
More general case of a loop moving in a magnetic field:

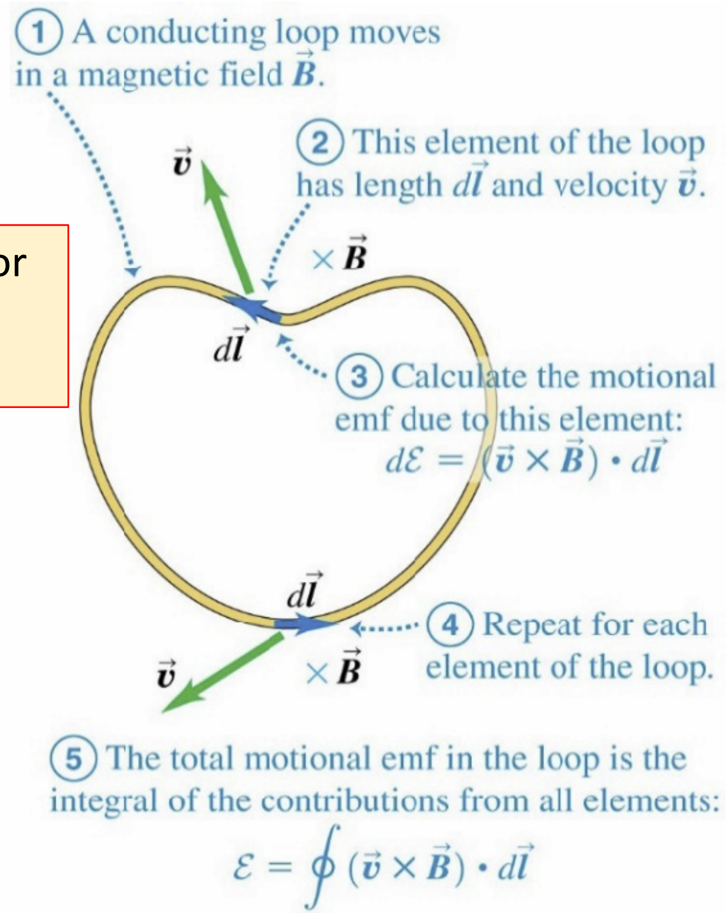
Magnetic force on $d\vec{l}$ is $Q(\vec{v} \times \vec{B})$, ∴ induced emf on $d\vec{l}$ is $d\mathcal{E} = (\vec{v} \times \vec{B}) \cdot d\vec{l}$

$$\Rightarrow \quad \mathcal{E} = \oint (\vec{v} \times \vec{B}) \cdot d\vec{l}$$

$\mathcal{E} = vBL$ for a straight conductor

e.g. in this loop, only the branch ab is moving





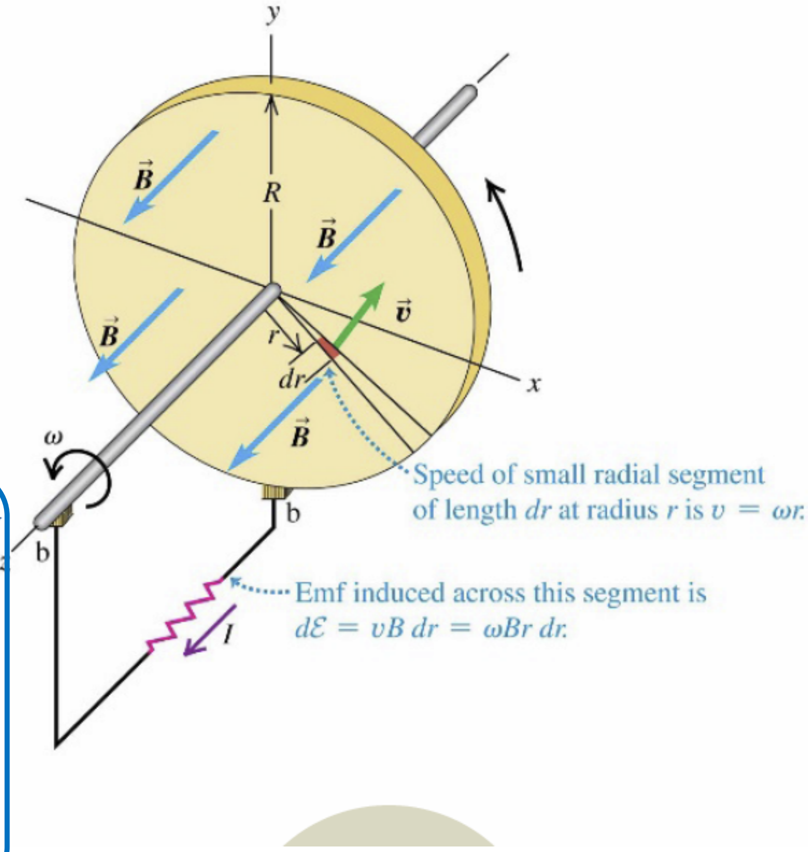
⚠ Although not obvious, $\mathcal{E} = \oint (\vec{v} \times \vec{B}) \cdot d\vec{l}$ is only a special case of the more general Faraday's law, $\mathcal{E} = -d\Phi_B/dt$. It adds no new content, but may be more convenient in applying to a conductor that moves in a magnetic field

