Electromagnetic wave

Maxwell's Equation of Electromagnetism

A unified description of electricity and magnetism

Gauss's law for $\vec{\boldsymbol{E}}$	$\oint \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{A}} = Q_{\text{encl}}/\epsilon_0$	Charge produces electrostatic electric field
Gauss's law for $\vec{\boldsymbol{B}}$	$\oint \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{A}} = 0$	No magnetic monopole
Faraday's law (for a stationary path)	$\oint \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{l}} = -\dfrac{d\Phi_B}{dt}$	Varying Φ_B produces non-electrostatic electric field
Ampere's law (for a stationary path)	$\oint \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{l}} = \mu_0 \left(i_C + \epsilon_0 \dfrac{d\Phi_E}{dt} \right)_{\text{encl}}$	Varying Φ_E produces magnetic field

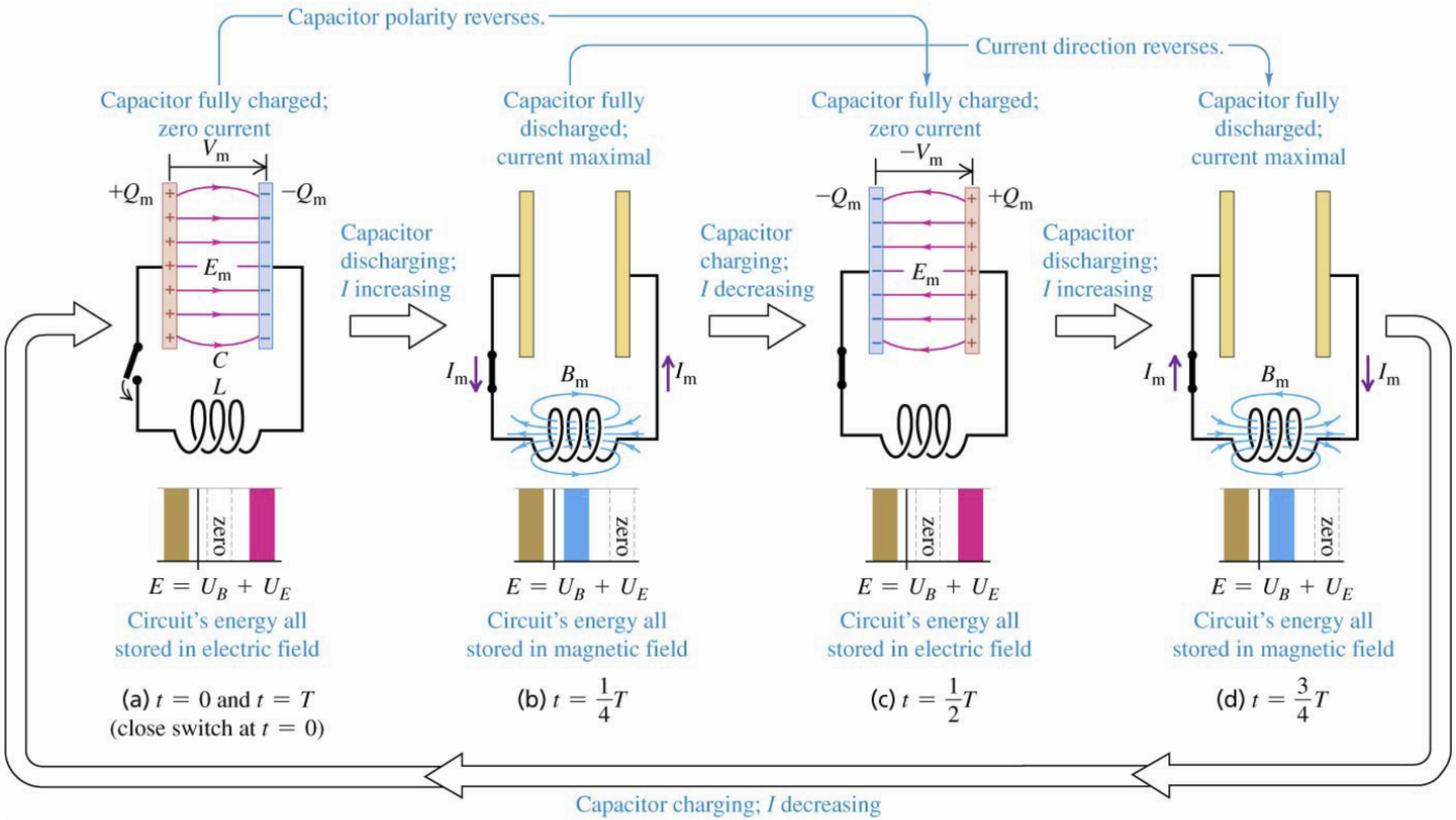
Together with the Lorentz force $\vec{\boldsymbol{F}} = q(\vec{\boldsymbol{E}} + \vec{\boldsymbol{v}} \times \vec{\boldsymbol{B}})$, these are the fundamental relationships in classical electromagnetism

Motivation:

The *L-C* Circuit – analogy of a harmonic oscillator

An electrical oscillation, energy transfer between electric and magnetic energy

c.f. a mechanical oscillation (spring and mass), energy transfer between PE and KE



Motivation:

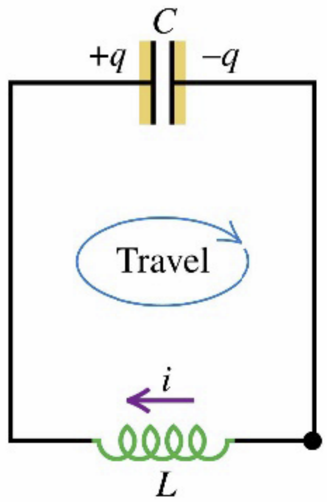
Time varying magnetic field → electric field (Faraday's law)

Time varying electric field → magnetic field (Ampere's law)

Can time varying $\vec{\boldsymbol{E}} \rightarrow \vec{\boldsymbol{B}} \rightarrow \vec{\boldsymbol{E}} \rightarrow \vec{\boldsymbol{B}} \rightarrow \cdots$ be a self-sustained system?

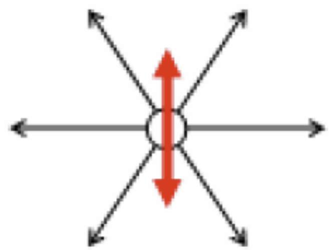
Yes, e.g. in *L*-*C* circuit

electromagnetic oscillation, $\omega = 1/\sqrt{LC}$, *confined* in the circuit



How about in vacuum with no charge? Yes, except the electromagnetic oscillation appears as a *propagating wave*!

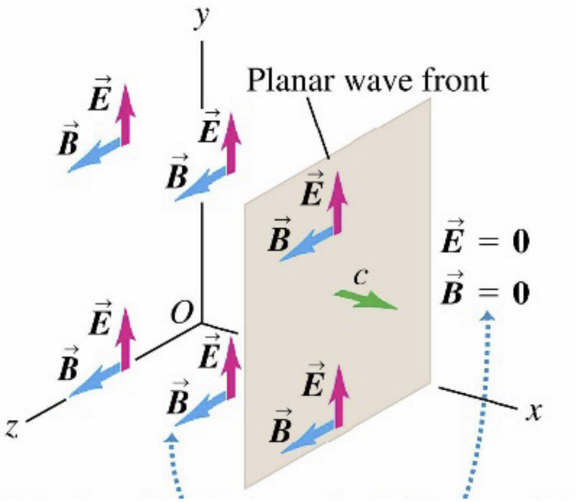
e.g. an electron oscillates together with its own $\vec{\boldsymbol{E}} \rightarrow \vec{\boldsymbol{B}} \rightarrow \vec{\boldsymbol{E}} \rightarrow \vec{\boldsymbol{B}} \rightarrow \cdots$ i.e., an EM wave



⚠ Any accelerating charge emits EM wave

A Simple (may be too simple) Plane Electromagnetic Wave

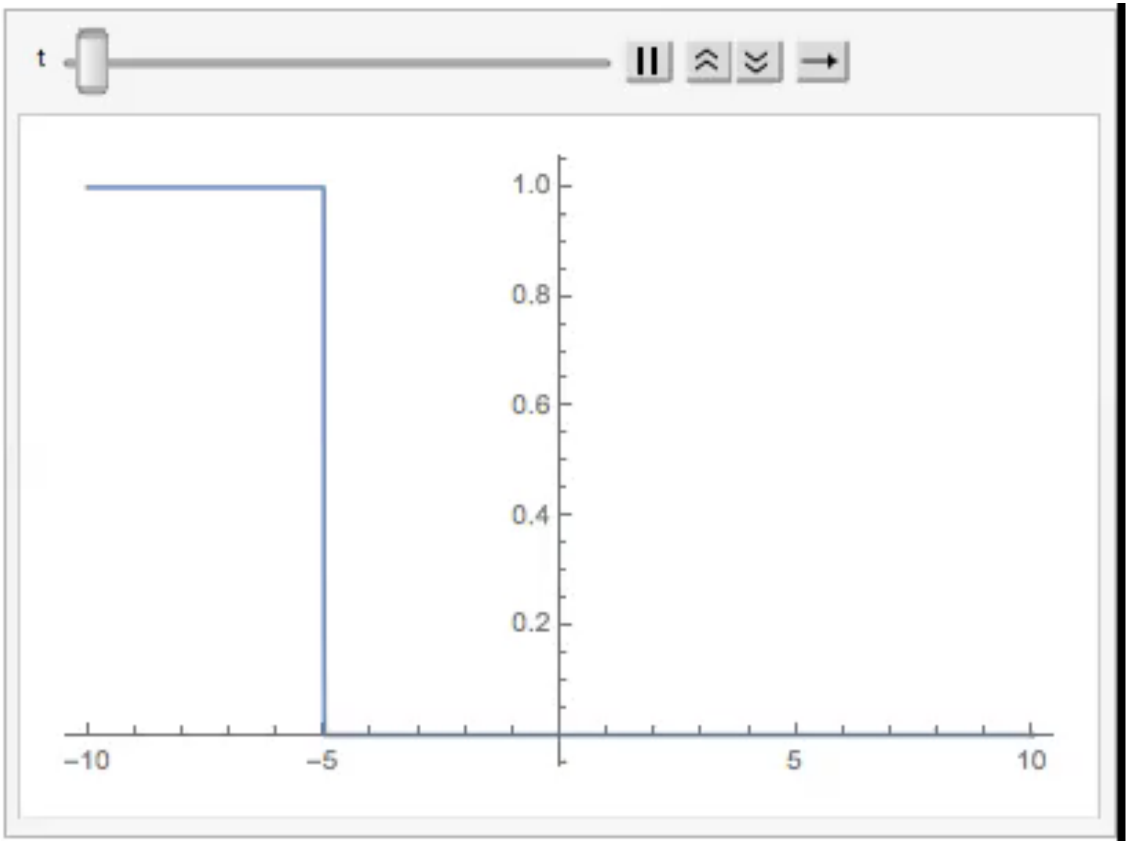
Scenario: take a *plane wave* (a 1D propagating wave)

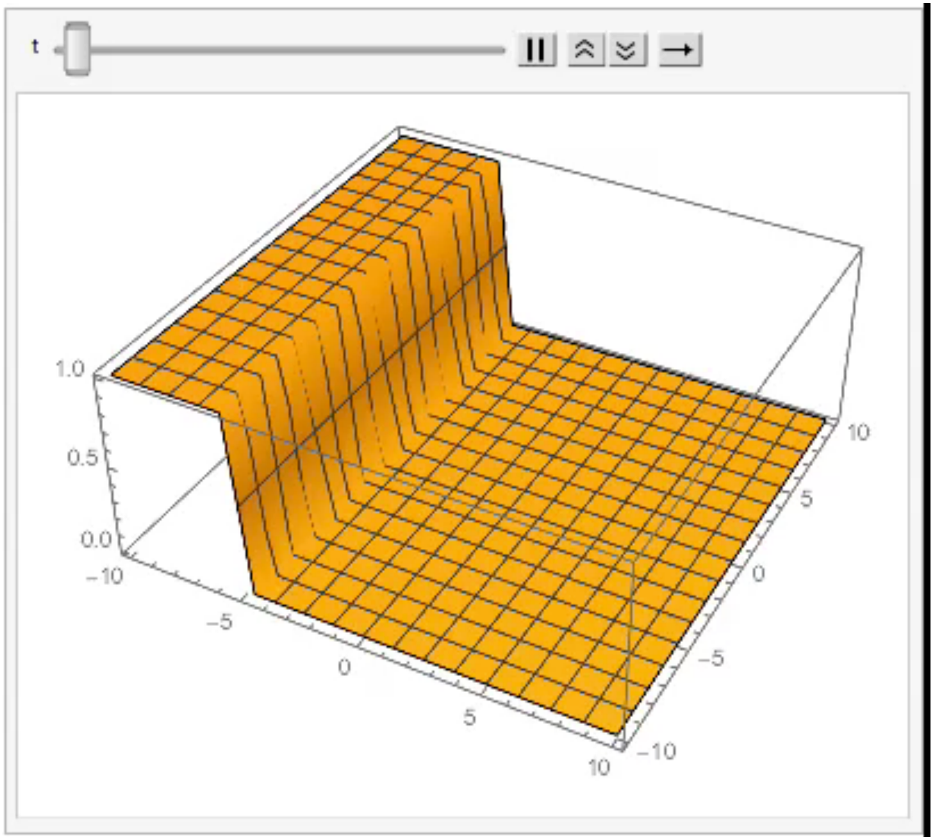


1. The *wavefront* is a plane ∥ to the *y*-*z* plane
2. Behind the wavefront, $\vec{\boldsymbol{E}}$ uniform (everywhere the same) and in *y* direction. $\vec{\boldsymbol{B}}$ also uniform and in *z* direction. $\vec{\boldsymbol{E}} \perp \vec{\boldsymbol{B}}$
3. In front of the wavefront, $\vec{\boldsymbol{E}} = 0$ and $\vec{\boldsymbol{B}} = 0$
4. Wavefront propagating in +ve *x* direction with constant speed *c*

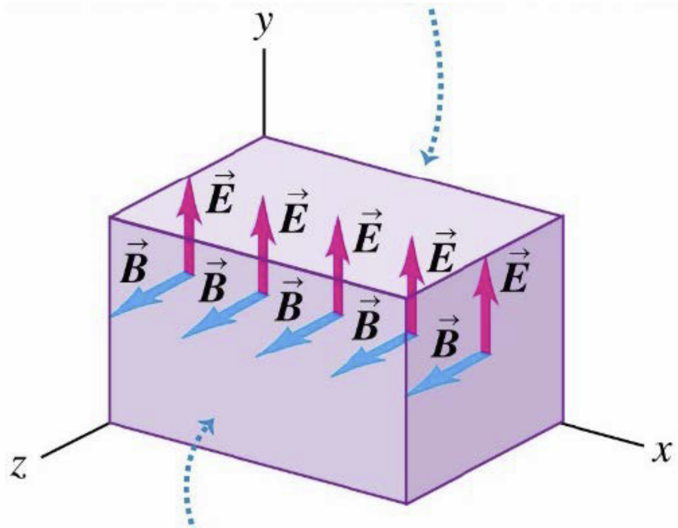
Want to check if this scenario is consistent with the Maxwell's equations, and at the same time find *c*

Propagating 1D plane wave





Gauss's laws for electric and magnetic fields



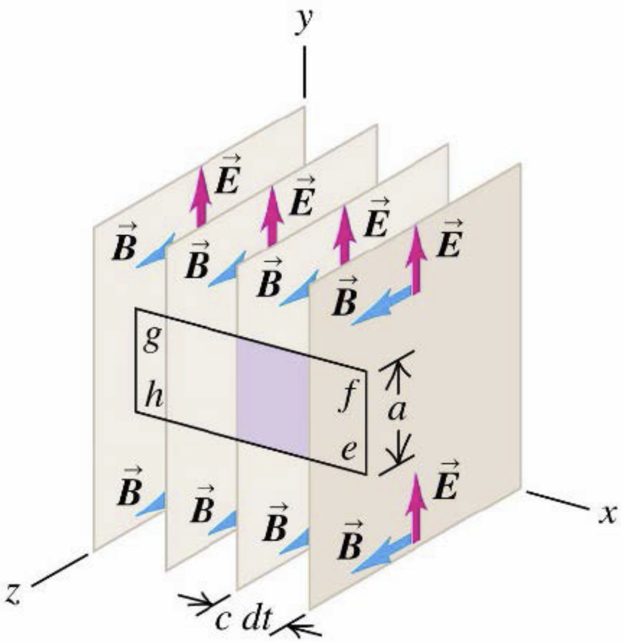
Since $\vec{\boldsymbol{E}}$ and $\vec{\boldsymbol{B}}$ uniform anywhere, obviously

$$\Phi_E = \oint \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{A}} = 0$$

