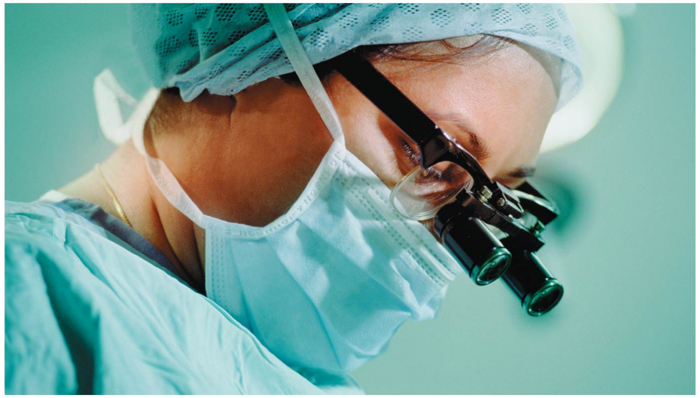
Geometric Optics

Learning Goals

Looking forward at …

- how a plane mirror forms an image, and why concave and convex mirrors form images of different kinds.
- how images can be formed by a curved interface between two transparent materials.
- what aspects of a lens determine the type of image that it produces.
- what causes various defects in human vision, and how they can be corrected.
- how microscopes and telescopes work.

Introduction



- This surgeon performing microsurgery needs a sharp, magnified view of the surgical site.
- To obtain this, she's wearing glasses with magnifying lenses.
- How do magnifying lenses work?
- How do lenses and mirrors form images?
- We shall use light rays to understand the principles behind optical devices such as camera lenses, the eye, microscopes, and telescopes.

Reflection at a plane surface

- Light rays from the object at point *P* are reflected from a plane mirror.
- The reflected rays entering the eye look as though they had come from image point *P'*

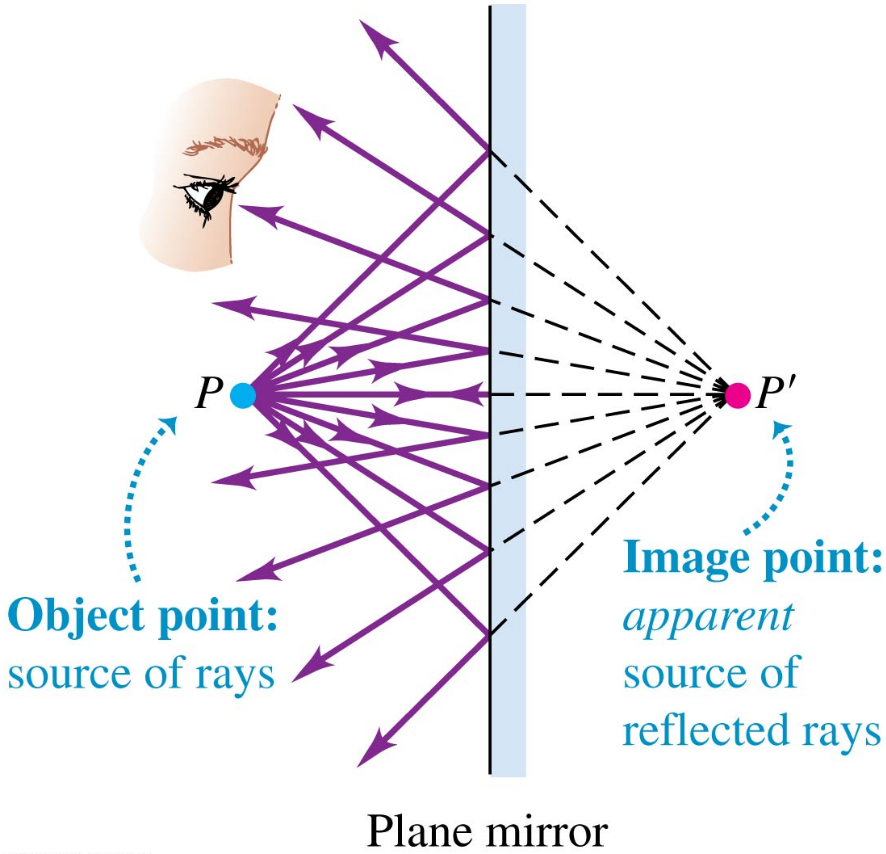


Image formation by a plane mirror

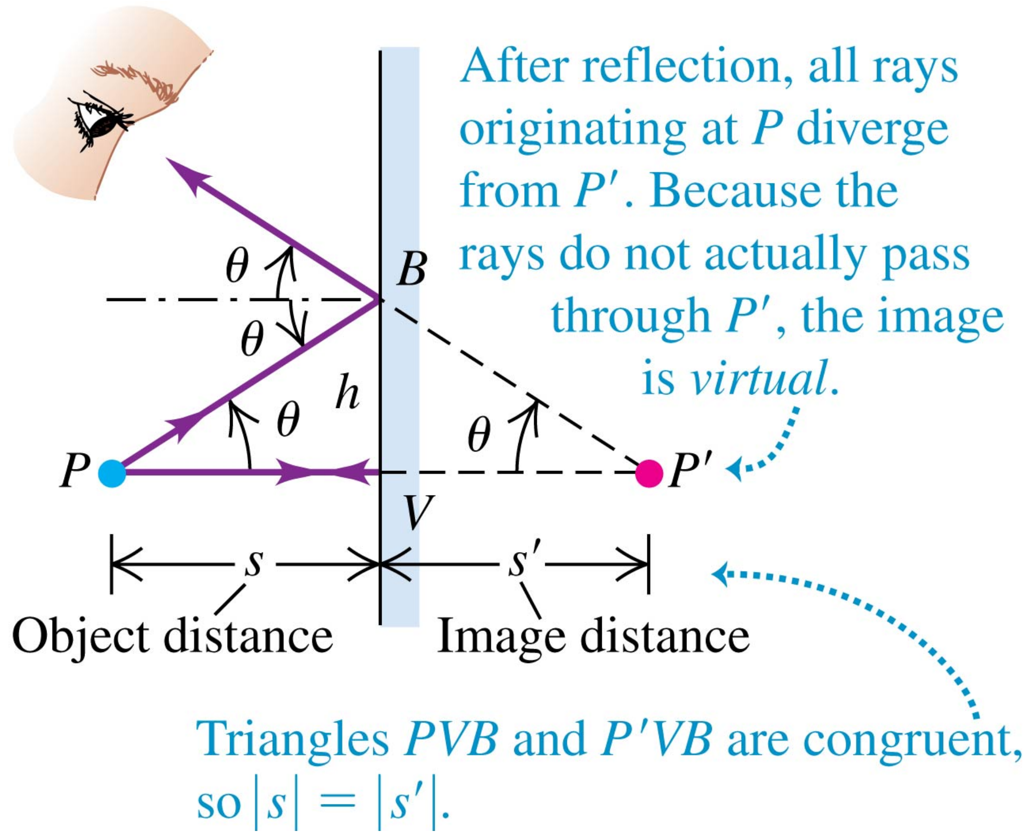
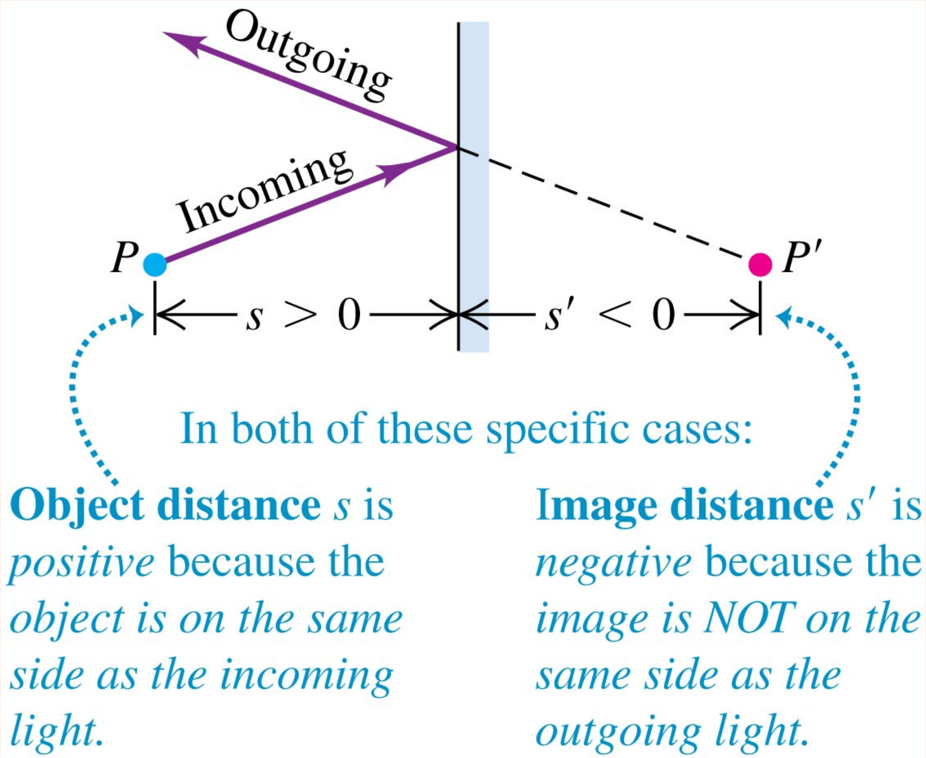


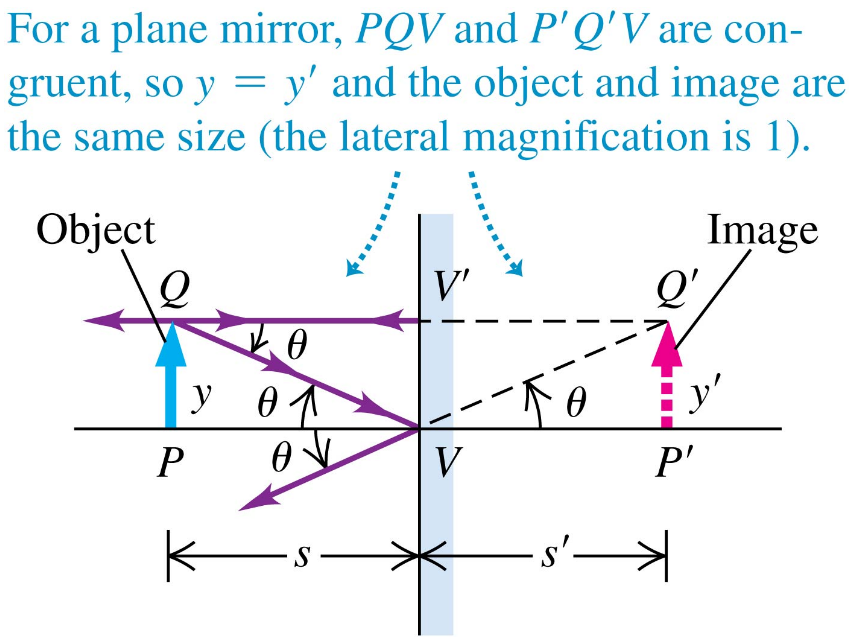
Image formation by a plane mirror: Sign rules



$$s = -s' \qquad \text{(plane mirror)}$$

Characteristics of the image from a plane mirror

- In a plane mirror, the image is virtual, erect, reversed, and the same size as the object.