The Faraday disk dynamo (Example 29.10)

Only the radial line in the complete circuit is moving.
Emf induced

$$\mathcal{E} = \oint (\vec{\boldsymbol{v}} \times \vec{\boldsymbol{B}}) \cdot d\vec{\boldsymbol{l}} = \int_0^R (\omega r) B dr = \frac{1}{2} \omega B R^2$$

Direction of $\mathcal{E}$ is obvious by considering the motion of charges in the magnetic field



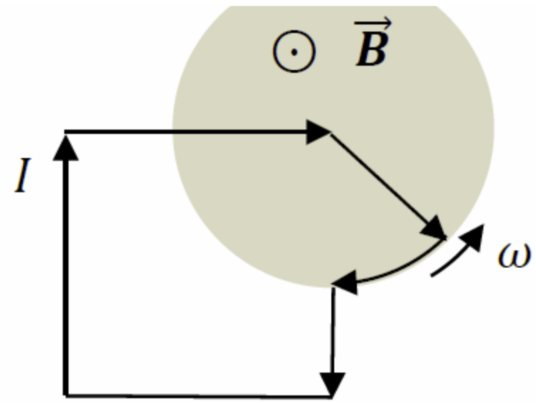
Alternatively, can get the same result from Faraday's law as follows:

Area sweep out by a radius in time dt is $dA = \frac{1}{2} R^2 d\theta$

$$\mathcal{E} = \frac{d\Phi_B}{dt} = B\frac{dA}{dt} = \frac{1}{2} B R^2 \frac{d\theta}{dt} = \frac{1}{2} \omega B R^2$$

Direction: loop increasing in area, flux increasing, induced current produces a magnetic field opposing $\vec{B}$, therefore radially outward

⚠ This Lenz's law argument in this case is greatly simplified and is not totally correct, but the conclusion is.



