Inductance

Mutual Inductance

- **If the current in coil 1 changes, the flux through coil 2 changes.**
- **According to Faraday's law, this induces an emf in coil 2.**

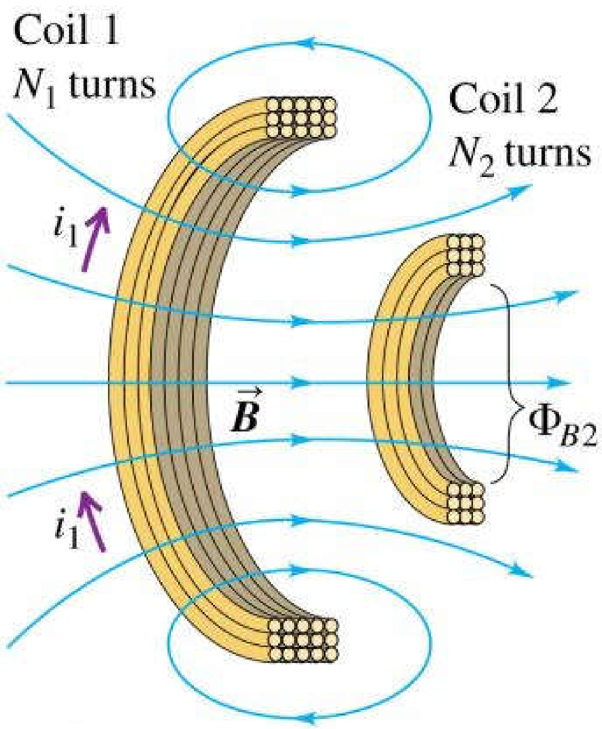
$$B \propto i_1 \Rightarrow \Phi_{B2} \propto i_1$$

$$\text{or} \quad \Phi_{B2} = (\text{constant})i_1$$

$$\text{or} \quad \Phi_{B2} = M_{21}i_1.$$

Mutual inductance

Mutual inductance: If the current in coil 1 is changing, the changing flux through coil 2 induces an emf in coil 2.



Mutual Inductance

$$\Phi_{B2} = M_{21} i_1$$

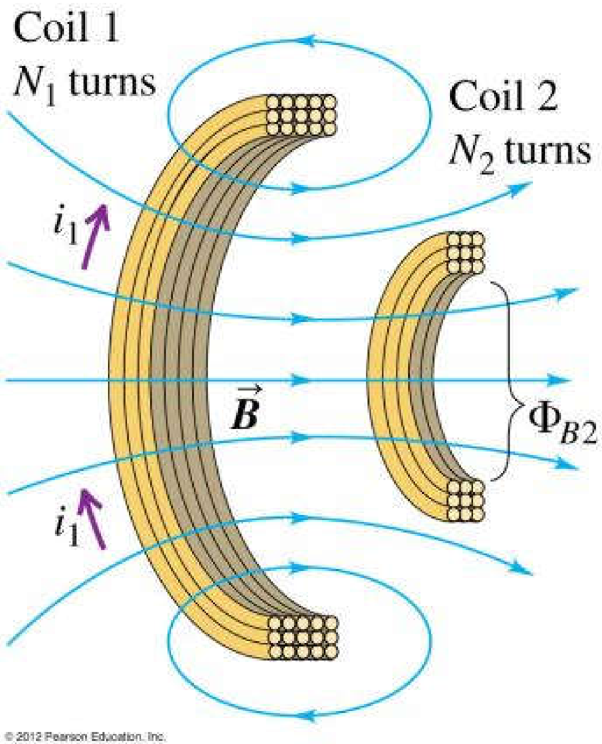
$$\Rightarrow \frac{d\Phi_{B2}}{dt} = M_{21} \frac{di_1}{dt}.$$

$$\Rightarrow \mathcal{E}_2 = -\frac{d\Phi_{B2}}{dt}$$

$$= -M_{21} \frac{di_1}{dt}.$$

Mutual inductance M_{21} depends only on geometry of coils (size, shape, number of turns, orientation, and separation).

Mutual inductance: If the current in coil 1 is changing, the changing flux through coil 2 induces an emf in coil 2.



Mutual Inductance

- **Opposite case: a changing current in coil 2 causes a changing flux and an emf in coil 1.**

$$\mathcal{E}_2 = -M_{12}\frac{di_2}{dt}$$

- **It can be shown using some integral tricks that *M*$_{12}$ is always equal to *M*$_{21}$.**
- **Call this common value simply the mutual inductance *M*.**

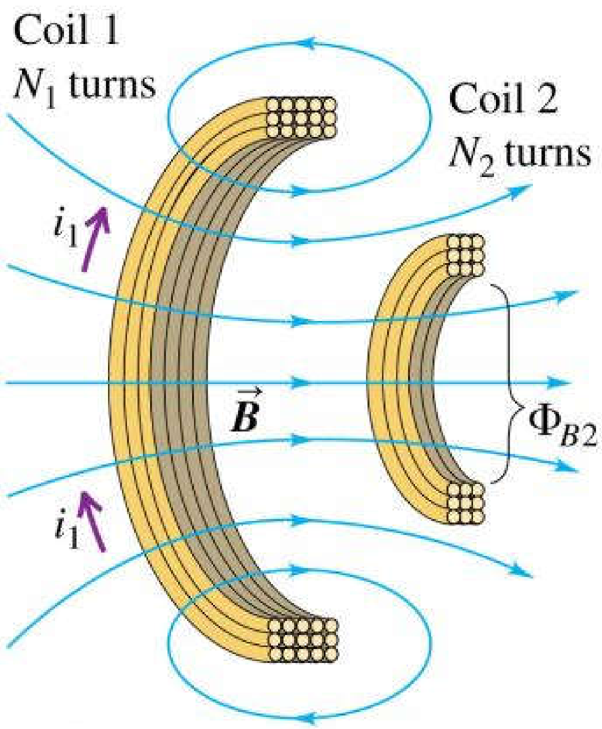
$$\mathcal{E}_2 = -M\frac{di_1}{dt},\ \mathcal{E}_1 = -M\frac{di_2}{dt}.$$

$$\text{where } M = \frac{\Phi_{B2}}{i_1} = \frac{\Phi_{B1}}{i_2}.$$

$$1\ \text{H} = 1\ \text{Wb/A} = 1\ \text{V}\cdot\text{s/A}$$

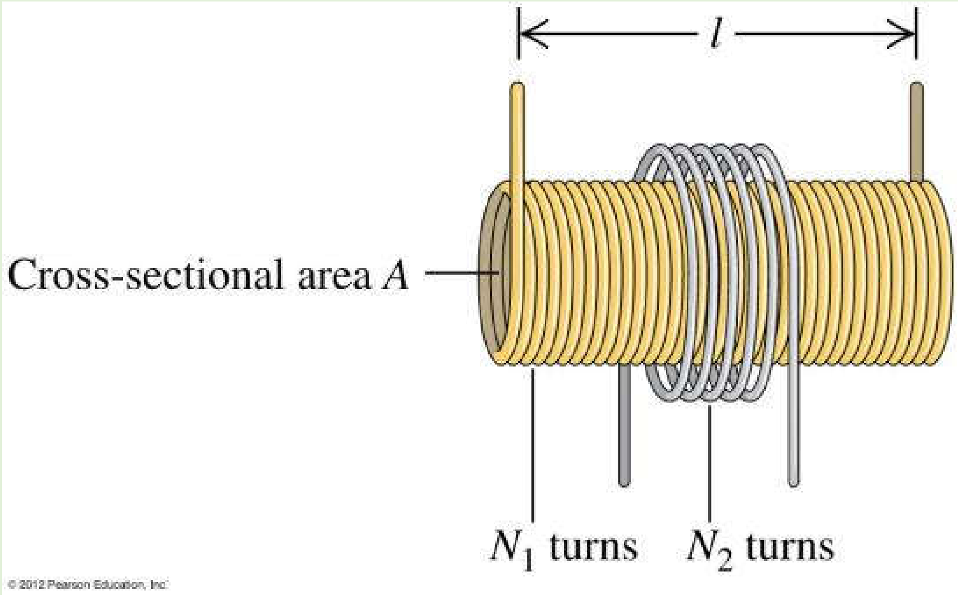
$$= 1\ \Omega\cdot\text{s} = 1\ \text{J/A}^2.$$

Mutual inductance: If the current in coil 1 is changing, the changing flux through coil 2 induces an emf in coil 2.



Example 30.1
Calculating mutual inductance

- **A long solenoid with length *L* and cross-sectional area *A* is closely wound with N_1 turns of wire. A coil with N_2 turns surrounds it at its center. Find the mutual inductance *M*.**



Example 30.1
Calculating mutual inductance

$$B_1 = \mu_0 n_1 i_1 = \frac{\mu_0 N_1 i_1}{l}.$$

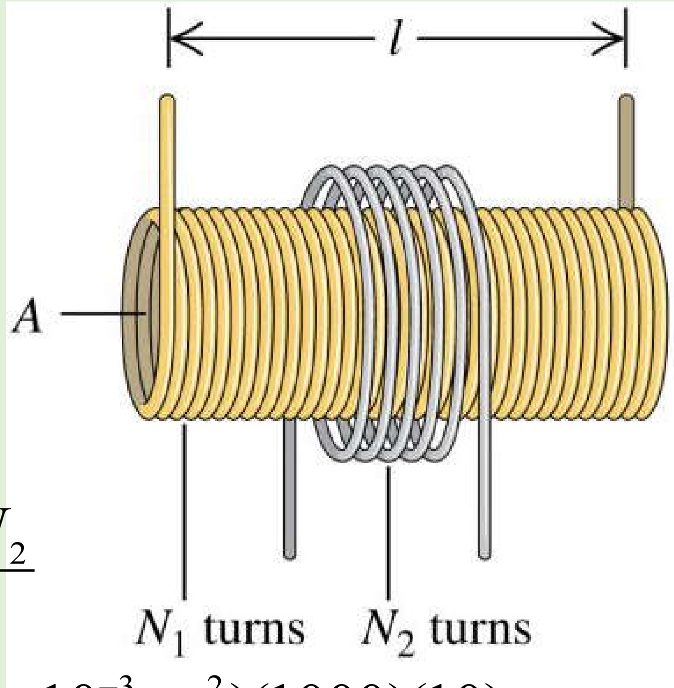
$$\Phi_{B2} = B_1 A.$$

$$M = \frac{N_2 \Phi_{B2}}{i_1} = \frac{N_2 (B_1 A)}{i_1}$$

$$= \frac{N_2 [(\frac{\mu_0 N_1 i_1}{l}) A]}{i_1} = \frac{\mu_0 A N_1 N_2}{l}$$

$$= \frac{(4\pi \times 10^{-7}\ \text{Wb/A} \cdot \text{m})(1.0 \times 10^{-3}\ \text{m}^2)(1000)(10)}{0.50\ \text{m}}$$

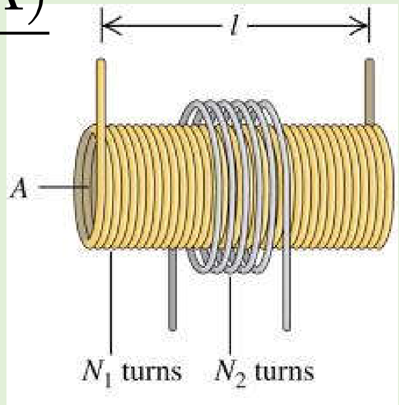
$$= 25 \times 10^{-6}\ \text{Wb/A} = 25 \times 10^{-6}\ \text{H} = 25\ \mu\text{H}.$$