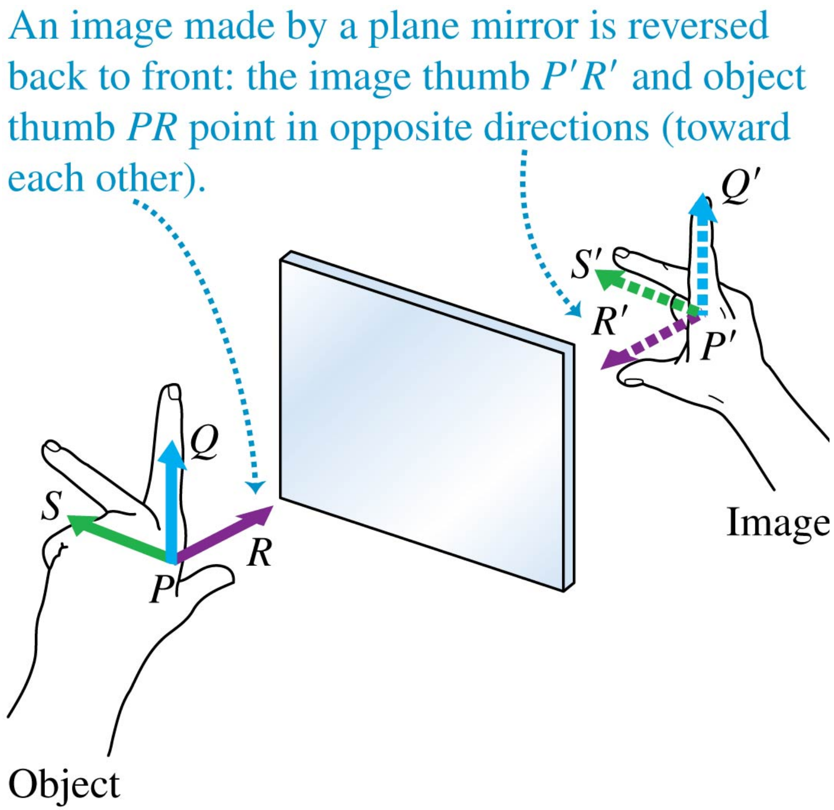
Lateral magnification in an image-forming situation

$$m = \frac{y'}{y}$$

y': Image height
y: Object height

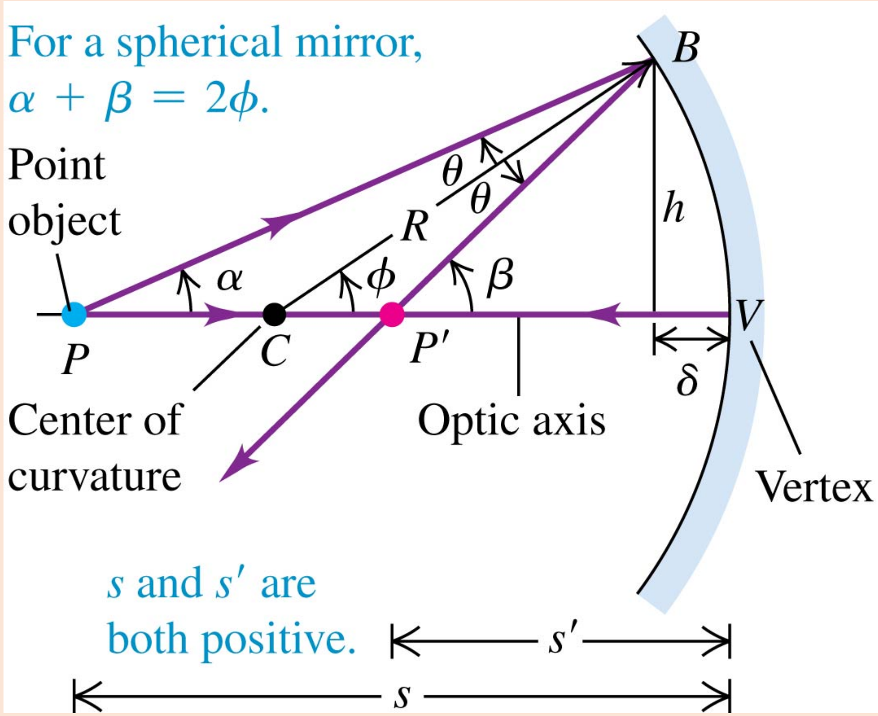
The image is reversed

- The image formed by a plane mirror is reversed; the image of a right hand is a left hand, and so on.



Spherical mirror with a point object

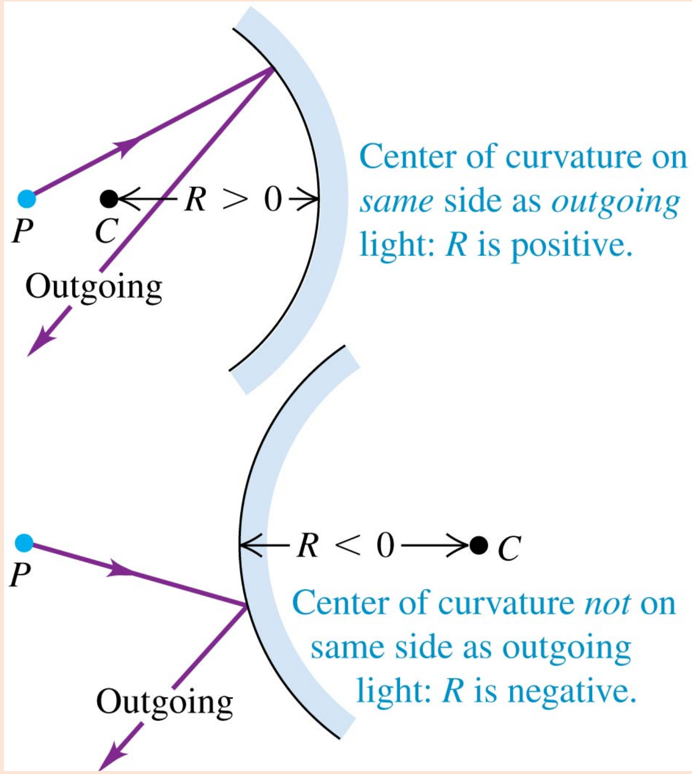
- A spherical mirror with radius of curvature *R* forms a real image *P'* of the point object *P*.



$$\frac{1}{s} + \frac{1}{s'} = \frac{2}{R} \qquad \text{(object–image relationship, spherical mirror)}$$

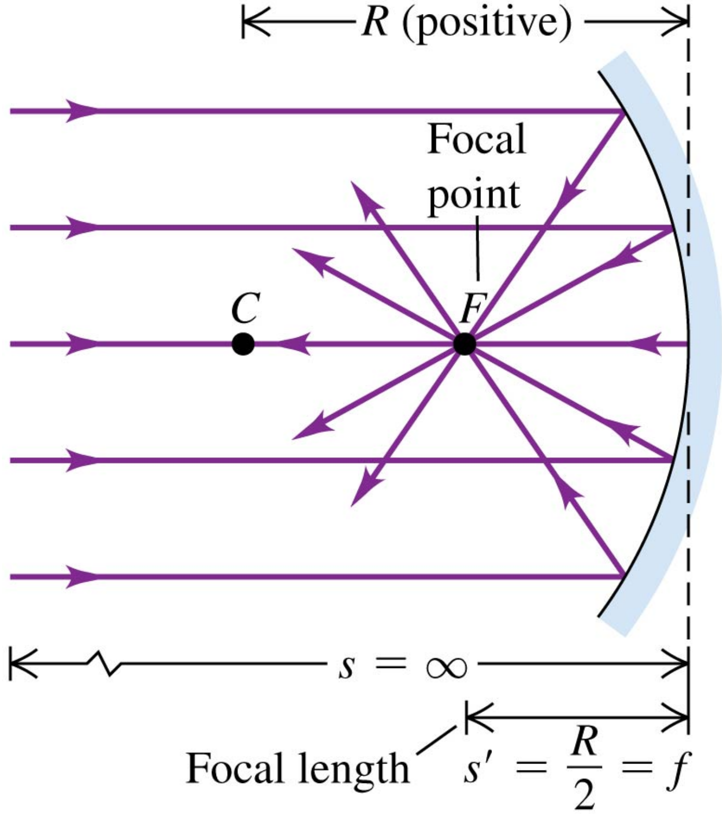
General Sign conventions (mirror & lens)

- If the object point *P* is on the same side as the incident light, then *s* is positive.
- If the image point *P'* is on the same side as the reflected light, then *s'* is positive.
- If the center of curvature *C* is on the same side as the outgoing light, then *R* is positive.



Focal point and focal length

- When the object is very far from the spherical mirror, the incoming rays are **parallel**.
- The beam of incident parallel rays converges, after reflection from the mirror, to a **focal point**, point *F*.
- The distance from the vertex to the focal point, denoted by *f*, is called the **focal length**.



Focal point and focal length

- With the object at the focal point, the reflected rays are parallel to the optic axis.
- The reflected rays meet only at a point infinitely far from the mirror, so the image is at infinity.