Case 2: **Induced Electric Field** when there is no relative motion

Magnetic field uniform inside solenoid

$$B = \mu_0 n I$$

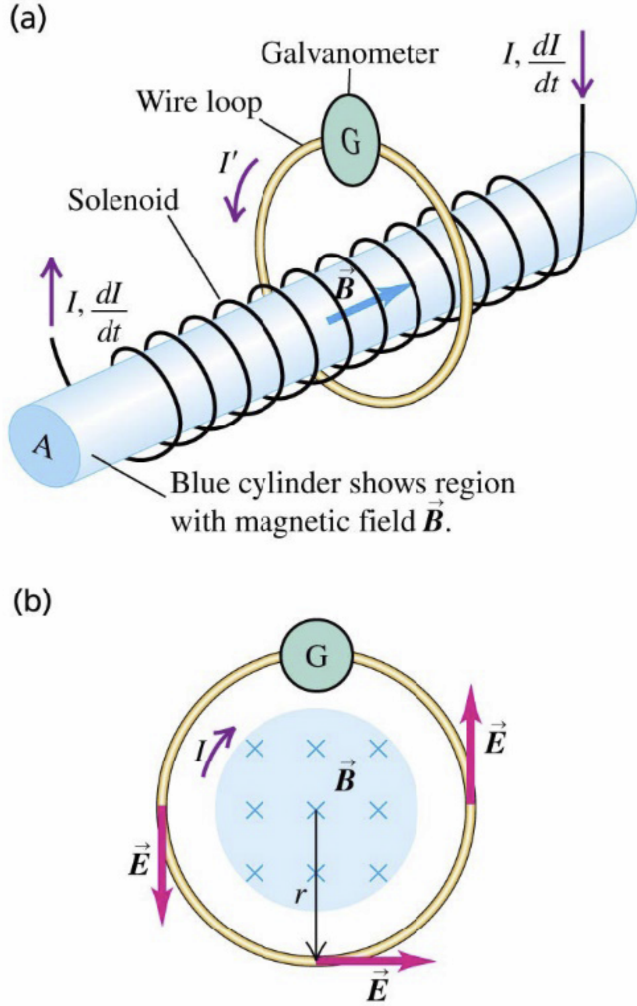
Magnitude of emf induced in the ring

$$\mathcal{E} = \frac{d\Phi_B}{dt} = \mu_0 n A \frac{dI}{dt}$$

Direction: assume I increasing, induced emf produce an effect to oppose the increasing magnetic field, i.e., $\mathcal{E}$ in the outer ring is counterclockwise

Suppose solenoid has 500 turns per meter and cross sectional area $A = 4.0\ \mathrm{cm}^2$. The current is increasing at $100\ \mathrm{A/s}$.

$$\mathcal{E} = (4\pi \times 10^{-7}\ \mathrm{Wb/Am})(500\ /\mathrm{m})(4.0 \times 10^{-4}\ \mathrm{m}^2)(100\ \mathrm{A/s})$$
$$= 25 \times 10^{-6}\ \mathrm{V}$$



☹ How can we make sense out of this? What force is driving charge carriers around the loop? Not magnetic because no magnetic field anywhere along the ring. How would charge carries of the ring even know that the magnetic field is changing?

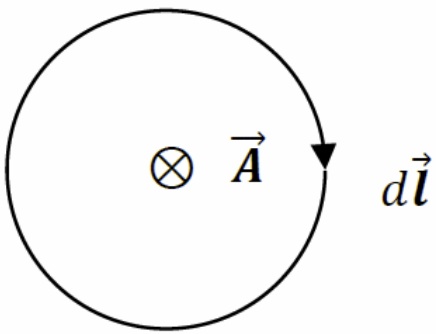
Conclusion: we are *forced* to believe that a changing flux produces an **induced electric field** that drives charge carriers to go round the ring.

Work done by this induced electric field in driving a charge around one loop

$$W = q\oint \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{l}} = q\mathcal{E}$$

Therefore an alternative form of Faraday's law is

$$\boxed{\oint \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{l}} = -\frac{d\Phi_B}{dt}} \qquad \text{for a stationary loop}$$



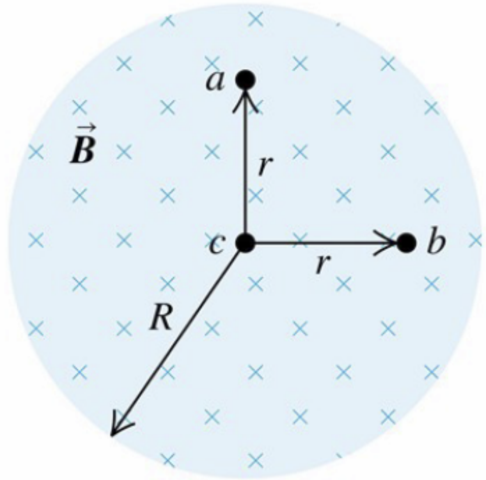
How does the sign work

1. $d\vec{\boldsymbol{l}}$ on the left and $\vec{\boldsymbol{A}}$ in Φ_B on the right *must* be consistent (through the right-hand rule)
2. For simplicity (although not compulsory) choose $\vec{\boldsymbol{A}}$ along $\vec{\boldsymbol{B}}$ to make $\Phi_B > 0$

Clicker Questions

Q29.10

The drawing shows the uniform magnetic field inside a long, straight solenoid. The field is directed into the plane of the drawing and is increasing. What is the direction of the *electric* force on a positive point charge placed at point *b*?