Example 30.2
Emf due to mutual inductance

- **Suppose i_2 = (2.0 x 10^6 A/s)t. (a) At t = 3.0 μs, what is the average magnetic flux through each turn of the solenoid (coil 1) due to the current in the outer coil? (b) What is the induced emf in the solenoid?**

- (a) $$\Phi_{B1} = \frac{Mi_2}{N_1} = \frac{(25\times10^{-6}\ \mathrm{H})(6.0\ \mathrm{A})}{1000} = 1.5\times10^{-7}\ \mathrm{Wb}.$$



(b) $$\mathcal{E}_1 = -M\frac{di_2}{dt} = -(25\times10^{-6}\ \mathrm{H})(2.0\times10^{6}\ \mathrm{A/s}) = -50\ \mathrm{V}.$$

Self-induction and Inductors

Typically: a solenoid with N turns

Varying current → varying B field → induced emf

This is called **self-induction**

Assuming vacuum, or in a magnetic material with constant $\boldsymbol{K_m}$ and $\overrightarrow{\boldsymbol{M}} \propto \overrightarrow{\boldsymbol{B}}$, then total flux through the coil

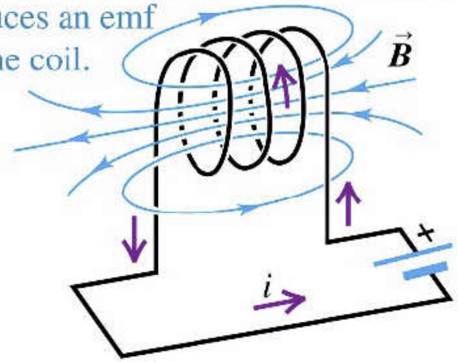
$$N\Phi_B \propto i$$

Define the **inductance** of the coil

$$\boxed{L = \frac{N\Phi_B}{i}}$$

L depends on the geometry of the coil

Self-inductance: If the current i in the coil is changing, the changing flux through the coil induces an emf in the coil.



SI unit: $\mathrm{Tm}^2/\mathrm{A} \equiv$ H (henry) ⚠ H is a large unit, typically mH or μH

From Faraday's law

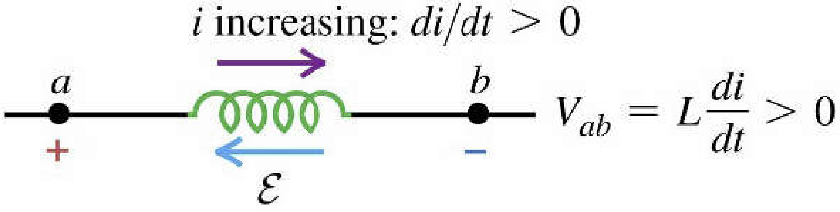
$$\boxed{\mathcal{E} = -N\underbrace{\frac{d\Phi_B}{dt}}_{\text{emf induced in one loop}} = -L\frac{di}{dt}} \qquad \text{self-induced emf}$$

N loops

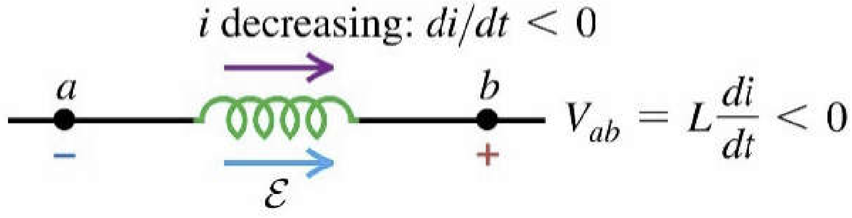
Inductors As Circuit Elements

Direction of $\mathcal{E}$ determined by Lenz's law

(c) Inductor with *increasing* current i flowing from a to b: potential drops from a to b.



(d) Inductor with *decreasing* current i flowing from a to b: potential increases from a to b.



Can be considered as a circuit element in, e.g., Kirchhoff's law where i from $a \rightarrow b$,

$$\boxed{V_{ab} = V_a - V_b = L\frac{di}{dt}}$$

⚠ no need to worry whether i is increasing/decreasing. The sign of di/dt will take care of the direction of $\mathcal{E}$

Magnetic Field Energy

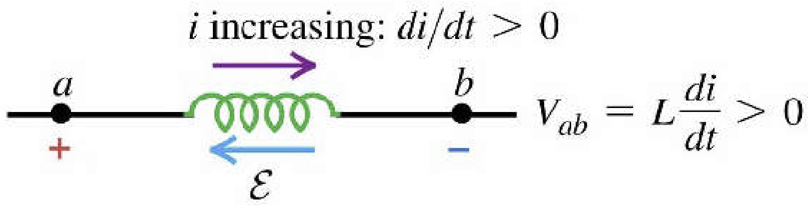
As i increases, induced emf resists it, therefore external source must supply energy. Can consider the energy used in building up the magnetic field, or magnetic flux, *c.f.* charging of a capacitor

Assume inductor has no resistance, all energy stored in the magnetic field inside the inductor

External power delivered to inductor

$$P = \frac{dU}{dt} = V_{ab} i = Li\frac{di}{dt}$$

$$\Rightarrow \quad U = L\int_0^I i\,di \quad \Rightarrow \quad \boxed{U = \frac{1}{2}LI^2}$$

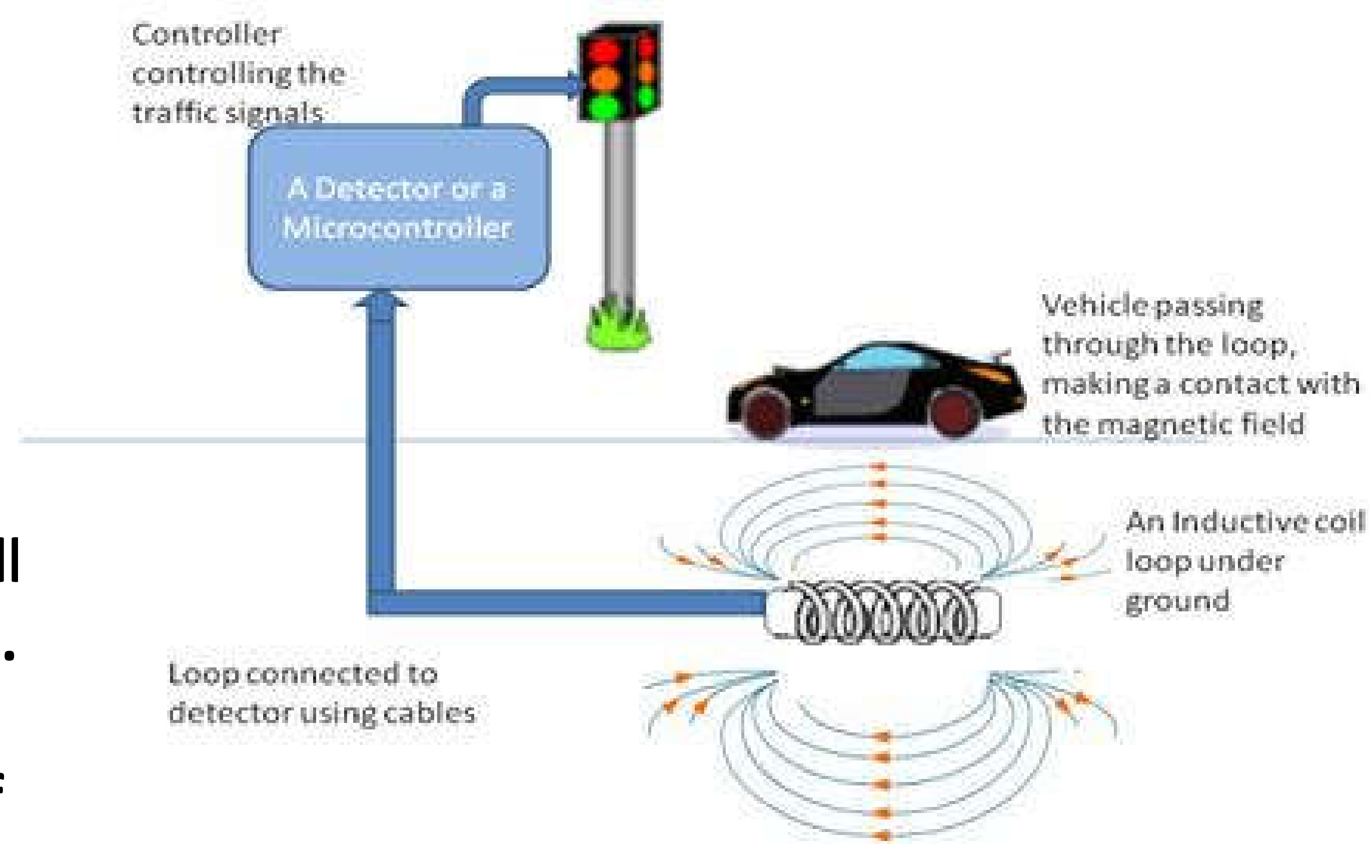


I is the final steady current

⚠ *c.f* energy stored in a capacitor is $\frac{1}{2}CV^2$

⚠ for a comparison between inductor and capacitor, see Summary I attached.

Inductor - application



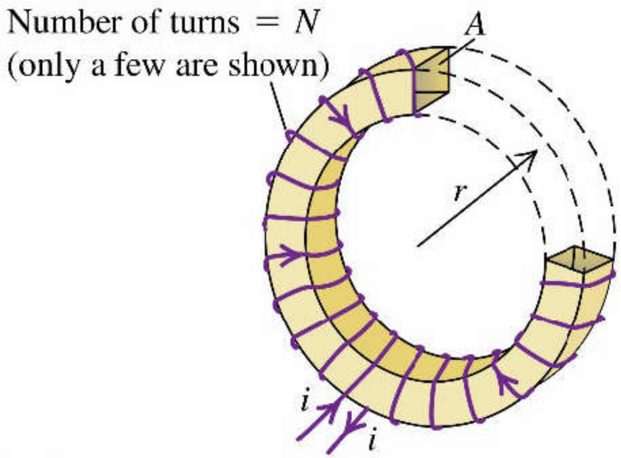
Many traffic lights change when a car roll up to the intersection. How does the light sense the presence of the car?

Magnetic Energy Density

Suppose the inductor is a thin toroidal solenoid whose thickness $\ll r$, cross sectional area A, and has N turns

Magnetic field inside is uniform

$$B = \frac{\mu_0 N i}{2\pi r}$$



Its inductance is

$$L = \frac{N\Phi_B}{i} = \frac{\mu_0 N^2 A}{2\pi r}$$

Total energy stored in inductor after the current settles to a constant value I is

$$U = \frac{1}{2}LI^2 = \frac{1}{2}\frac{\mu_0 N^2 A}{2\pi r}I^2$$

Magnetic energy density (total energy per unit volume) is

$$u = \frac{U}{2\pi r A} = \frac{1}{2}\mu_0\left(\frac{NI}{2\pi r}\right)^2 \quad \Longrightarrow \quad \boxed{u = \frac{B^2}{2\mu_0}}$$

- *c.f* electric energy density is $\frac{1}{2}\epsilon_0 E^2$
- For a magnetic material with constant permeability $\mu = K_m \mu_0$, $u = B^2/2\mu$
- Just like the electric energy density, this result not only true for an ideal solenoid. It is true in general for any magnetic field provided μ is constant

Example

A thin toroidal solenoid has $N = 200$ turns, cross sectional area $A = 5.0\ \text{cm}^2$, and $r = 0.10$ m

$$L = \frac{(4\pi \times 10^{-7}\ \text{Wb/Am})(200)^2(5.0 \times 10^{-4}\ \text{m}^2)}{2\pi(0.10\ \text{m})} = 40\ \mu\text{H}$$

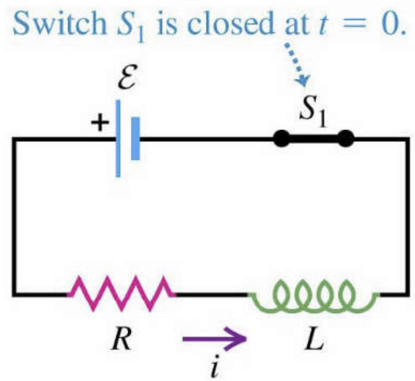
With a final current $I = 200$ A, the total magnetic energy stored is

$$U = \frac{1}{2}(40 \times 10^{-6}\ \text{H})(200\ \text{A})^2 = 0.8\ \text{J}$$

Not practical as an energy storage!