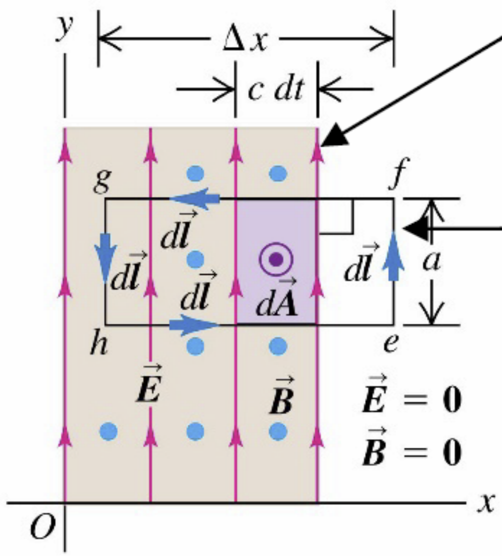
$$\Phi_B = \oint \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{A}} = 0$$

(whatever goes in = whatever goes out)

Faraday's law



side view



In time interval dt

$$d\Phi_B = Ba(cdt) \Rightarrow \frac{d\Phi_B}{dt} = acB$$

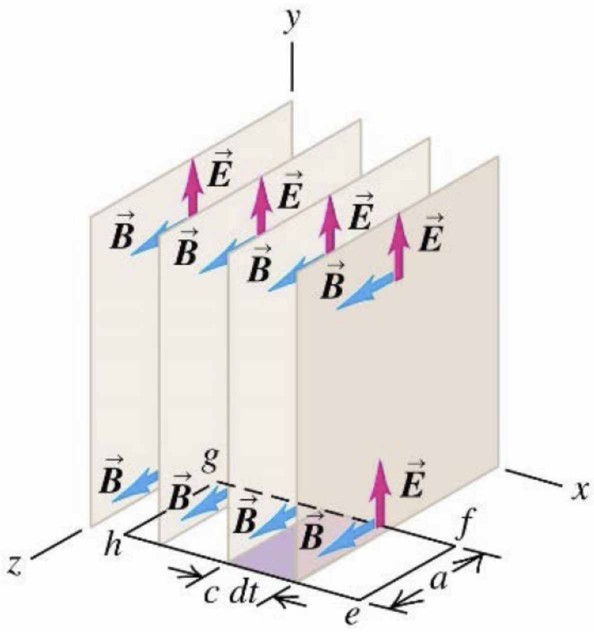
$$\oint \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{l}} = -Ea$$

$$\oint \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{l}} = -\frac{d\Phi_B}{dt} \Rightarrow \boxed{E = cB}$$

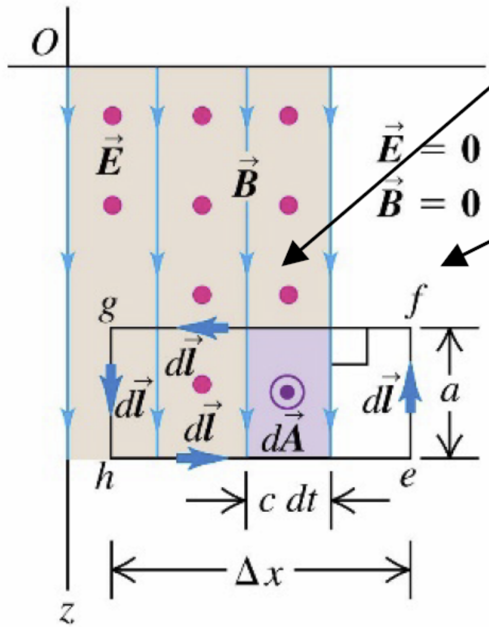
⚠ If $\vec{\boldsymbol{B}}$ has y component, it doesn't contribute to $d\Phi_B$, ∴ irrelevant to the wave

⚠ $\vec{\boldsymbol{B}}$ cannot be in –ve z direction, otherwise get $E = -cB$, contradict the assumption of propagation direction

Ampere's law



top view



In time interval dt

$$d\Phi_E = Ea(cdt) \Rightarrow \frac{d\Phi_E}{dt} = acE$$

$$\oint \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{l}} = Ba$$

$$\oint \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{l}} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt} \Rightarrow B = \mu_0 \epsilon_0 cE$$

Since $E = cB$,

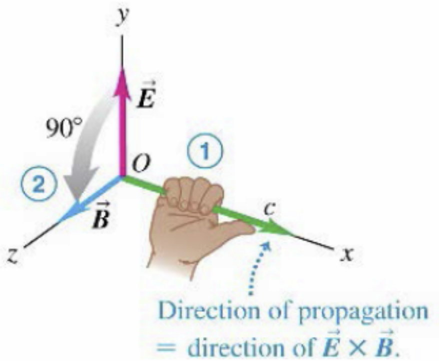
$$\boxed{c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}}$$

⚠ With $\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{C^2/Nm^2}$ and $\mu_0 = 4\pi \times 10^{-7}\ \mathrm{N/A^2}$, $c = 3.00 \times 10^8\ \mathrm{m/s}$

Conclusion: this simple plane EM wave is consistent with the Maxwell's equations

Summary of Properties of EM Waves

1. Require no medium for propagation
2. Propagate in vacuum with a constant speed $c = 1/\sqrt{\epsilon_0 \mu_0}$
3. Propagation direction is $\vec{\boldsymbol{E}} \times \vec{\boldsymbol{B}}$, i.e., wave is *tranverse*. Also, $\vec{\boldsymbol{E}} \perp \vec{\boldsymbol{B}}$, and $E = cB$



Question:

For an EM wave propagating in the +ve y direction and $\vec{\boldsymbol{E}}$ is along the –ve z direction, $\vec{\boldsymbol{B}}$ is along the ___________ direction.

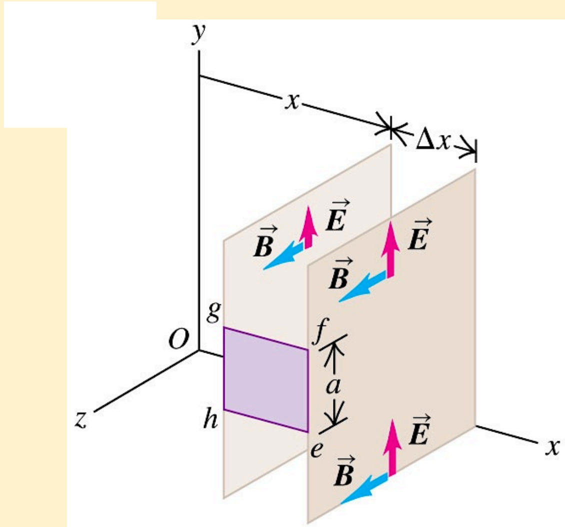
Derivation of Electromagnetic Wave Equation - 1

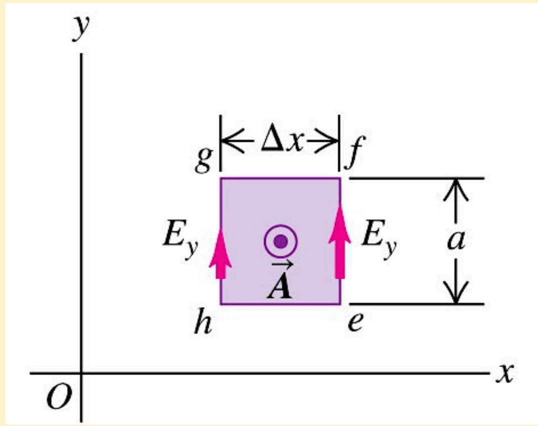
$$\oint \vec{E} \cdot d\vec{l} = -E_y(x,t)a + E_y(x+\Delta x,t)a$$

$$= a\big[E_y(x+\Delta x,t) - E_y(x,t)\big].$$

$$\Phi_B = B_z(x,t)A = B_z(x,t)a\Delta x,$$

$$\frac{d\Phi_B}{dt} = \frac{\partial B_z(x,t)}{\partial t} a\Delta x.$$





Derivation of Electromagnetic Wave Equation - 1

- Apply Faraday's law:

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

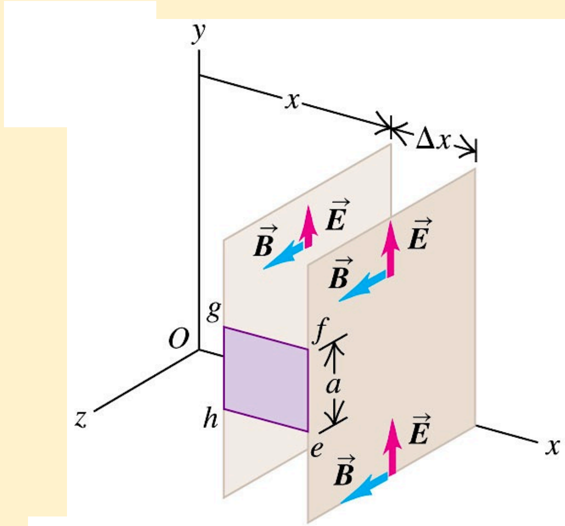
$$a\left[E_y(x+\Delta x, t) - E_y(x,t)\right] = -\frac{\partial B_z(x,t)}{\partial t} a\Delta x$$

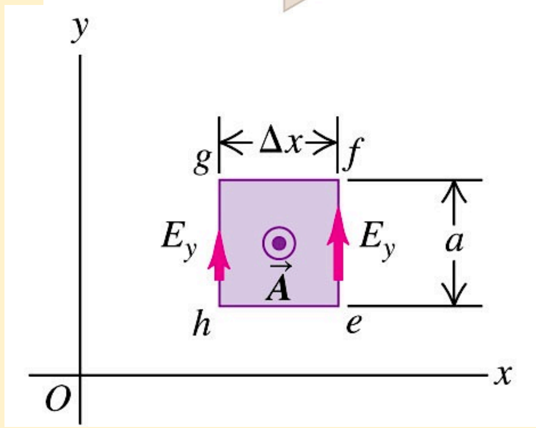
$$\frac{E_y(x+\Delta x, t) - E_y(x,t)}{\Delta x} = -\frac{\partial B_z(x,t)}{\partial t}$$

$$\Downarrow \text{ as } \Delta x \to 0$$

$$\frac{\partial E_y(x,t)}{\partial x} = -\frac{\partial B_z(x,t)}{\partial t}.$$

- This shows that if there is a time-varying component B_z, there must also be a component E_y that varies with x, and conversely.



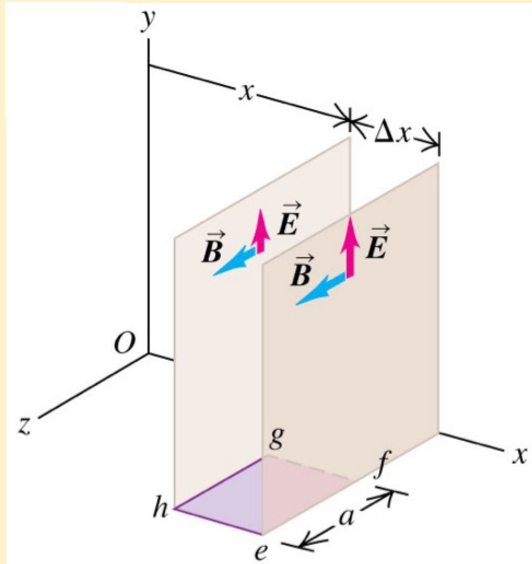


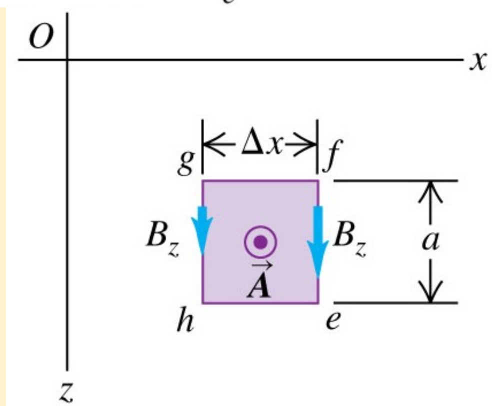
Derivation of Electromagnetic Wave Equation - 2

$$\oint \vec{B} \cdot d\vec{l} = -B_z(x + \Delta x, t)a + B_z(x, t)a.$$

$$\Phi_E = E_y(x,t)A = E_y(x,t)a\Delta x,$$

$$\frac{d\Phi_E}{dt} = \frac{\partial E_y(x,t)}{\partial t} a\Delta x.$$





Derivation of Electromagnetic Wave Equation - 2

- Apply Ampere's law:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

$$-B_z(x+\Delta x, t)a + B_z(x,t)a = \varepsilon_0 \mu_0 \frac{\partial E_y(x,t)}{\partial t} a \Delta x$$

$$\Downarrow \text{ as } \Delta x \to 0$$

$$-\frac{B_z(x+\Delta x, t) - B_z(x,t)}{\Delta x} = \varepsilon_0 \mu_0 \frac{\partial E_y(x,t)}{\partial t}$$

$$-\frac{\partial B_z(x,t)}{\partial x} = \varepsilon_0 \mu_0 \frac{\partial E_y(x,t)}{\partial t}.$$

Derivation of Electromagnetic Wave Equation - 3

$$\frac{\partial E_y(x,t)}{\partial x} = -\frac{\partial B_z(x,t)}{\partial t}$$

$$\Downarrow \partial / \partial x$$

$$\frac{\partial^2 E_y(x,t)}{\partial x^2} = -\frac{\partial^2 B_z(x,t)}{\partial x \partial t}.$$

$$-\frac{\partial B_z(x,t)}{\partial x} = \varepsilon_0 \mu_0 \frac{\partial E_y(x,t)}{\partial t}$$

$$\Downarrow \partial / \partial t$$

$$-\frac{\partial^2 B_z(x,t)}{\partial x \partial t} = \varepsilon_0 \mu_0 \frac{\partial^2 E_y(x,t)}{\partial t^2}.$$

EM wave equation in vacuum:

$$\boxed{\frac{\partial^2 E_y(x,t)}{\partial x^2} = \varepsilon_0 \mu_0 \frac{\partial^2 E_y(x,t)}{\partial t^2}.}$$

$$v = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

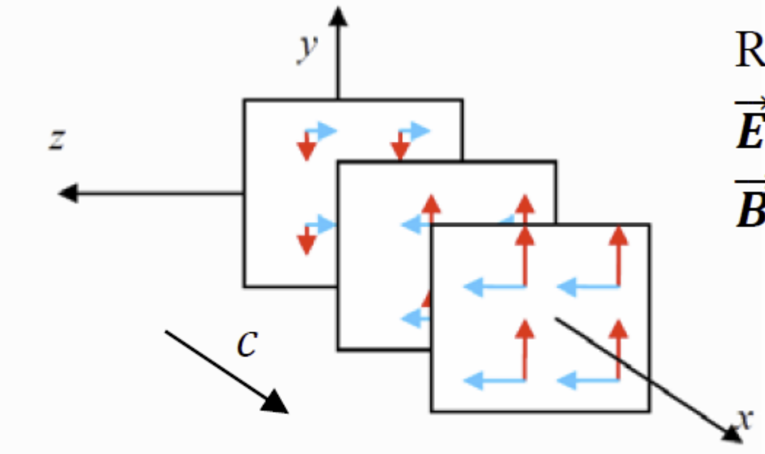
General wave equation:

$$\frac{\partial^2 y(x,t)}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y(x,t)}{\partial t^2}.$$

Sinusoidal Electromagnetic Waves

Previous argument assume $\vec{E}$ and $\vec{B}$ uniform behind wavefront. Can generalize to $\vec{E}$ and $\vec{B}$ continuous varying along x, by considering a narrow range of x within which $\vec{E}$ and $\vec{B}$ are approximately constant.