A. to the left

B. to the right

C. straight up

D. straight down

E. Misleading question—the electric force at this point is zero.

A29.10

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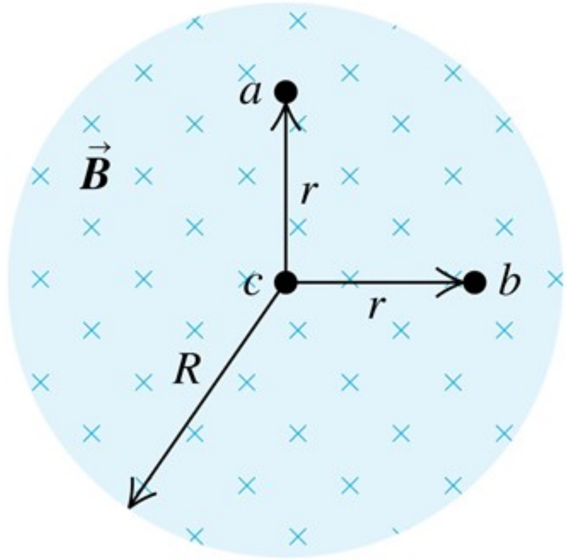


A. to the left

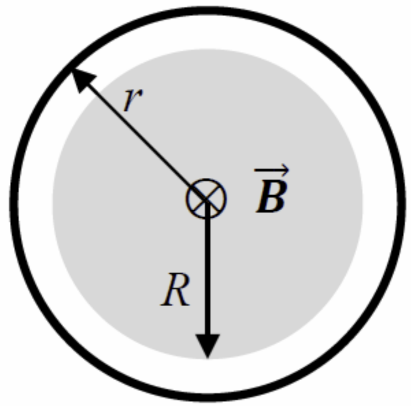
B. to the right

C. straight up

D. straight down

E. Misleading question—the electric force at this point is zero.

⚠ Induced electric field spreads over space and is there even when there is no conductor



Consider an imaginary (not conducting) ring with radius r

Outside solenoid, $r > R$

$\vec{E}$ cannot have radial component (∵ Gaussian surface encloses no net charge), ∴ $\vec{E}$ must be tangential

$$\oint \vec{E} \cdot d\vec{l} = E(2\pi r) = -(\pi R^2)\frac{dB}{dt}$$

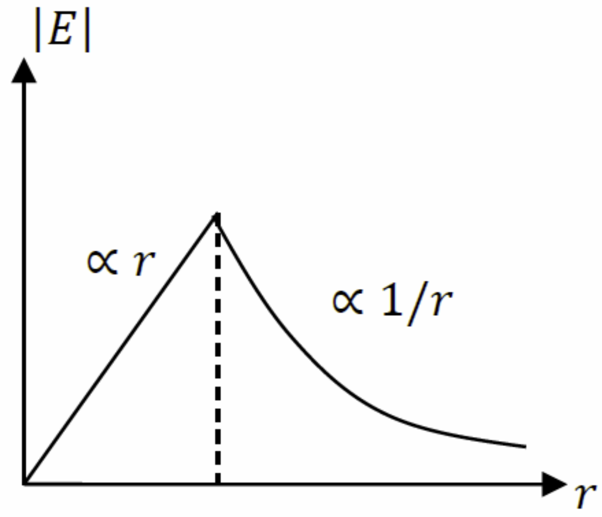
$$\Rightarrow \quad E = -\left(\frac{R^2}{2r}\frac{dB}{dt}\right)$$

-ve sign means $\vec{E}$ is (clockwise / counterclockwise)