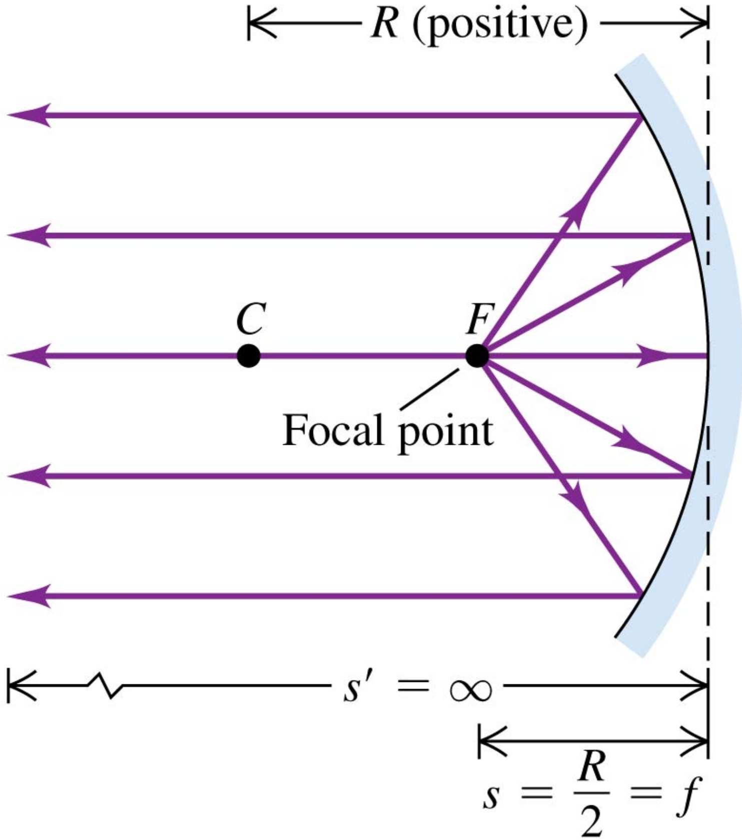


Image of an extended object: Spherical mirror

- Shown is how to determine the position, orientation, and height of an image formed by a concave spherical mirror.

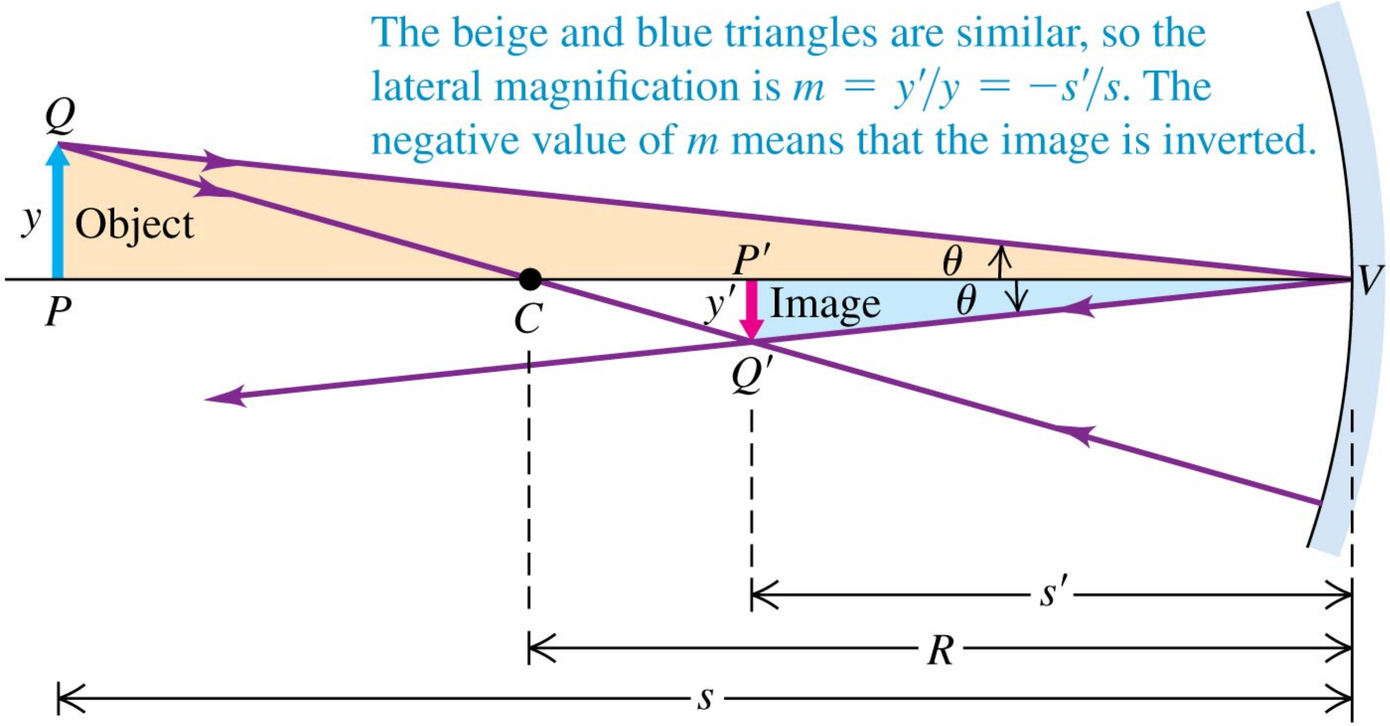
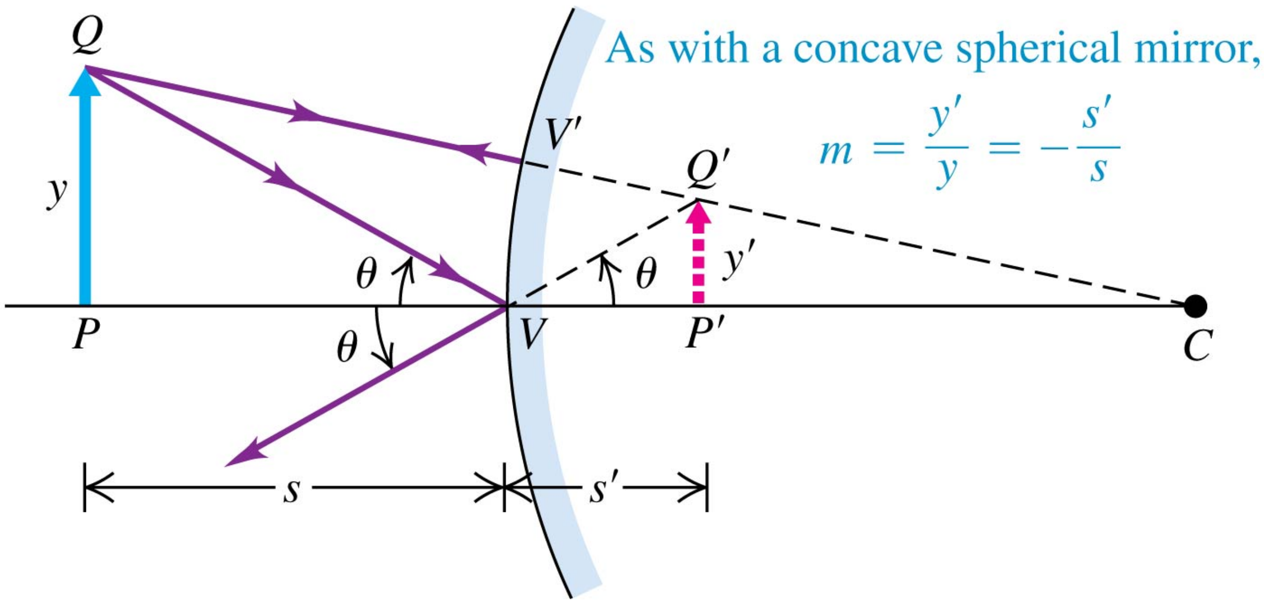


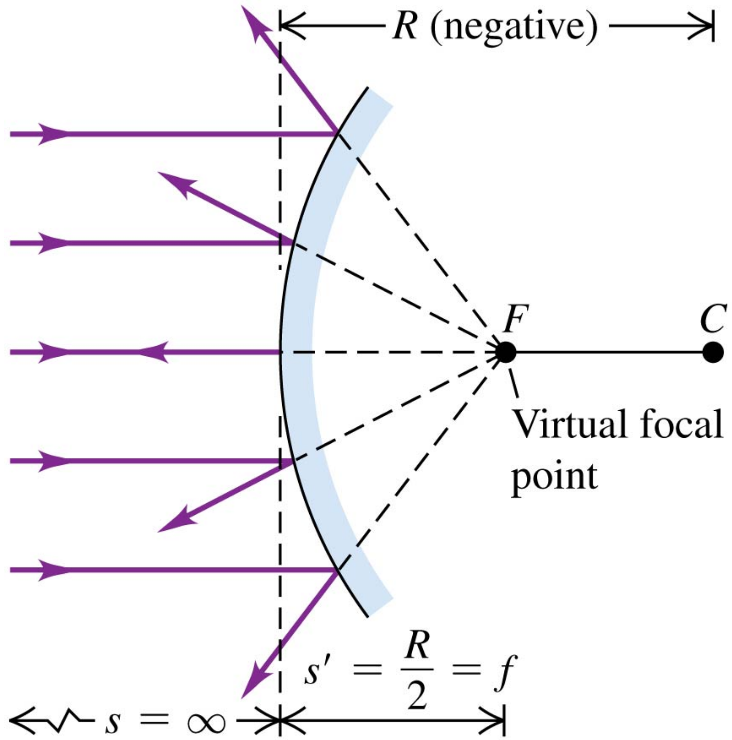
Image formation by a convex mirror

- If the mirror is convex, so that *R* is negative, the resulting image is virtual (that is, the image point is on the opposite side of the mirror from the object), erect, and smaller than the object.