The *R-L* Circuit

Current Growth (building up flux inside inductor, *c.f.* charging in a *R-C* circuit)



Kirchhoff's loop rule:

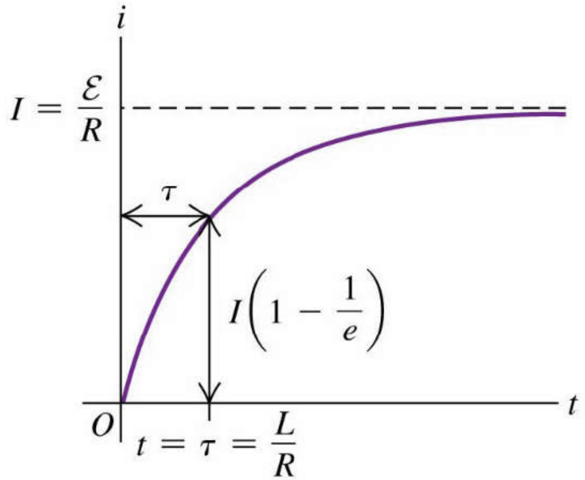
$$\mathcal{E} - iR - L\frac{di}{dt} = 0$$

For a summary of voltage change across a circuit element, see Summary II attached

Qualitatively:

1. At $t = 0$, $i = 0$, $di/dt = \mathcal{E}/L$ a maximum
2. As i grows, di/dt decreases, eventually to zero
3. Therefore i increases from 0 to $I = \mathcal{E}/R$ (max when no emf is induced in L, must be of the form

$$i(t) = I(1 - e^{-t/\tau})$$



Quantitatively:

$$\frac{di}{dt} = \frac{\mathcal{E}}{L} - \frac{R}{L}i$$

With initial condition $i(0) = 0$

$$i(t) = \frac{\mathcal{E}}{R}\left(1 - e^{-\frac{t}{L/R}}\right)$$

Define time constant $\tau \equiv L/R$

$$i(t) = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau}\right)$$

Power delivery and dissipation

Instantaneous power
(multiply i to Kirchhoff's rule)

$$i(t)\mathcal{E} - i^2(t)R - i(t)\left(L\frac{di}{dt}\right) = 0$$

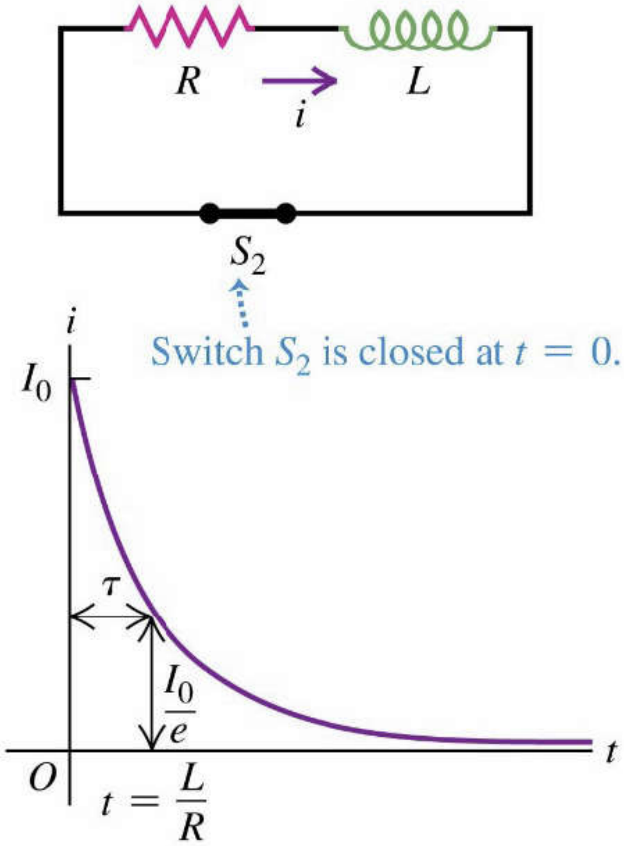
Delivered by $\mathcal{E}$

Dissipated in R

Stored in L, $= \frac{d}{dt}\left(\frac{1}{2}Li^2\right)$

Current Decay (draining flux inside inductor, *c.f.* discharging in a *R*-*C* circuit)

After the current reaches the constant value I_0 in the previus case, then remove $\mathcal{E}$



Qualitatively:

i must decrease from initial value I_0 to zero (when all energy stored in L is drained), therefore must be of the form

$$i(t) = I_0 e^{-t/\tau}$$

Quantitatively:

Put $\mathcal{E} = 0$ in loop rule

$$-iR - L\frac{di}{dt} = 0$$

$$\Rightarrow \quad i(t) = I_0 e^{-t/\tau}$$

Instantaneous power

$$i^2(t)R + i(t)\left(L\frac{di}{dt}\right) = 0$$

Dissipated in R

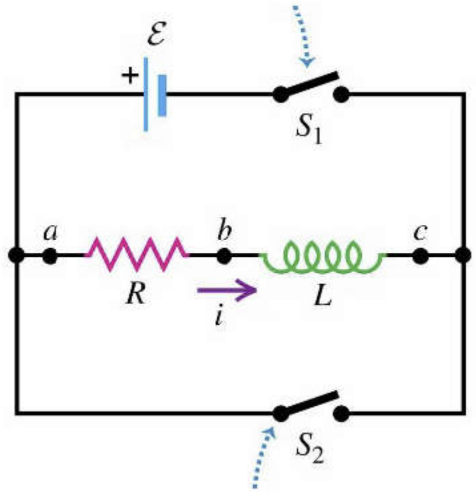
< 0, energy stored in L decreasing

Question

In the circuit, before the current settles to a constant value

a) when S_1 is closed and S_2 is open,
   V_{ab} (> / <) 0 and V_{bc} (> / <) 0

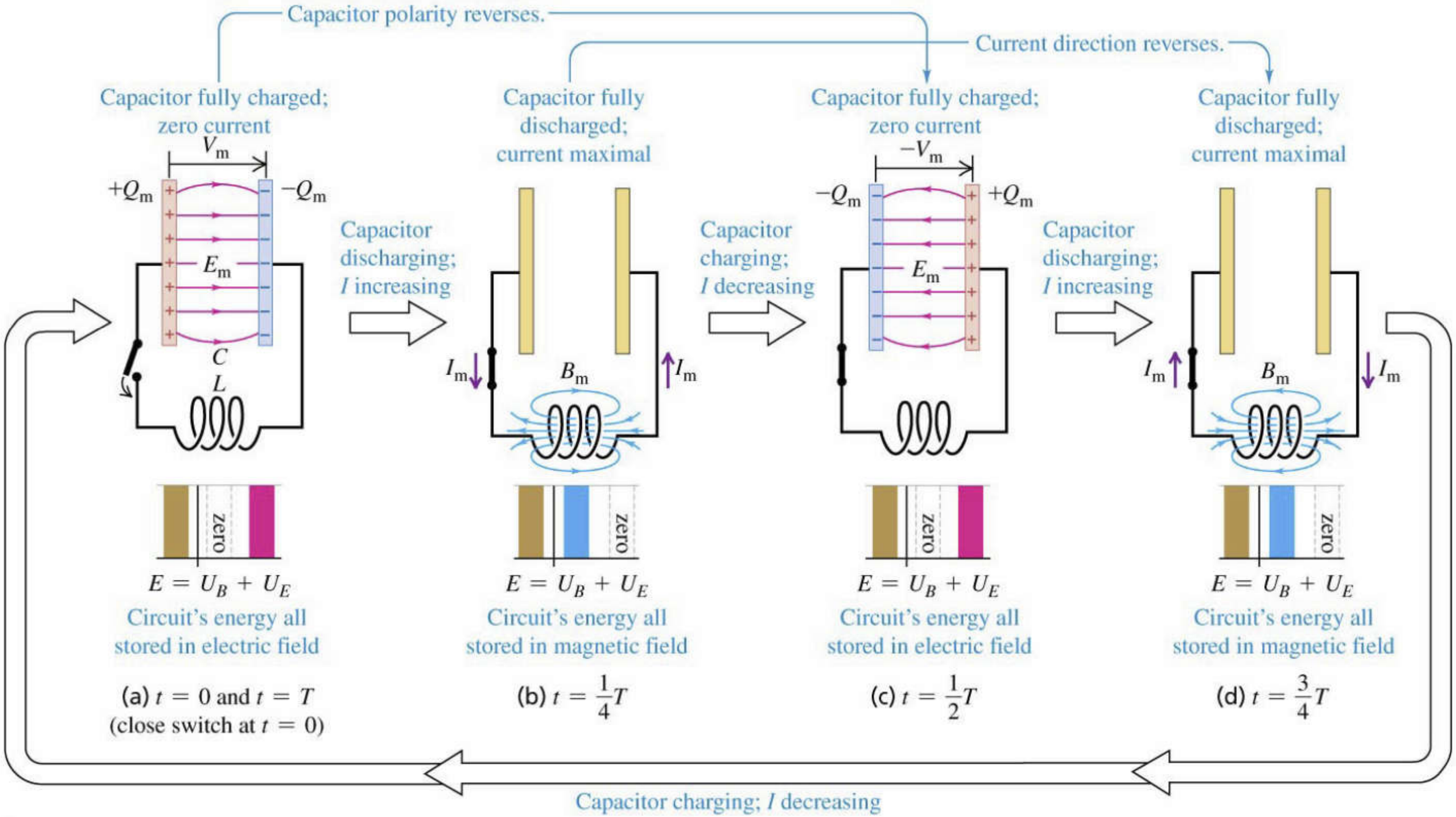
b) when S_1 is open and S_2 is closed, and current is flowing in the direction shown,
   V_{ab} (> / <) 0 and V_{bc} (> / <) 0

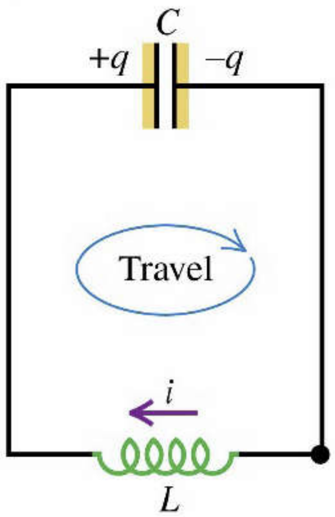


The *L-C* Circuit – analogy of a harmonic oscillator

An electrical oscillation, energy transfer between electric and magnetic energy

c.f. a mechanical oscillation (spring and mass), energy transfer between PE and KE