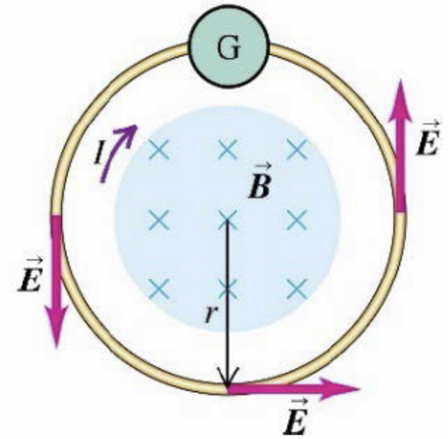
Inside solenoid, $r < R$

$$\oint \vec{E} \cdot d\vec{l} = E(2\pi r) = -(\pi r^2)\frac{dB}{dt}$$

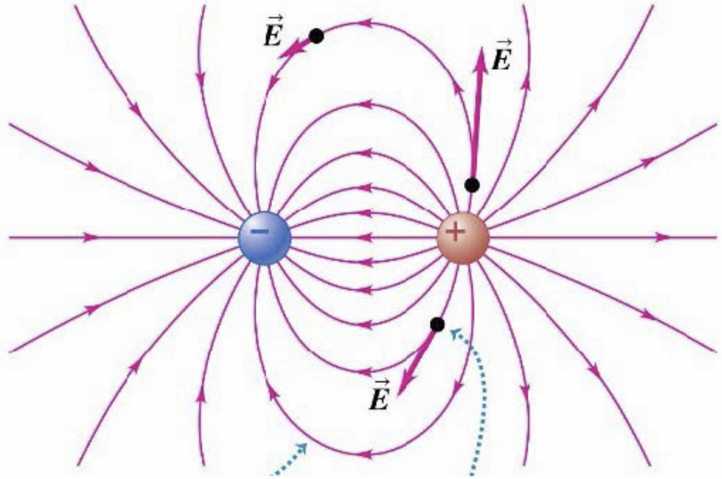
$$\Rightarrow \quad E = -\left(\frac{r}{2}\frac{dB}{dt}\right)$$



⚠ Induced electric field is **non-electrostatic,** field lines have no beginning or end



Induced, $\vec{E}$ goes around a loop, no beginning or end



Electrostatic, $\vec{E}$ starts at +ve charge and end at –ve charge

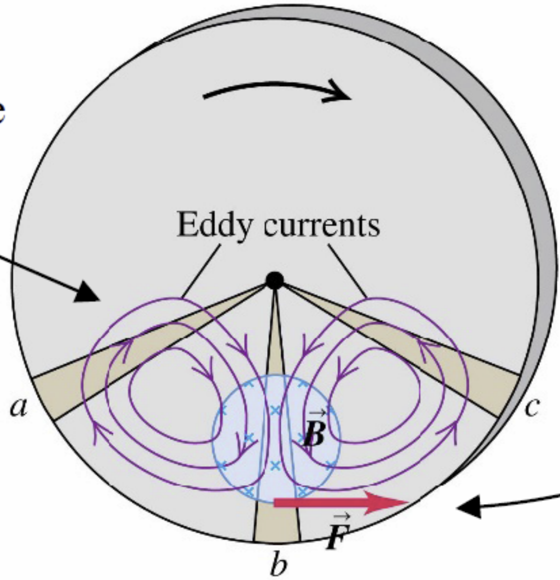
Around a loop, $\oint \vec{E} \cdot d\vec{l} = \mathcal{E} \neq 0$, therefore

1. **Induced electric field is not conservative**
2. **Electric potential is not defined for induced electric field**
3. **The conducting ring above is not an equipotential surface**

Eddy Currents – induced current in a bulk conductor and is not confined to a single path
(An eddy is a current in a fluid (e.g. a river) moving contrary to the direction of the main current, especially in a circular motion.)
Two possible ways (both according to Lenz's law) to determine eddy current direction

1. current loop *leaving* magnetic field, must save the decreasing Φ_B. Likewise on the other side which is *entering* magnetic field

(b) Resulting eddy currents and braking force

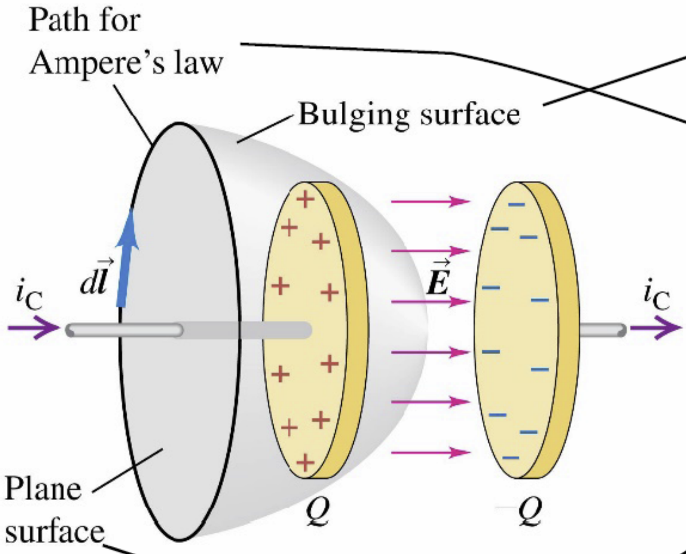


2. Strip Ob passes the magnetic field, induce current produces a force $\vec{F} = I\vec{L} \times \vec{B}$ to oppose the motion, i.e., I downwards in strip

For other examples and applications of eddy currents, see your textbook or your HKDSE physics textbook.