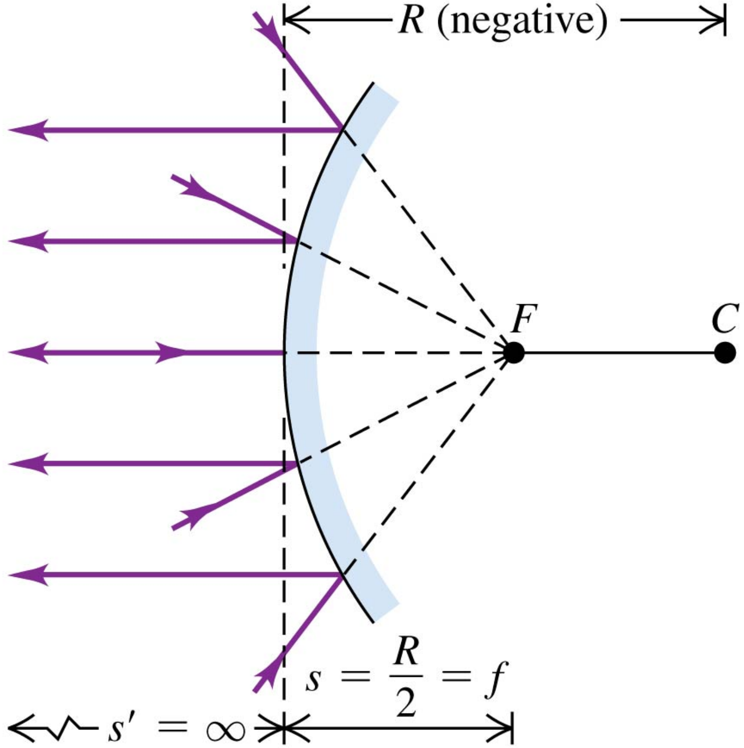
Focal point and focal length of a convex mirror

- When incoming rays that are parallel to the optic axis are reflected from a convex mirror, they diverge as though they had come from the **virtual focal point** *F* at a distance *f* behind the mirror.
- The corresponding image distance *s* is negative.

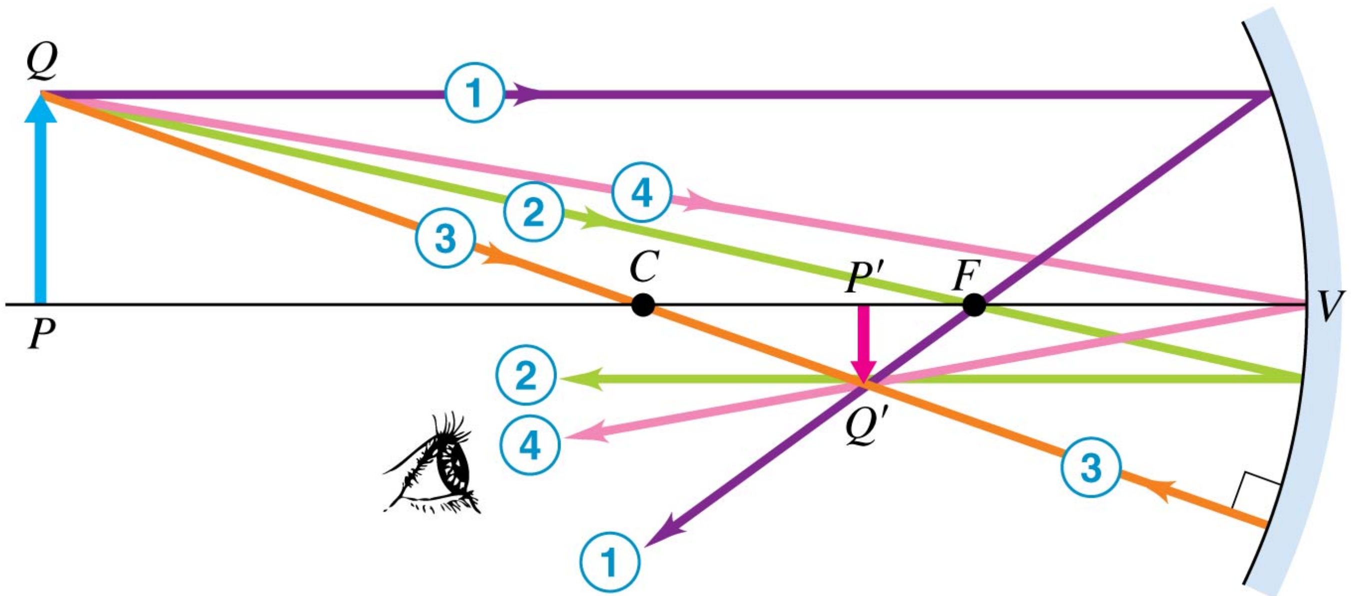


Focal point and focal length of a convex mirror

- When the incoming rays are converging as though they would meet at the virtual focal point *F*, then they are reflected parallel to the optic axis.



Graphical method of locating images



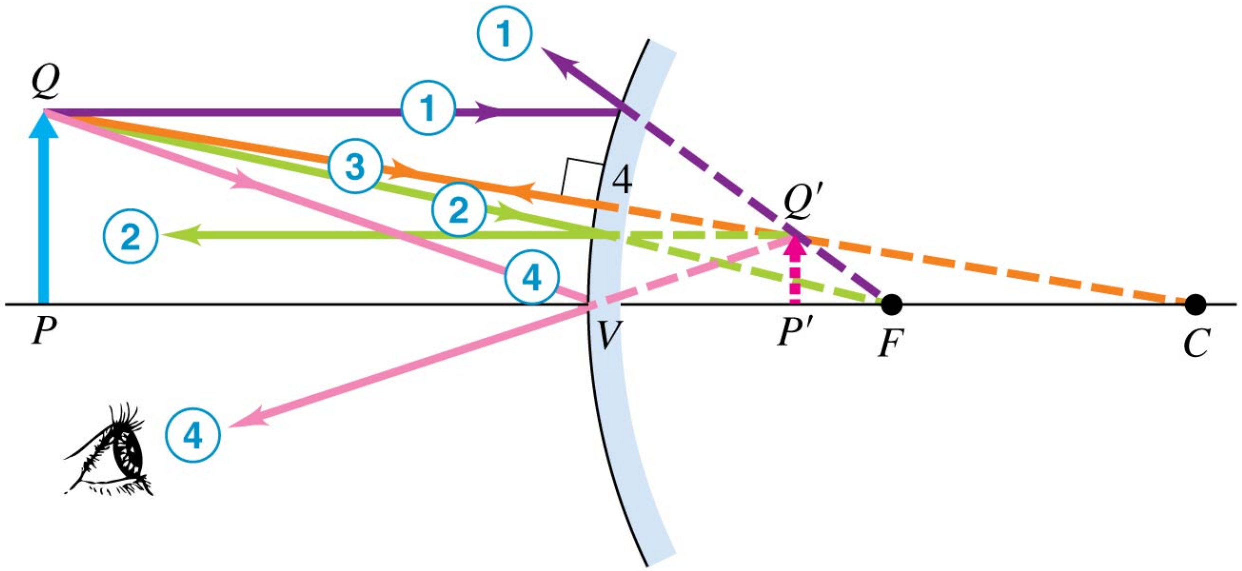
(1) Ray parallel to axis reflects through focal point.

(2) Ray through focal point reflects parallel to axis.

(3) Ray through center of curvature intersects the surface normally and reflects along its original path.

(4) Ray to vertex reflects symmetrically around optic axis.

Graphical method of locating images



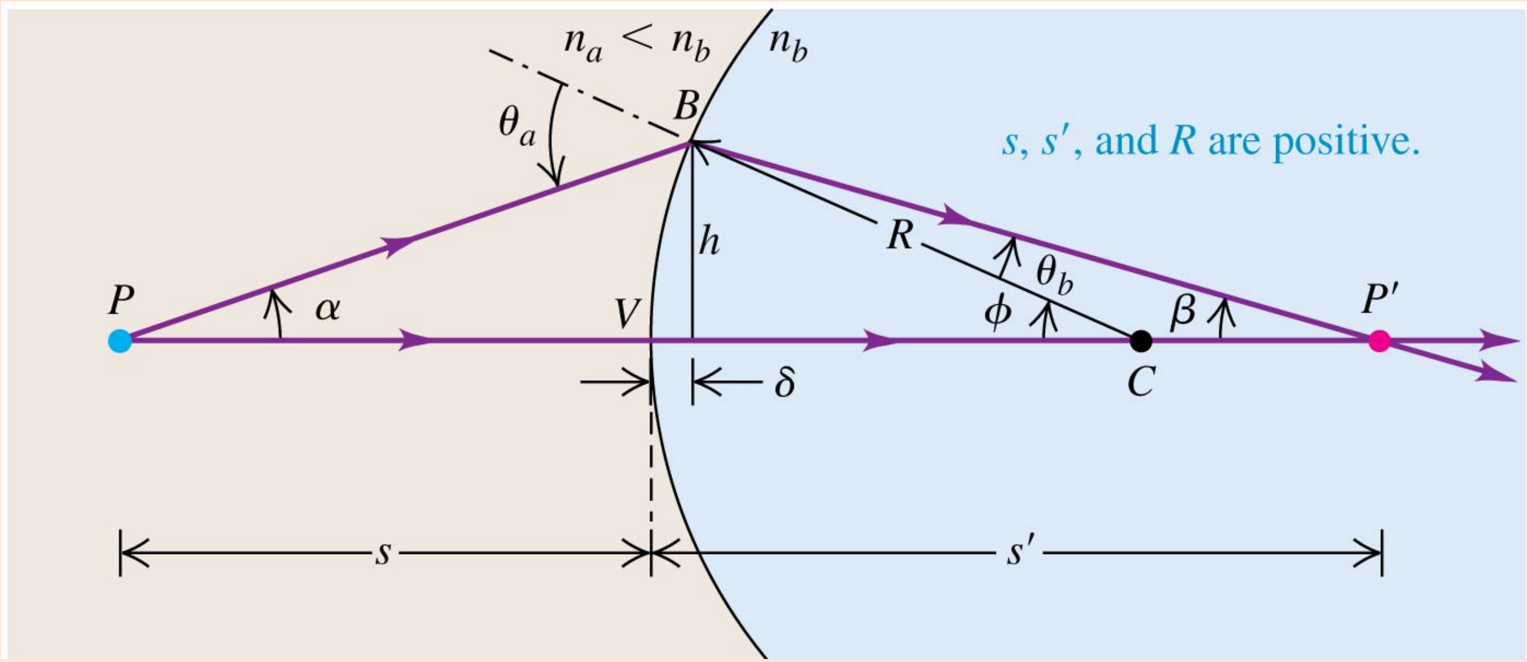
1 Reflected parallel ray appears to come from focal point.

2 Ray toward focal point reflects parallel to axis.

3 As with concave mirror: Ray radial to center of curvature intersects the surface normally and reflects along its original path.