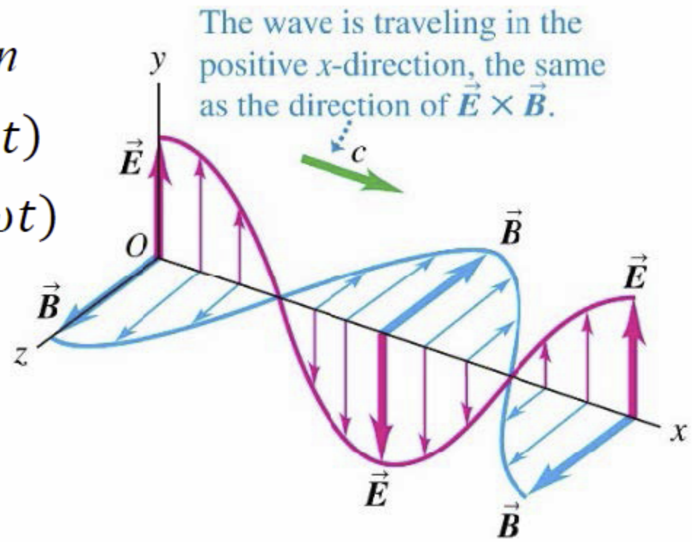
In particular, a plane sinusoidal wave

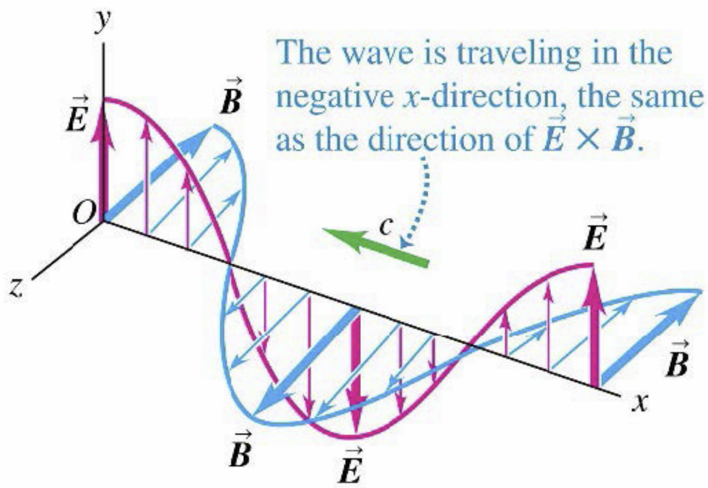


Represented as 1D *wave train*

$$\vec{E}(x,t) = \hat{\boldsymbol{j}}\, E_{\max} \cos(kx - \omega t)$$

$$\vec{B}(x,t) = \hat{\boldsymbol{k}}\, B_{\max} \cos(kx - \omega t)$$





Wave train travelling in –ve x direction

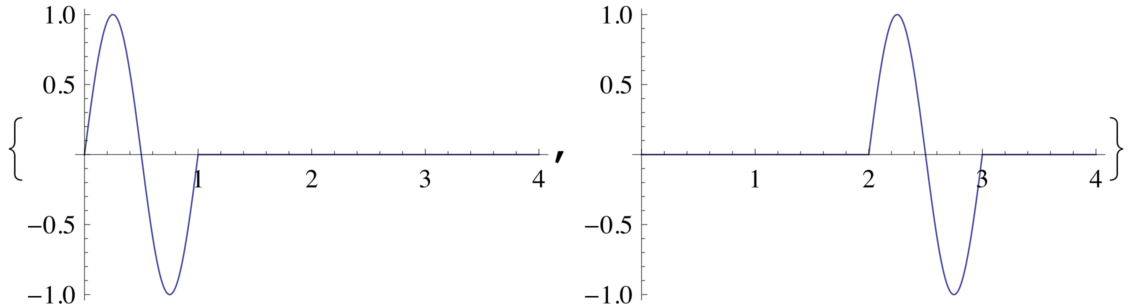
$$\vec{\boldsymbol{E}}(x,t) = \hat{\boldsymbol{\jmath}}\, E_{\max} \cos(kx + \omega t)$$
$$\vec{\boldsymbol{B}}(x,t) = -\hat{\boldsymbol{k}}\, B_{\max} \cos(kx + \omega t)$$

See simulation of EM wave

- $\vec{\boldsymbol{E}}$ and $\vec{\boldsymbol{B}}$ always *in phase* (reach max and min at the same time), actually a 1D wave
- $\vec{\boldsymbol{E}} \times \vec{\boldsymbol{B}}$ is the direction of propagation
- $k = 2\pi/\lambda$ and $\omega = 2\pi f$, $c = \lambda f = \omega/k$

Why does it propagate in +x direction?

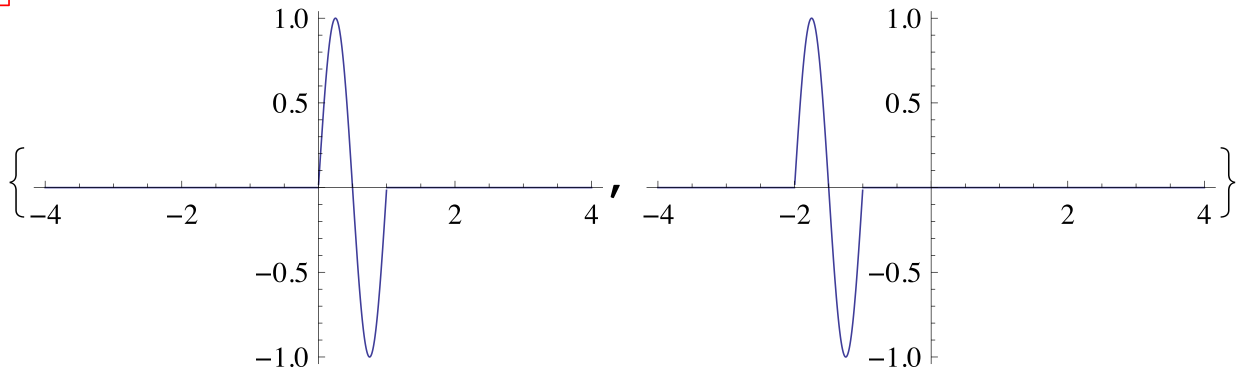
$\boxed{\sin(x - vt)}$



t=0

t=2/v

$\boxed{\sin(x + vt)}$



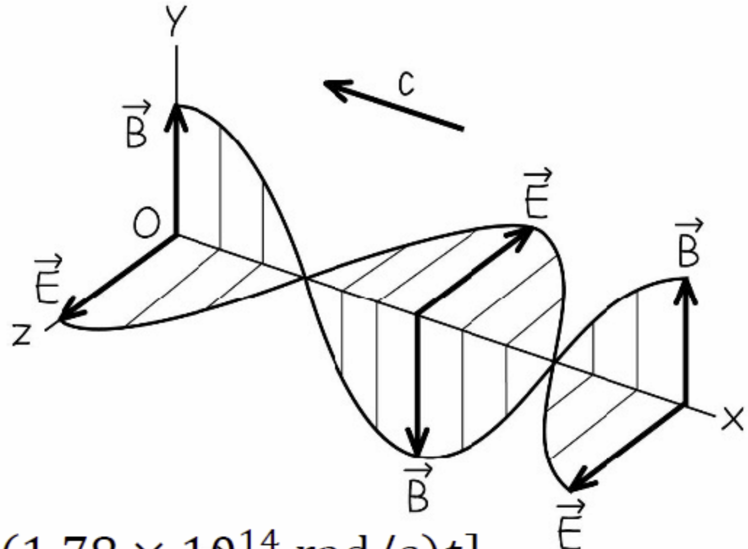
Example 32.1

A CO_2 laser emits a monochromatic (single frequency) EM wave that travel in vacuum in the –ve x direction. Its wavelength is 10.6 μm (infrared). $\vec{\boldsymbol{E}}$ is parallel to the z axis with $E_{\max} = 1.5\ \mathrm{MV/m}$.

$$B_{\max} = \frac{E_{\max}}{c} = \frac{1.5 \times 10^6\ \mathrm{V/m}}{3.0 \times 10^8\ \mathrm{m/s}} = 5.0 \times 10^{-3}\ \mathrm{T}$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{10.6 \times 10^{-6}\ \mathrm{m}} = 5.93 \times 10^5\ \mathrm{rad/m}$$

$$\omega = ck = (3.0 \times 10^8\ \mathrm{m/s})(5.93 \times 10^5\ \mathrm{rad/m})$$
$$= 1.78 \times 10^{14}\ \mathrm{rad/s}$$



$$\vec{\boldsymbol{E}}(x,t) = \hat{\boldsymbol{k}}\ (1.5 \times 10^6\ \mathrm{V/m}) \cos[(5.93 \times 10^5\ \mathrm{rad/m})x + (1.78 \times 10^{14}\ \mathrm{rad/s})t]$$
$$\vec{\boldsymbol{B}}(x,t) = \hat{\boldsymbol{j}}\ (5.0 \times 10^{-3}\ \mathrm{T}) \cos[(5.93 \times 10^5\ \mathrm{rad/m})x + (1.78 \times 10^{14}\ \mathrm{rad/s})t]$$

Q32.4

At a certain point in space, the electric and magnetic fields of an electromagnetic wave at a certain instant are given by

$$\vec{E} = \hat{i}\left(6\times10^{3}\ \text{V/m}\right)$$

$$\vec{B} = \hat{k}\left(2\times10^{-5}\ \text{T}\right)$$

This wave is propagating in the

A. positive *x*-direction.

B. negative *x*-direction.

C. positive *y*-direction.

D. negative *y*-direction.

E. unknown direction.

A32.4

At a certain point in space, the electric and magnetic fields of an electromagnetic wave at a certain instant are given by

$$\vec{\boldsymbol{E}} = \hat{\boldsymbol{i}}\left(6\times10^{3}\ \text{V/m}\right)$$

$$\vec{\boldsymbol{B}} = \hat{\boldsymbol{k}}\left(2\times10^{-5}\ \text{T}\right)$$

This wave is propagating in the

A. positive x-direction.

B. negative x-direction.

C. positive y-direction.

D. negative y-direction.

E. unknown direction.

Q32.7

In a sinusoidal electromagnetic wave in a vacuum, the electric field has only an x-component. This component is given by

$$E_x = E_{max} \cos(ky + \omega t)$$

The magnetic field of this wave

A. has only an x-component.

B. has only a y-component.

C. has only a z-component.

D. has components along two of the x-, y-, and z-axes.

E. not enough information is given to decide.

A32.7

In a sinusoidal electromagnetic wave in a vacuum, the electric field has only an x-component. This component is given by

$$E_x = E_{\max} \cos (ky + \omega t)$$

The magnetic field of this wave

A. has only an x-component.

B. has only a y-component.

C. has only a z-component.

D. has components along two of the x-, y-, and z-axes.

E. not enough information is given to decide.

Electromagnetic Waves in Matter

In a dielectric with dielectric constant K and magnetic constant K_m (⚠ usually $K_m \approx 1$ unless the dielectric is ferromagnetic). Speed of light in this material

$$v = \frac{1}{\sqrt{\epsilon\mu}} = \frac{1}{\sqrt{K\epsilon_0 K_m \mu_0}} = \frac{c}{\sqrt{KK_m}} \approx \frac{c}{\sqrt{K}}$$

Refractive index of the dielectric medium is

$$\boxed{n = \frac{c}{v} = \sqrt{KK_m} \approx \sqrt{K}}$$

⚠ The "dielectric constant is frequency dependent, $K(\omega)$, called *dielectric function*. The previous K values used in capacitors corresponding to the static situation ($\omega = 0$) and is not applicable here.

⚠ Speed of light in transparent material is typically $0.2c$ to c, e.g., $0.75c$ in water, $0.41c$ in diamond (see Example 32.2)

Energy and Momentum in Electromagnetic Waves

- In a region of empty space where **E** and **B** fields are present, the total energy density *u* is

$$u = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2\mu_0} B^2.$$

- For electromagnetic waves in vacuum,

$$B = \frac{E}{c} = \sqrt{\varepsilon_0 \mu_0} E.$$

- Combining gives

$$u = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2\mu_0}\left(\sqrt{\varepsilon_0 \mu_0} E\right)^2 = \varepsilon_0 E^2.$$

- This shows that in vacuum, energy density of **E** field is equal to energy density of **B** field.

Electromagnetic Energy Flow and the Poynting Vector

- Energy *dU* in the volume is

$$dU = u\,dV = \left(\varepsilon_0 E^2\right)\left(A\,c\,dt\right).$$

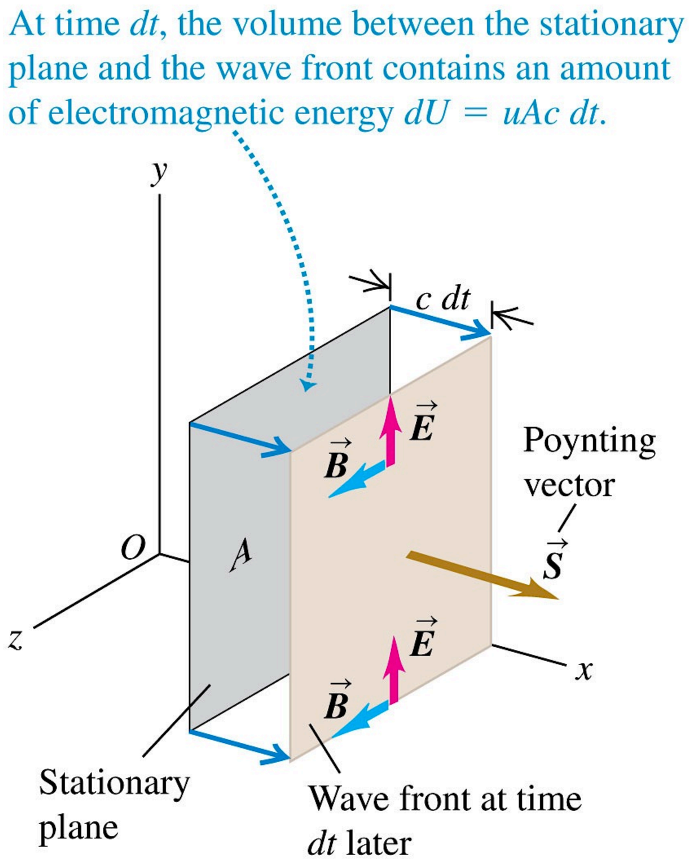
- Energy flow per unit time per unit area *A* is

$$\boxed{S = \frac{1}{A}\frac{dU}{dt}} = \varepsilon_0 c E^2$$

$$= \frac{\varepsilon_0}{\sqrt{\varepsilon_0 \mu_0}} E^2 = \sqrt{\frac{\varepsilon_0}{\mu_0}} E^2 = \frac{EB}{\mu_0}.$$

- SI unit of *S*:

  1 J/(sm^2) or 1 W/m^2.



Electromagnetic Energy Flow and the Poynting Vector

- The energy the EM wave carries per unit time per unit area is given by the Poynting vector

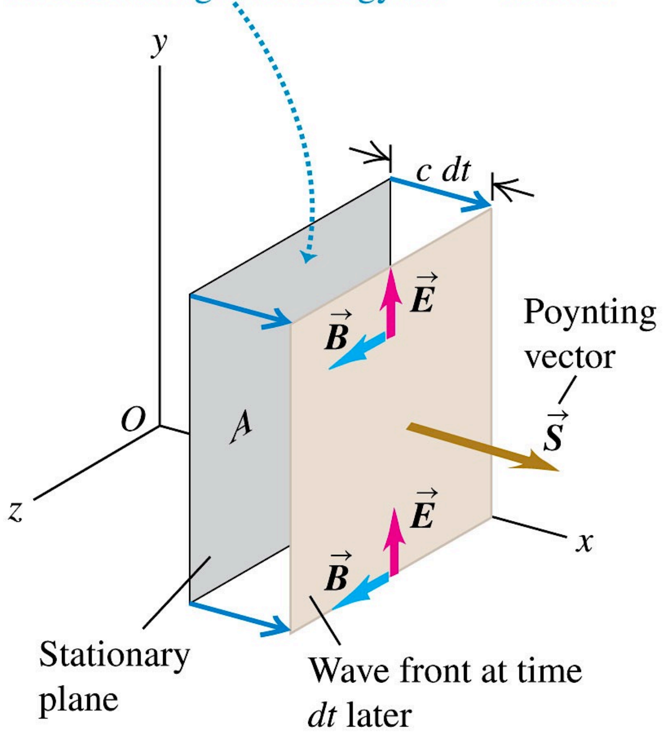
$$\boxed{\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}}$$

$$S = \frac{1}{\mu_0} EB \sin 90^\circ = \frac{1}{\mu_0} EB.$$

- Its direction is in the direction of propagation of the wave.
- SI unit of *S*:

  1 J/(sm^2) or 1 W/m^2.

At time dt, the volume between the stationary plane and the wave front contains an amount of electromagnetic energy $dU = uAc\,dt$.