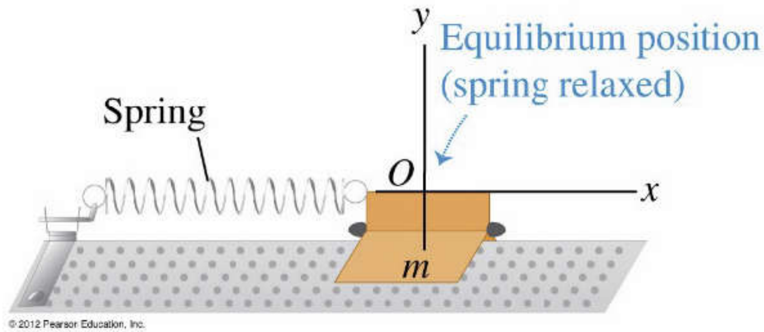


Quantitatively: from loop rule

$$-L\frac{di}{dt} - \frac{q}{C} = 0$$

Since $i = dq/dt$

$$\Rightarrow \frac{d^2q}{dt^2} + \frac{1}{LC}q = 0$$



c.f. in a mass and spring system with spring constant k

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

solution is $x = A\cos(\omega t + \phi)$, $\omega = \sqrt{k/m} = 2\pi f$

$$q = Q\cos(\omega t + \phi)$$

$$i = -\omega Q\sin(\omega t + \phi)$$

$$\omega = \sqrt{\frac{1}{LC}}$$

Amplitude Q and phase ϕ determined by initial conditions, e.g,

If $q(0) = Q$ and $i(0) = 0$, then $\phi = 0$

If $q(0) = 0$, then $\phi = \pm\pi/2$

Analogy between the mass-spring and inductor-capacitor system

Mass-Spring System	**Inductor-Capacitor Circuit**
Kinetic energy $= \frac{1}{2}mv_x^2$	Magnetic energy $= \frac{1}{2}Li^2$
Potential energy $= \frac{1}{2}kx^2$	Electrical energy $= q^2/2C$
$\frac{1}{2}mv_x^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$	$\frac{1}{2}Li^2 + q^2/2C = Q^2/2C$
$v_x = \pm\sqrt{k/m}\sqrt{A^2 - x^2}$	$i = \pm\sqrt{1/LC}\sqrt{Q^2 - q^2}$
$v_x = dx/dt$	$i = dq/dt$
$\omega = \sqrt{\frac{k}{m}}$	$\omega = \sqrt{\frac{1}{LC}}$
$x = A\cos(\omega t + \phi)$	$q = Q\cos(\omega t + \phi)$

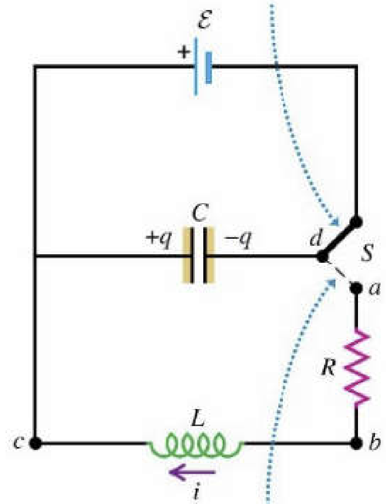
⚠ **total energy in a *L-C* circuit is conserved**, just like a harmonic oscillator

The *L-R-C* Circuit – analogy of a damped harmonic oscillator

$$-iR - L\frac{di}{dt} - \frac{q}{C} = 0$$

$$\Rightarrow \quad \frac{d^2q}{dt^2} + \frac{R}{L}\frac{dq}{dt} + \frac{1}{LC}q = 0$$

$$q(t) = Ae^{-(R/2L)t}\cos\left(\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}\,t + \phi\right)$$



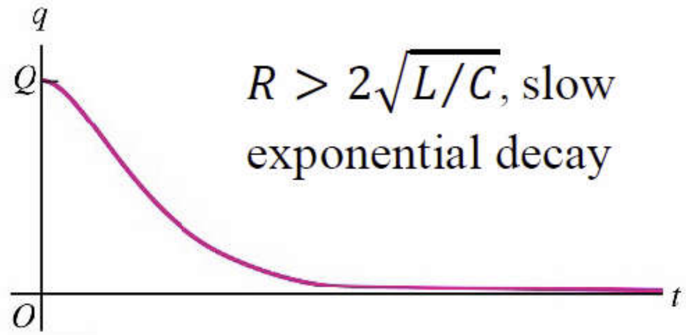
Oscillation frequency is no longer $\omega = 1/\sqrt{LC}$, but

$$\omega' = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$$

$$\frac{1}{LC} - \frac{R^2}{4L^2} > 0$$

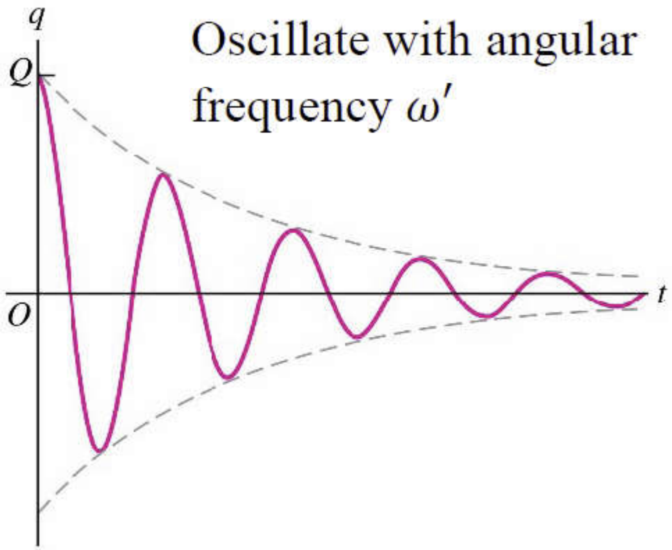
$$\Rightarrow \; R < 2\sqrt{L/C}$$

(c) Overdamped circuit (very large resistance R)



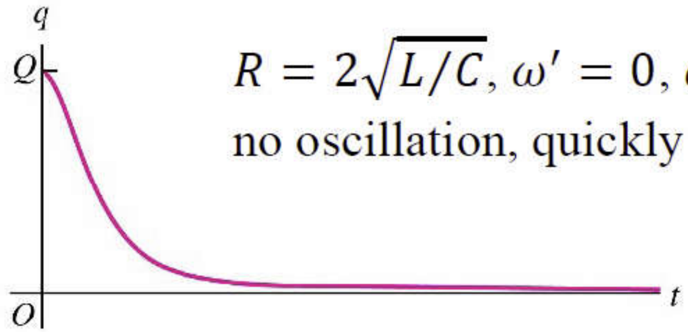
$R > 2\sqrt{L/C}$, slow exponential decay

(a) Underdamped circuit (small resistance R)



Oscillate with angular frequency ω'

(b) Critically damped circuit (larger resistance R)



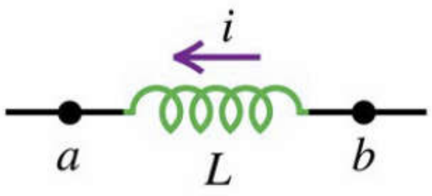
$R = 2\sqrt{L/C}$, $\omega' = 0$, $q \propto e^{-(R/2L)t}$, no oscillation, quickly damped to zero

⚠ For an analogy of forced harmonic oscillator, need an AC emf

Clicker Questions

Q30.2

A current i flows through an inductor L in the direction from point b toward point a. There is zero resistance in the wires of the inductor. If the current is *decreasing*,



A. the potential is greater at point a than at point b.

B. the potential is less at point a than at point b.

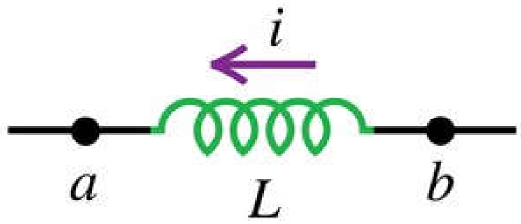
C. the answer depends on the magnitude of di/dt compared to the magnitude of i.

D. the answer depends on the value of the inductance L.

E. both C and D are correct.

A30.2

A current i flows through an inductor L in the direction from point b toward point a. There is zero resistance in the wires of the inductor. If the current is *decreasing*,



A. the potential is greater at point a than at point b.

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C. the answer depends on the magnitude of di/dt compared to the magnitude of i.

D. the answer depends on the value of the inductance L.

E. both C and D are correct.

Q30.6

An inductance L and a resistance R are connected to a source of emf as shown. Initially, switch S_1 is closed, switch S_2 is open, and current flows through L and R. When S_1 is opened and S_2 is simultaneously closed, the *rate* at which this current decreases

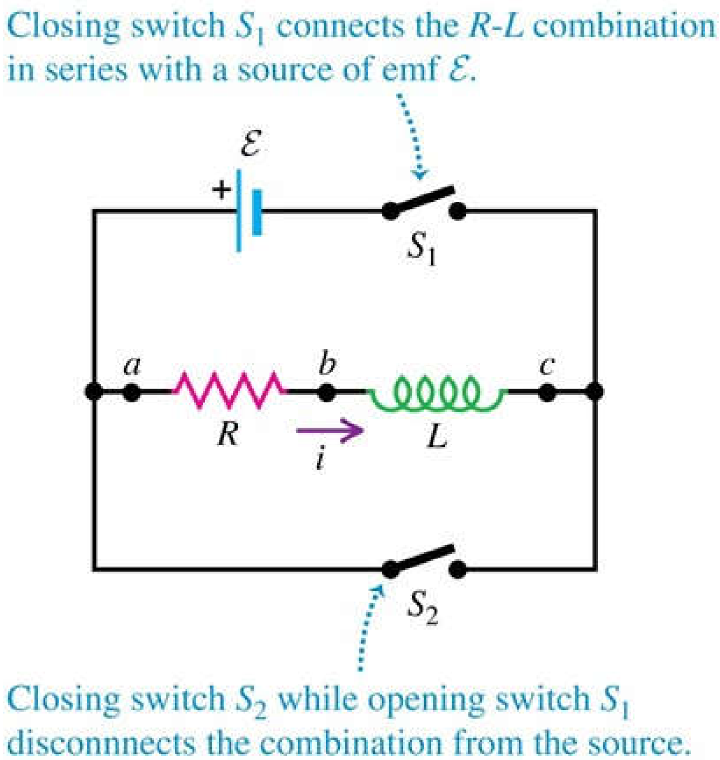
A. remains constant.