4 As with concave mirror: Ray to vertex reflects symmetrically around optic axis.

Image of a point object at a spherical surface

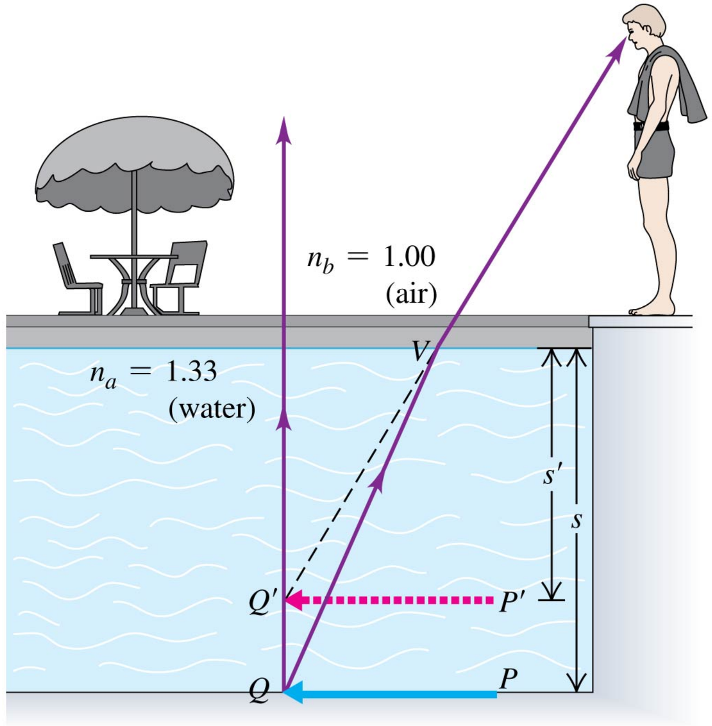


$$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$$ (object–image relationship, spherical refracting surface)

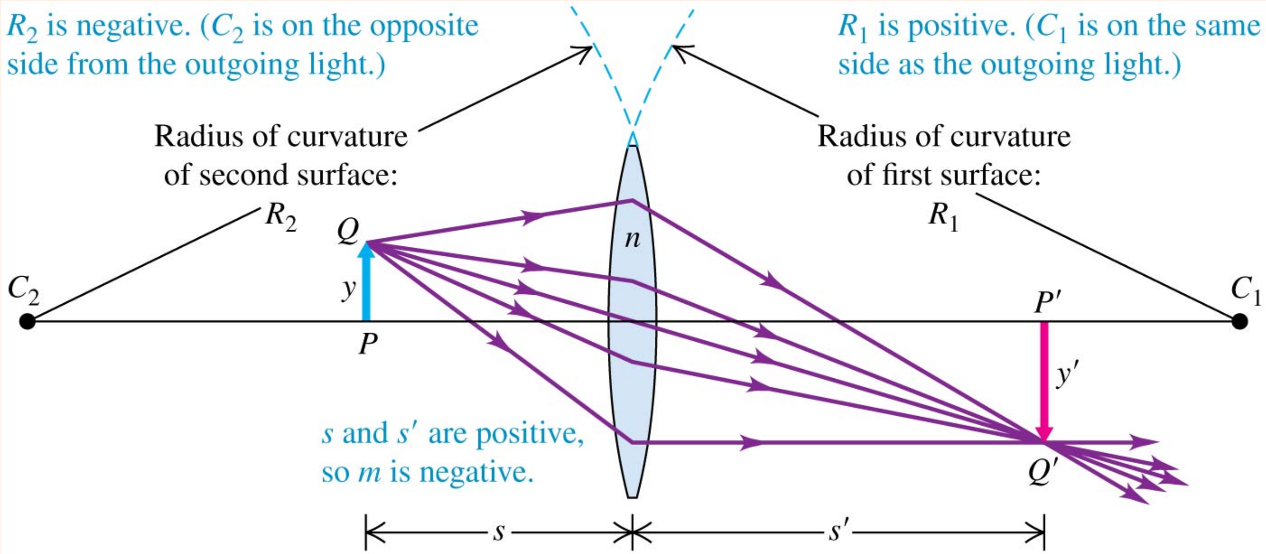
Apparent depth of a swimming pool

$$\frac{n_a}{s} + \frac{n_b}{s'} = 0 \qquad \text{(plane refracting surface)} \qquad R \rightarrow \infty$$

- When light travels through a plane surface between two optical materials, the image has the same lateral size (m = 1) and is always erect.
- s'=-s/1.33
- The apparent depth of a pool is less than its actual depth.



Lensmaker's equation



Object–image relationship, thin lens:

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

Focal length of lens

Object distance

Image distance

Lensmaker's equation for a thin lens:

Index of refraction of lens material

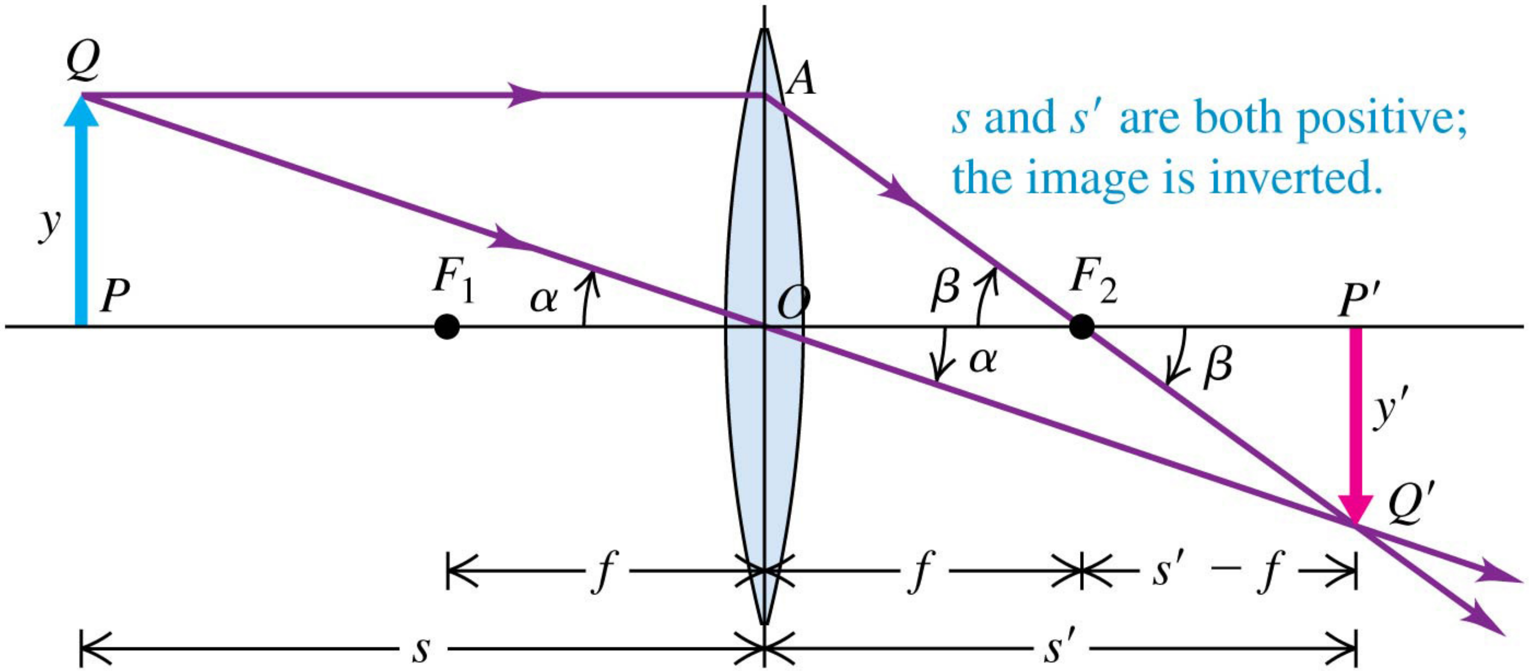
$$\frac{1}{f} = (n - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Focal length

Radius of curvature of first surface

Radius of curvature of second surface

Magnification

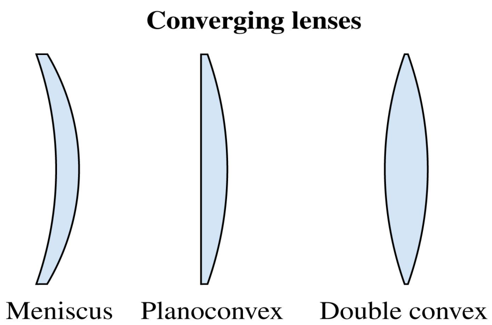


$$m = \frac{y'}{y} = -\frac{s'}{s}$$

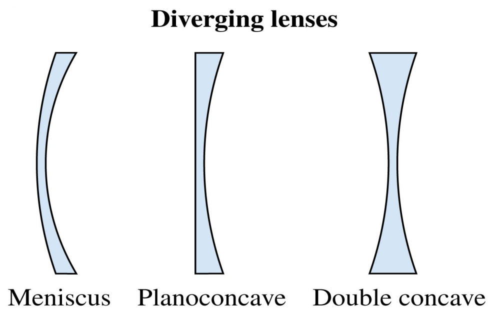
Types of lenses

- Shown below are various types of lenses, both converging and diverging.
- Any lens that is thicker at its center than at its edges is a ***converging* lens with positive *f*** ; and any lens that is thicker at its edges than at its center is **a *diverging* lens with negative *f*.**

$$f > 0 \qquad \frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad f < 0$$