Electromagnetic Energy Flow and the Poynting Vector

$$\vec{S}(x,t) = \frac{1}{\mu_0}\vec{E}(x,t)\times\vec{B}(x,t)$$

$$= \frac{1}{\mu_0}\hat{j}E_{\max}\cos(kx-\omega t)\times\hat{k}B_{\max}\cos(kx-\omega t)$$

$$= \frac{1}{\mu_0}E_{\max}B_{\max}\underbrace{\cos^2(kx-\omega t)}_{\geq 0}\underbrace{\hat{j}\times\hat{k}}_{\hat{i}}.$$

$$S_x(x,t) = \frac{E_{\max}B_{\max}}{\mu_0}\cos^2(kx-\omega t)$$

$$= \frac{E_{\max}B_{\max}}{2\mu_0}\left[1+\cos 2(kx-\omega t)\right].$$

Intensity of EM wave I = time-average Poynting vector = $\frac{E_{max}B_{max}}{2\mu_0}$

Electromagnetic
Momentum Flow and Radiation Pressure

- EM waves carry momentum *p*, with a corresponding momentum density

$$\frac{dp}{dV} = \frac{EB}{\mu_0 c^2} = \varepsilon_0 EB = \frac{S}{c^2}.$$

$$\boxed{\begin{array}{l} \text{momentum Density} = \dfrac{EB}{\mu_0 c^2} = \dfrac{\varepsilon_0 E^2}{c} \\ \text{Energy Density} = \varepsilon_0 E^2 = c \times (\text{momentum Density}) \end{array}}$$

- The momentum flow rate per unit area is

$$\frac{1}{A}\frac{dp}{dt} = \frac{S}{c} = \frac{EB}{\mu_0 c}.$$

$$\boxed{dV = Acdt}$$

$$\left[\because \frac{dp}{dV} = \frac{S}{c^2} \Rightarrow \frac{dp}{A\cancel{c}dt} = \frac{S}{c^{\cancel{2}}}.\right]$$

Electromagnetic
Momentum Flow and Radiation Pressure

- The average rate of momentum transfer per unit area is $S_{av}/c = I/c$.
- Notice that the rate of momentum transfer is the force
- When an EM wave is completely absorbed by a surface, the wave's momentum is also transferred to the surface. The radiation pressure p_{rad} (=Force/Area) is

$$\boxed{\begin{aligned} p_{\text{rad}} &= \left(\frac{1}{A}\frac{dp}{dt}\right)_{\text{av}} = \left(\frac{S}{c}\right)_{\text{av}} \\ \text{or}\quad p_{\text{rad}} &= \frac{S_{\text{av}}}{c} = \frac{I}{c}. \end{aligned}}$$

Electromagnetic
Momentum Flow and Radiation Pressure

- When an EM wave is totally reflected, the momentum change is twice as great:

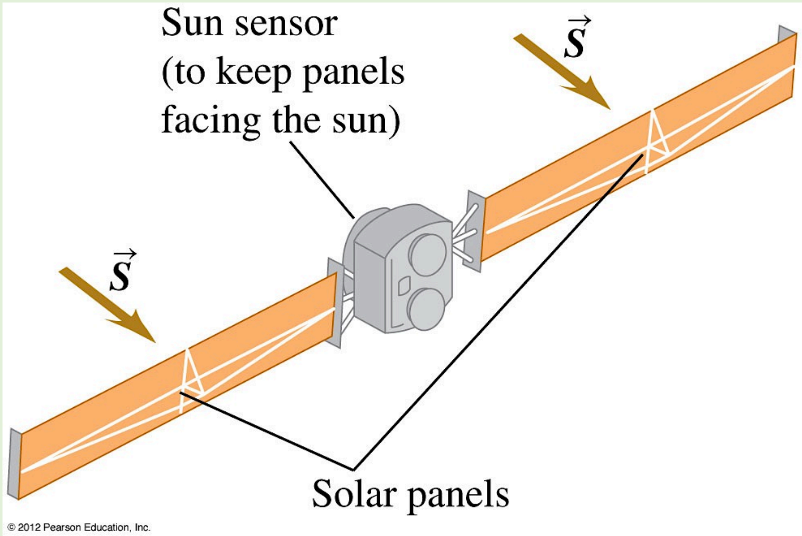
$$p_{\mathrm{rad}} = \frac{2S_{\mathrm{av}}}{c} = \frac{2I}{c}.$$

- For example, for direct sunlight, I = 1.4 kW/m^2 (approx.), average pressure on a completely absorbing surface is

$$p_{\mathrm{rad}} = \frac{I}{c} = \frac{1.4\times 10^{3}\ \mathrm{W/m^{2}}}{3.0\times 10^{8}\ \mathrm{m/s}} = 4.7\times 10^{-6}\ \mathrm{Pa}.$$

Example 32.5
Power and pressure from sunlight

An earth-orbiting satellite has solar energy–collecting panels with a total area of 4.0 m^2 (Fig. 32.21). If the sun's radiation is perpendicular to the panels and is completely absorbed, find the average solar power absorbed and the average radiation-pressure force.



Example 32.5
Power and pressure from sunlight

EXECUTE: The intensity I (power per unit area) is $1.4 \times 10^3\ \mathrm{W/m^2}$. Although the light from the sun is not a simple sinusoidal wave, we can still use the relationship that the average power P is the intensity I times the area A:

$$P = IA = (1.4 \times 10^3\ \mathrm{W/m^2})(4.0\ \mathrm{m^2})$$
$$= 5.6 \times 10^3\ \mathrm{W} = 5.6\ \mathrm{kW}$$

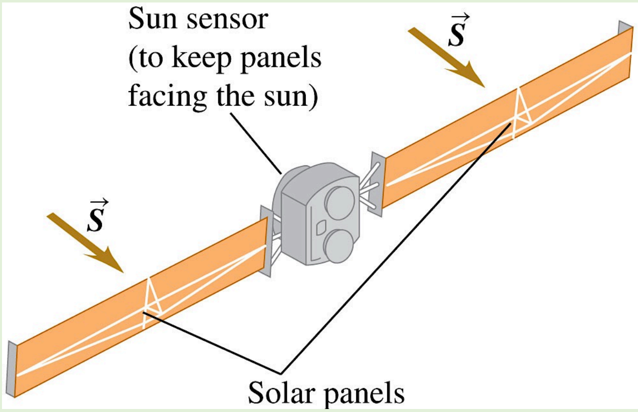
The radiation pressure of sunlight on an absorbing surface is $p_{\mathrm{rad}} = 4.7 \times 10^{-6}\ \mathrm{Pa} = 4.7 \times 10^{-6}\ \mathrm{N/m^2}$. The total force F is the pressure p_{rad} times the area A:

$$F = p_{\mathrm{rad}}A = (4.7 \times 10^{-6}\ \mathrm{N/m^2})(4.0\ \mathrm{m^2}) = 1.9 \times 10^{-5}\ \mathrm{N}$$

Example 32.5
Power and pressure from sunlight

EVALUATE: The absorbed power is quite substantial. Part of it can be used to power the equipment aboard the satellite; the rest goes into heating the panels, either directly or due to inefficiencies in the photocells contained in the panels.

The total radiation force is comparable to the weight (on earth) of a single grain of salt. Over time, however, this small force can have a noticeable effect on the orbit of a satellite like that in Fig. 32.21, and so radiation pressure must be taken into account.



Standing Electromagnetic Waves

- An EM wave traveling in –ve x direction incident on a perfect conductor in the yz plane
- On conductor surface, $E_{\parallel} = 0$ (equipotential surface), how to achieve this?
- Charges on conductor surface accelerated by incident field, themselves radiate and produce a reflected wave
- Incident and reflected $\vec{\boldsymbol{E}}$ cancel on conductor surface to make sure $E_{\parallel} = 0$

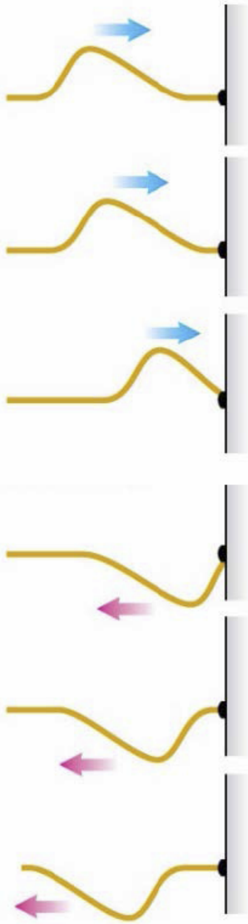
Incident wave

$$\vec{\boldsymbol{E}}(x,t) = \hat{\boldsymbol{j}}\, E_{\max} \cos(kx + \omega t)$$
$$\vec{\boldsymbol{B}}(x,t) = -\hat{\boldsymbol{k}}\, B_{\max} \cos(kx + \omega t)$$

Reflected wave

$$\vec{\boldsymbol{E}}(x,t) = -\hat{\boldsymbol{j}}\, E_{\max} \cos(kx - \omega t)$$
$$\vec{\boldsymbol{B}}(x,t) = -\hat{\boldsymbol{k}}\, B_{\max} \cos(kx - \omega t)$$

Analogy of a wave in a string with a fixed end

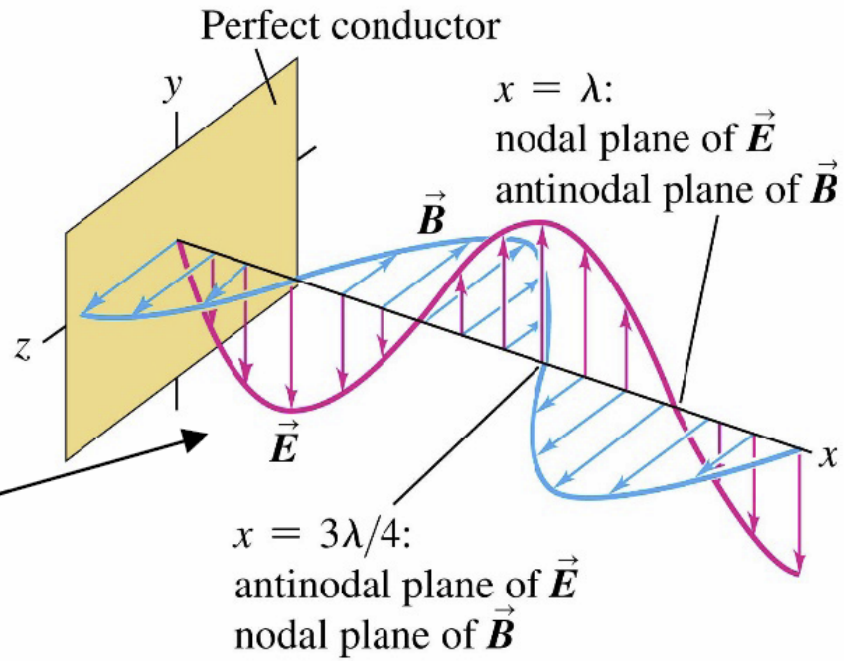


Superposition:

$$E_y(x,t) = E_{\max}[\cos(kx+\omega t) - \cos(kx-\omega t)]$$
$$B_z(x,t) = B_{\max}[-\cos(kx+\omega t) - \cos(kx-\omega t)]$$

Using $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$$\boxed{\begin{aligned} E_y(x,t) &= -2E_{\max} \sin kx \sin \omega t \\ B_z(x,t) &= -2B_{\max} \cos kx \cos \omega t \end{aligned}}$$



⚠ The superimposed $\vec{E}$ and $\vec{B}$ are *not* in phase. At a fixed instant t

$$E_y(x,t) = -2E_{\max} \sin kx \sin \omega t = -2E_{\max} \cos(kx - 90°) \sin \omega t$$

Phase difference is 90°, i.e., **nodal planes** of $\vec{E}$ are **antinodal planes** of $\vec{B}$ and vice versa

where wave is zero   where wave at max. amplitude

⚠ These E_y and B_z also satisfy the EM wave equations, even though they are not in phase

Standing waves in a cavity

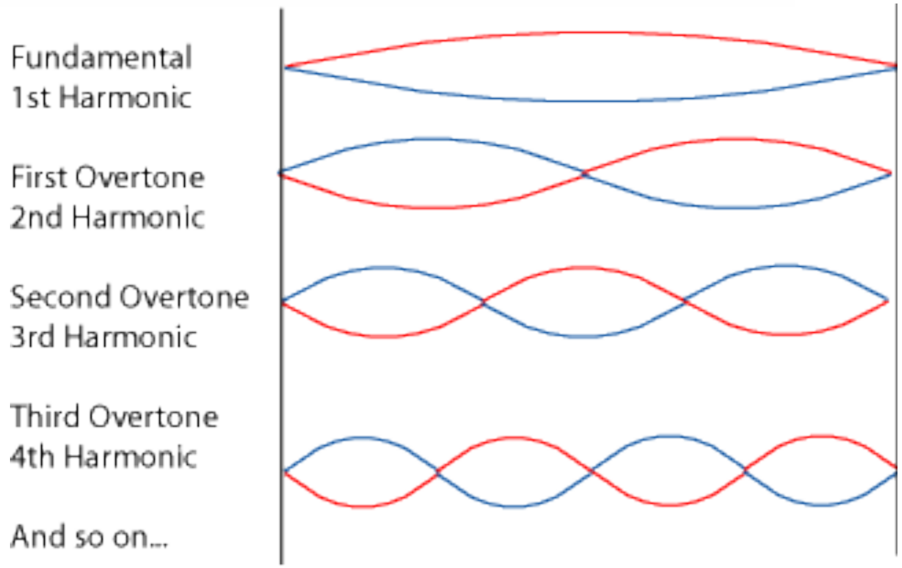
If put another conductor at $x = L$, those with wavelength λ_n form *standing wave*, where

$$L = n\frac{\lambda_n}{2}, \quad n = 1, 2, 3, \ldots$$

The corresponding frequencies are

$$f_n = \frac{c}{\lambda_n} = n\frac{c}{2L}$$

Application: a typical microwave oven sets up a standing wave with $\lambda = 12.2$ cm. This EM wave is in the microwave region and is strongly absorbed by water in food.

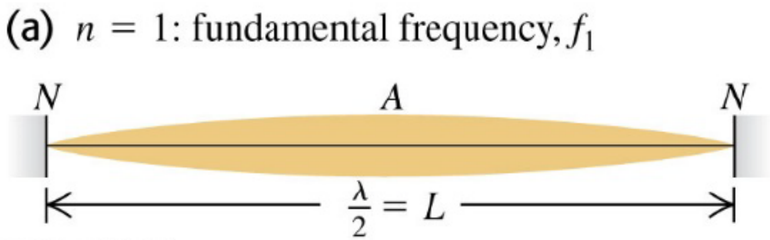


Example 32.7

A cavity has two parallel conducting walls at 1.50 cm apart.

The standing wave with longest wavelength has $\lambda = 2L = 3.00$ cm, corresponding frequency is

$$f = \frac{c}{2L} = \frac{3.00 \times 10^{8}\ \mathrm{m/s}}{3.00 \times 10^{-2}\ \mathrm{m}} = 1.00 \times 10^{10}\ \mathrm{Hz}$$



Since $n = 1$, $\vec{E}$ has nodes (zero) at the two walls and, and antinodes (maximum) at midway between the walls 0.750 cm

$\vec{B}$ is 90° out of phase with $\vec{E}$, therefore zero when $\vec{E}$, and maximum when $\vec{E}$ is zero

Final Comment

A sequence of events that leads to the birth of modern physics:

Faraday: A brilliant experimentalist with strong physical intuition. E field as a way to visualize force, but lacked the theoretical and mathematical background

→ mathematical instinct →

Maxwell: E field is real, and gave a unified mathematical description for E and B fields

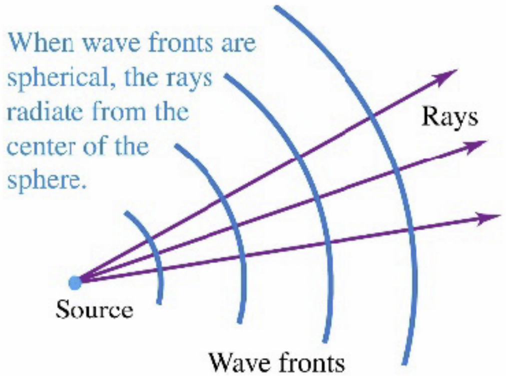
→ physical instinct →

Einstein: What if we ride on an EM wave? → special theory of relativity → unified theory for electricity and magnetism

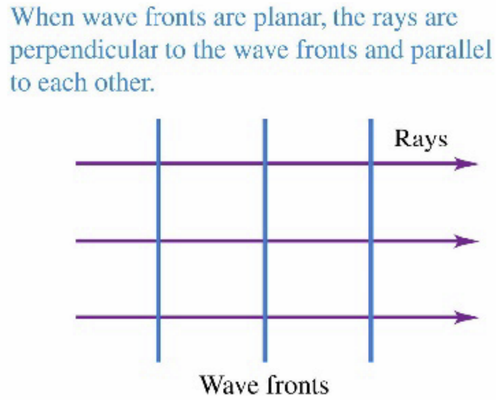
The Nature and propagation of light

Light as Traverse Wave

Spherical wave:



Plane wave:

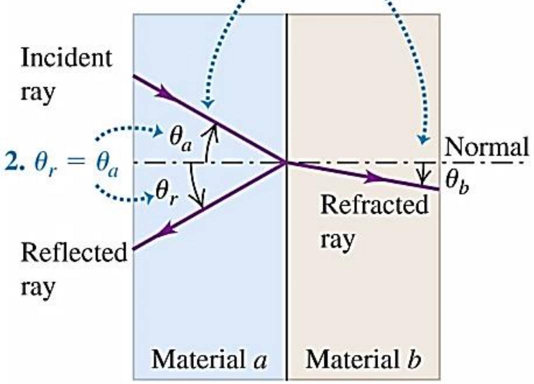


Why is it traverse?