B. increases with time.

C. decreases with time.

D. Any of A, B, or C is possible.

E. Misleading question — the current does not decrease.



A30.6

An inductance L and a resistance R are connected to a source of emf as shown. Initially, switch S_1 is closed, switch S_2 is open, and current flows through L and R. When S_1 is opened and S_2 is simultaneously closed, the *rate* at which this current decreases

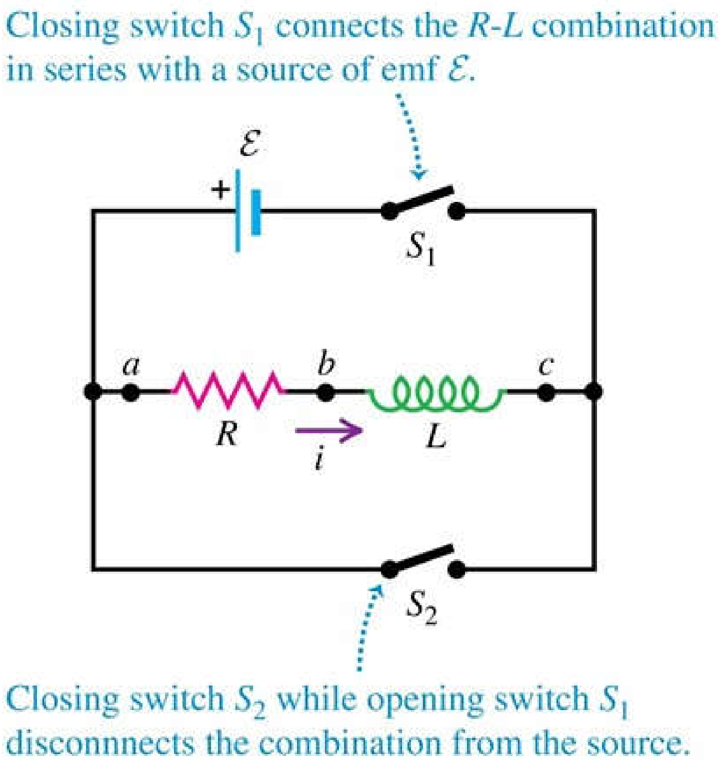
A. remains constant.

B. increases with time.

✔ C. decreases with time.

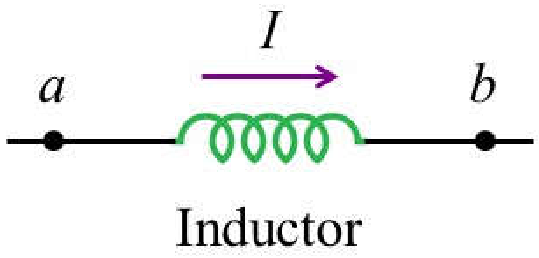
D. Any of A, B, or C is possible.

E. Misleading question — the current does not decrease.



Q-RT30.1

Rank the following inductors in order of the potential difference $V_{ab} = V_a - V_b$, from most positive to most negative. In each case the inductor has zero resistance and the current flows from point a through the inductor to point b.



A. The current through a 4.0-μH inductor increases from 3.0 A to 4.0 A in 2.0 s.

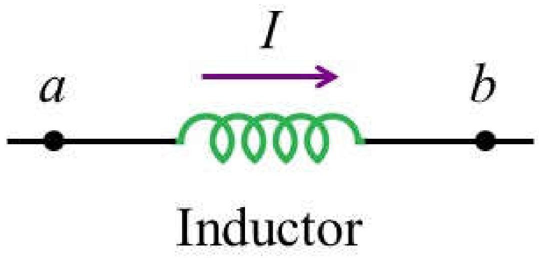
B. The current through a 1.0-μH inductor remains constant at 4.0 A.

C. The current through a 4.0-μH inductor decreases from 3.0 A to 0 in 2.0 s.

D. The current through a 2.0-μH inductor increases from 1.0 A to 2.0 A in 0.50 s.

A-RT30.1

Rank the following inductors in order of the potential difference $V_{ab} = V_a - V_b$, from most positive to most negative. In each case the inductor has zero resistance and the current flows from point a through the inductor to point b.



A. The current through a 4.0-μH inductor increases from 3.0 A to 4.0 A in 2.0 s.

B. The current through a 1.0-μH inductor remains constant at 4.0 A.

C. The current through a 4.0-μH inductor decreases from 3.0 A to 0 in 2.0 s.

D. The current through a 2.0-μH inductor increases from 1.0 A to 2.0 A in 0.50 s.

✔ Answer: DABC

Alternating current

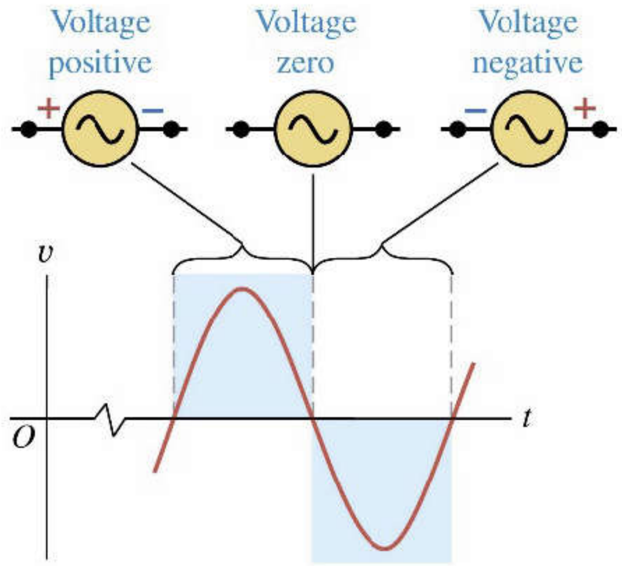
An alternating current (ac) source provides a sinusoidal voltage

$$v = V\cos\omega t$$

Convention: lower case (v) is the *instantaneous* value, while upper case (V) is the maximum value

In HK and most of the world, $f = 50$ Hz, i.e., $\omega = 2\pi f = 314$ /s, whereas in North America, $f = 60$ Hz

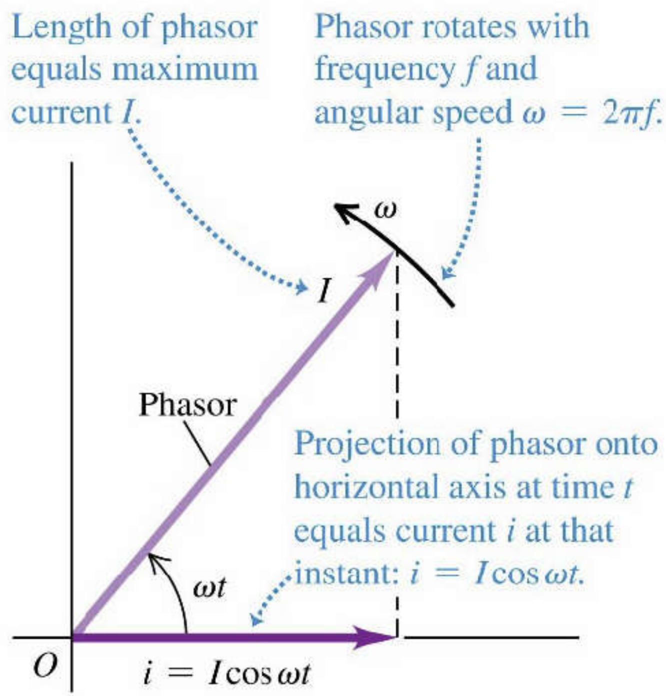
Period of the signal is $T = 1/f = 2\pi/\omega$



Phasor diagram – represent a sinusoidal signal (e.g. $i = I\cos\omega t$) as a rotating vector in a 2D plane. The projection on the x axis gives the signal

Advantage:

- while adding multiple signals, use vector addition and then project to x axis to get the final answer.
- Avoid drawing multiple sinusoidal graphs.

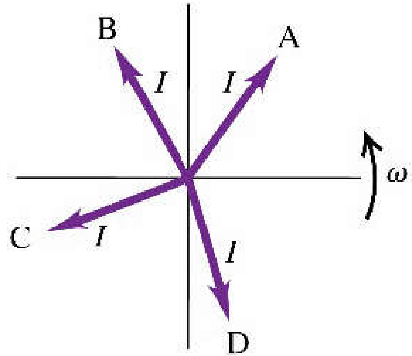


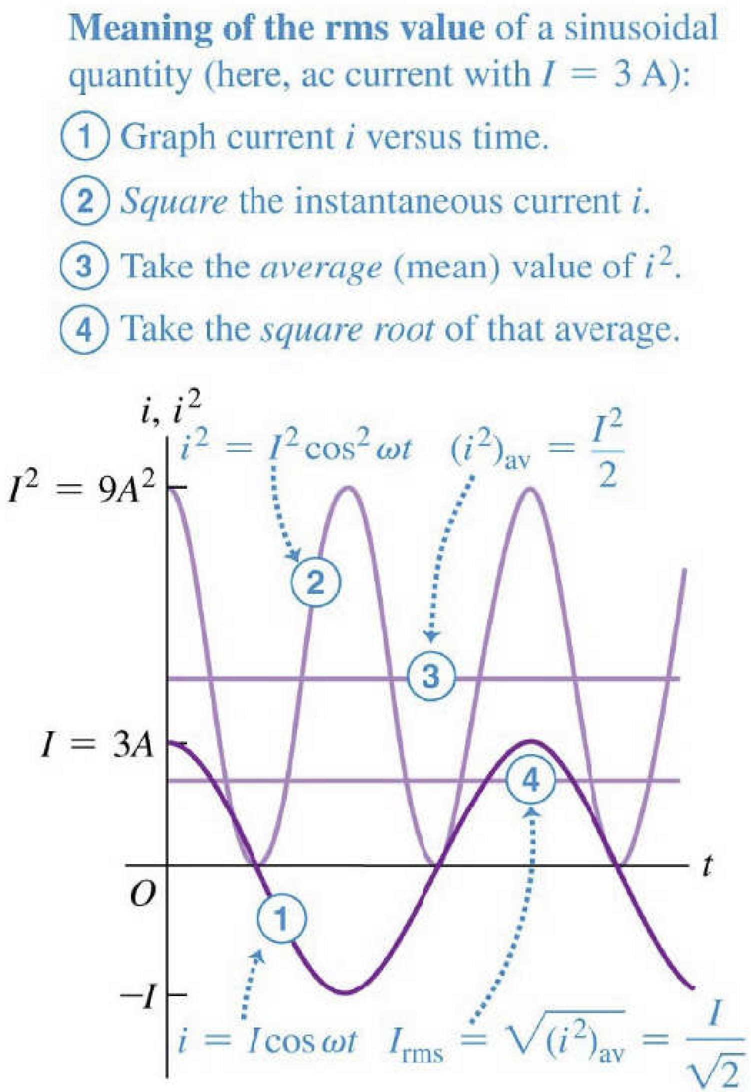
Question: For the phasor diagram for a sinusoidal current:

(A / B / C / D) is a +ve current that is becoming more +ve

(A / B / C / D) is a +ve current that is decreasing

(A / B / C / D) is a –ve current that is becoming more -ve





Average a sinusoidal signal over period T is zero and makes no sense

$$I_{av} \equiv \frac{1}{T}\int_0^T i\,dt = \frac{\omega I}{2\pi}\int_0^{\frac{2\pi}{\omega}} \cos\omega t\,dt = 0$$

A meaningful way – **root-mean-square** value defined as

$$I_{rms} = \sqrt{(i^2)_{av}}$$

$$(i^2)_{av} \equiv \frac{1}{T}\int_0^T i^2 dt = \frac{\omega I^2}{2\pi}\underbrace{\int_0^{\frac{2\pi}{\omega}} \cos^2\omega t\,dt}_{\int_0^{\frac{2\pi}{\omega}} \frac{1}{2}(1+\cos 2\omega t)dt = \frac{\pi}{\omega}} = \frac{I^2}{2}$$

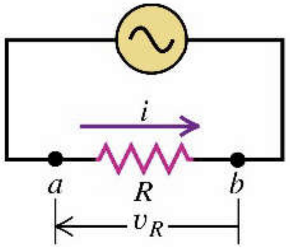
$$\Rightarrow \quad \boxed{I_{rms} = \frac{I}{\sqrt{2}}}$$

Relationship between the (AC) voltage and (AC) current in a circuit

Resistor in an ac Circuit

Take the current $i = I\cos\omega t$ as the reference in drawing phasor diagram. i counterclockwise in the following circuit is +ve