Displacement Current

Charging of a capacitor contradicts the Ampere's law



enclose no current

enclose a real current i_C

$$\oint \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{l}} = \mu_0 i_C \text{ but } \neq 0$$

A contradiction ☹

Charge in capacitor is

$$q = CV = \left(\frac{\epsilon_0 A}{d}\right)(Ed) = \epsilon_0 \Phi_E$$

$$i_C = \frac{dq}{dt} = \epsilon_0 \frac{d\Phi_E}{dt}$$

⚠ Recall the Ampere's law $\oint \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{l}} = \mu_0 I_{\text{encl}}$ holds for continuous current only, but not here

⚠ Fix it by "continuing" the real current i_C into the region inside the capacitor, become a fictitious current between the plates called **displacement current** i_D

$$i_D(\text{between plates}) = i_C(\text{outside}) = \epsilon_0 \frac{d\Phi_E}{dt}$$

Generalized Ampere's law
Sometimes called Ampere-Maxwell law

$$\boxed{\oint \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{l}} = \mu_0 (i_C + i_D)_{\text{encl}}}$$

nonzero in outside circuit only

nonzero between plates only

⚠ Again sign convention of $d\vec{\boldsymbol{l}}$ on the left and i_C and $\vec{\boldsymbol{A}}$ in Φ_E on the right must be consistent

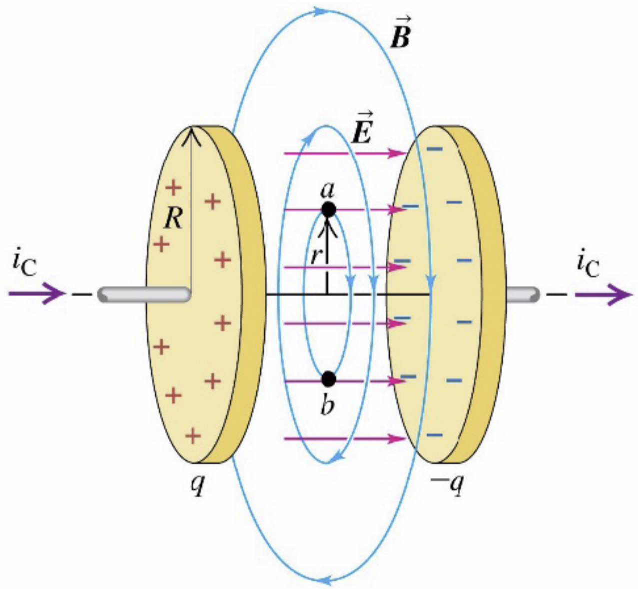
As oppose to a changing Φ_B induces an electric field, a **changing Φ_E induces a magnetic field** between the plates

$$\oint \vec{B} \cdot d\vec{l} = 2\pi r B = \mu_0 \left(\frac{\pi r^2}{\pi R^2}\right) i_C$$

$$\Rightarrow \quad B = \left(\frac{\mu_0}{2\pi}\frac{i_C}{R^2}\right) r$$

Likewise, for $r > R$,

$$B = \left(\frac{\mu_0 i_C}{2\pi}\right)\frac{1}{r}$$



The real current i_C is *"converted to"* the displacement current and distributed uniformly between two conducting plates.