Laws of Reflection and Refraction

1. The incident, reflected, and refracted rays and the normal to the surface all lie in the same plane.

Angles θ_a, θ_b, and θ_r are measured *from the normal*.

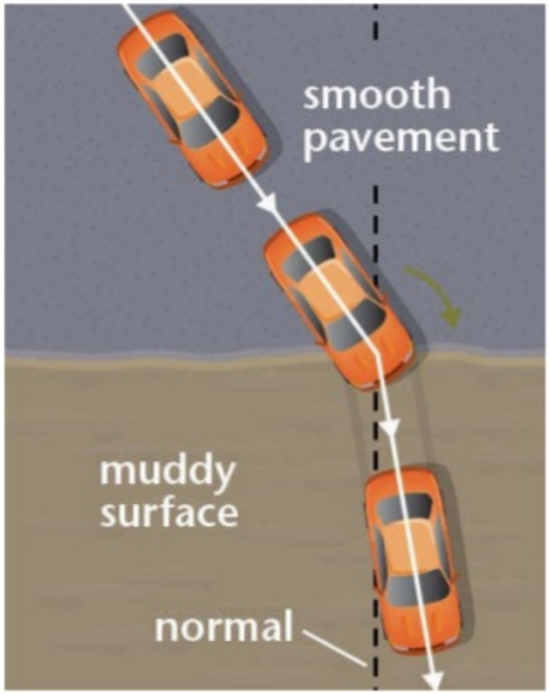


- **Incident angle =** θ_a
- **Reflected angle =** θ_r
- **Refracted angle =** θ_b

Snell's law:

3. When a monochromatic light ray crosses the interface between two given materials a and b, the angles θ_a and θ_b are related to the indexes of refraction of a and b by

$$\frac{\sin\theta_a}{\sin\theta_b} = \frac{n_b}{n_a}$$



Total Internal Reflection

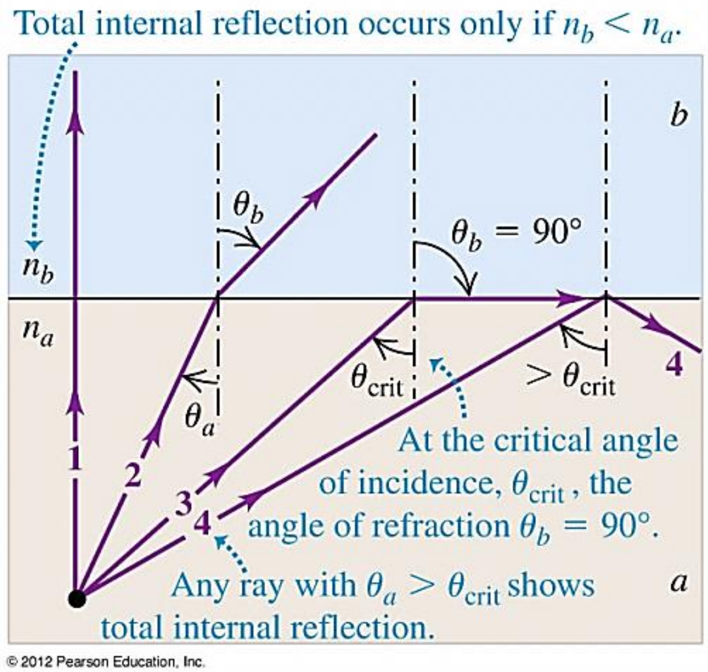
- **From Snell's law,**

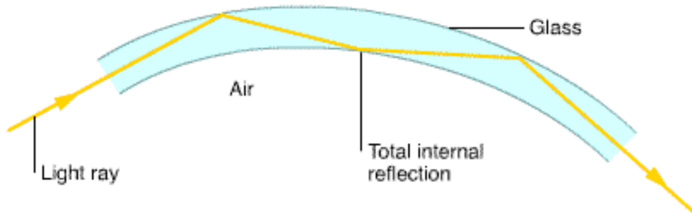
$$\sin\theta_b = \frac{n_a}{n_b}\sin\theta_a .$$

When $\theta_b = 90°$,

$\theta_a = \theta_{\text{crit}}$:

$$\sin\theta_{\text{crit}} = \frac{n_b}{n_a} .$$





Light trapped by water (~ optical fiber)

https://youtu.be/Z9O5xY3Z1WE

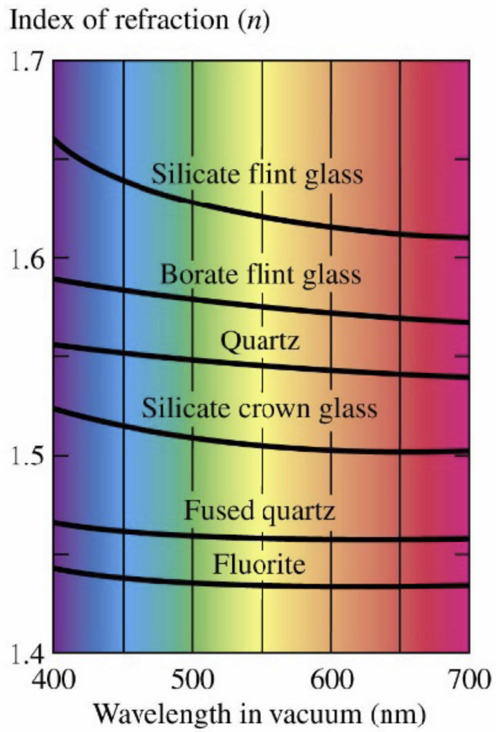
Dispersion

$$1 < n_{\text{red}} < n_{\text{yellow}} < n_{\text{blue}}$$

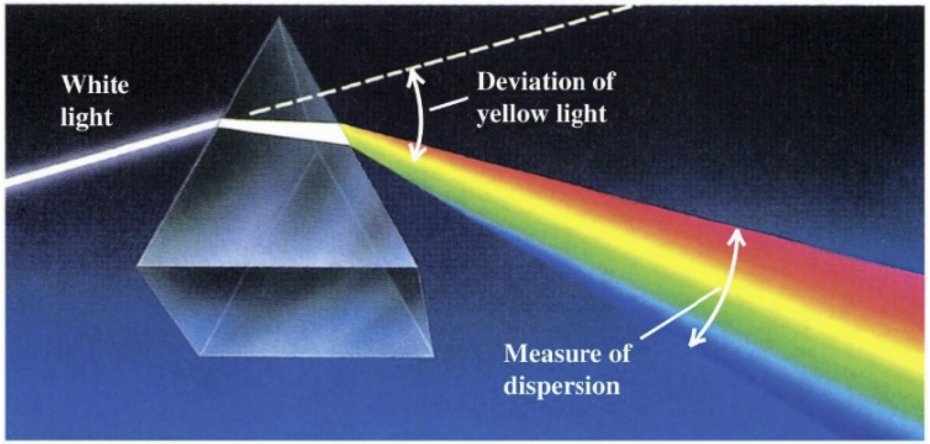
As EM wave enter a material through a boundary:

1. Frequency doesn't change (number of cycles entering the boundary per unit time same as going out)
2. Speed smaller, $v < c$, therefore wavelength shorter, $\lambda < \lambda_0$

$$\boxed{n = \frac{c}{v} = \frac{\lambda_0}{\lambda}} \quad \text{refractive index}$$



Usually n larger for smaller λ_0. When white light (even mixture of all colors) enter a boundary, blue (smaller λ_0) bends more than red → **chromatic dispersion**



⚠ Light of single wavelength entering a boundary shows no dispersion, called **monochromatic**

Clicker Questions

Q33.1

When light passes from vacuum (index of refraction $n = 1$) into water ($n = 1.333$),

A. the wavelength increases and the frequency is unchanged.

B. the wavelength decreases and the frequency is unchanged.

C. the wavelength is unchanged and the frequency increases.

D. the wavelength is unchanged and the frequency decreases.

E. both the wavelength and the frequency change.

A33.1

When light passes from vacuum (index of refraction $n = 1$) into water ($n = 1.333$),

A. the wavelength increases and the frequency is unchanged.

✔ B. the wavelength decreases and the frequency is unchanged.

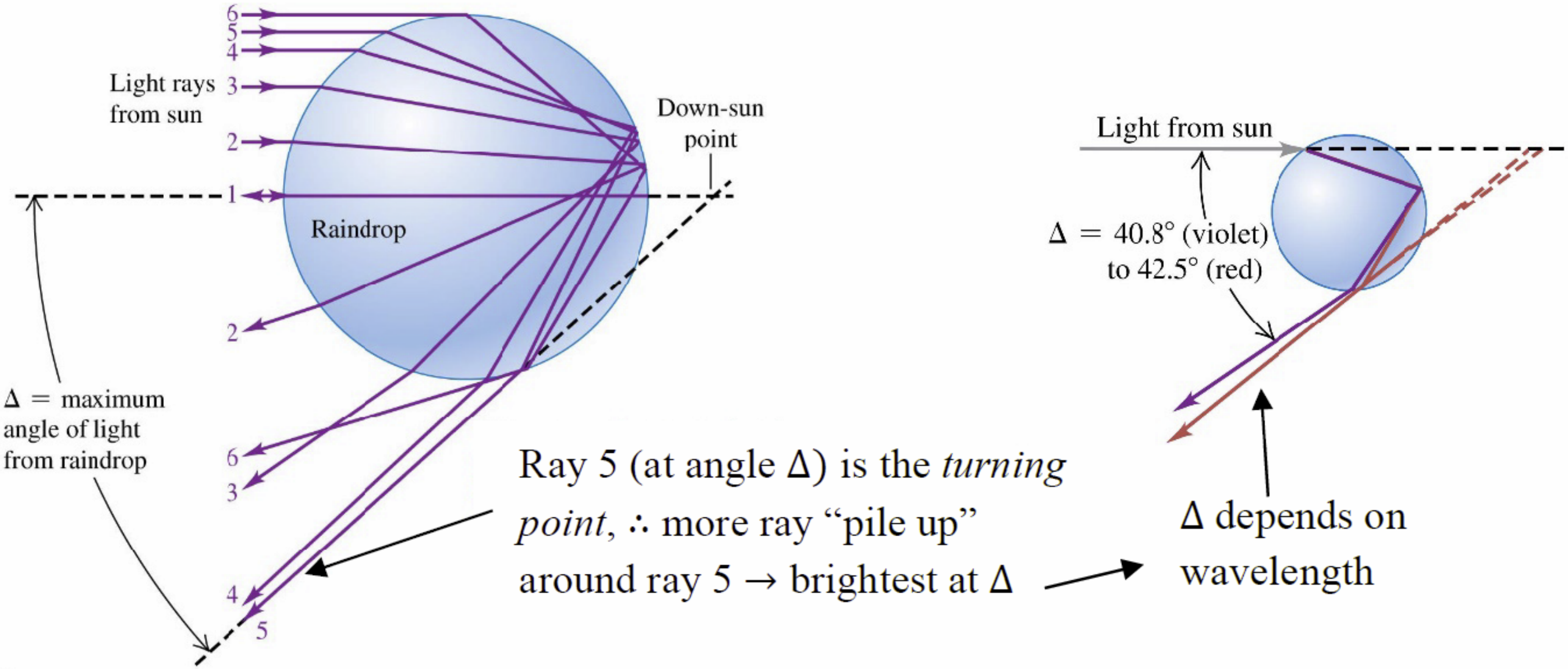
C. the wavelength is unchanged and the frequency increases.

D. the wavelength is unchanged and the frequency decreases.

E. both the wavelength and the frequency change.

Formation of Rainbow

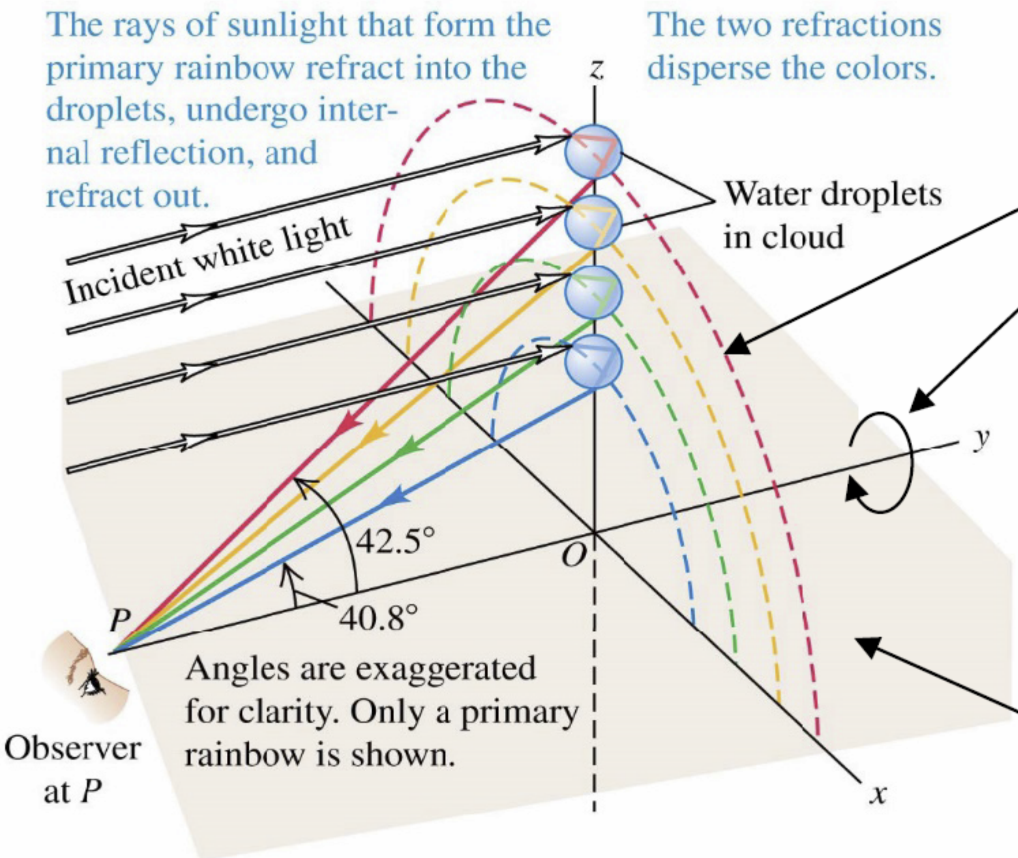
(b) The paths of light rays entering the upper half of a raindrop



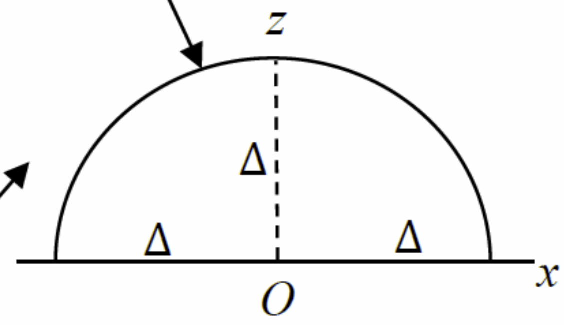
Ray 5 (at angle Δ) is the *turning point*, ∴ more ray "pile up" around ray 5 → brightest at Δ

Δ depends on wavelength

(c) Forming a rainbow. The sun in this illustration is directly behind the observer at P.