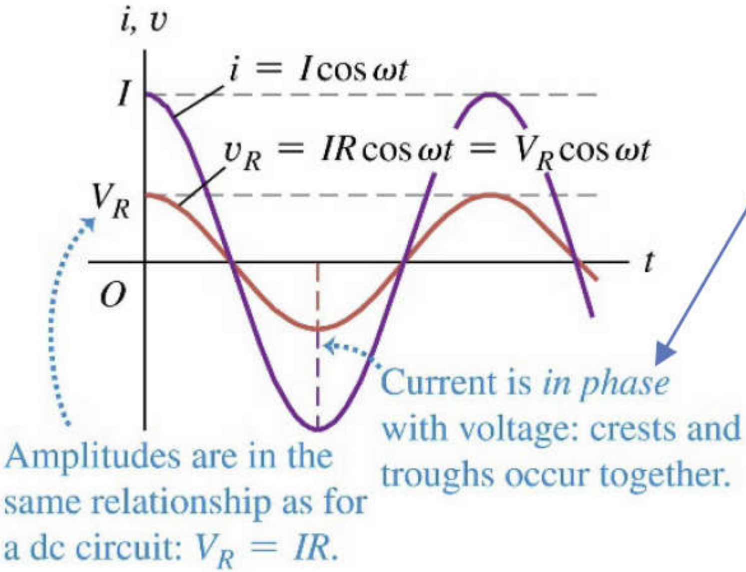
$$v_R \equiv V_a - V_b = iR = IR\cos\omega t \equiv V_R\cos\omega t$$

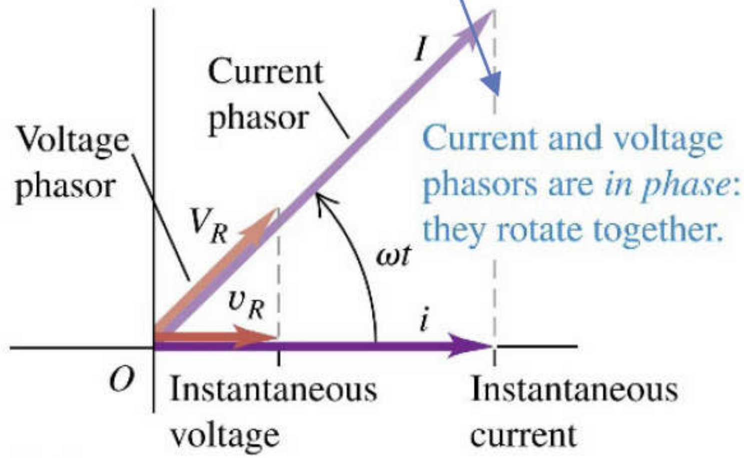


potential *drop* after tranversing along i

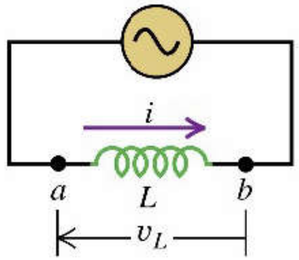
Maximum of v_R is $\boxed{V_R = IR}$

i and v are *in phase*, i.e. they have the same *phase* ωt, their phasors are collinear

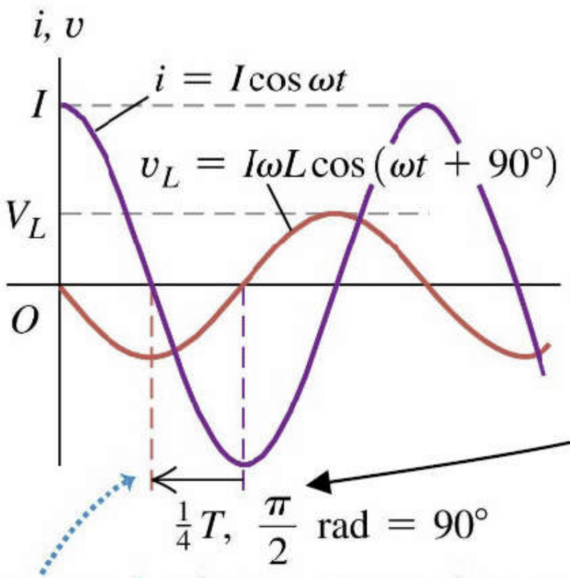




Inductor in an ac Circuit



$$v_L = L\frac{di}{dt} = -I\omega L\sin\omega t = \underbrace{(I\omega L)}_{V_L}\cos(\omega t + 90°)$$



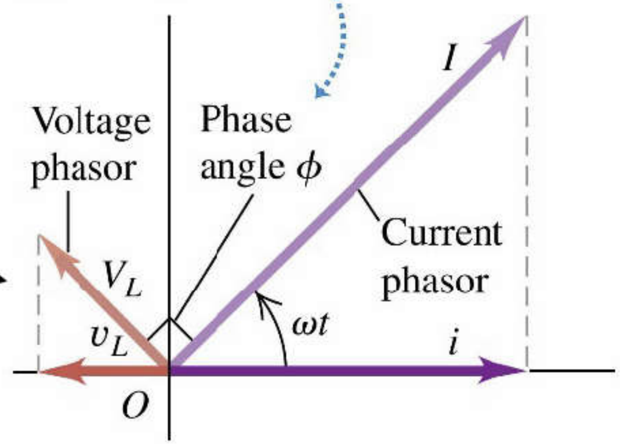
Voltage curve *leads* current curve by a quarter-cycle (corresponding to $\phi = \pi/2$ rad = 90°).

v_L *leads* i by 90°

Geometric meaning:

v_L reaches its min. (or max.) ¼ of a period *earlier* than i

Voltage phasor *leads* current phasor by $\phi = \pi/2$ rad = 90°.

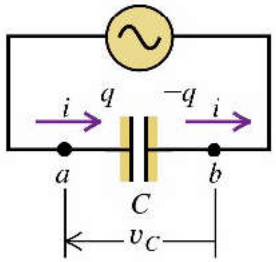


Define the **inductive reactance** $\boxed{X_L \equiv \omega L}$

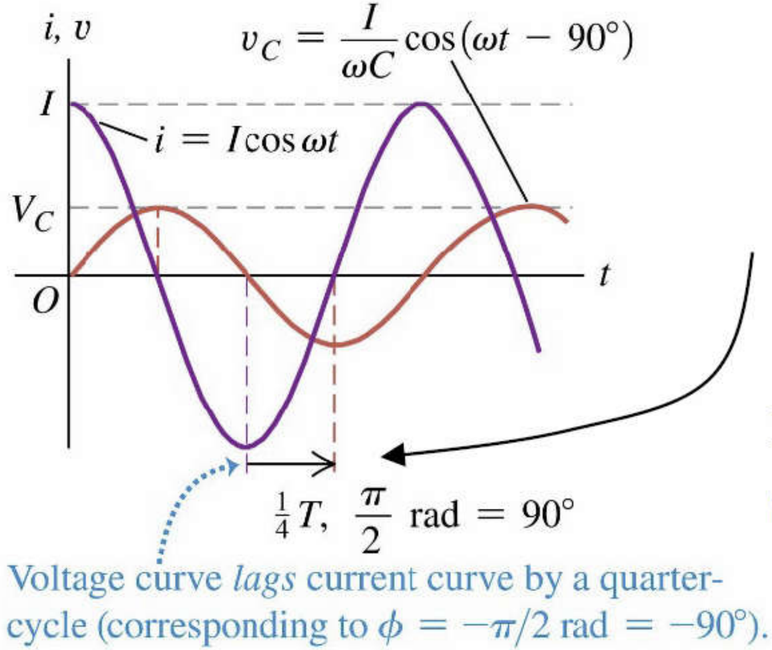
SI unit: Ω

$$\boxed{V_L = IX_L}$$

Capacitor in an ac Circuit



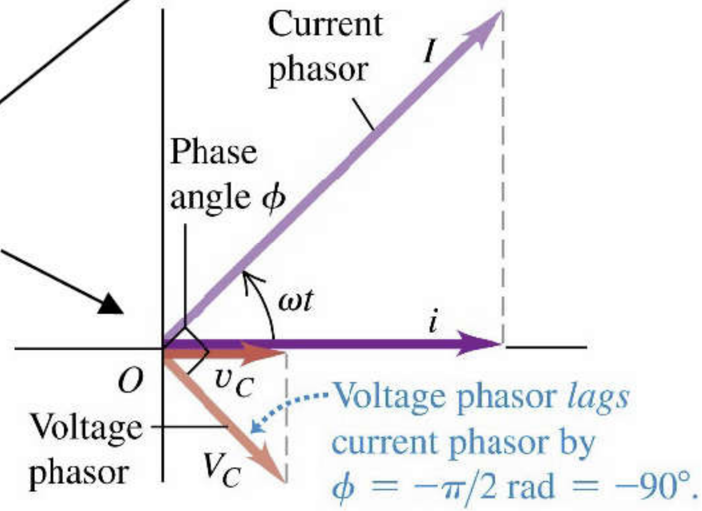
$$v_C = \frac{q}{C} = \frac{I}{C}\int \cos\omega t\,dt = \frac{I}{\omega C}\sin\omega t = \underbrace{\frac{I}{\omega C}}_{V_C}\cos(\omega t - 90°)$$



v_C *lags* i by 90°

Geometric meaning:

v_C reaches its min. (or max.) ¼ of a period *later* than i



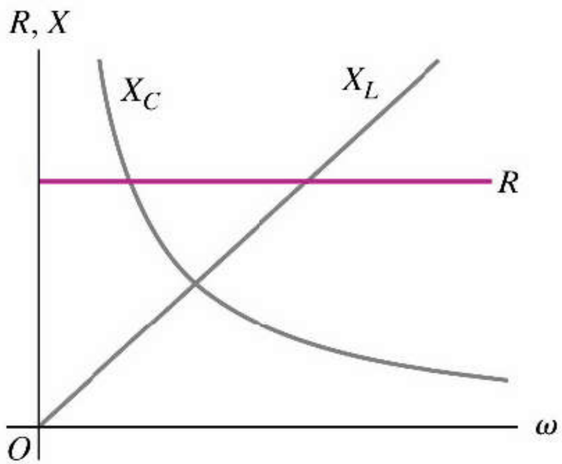
Define the **capacitive reactance** $\boxed{X_C \equiv 1/\omega C}$

SI unit: Ω

$$\boxed{V_C = I/\omega C}$$

Summary:

Circuit Element	Amplitude Relationship	Circuit Quantity	Phase of v
Resistor	$V_R = IR$	R	In phase with i
Inductor	$V_L = IX_L$	$X_L = \omega L$	Leads i by 90°
Capacitor	$V_C = IX_C$	$X_C = 1/\omega C$	Lags i by 90°



For a fixed applied V

- ⚠ For an inductor, higher f means larger X_L and smaller I → **low-pass filter**
- ⚠ For a capacitor, lower f means larger X_C and smaller I → **high-pass filter**

See [Simulation of Resistor, Inductor, and Capacitor in an ac Circuit]

Question:

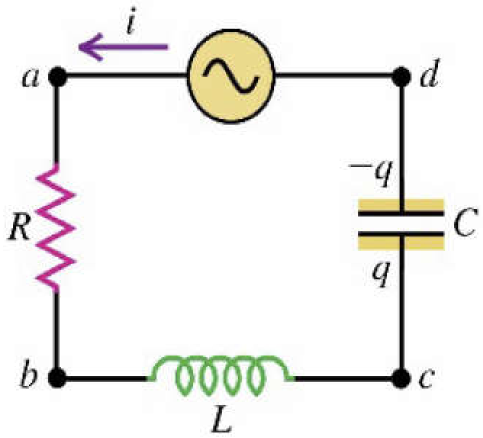
An ac voltage of fixed amplitude is applied across a circuit element. The frequency f of the voltage is increase. The amplitude of the current will:

(increase / decrease / remain the same) if the circuit element is a resistor

(increase / decrease / remain the same) if the circuit element is a inductor

(increase / decrease / remain the same) if the circuit element is a capacitor

The *L-R-C* Series Circuit



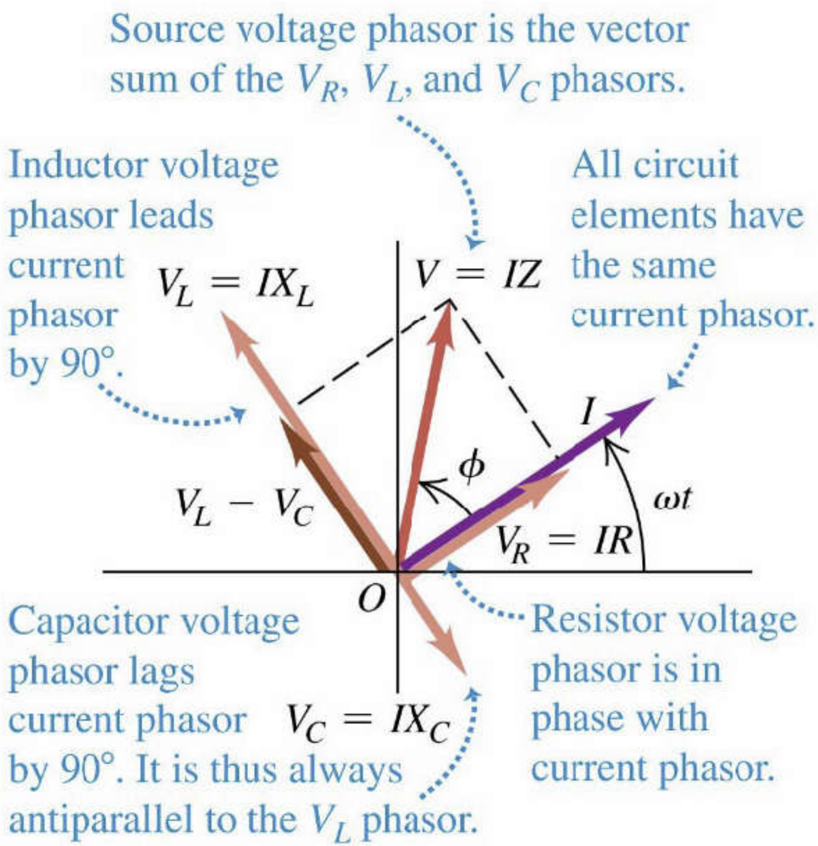
From Kirchhoff's loop rule, the external emf source v is

$$v = v_R + v_L + v_C$$

Use phasor diagram to perform this addition.

Purpose: use $i = I\cos\omega t$ as reference, find the external emf in the form $v = V\cos(\omega t + \phi)$, i.e., to find the amplitude V and phase angle ϕ

(b) Phasor diagram for the case $X_L > X_C$



Amplitude:

$$V = \sqrt{V_R^2 + (V_L - V_C)^2} = I\underbrace{\sqrt{R^2 + (X_L - X_C)^2}}$$

impedance of the ac circuit

$$\boxed{Z \equiv \sqrt{R^2 + (X_L - X_C)^2}} = \sqrt{R^2 + [\omega L - (1/\omega C)]^2}$$

Phase angle:

$$\tan\phi = \frac{X_L - X_C}{R}$$

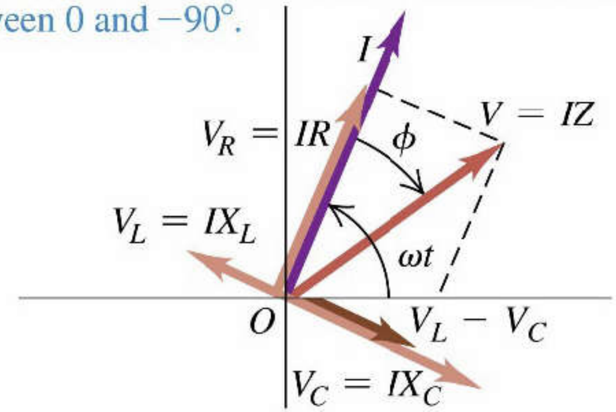
Therefore

$$v = IZ\cos\left(\omega t + \tan^{-1}\frac{X_L - X_C}{R}\right)$$

If $X_L > X_C$, *more inductive than capacity*, $\phi > 0$, i.e., v *leads* i

(c) Phasor diagram for the case $X_L < X_C$

If $X_L < X_C$, the source voltage phasor lags the current phasor, $X < 0$, and ϕ is a negative angle between 0 and $-90°$.

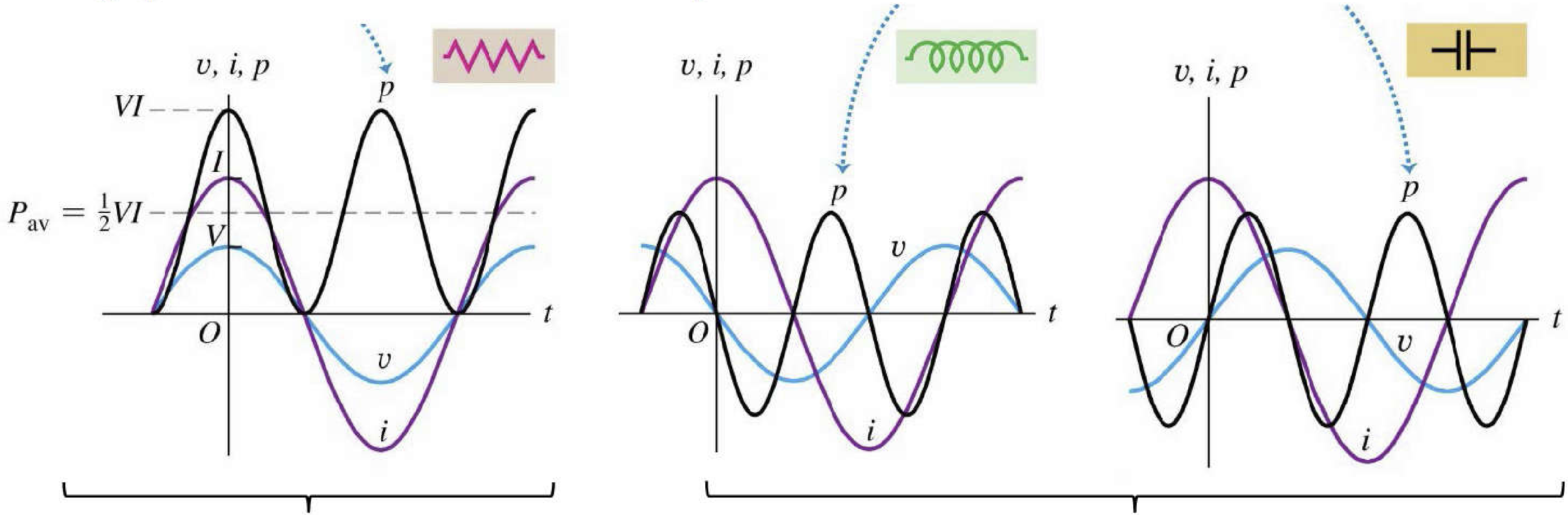


If $X_L < X_C$, *more capacitive than inductive,*
$\phi < 0$, i.e., i *leads* v

$$v = IZ\cos\left(\omega t - \tan^{-1}\frac{X_C - X_L}{R}\right)$$

Power in an ac Circuit

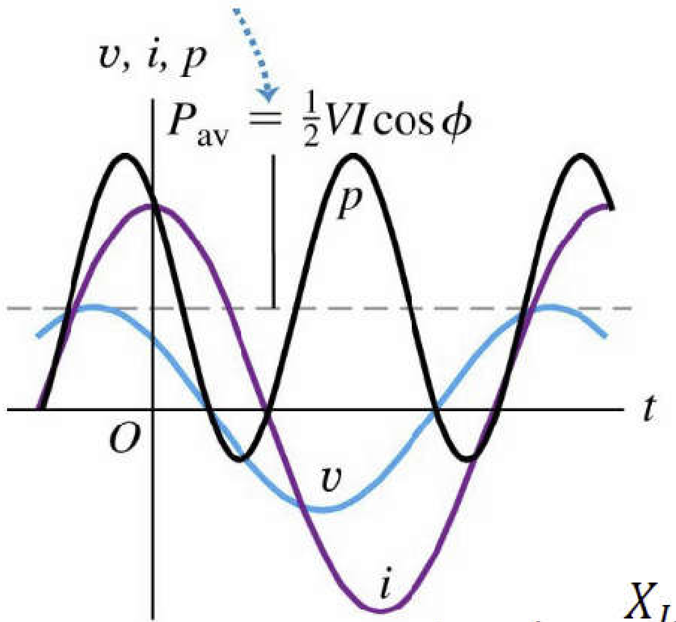
Average power delivered to inductor and capacitor are zero



$$p_R = v_R i = VI\cos^2\omega t$$
$$(p_R)_{av} = VI(\cos^2\omega t)_{av}$$
$$= \tfrac{1}{2}VI = V_{rms}I_{rms}$$

$$p = vi = VI\cos\omega t\cos(\omega t \pm 90°)$$
$$= \frac{1}{2}VI\cos(2\omega t \pm 90°)$$
$$(p)_{av} = 0$$

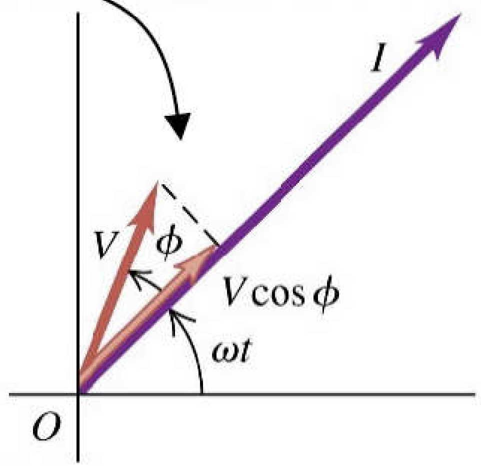
For an arbitrary ac circuit



$$p = vi = VI\cos\omega t\cos(\omega t + \phi)$$
$$= \frac{1}{2}VI[\underbrace{\cos(2\omega t + \phi)}_{\text{average out to zero}} + \cos\phi]$$