Maxwell's Equation of Electromagnetism

A unified description of electricity and magnetism

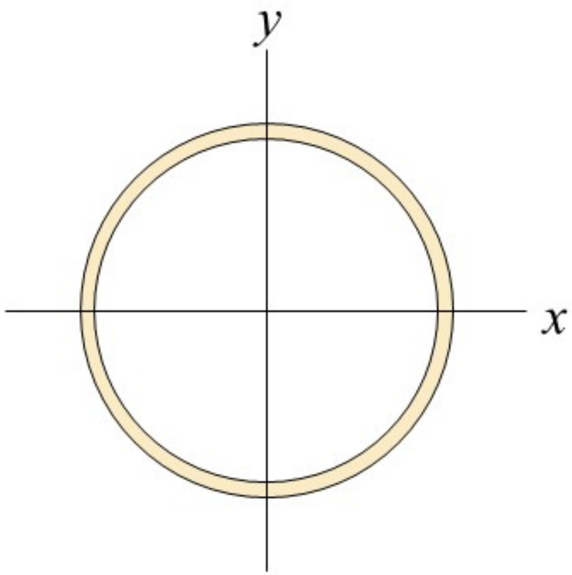
Gauss's law for $\vec{\boldsymbol{E}}$	$\oint \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{A}} = Q_{\text{encl}}/\epsilon_0$	Charge produces electrostatic electric field
Gauss's law for $\vec{\boldsymbol{B}}$	$\oint \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{A}} = 0$	No magnetic monopole
Faraday's law (for a stationary path)	$\oint \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{l}} = -\dfrac{d\Phi_B}{dt}$	Varying Φ_B produces non-electrostatic electric field
Ampere's law (for a stationary path)	$\oint \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{l}} = \mu_0 \left(i_C + \epsilon_0 \dfrac{d\Phi_E}{dt}\right)_{\text{encl}}$	Varying Φ_E produces magnetic field

Together with the Lorentz force $\vec{\boldsymbol{F}} = q(\vec{\boldsymbol{E}} + \vec{\boldsymbol{v}} \times \vec{\boldsymbol{B}})$, these are the fundamental relationships in classical electromagnetism

⚠ First example of unification of physical laws – theories for electricity and magnetism are unified into a single theory called electromagnetism. Possible to unify with others, strong, weak, and gravitational interactions? Grand Unified Theory (GUT)? A bit far off for this course? ☺

Q-RT29.1

A circular loop of wire lies in the xy-plane. The loop is exposed to a uniform, time-varying magnetic field. ***Rank*** the following uniform magnetic fields in order of the magnitude of emf that they produce in the loop at $t = 0$, from largest to smallest value.



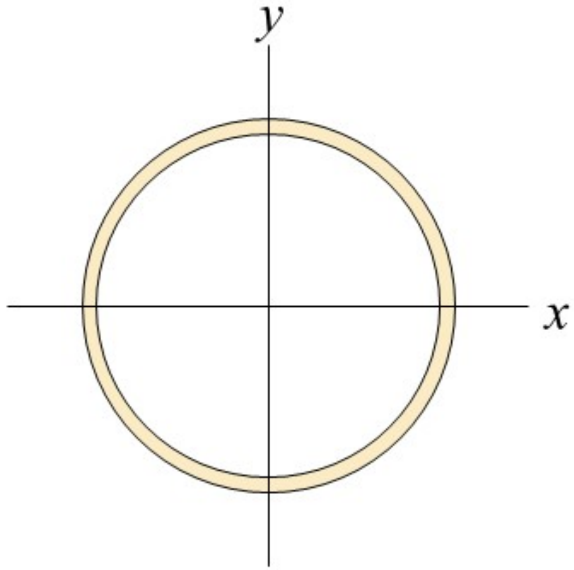
A. $\vec{B} = (3.00\ \mu\text{T})\cos\left[(4.00\ \text{rad/s})t\right]\hat{k}$

B. $\vec{B} = (1.00\ \mu\text{T})\left[(3.00\ \text{s}^{-2})t^2 - (5.00\ \text{s}^{-1})t\right]\hat{k}$

C. $\vec{B} = (1.50\ \mu\text{T})e^{-\alpha t}\hat{k}$, where $\alpha = 2.00\ \text{s}^{-1}$

A-RT29.1

A circular loop of wire lies in the xy-plane. The loop is exposed to a uniform, time-varying magnetic field. ***Rank*** the following uniform magnetic fields in order of the magnitude of emf that they produce in the loop at $t = 0$, from largest to smallest value.



A. $\vec{\boldsymbol{B}} = (3.00\ \mu\text{T})\cos\left[(4.00\ \text{rad/s})t\right]\hat{\boldsymbol{k}}$

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C. $\vec{\boldsymbol{B}} = (1.50\ \mu\text{T})e^{-\alpha t}\hat{\boldsymbol{k}}$, where $\alpha = 2.00\ \text{s}^{-1}$

✔ Answer: BCA