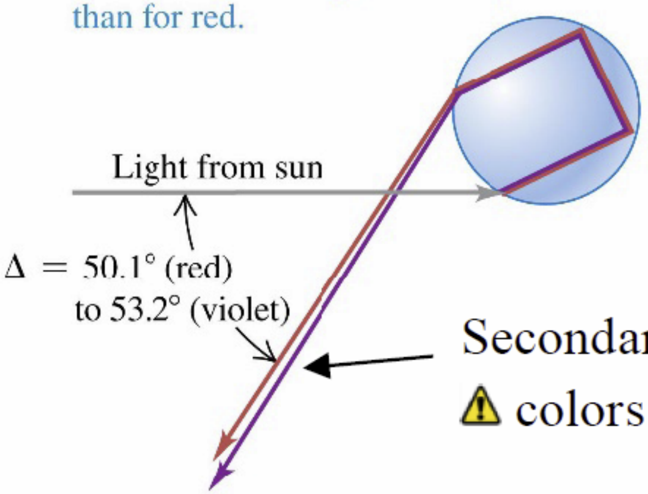


Axial symmetry creates a semi-circular disk of bright region with "angular radius" Δ, *brightest at the edge*



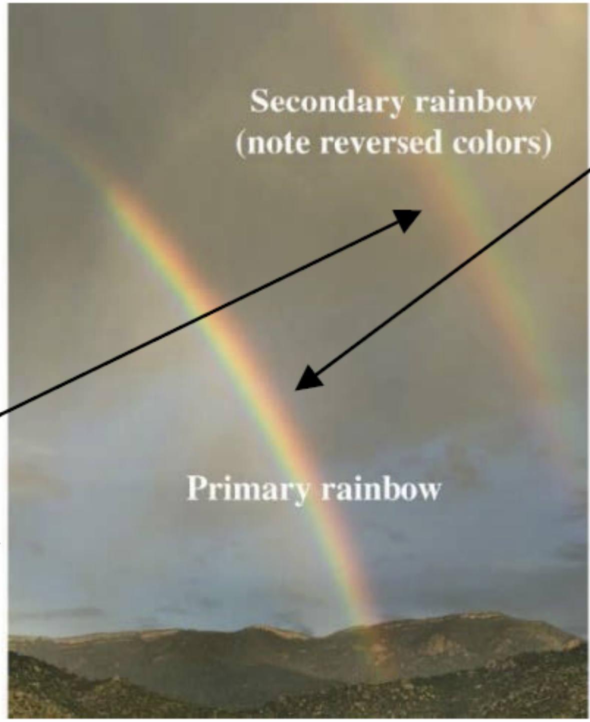
Region outside disk, droplets cannot send light to your eyes, $\therefore$ appears darker

(e) A secondary rainbow is formed by rays that undergo two refractions and *two* internal reflections. The angle Δ is larger for violet light than for red.



Secondary rainbow

⚠ colors reversed



Darker outside bright disk

⚠ rainbow is *not* an object. Different people see *different* rainbows depending on location and conditions

Polarization

An EM wave is **linearly polarized** if $\vec{\boldsymbol{E}}$ is along one direction only, called the **polarization direction**, e.g. linearly polarized in the *y* direction:

$$\vec{\boldsymbol{E}}(x,t) = \hat{\boldsymbol{j}}\, E_{\text{max}} \cos(kx - \omega t)$$

$$\vec{\boldsymbol{B}}(x,t) = \hat{\boldsymbol{k}}\, B_{\text{max}} \cos(kx - \omega t)$$

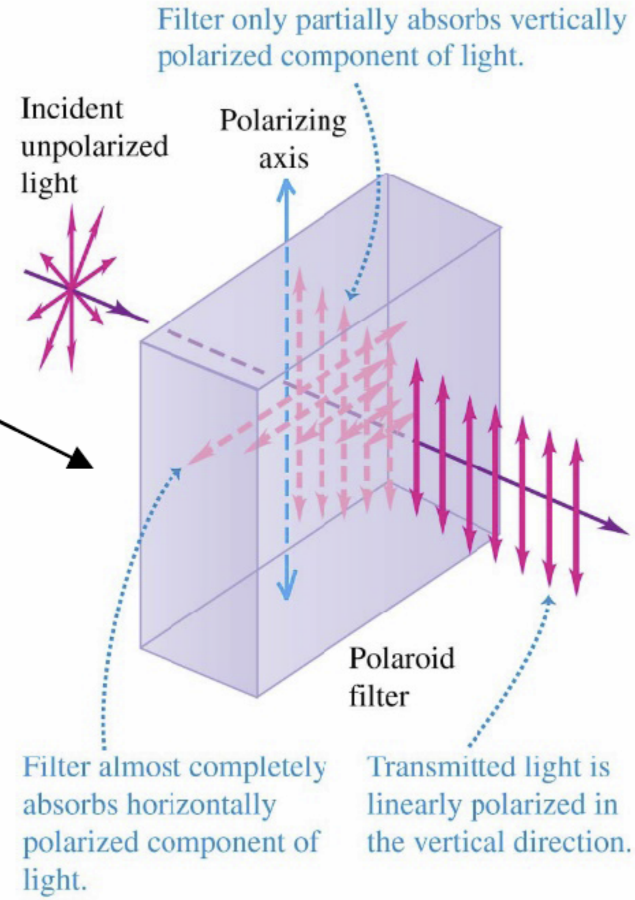
⚠ Polarization *always* refer to $\vec{\boldsymbol{E}}$, *not* $\vec{\boldsymbol{B}}$ because comment EM wave detectors respond to electric, not magnetic field

⚠ Usually light source produces many waves with $\vec{\boldsymbol{E}}$ in random directions, **unpolarized**

Polarization by Polarizing Filters

A **dichroic material** only allows component of $\vec{E}$ along its **polarizing axis** to pass through. Emerging light is linearly polarized.

Called a polarizing filter, or **polarizer**, e.g., Polaroid

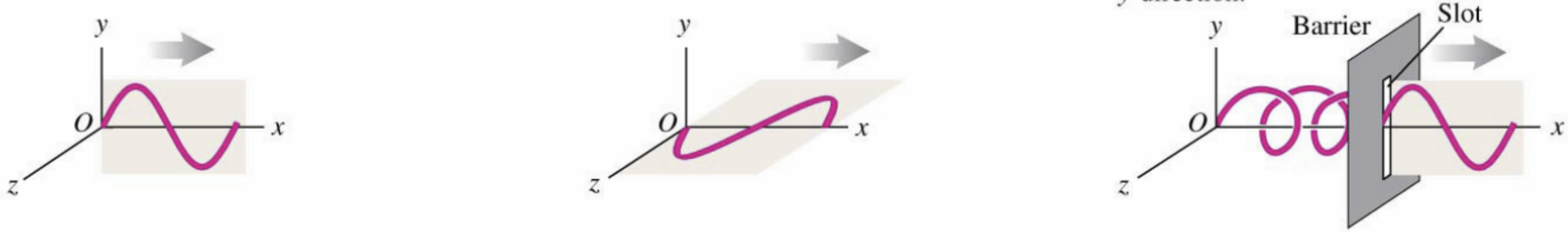


Analogy of a slot allowing one linearly polarized wave in a string to go through

(a) Transverse wave linearly polarized in the y-direction

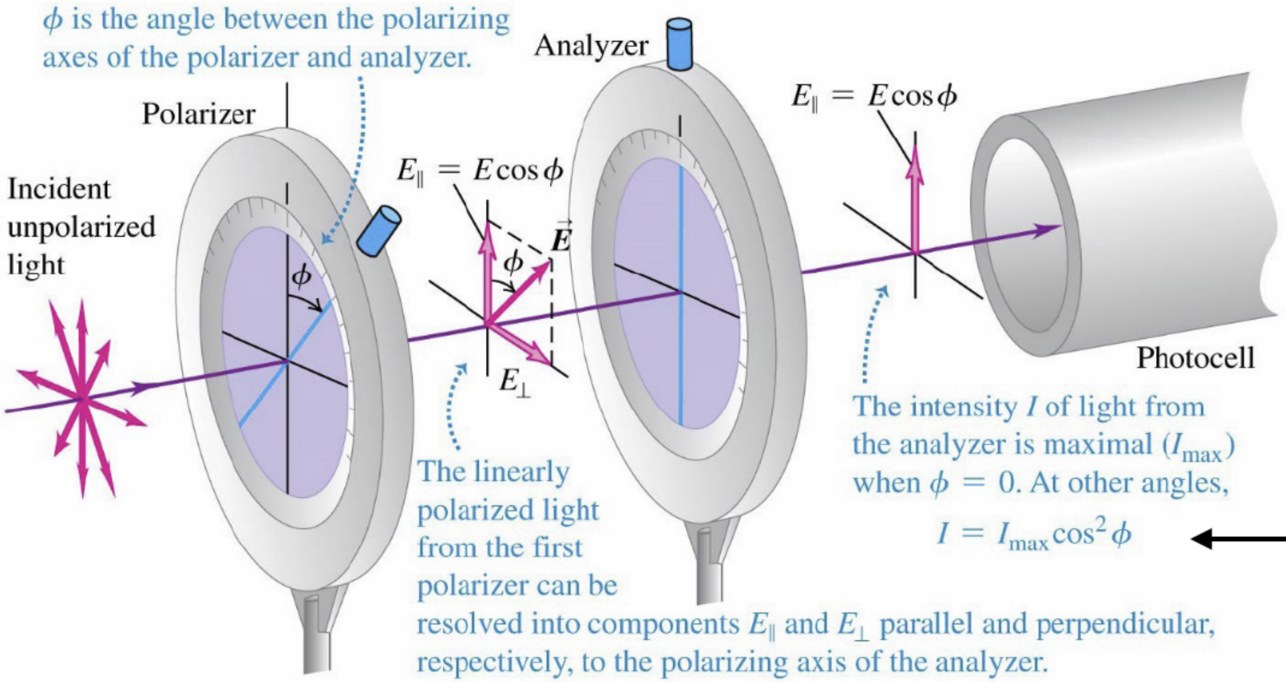
(b) Transverse wave linearly polarized in the z-direction

(c) The slot functions as a polarizing filter, passing only components polarized in the y-direction.



Intensity $I \propto |\vec{E}|^2 = (E_y^2 + E_z^2)$. Unpolarized light ($E_y = E_z$), after passing through an ideal polarizer, I reduced by 50%

Light passes through two consecutive polarizers, second one is also called an **analyzer**



Malus's law

$$\boxed{I = I_{max}\cos^2\phi}$$

Linearly polarized light emerge from polarizer, intensity cut off by 50%

Q33.5

Three polarizing filters are stacked with the polarizing axes of the second and third filters oriented at 45° and 90°, respectively, relative to the polarizing axis of the first filter. Unpolarized light of intensity I_0 is incident on the first filter. The intensity of light emerging from the third filter is

A. I_0.

B. $I_0/\sqrt{2}$.

C. $I_0/2$.

D. $I_0/4$.

E. $I_0/8$.

A33.5

Three polarizing filters are stacked with the polarizing axes of the second and third filters oriented at 45° and 90°, respectively, relative to the polarizing axis of the first filter. Unpolarized light of intensity I_0 is incident on the first filter. The intensity of light emerging from the third filter is

A. I_0.

B. $I_0/\sqrt{2}$.

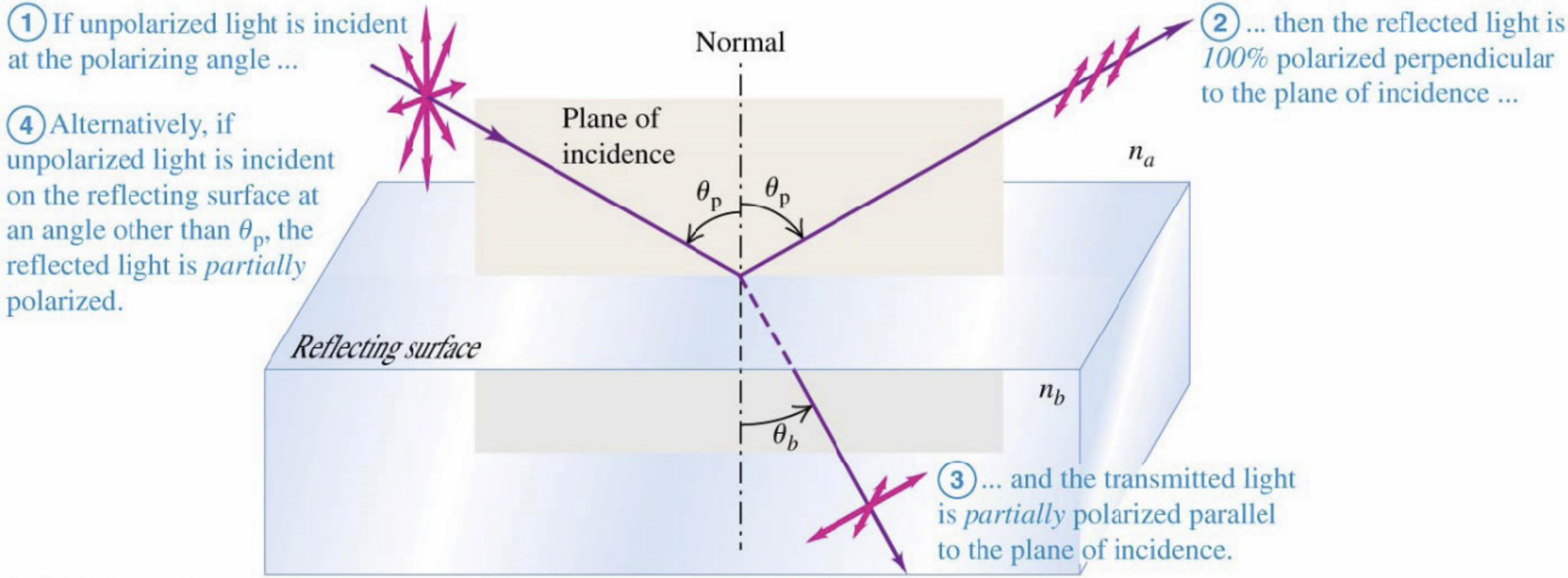
C. $I_0/2$.

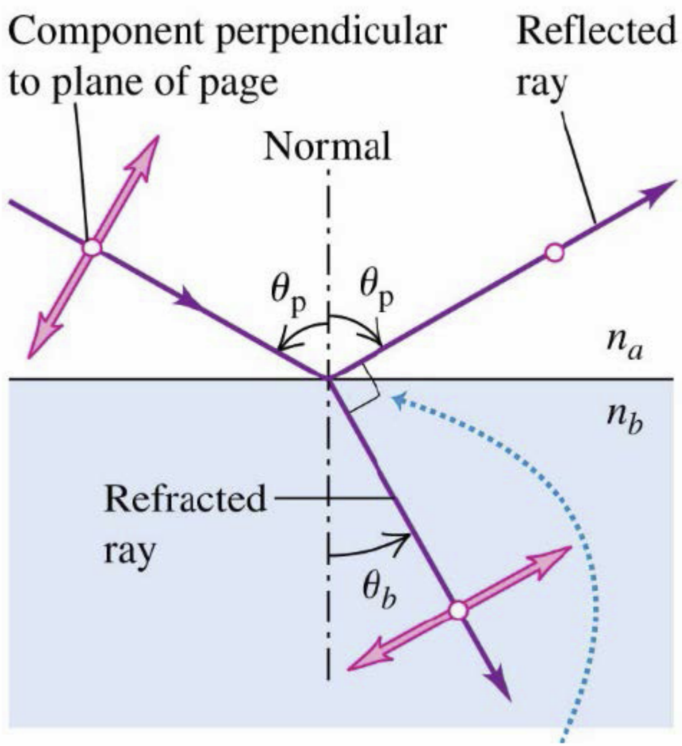
D. $I_0/4$.

✔ E. $I_0/8$.