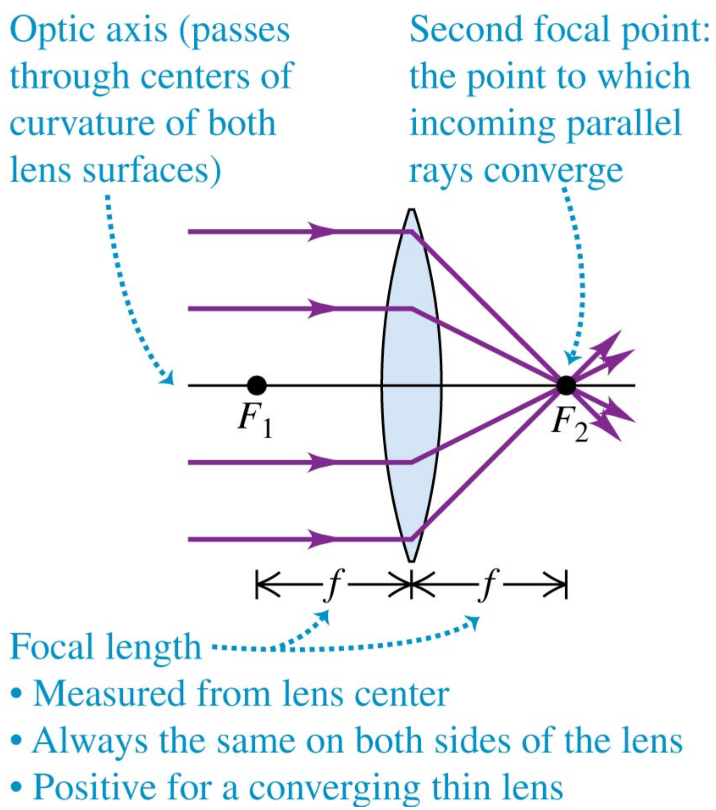
$0 > R_2 > R_1$ $R_1 = \infty$, $0 > R_2$ $R_1 > 0 > R_2$



$R_1 > R_2 > 0$ $R_1 = \infty$, $R_2 > 0$ $R_2 > 0 > R_1$

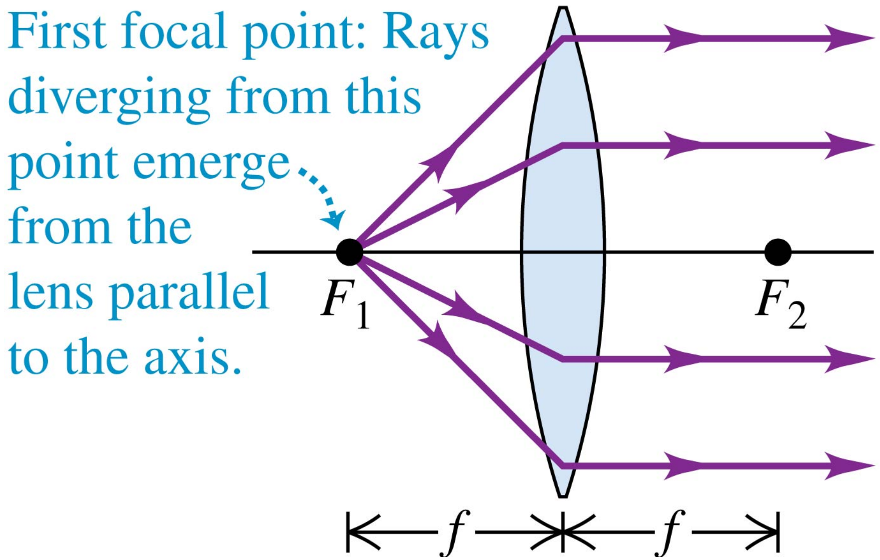
Thin converging lens

- A **lens** is an optical system with two refracting surfaces.
- The simplest lens has two spherical surfaces close enough together that we can ignore the distance between them; we call this a **thin lens**.



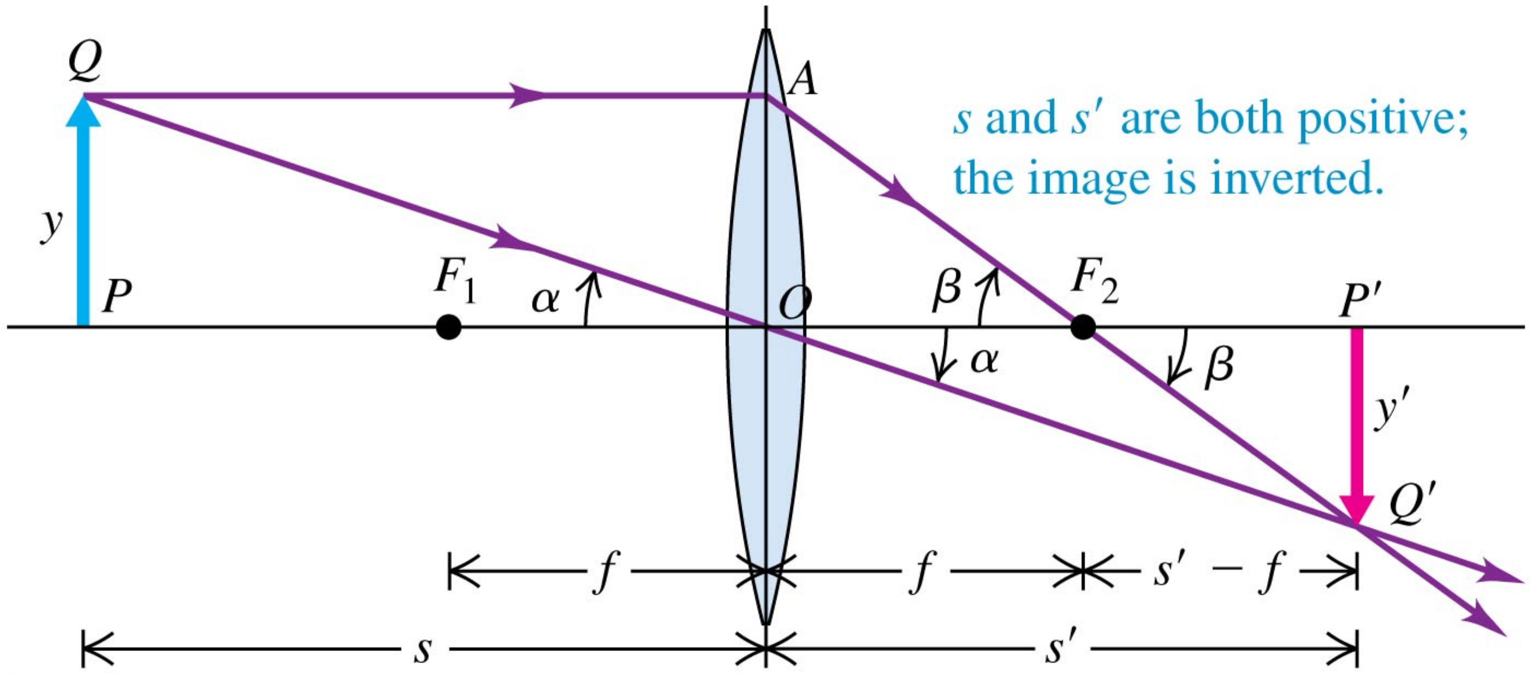
Thin converging lens

- Rays passing through the first focal point F_1 emerge from a converging lens as a beam of parallel rays.



- f is called the **focal length**.

Image formed by a thin converging lens



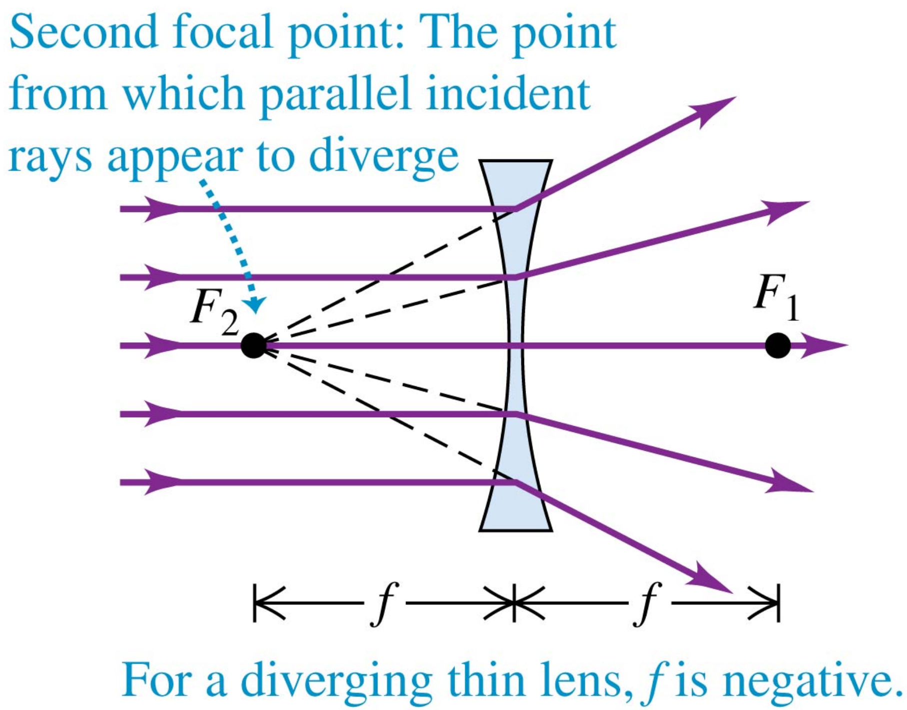
Object–image relationship, thin lens:

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

Object distance (s), Image distance (s'), Focal length of lens (f)

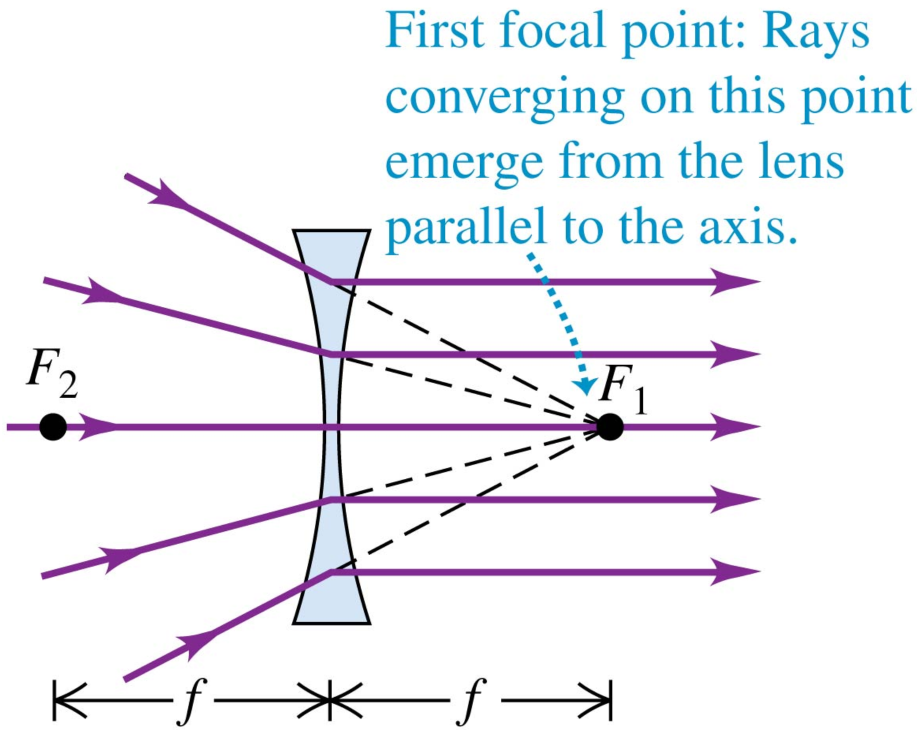
Thin diverging lens

- When a beam of parallel rays is incident on a **diverging lens**, the rays diverge after refraction.
- The focal length of a diverging lens is a negative quantity, and the lens is also called a *negative lens*.