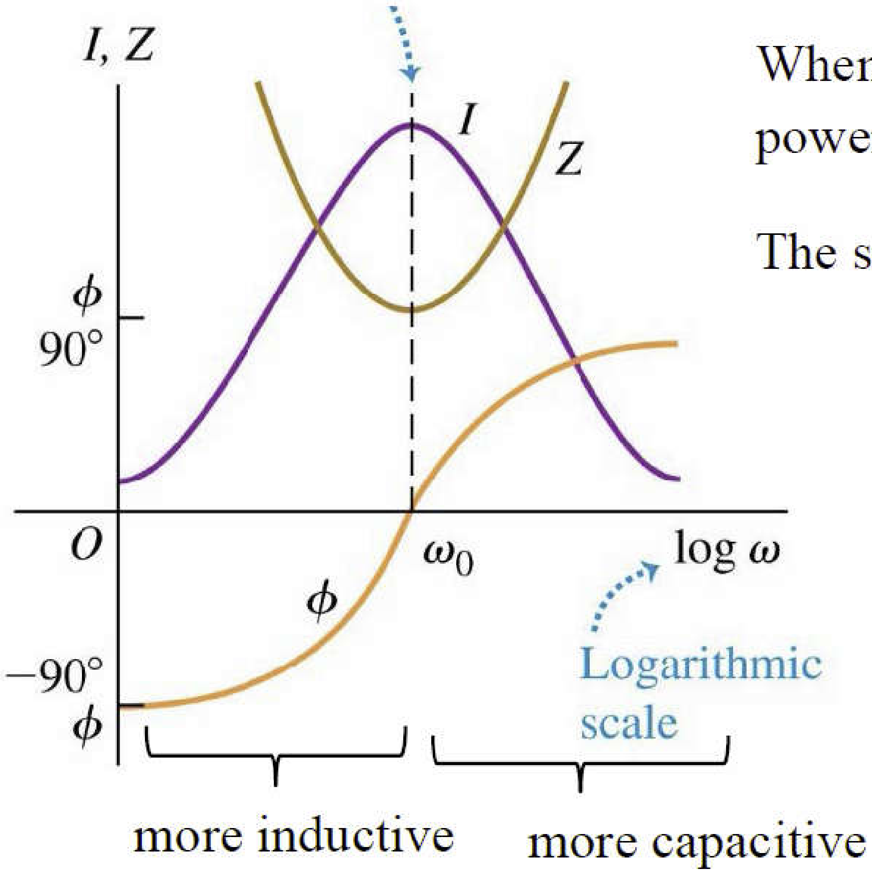
$$(p)_{av} = \frac{1}{2}VI\cos\phi$$
$$= V_{rms}I_{rms}\underbrace{\cos\phi}_{\textbf{power factor}}$$

Like a projection of V along I



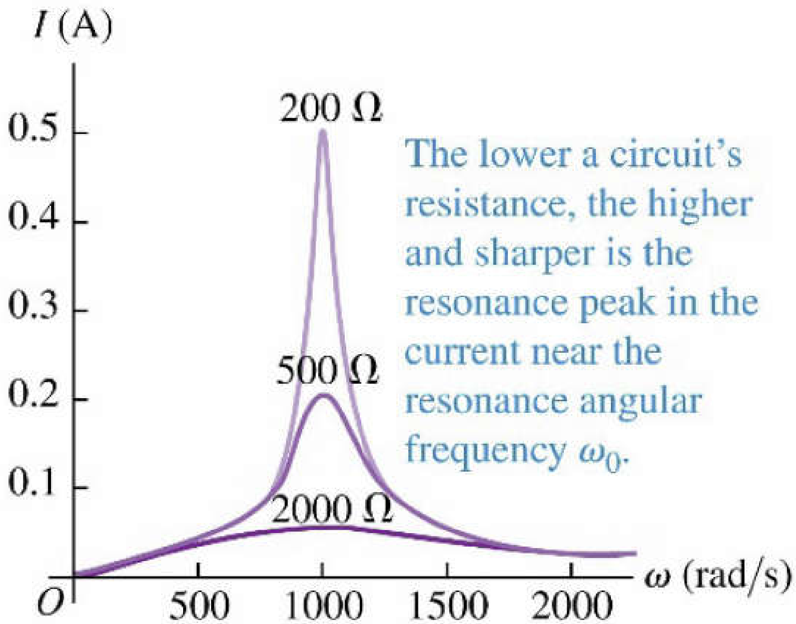
$$\tan\phi = \frac{X_L - X_C}{R} \Rightarrow \boxed{\cos\phi = \frac{R}{Z}}$$

Resonance



When ω is tuned to $\omega_0 = 1/\sqrt{LC}$, $X_L = X_C$ and the power factor $\cos\phi = 1$ is a maximum

The smaller R, the sharper the resonance



$$I = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}}$$

Tuning a radio (Example 31.8)

At resonance

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{(0.4\times10^{-3}\text{H})(10^{-10}\text{ F})}} = 800\text{ kHz}$$

$$X_L = X_C = \omega_0 L = 2000\ \Omega$$

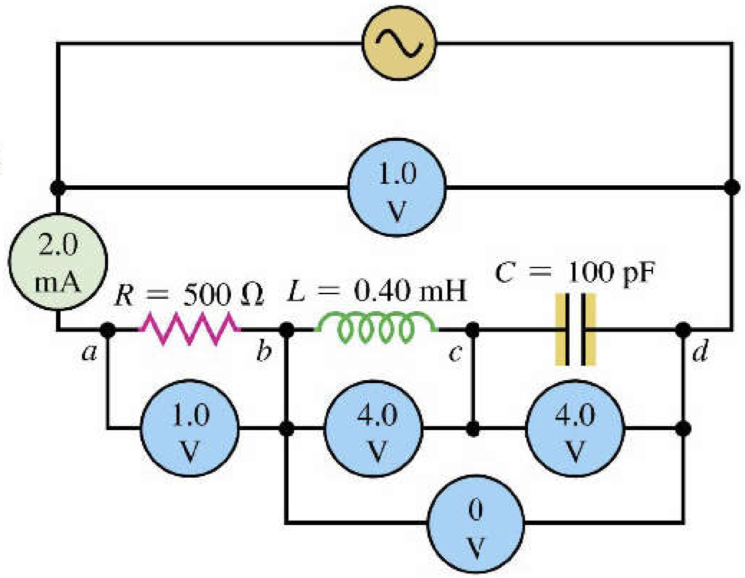
$$Z = R = 500\ \Omega$$

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{1.0\text{ V}}{500\ \Omega} = 2.0\text{ mA}$$

$$(V_R)_{\text{rms}} = I_{\text{rms}}R = (0.0020\text{ A})(500\ \Omega) = 1.0\text{ V}$$

$$(V_L)_{\text{rms}} = I_{\text{rms}}X_L = I_{\text{rms}}\omega_0 L = (0.0020\text{ A})(2000\ \Omega) = 4.0\text{ V}$$

$$(V_R)_{\text{rms}} = I_{\text{rms}}X_C = (0.0020\text{ A})(2000\ \Omega) = 4.0\text{ V}$$



Clicker Questions

Q31.4

An oscillating voltage of fixed amplitude is applied across a circuit element. If the frequency of this voltage is increased, the amplitude of the current will

A. increase if the circuit element is either an inductor or a capacitor.

B. decrease if the circuit element is either an inductor or a capacitor.

C. increase if the circuit element is an inductor, but decrease if the circuit element is a capacitor.

D. decrease if the circuit element is an inductor, but increase if the circuit element is a capacitor.

E. More than one of the above is possible, depending on circumstances.

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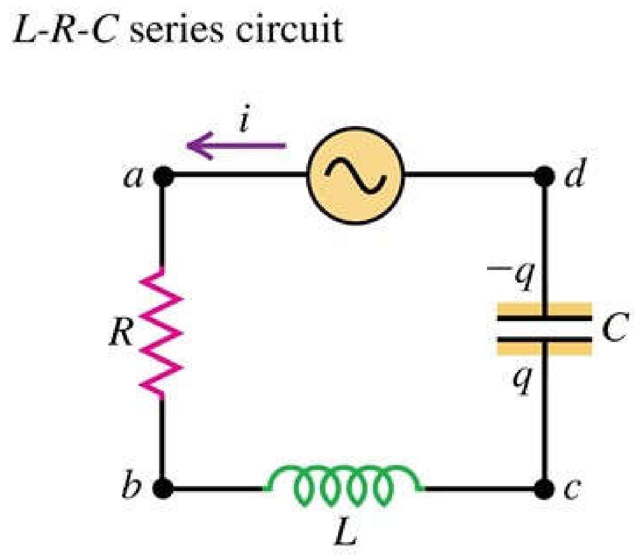
Q31.6

In an L-R-C series circuit as shown, the current has a very small amplitude if the ac source oscillates at a very high frequency. Which circuit element causes this behavior?



A. the resistor R

B. the inductor L

C. the capacitor C

D. Two of these elements acting together are necessary.

E. Misleading question—the current actually has a very *large* amplitude if the frequency is very high.

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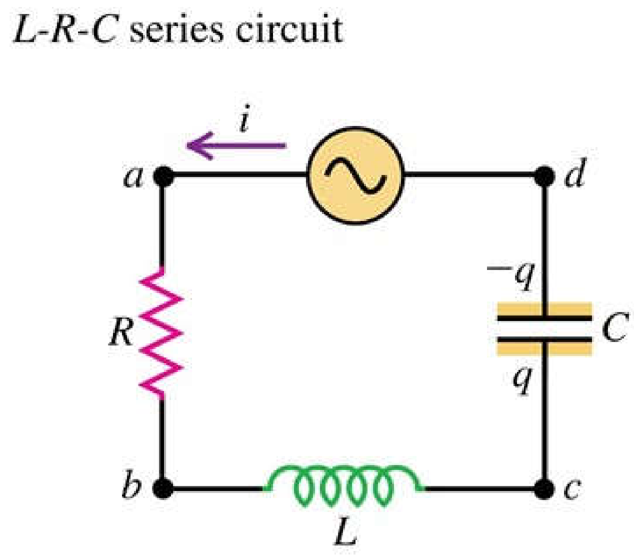

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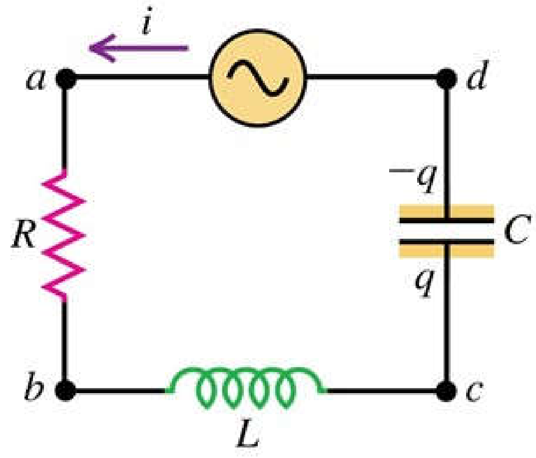
In an L-R-C series circuit as shown, suppose that the angular frequency of the ac source equals the resonance angular frequency. In this case, the circuit impedance

L-R-C series circuit



A. is maximum.

B. is minimum, but not zero.

C. is zero.

D. is neither a maximum nor a minimum.

E. could be anything; not enough information is given to decide.

A31.8

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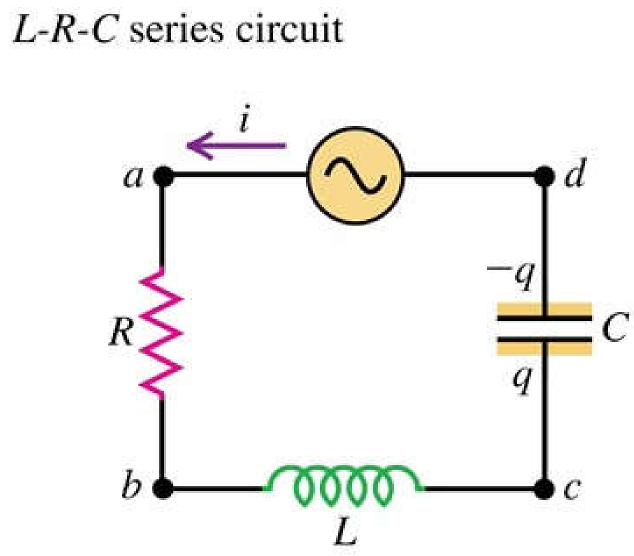

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