Polarization by Reflection

On a planar surface, reflection favors polarization ⊥ to the incident plane, refraction favors ∥





Brewster's law:

When incident angle is at the **polarizing angle** θ_p such that reflected and refracted rays are ⊥, reflected ray is linearly polarized ⊥ to the incident plane

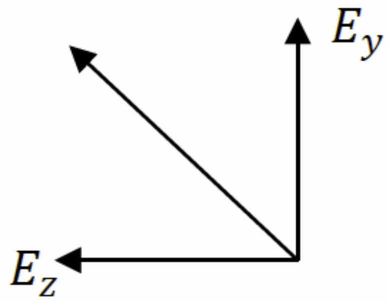
From Snell's law, $n_a \sin\theta_p = n_b \sin\theta_b = n_b \cos\theta_p$

$$\therefore \quad \boxed{\tan\theta_p = \frac{n_b}{n_a}}$$

⚠ Strong sunlight reflected from water surface, should wear sunglasses with polarization axis in the (horizontal / vertical) direction

Circular and Elliptical Polarization

If two linearly polarized light are in phase, add up to another linearly polarized light, e.g.



$$\vec{\boldsymbol{E}}_1 = \hat{\boldsymbol{j}} E_y \cos(kx - \omega t)$$

$$\vec{\boldsymbol{E}}_2 = \hat{\boldsymbol{k}} E_z \cos(kx - \omega t)$$

$$\vec{\boldsymbol{E}} = \vec{\boldsymbol{E}}_1 + \vec{\boldsymbol{E}}_2 = (E_y \hat{\boldsymbol{j}} + E_z \hat{\boldsymbol{k}}) \cos(kx - \omega t)$$

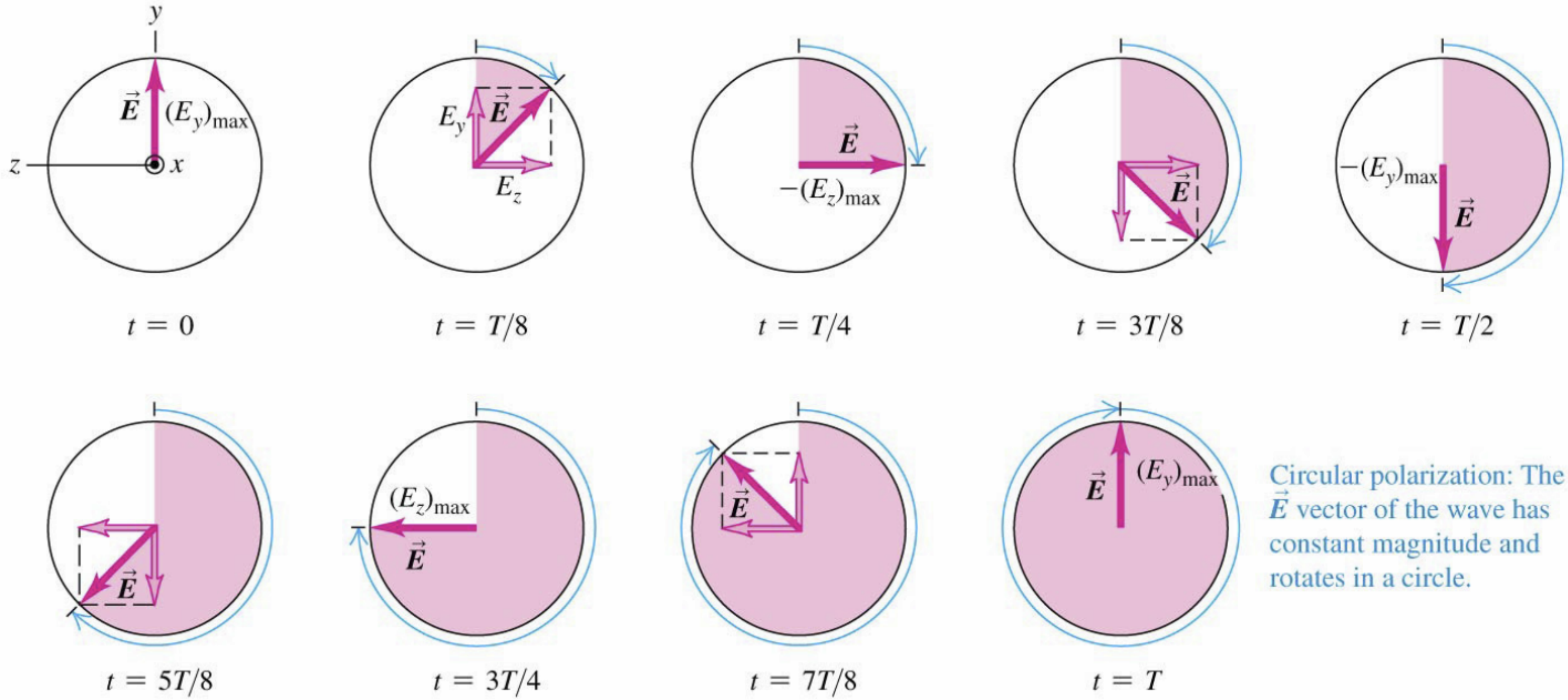
If $E_y = E_z$, polarization direction is at $45°$

But if they are 90° (quarter cycle) out of phase, add up to **circular polarized light** if $E_y = E_z$, e.g. $\vec{\boldsymbol{E}}_1$ *lags* $\vec{\boldsymbol{E}}_2$ by 90°

$$\vec{\boldsymbol{E}}_1 = \hat{\boldsymbol{j}} E_y \cos(kx - \omega t)$$

$$\vec{\boldsymbol{E}}_2 = \hat{\boldsymbol{k}} E_z \cos(kx - \omega t + 90°)$$

$\vec{\boldsymbol{E}}$ traces out a circle in clockwise direction when viewed from the front



Circular polarization: The $\vec{E}$ vector of the wave has constant magnitude and rotates in a circle.

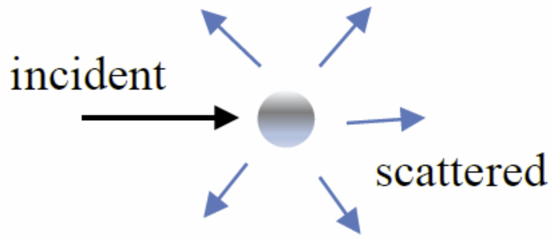
See https://www.youtube.com/watch?v=Fu-aYnRkUgg

- ⚠ If $\vec{\boldsymbol{E}}_1$ *leads* $\vec{\boldsymbol{E}}_2$ by 90°, trace out a circle in *counterclockwise* direction when viewed from the front
- ⚠ If $E_y \neq E_z$, traces out an ellipse instead of a circle, called **elliptical polarization**

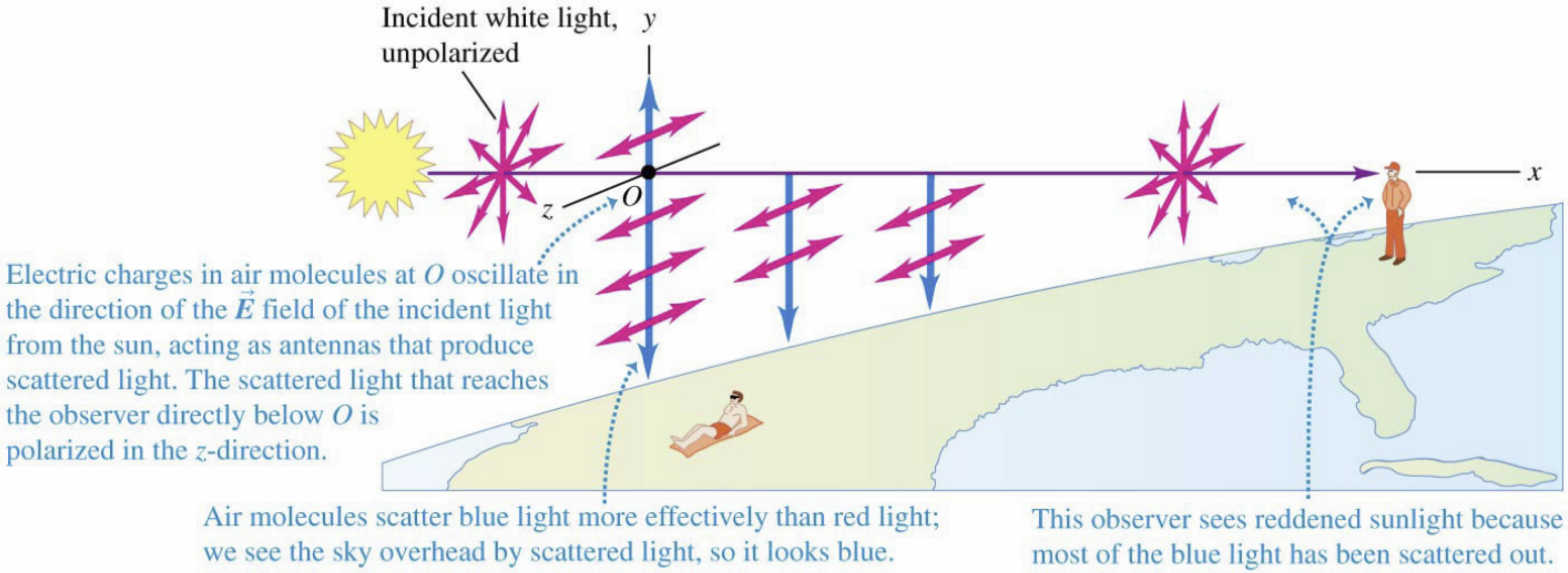
Scattering of Light

Scattering means when light hits a molecule:

1. Incident field sets charge of molecule in motion (absorption)
2. Charge in turns emit radiation in different directions (re-radiation)



Some atmospheric phenomena:

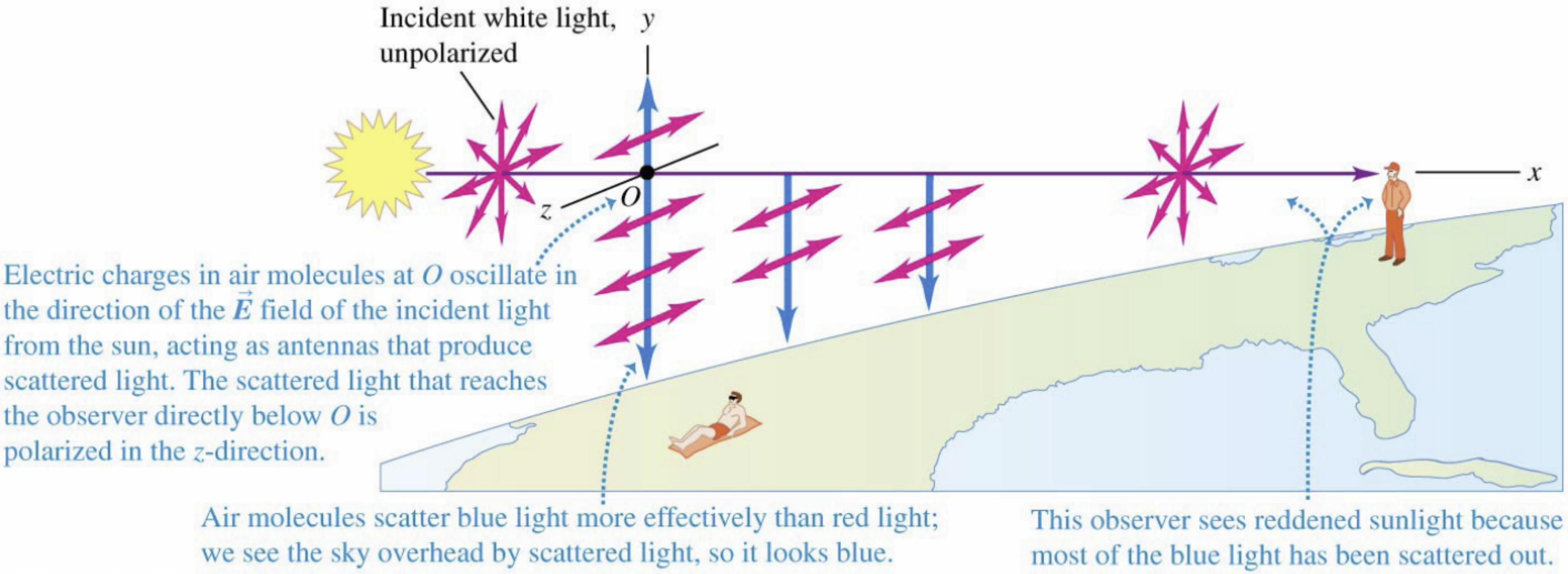


1. Why is the sky bright in daytime?
   Light from sun scattered by air molecule before reaching us.

2. Why is sunlight partially polarized?
   Sunlight scattered downwards from O cannot have polarization along y direction.

3. Why is sky blue in daytime?
   Light scattered by air molecule strongly favors blue, leaving red light going through unscattered.

Some atmospheric phenomena:

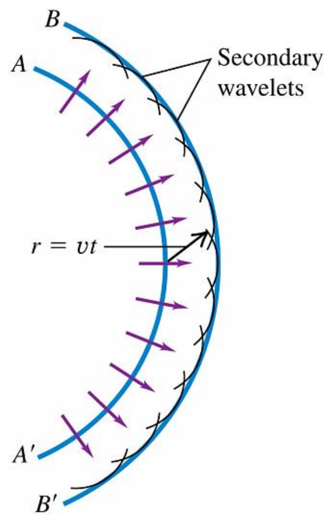


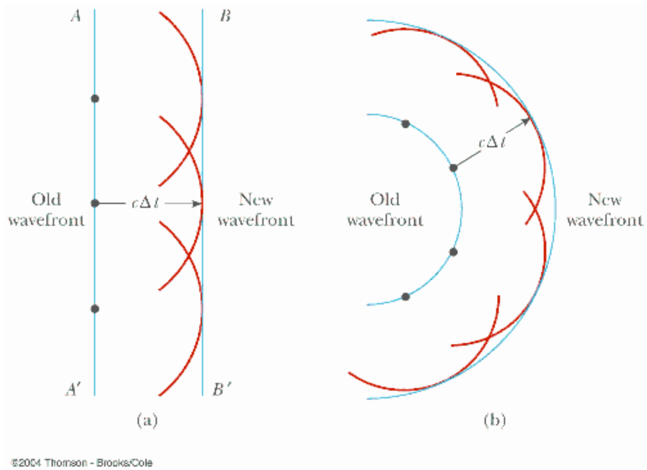
4. Why is sky reddish during sun rise/set?
   Sunlight travelled a long distance in atmosphere before reaching us, blue mostly scattered away, leaving reddish light.

5. Why is cloud (and milk too) white?
   Water droplets in cloud (colloids in milk) at high density scatter light of all wavelengths, mixing different colors into white.

Huygens's Principle

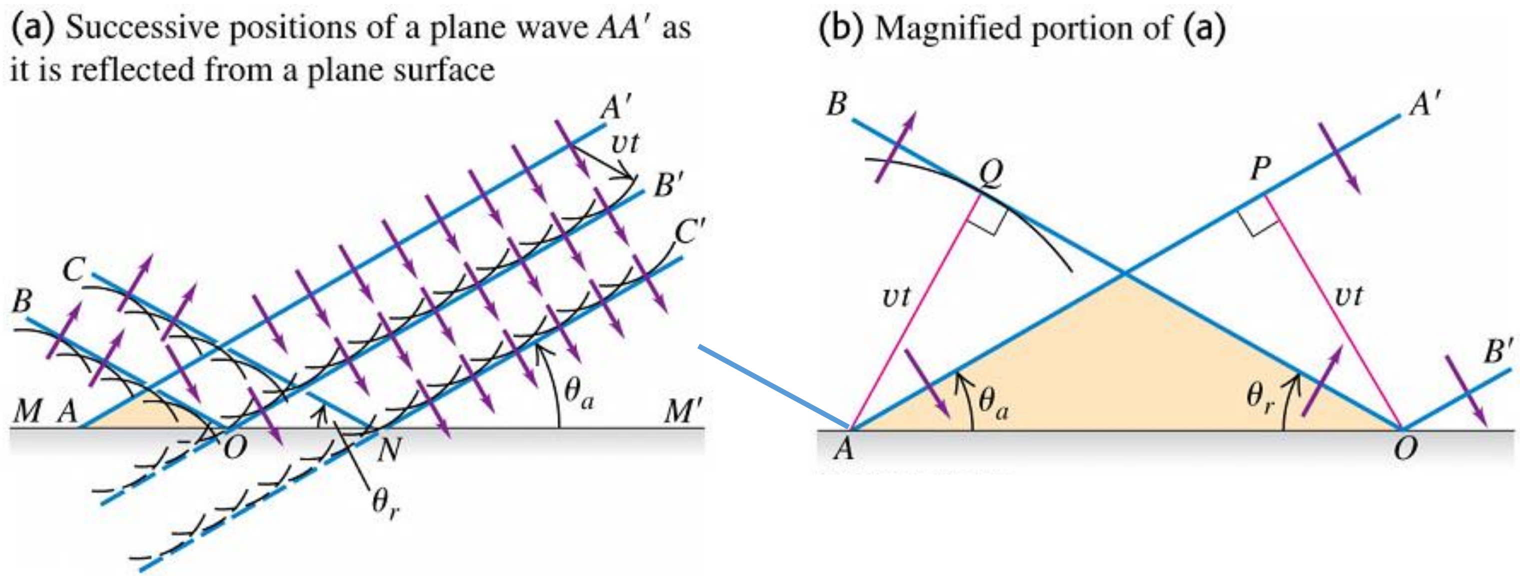
- A geometrical method for finding, from the known shape of a wave front at some instant, the shape of the wave front at some later time.
- Assume every point of a wave front may be considered the source of secondary wavelets that spread out in all directions with a speed equal to the speed of propagation of the wave.
- The new wave front is found by constructing a surface tangent to the secondary wavelets (envelope of wavelets).





https://www.youtube.com/watch?v=wENZmBofwSw

Reflection and Huygens's Principle



$$\boxed{\Delta AOP = \Delta OAQ \Rightarrow AP = OQ}$$

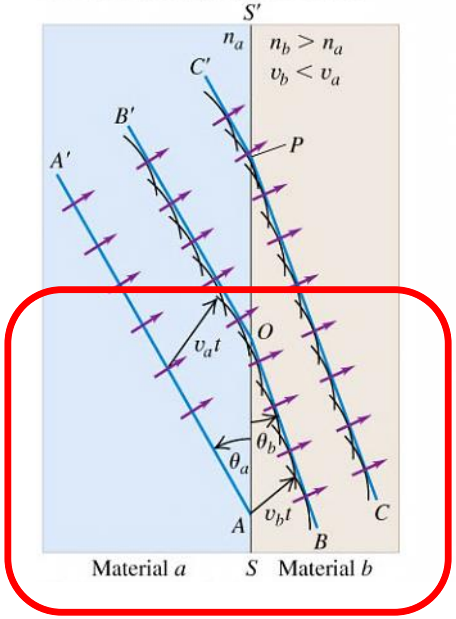
$$\overleftrightarrow{AP} = \overleftrightarrow{AO}\cos\theta_a,$$