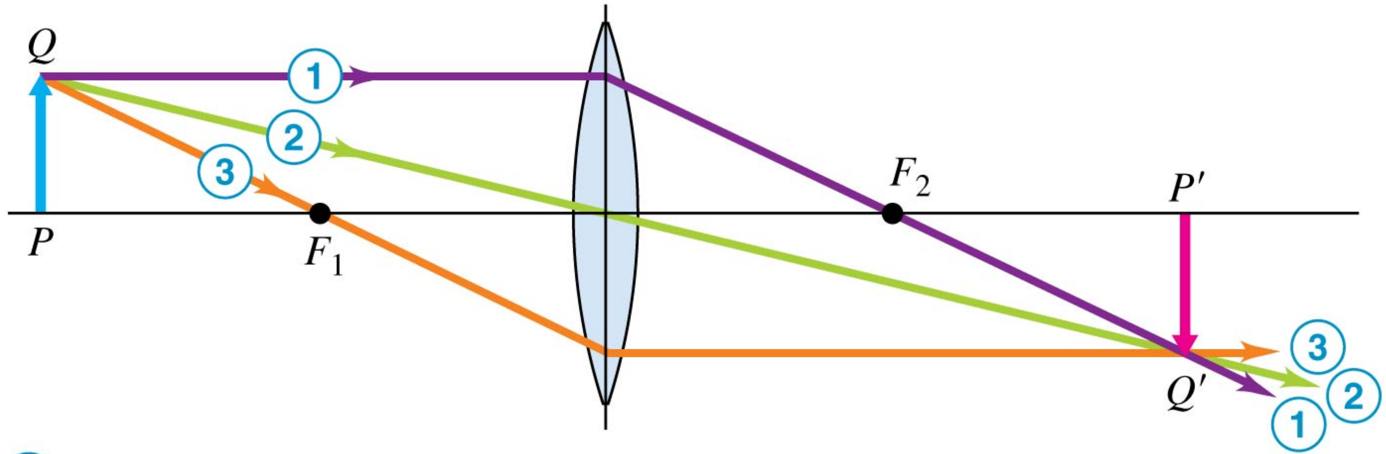
Thin diverging lens

- **Incident** rays converging toward the first focal point F_1 of a diverging lens emerge from the lens *parallel* to its axis.



Graphical methods for lenses

- Shown below is the method for drawing the three **principal rays** for a real image formed by a converging lens.

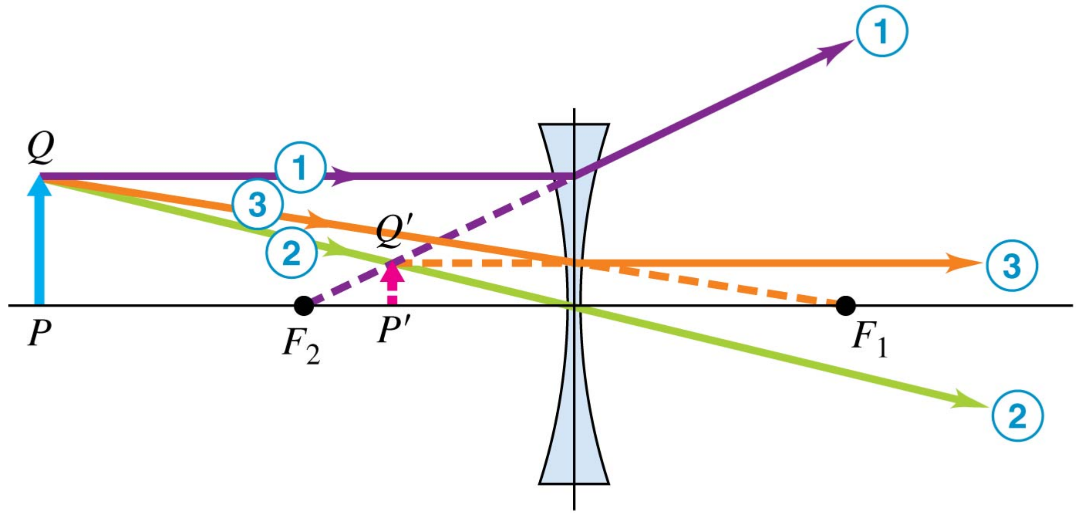


(1) Parallel incident ray refracts to pass through second focal point F_2.

(2) Ray through center of lens does not deviate appreciably.

(3) Ray through the first focal point F_1 emerges parallel to the axis.

Graphical methods for a diverging lens

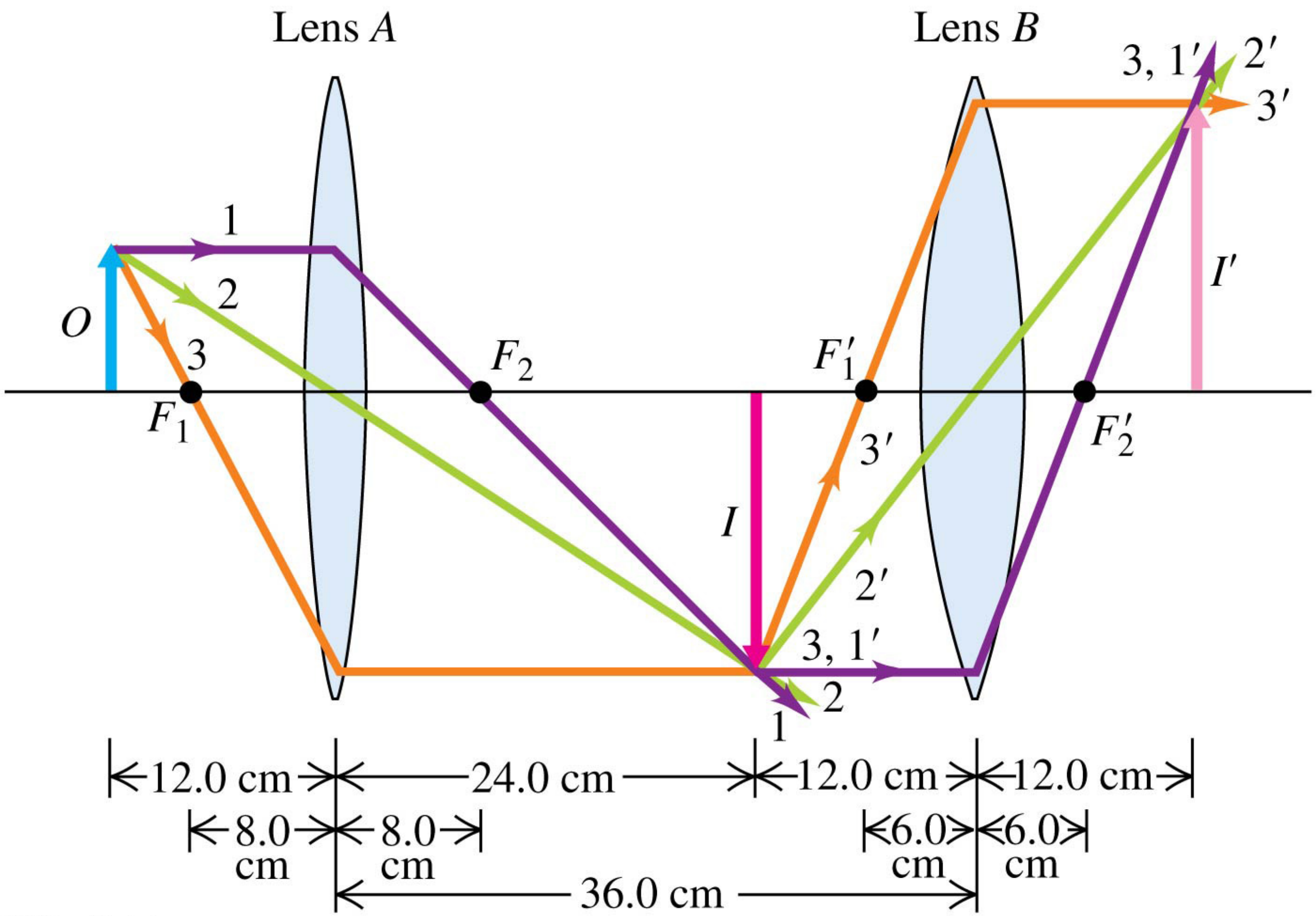


(1) Parallel incident ray appears after refraction to have come from the second focal point F_2.

(2) Ray through center of lens does not deviate appreciably.

(3) Ray aimed at the first focal point F_1 emerges parallel to the axis.