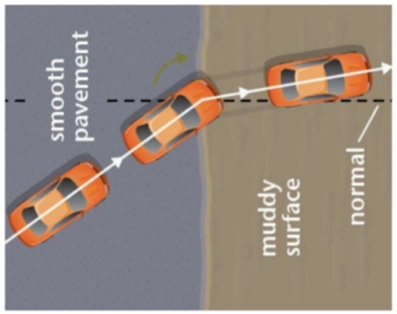
$$\overleftrightarrow{OQ} = \overleftrightarrow{AO}\cos\theta_r,$$

Therefore $\theta_a = \theta_r$

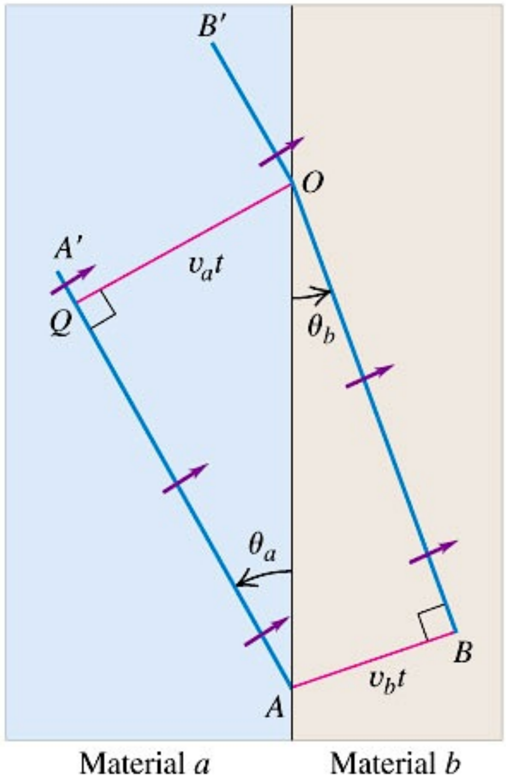
Refraction and Huygens's Principle

(a) Successive positions of a plane wave AA' as it is refracted by a plane surface





(b) Magnified portion of (a)



Refraction and Huygens's Principle

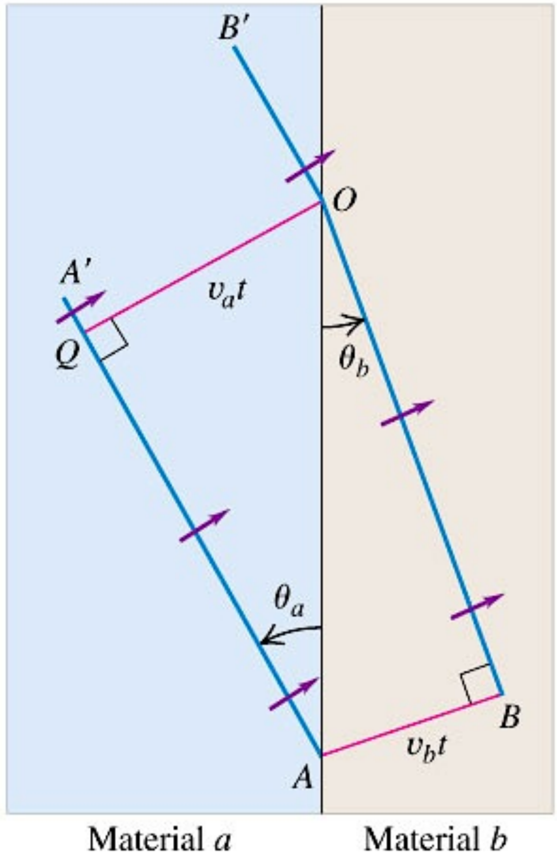
- **Derive Snell's law using Huygens's principle:**

$$\left.\begin{aligned}&\text{From } \Delta AOQ,\ \sin\theta_a = \frac{v_a t}{AO}\\ &\text{From } \Delta AOB,\ \sin\theta_b = \frac{v_b t}{AO}\end{aligned}\right\}$$

$$\Rightarrow \frac{\sin\theta_a}{\sin\theta_b} = \frac{v_a}{v_b} = \frac{n_b}{n_a}$$

$$\because \frac{n_b}{n_a} = \frac{c/v_b}{c/v_a} = \frac{v_a}{v_b}$$

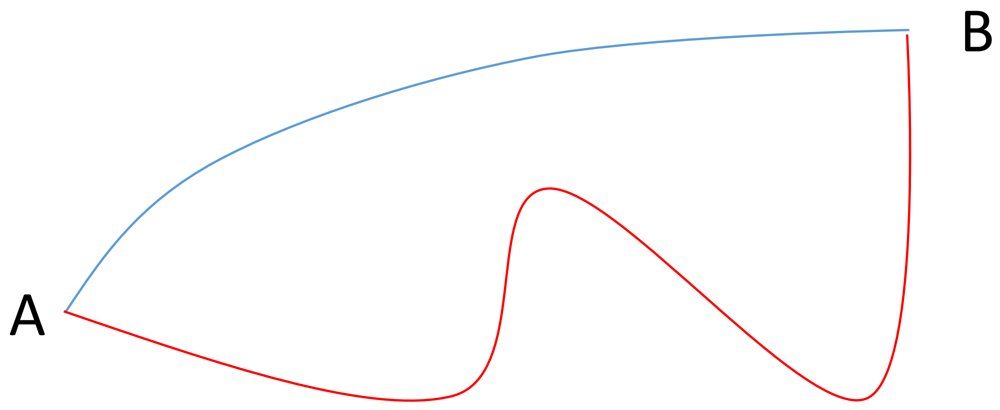
or $n_a \sin\theta_a = n_b \sin\theta_b$.



Fermat's principle

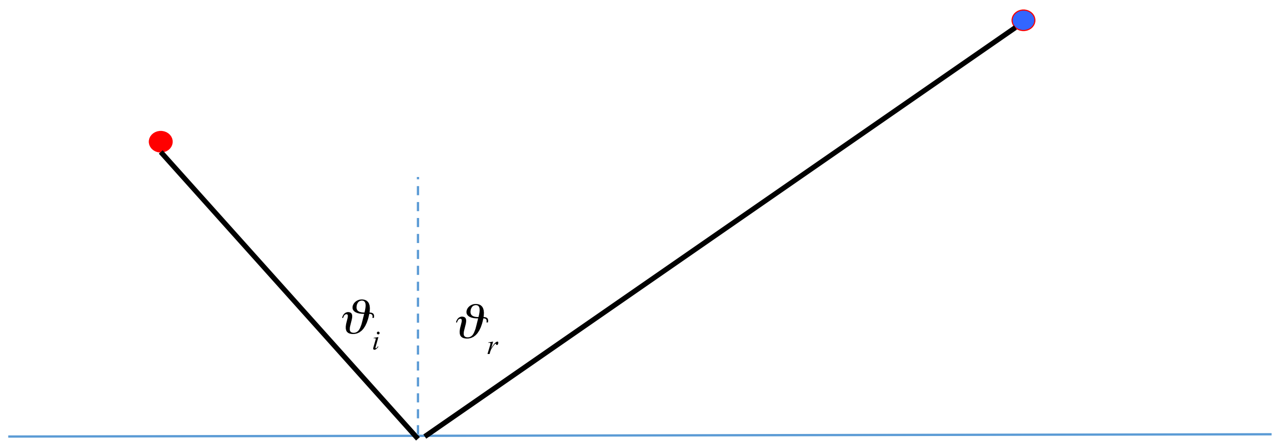
Principle of least time – path taken between two points by a ray of light is the path that can be traveled in the least time.

$$\boxed{T = \int_{t_0}^{t_1} dt = \frac{1}{c}\int_{t_0}^{t_1} \frac{c}{v}\frac{ds}{dt}\,dt = \frac{1}{c}\int_{A}^{B} n\,ds}$$



-> Light travels in a straight (shortest path) inside any homogeneous medium.

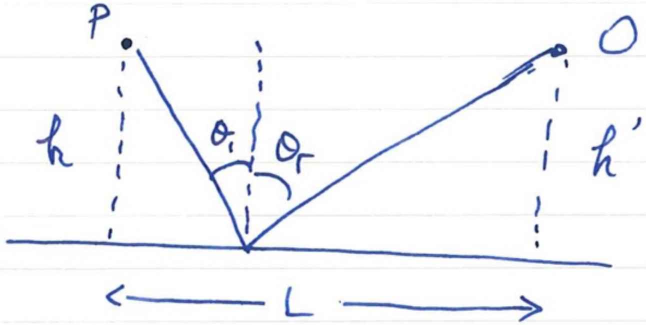
Example: (Law of Reflection)



$\theta_r = \theta_r(\theta_i)$.

1.) Reflection.

$$h \tan\theta_i + h' \tan\theta_r = L \quad — ①$$



$$T = \left(\frac{h}{\cos\theta_i} + \frac{h'}{\cos\theta_r}\right)\frac{1}{c} \quad — ②$$

From ①, $\rightarrow T = T(\theta_i)$

$$h \sec^2\theta_i + h' \sec^2\theta_r \frac{d\theta_r}{d\theta_i} = 0. \Rightarrow \frac{d\theta_r}{d\theta_i} = -\frac{h \cos^2\theta_r}{h' \cos^2\theta_i}. \quad — ③$$

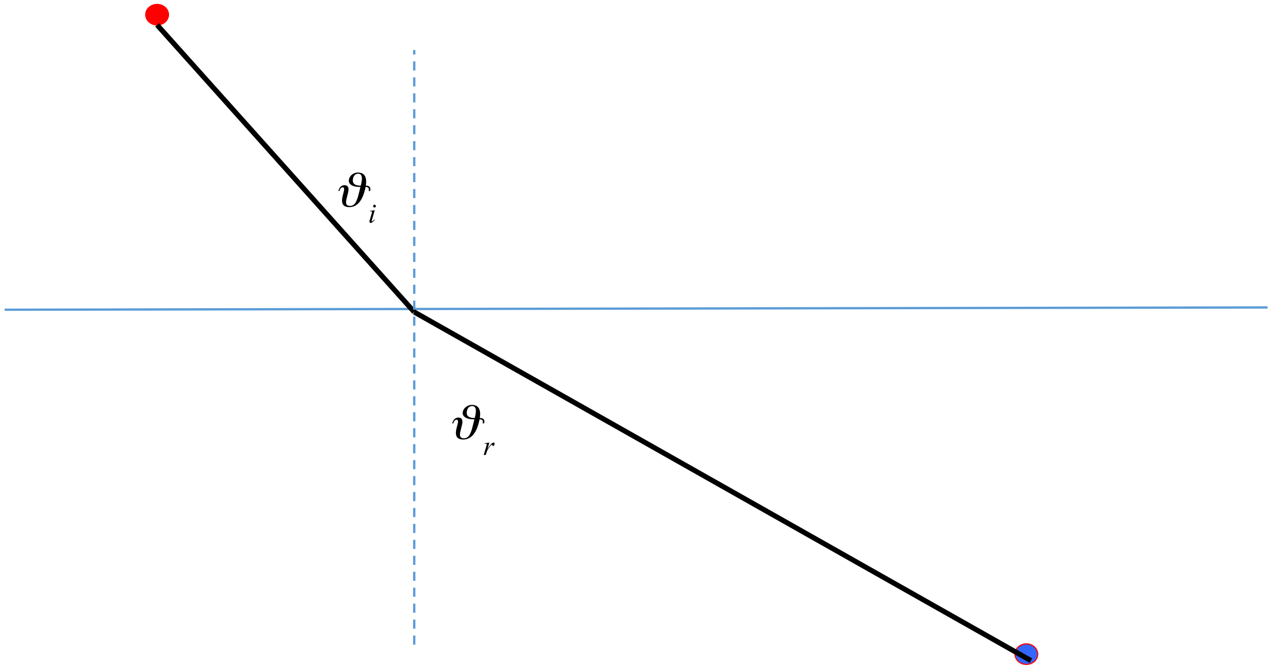
$$\frac{dT}{d\theta_i} = 0 \Rightarrow h \sec\theta_i \tan\theta_i + h' \sec\theta_r \tan\theta_r \left(\frac{d\theta_r}{d\theta_i}\right) = 0. \quad — ④$$

$$\Rightarrow h \sec\theta_i \tan\theta_i = + h' \sec\theta_r \tan\theta_r \left(\frac{h}{h'}\right) \frac{\cos^2\theta_r}{\cos^2\theta_i}$$

$$\Rightarrow h \cos\theta_i \tan\theta_i = h \cancel{\sec\theta_r} \cos\theta_r \tan\theta_r.$$

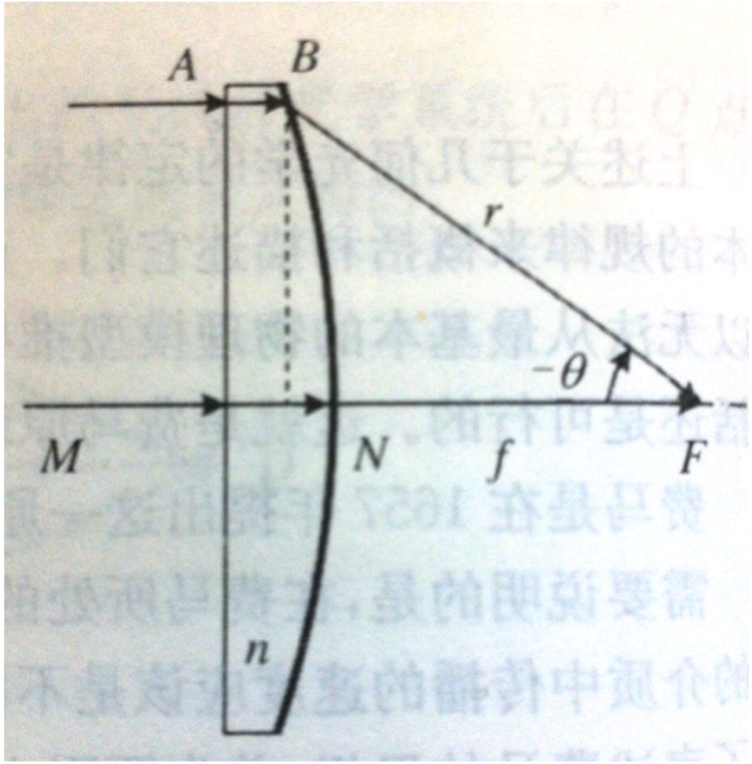
$$\Rightarrow \boxed{\theta_i = \theta_r.}$$ (law of reflection).

Example: (Law of Refraction)



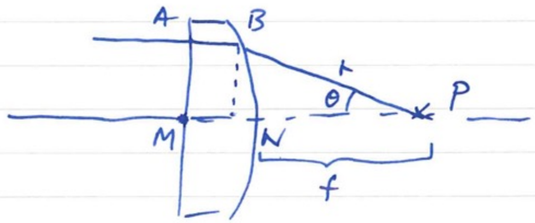
Exercise:

Design a planoconvex lens such that all light which are parallel to the optical axis which converge to a single point.



2. Planoconvex lens.

1. Along the ~~we~~ optical ~~axes~~ axis,



light path $= n\overline{MN} + NP$

$= n\overline{MN} + f.$

2. Along ABP,

light path $= n\overline{AB} + r$

Equating the two light path, we have.

$$n(r\cos\theta - f) + f = r$$

$$\Rightarrow \quad r = \frac{(n-1)f}{n\cos\theta - 1}$$

. (N.B. (r, θ) is the polar coordinate from the origin at P.)

∴ it is a ~~parabola!!~~ hyperbola!

$$r = \frac{C}{\cos\theta - a} \Rightarrow r\cos\theta - ra = C$$

$$\Rightarrow x - C = a\sqrt{x^2+y^2}.$$

$$\Rightarrow x^2 - 2Cx + C^2 = a^2x^2 + a^2y^2$$

~~$\neq Y$ ...~~

$$\Rightarrow (1-a^2)x^2 - 2Cx - a^2y^2 + C^2 = 0.$$

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

$$\Delta = B^2 - 4AC$$

$$= 0 - 4(1-a^2)(-a^2) > 0 \text{ is hyperbola !!}$$