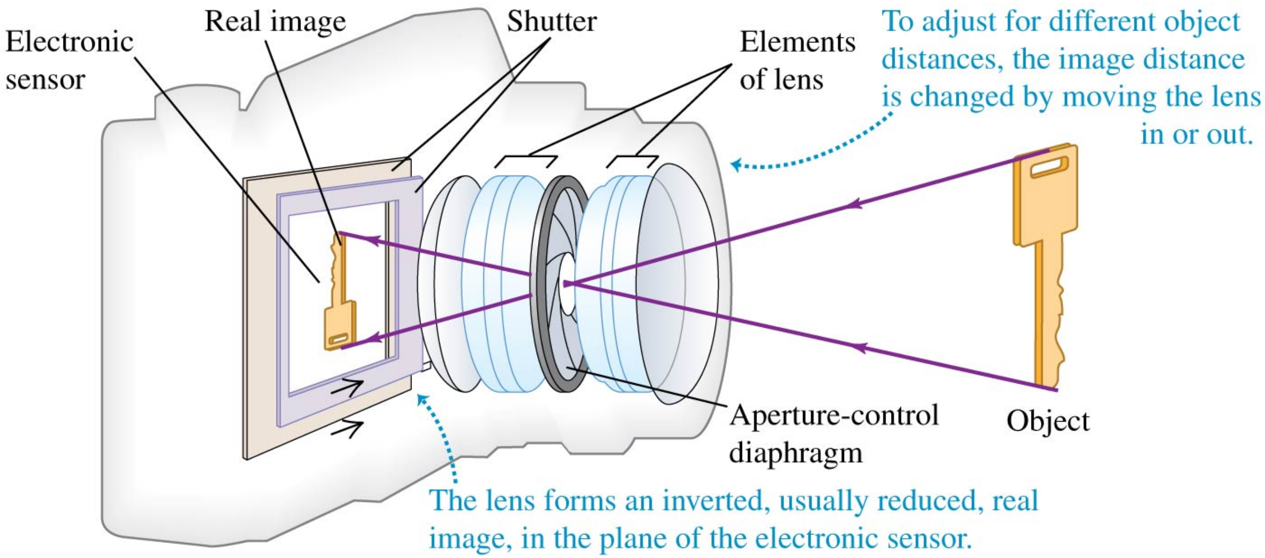
Example



Lens A
Lens B
3, 1′
2′
3′
1
2
O
3
F1
F2
F′1
F′2
I′
I
3′
2′
3, 1′
2
1
12.0 cm
24.0 cm
12.0 cm
12.0 cm
8.0 cm
8.0 cm
6.0 cm
6.0 cm
36.0 cm

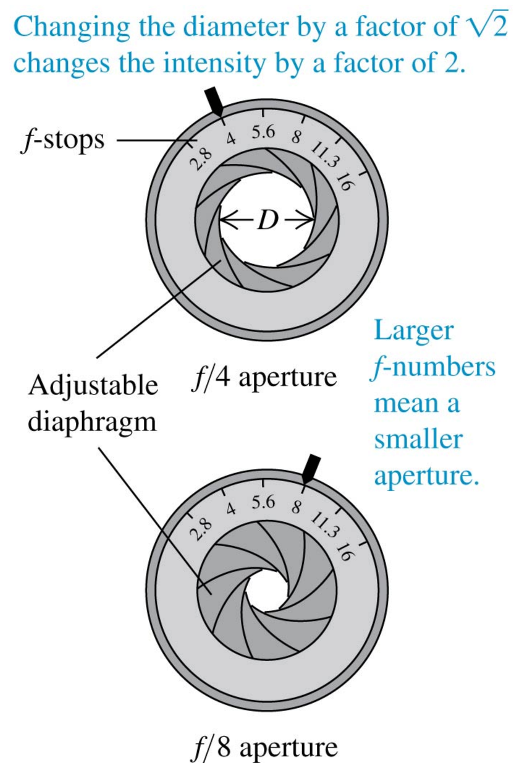
Cameras

- When a **camera** is in proper focus, the position of the electronic sensor coincides with the position of the real image formed by the lens.



Camera lens basics

- The focal length f of a camera lens is the distance from the lens to the image when the object is infinitely far away.
- The effective area of the lens is controlled by means of an adjustable lens aperture, a nearly circular hole with diameter D.
- Photographers commonly express the light-gathering capability of a lens in terms of the ratio f/D, called the f-number of the lens:



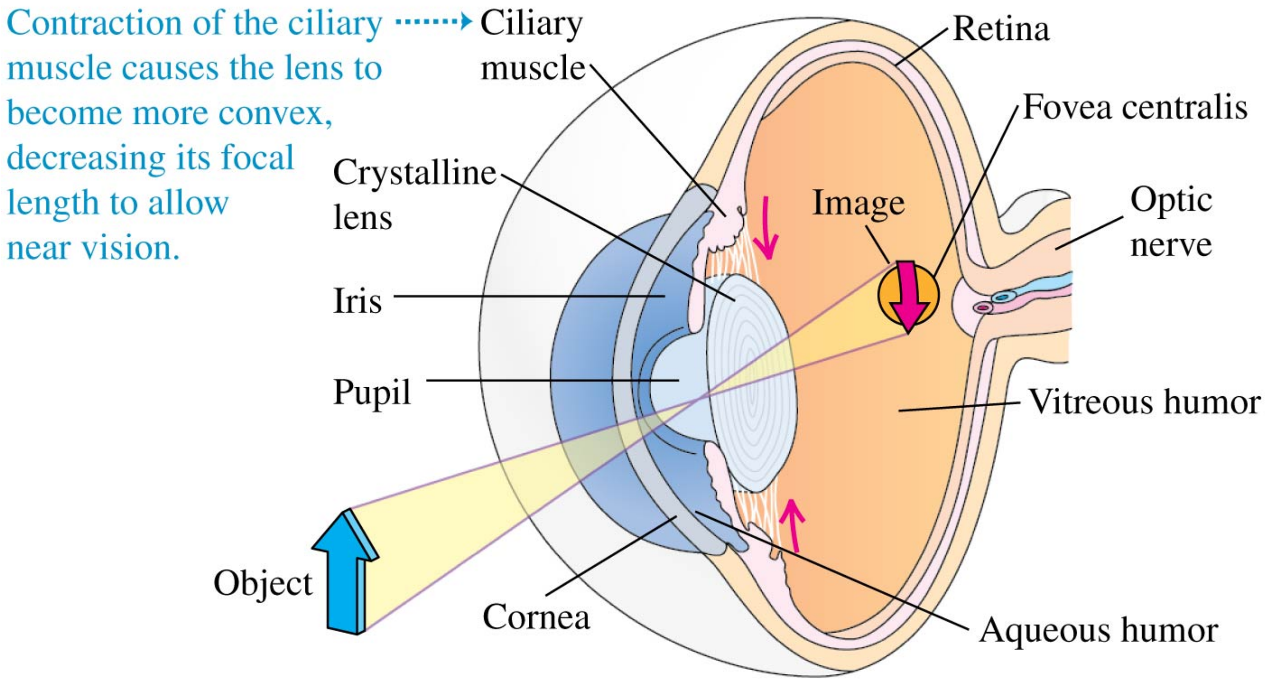
$$f\text{-number of a lens} = \frac{f}{D}$$

f: Focal length of lens

D: Aperture diameter

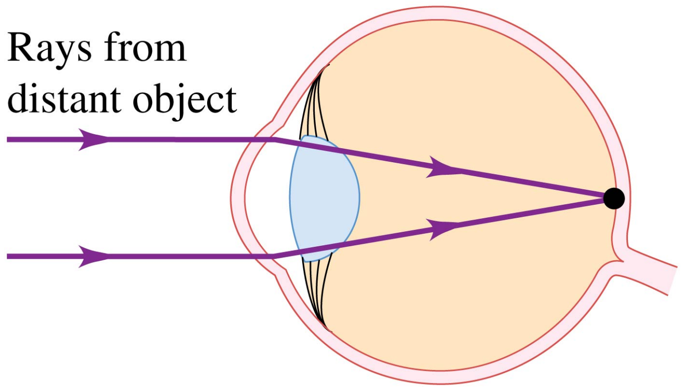
The eye

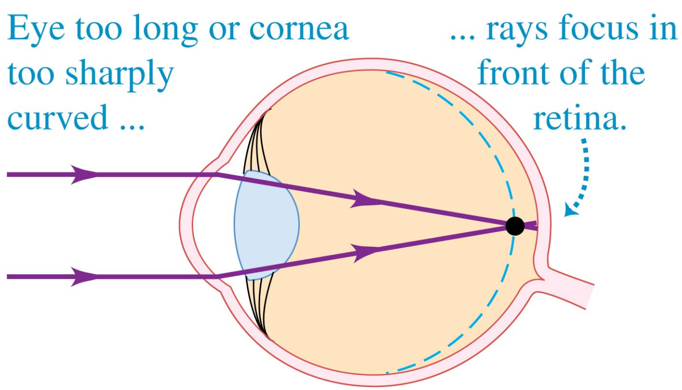
- The optical behavior of the eye is similar to that of a camera.



Defects of vision

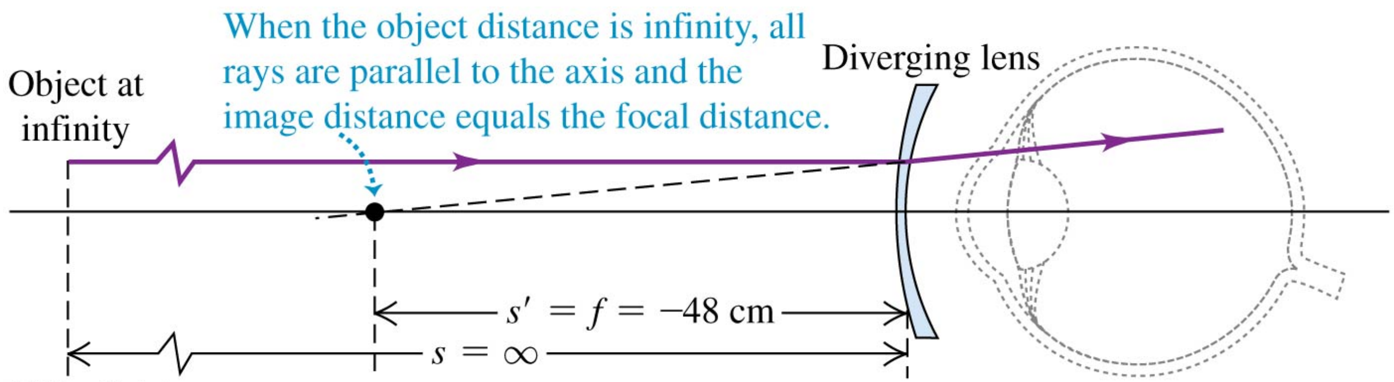
- A normal eye forms an image on the retina of an object at infinity when the eye is relaxed.
- In the **myopic** (**nearsighted**) eye, the eyeball is too long from front to back in comparison with the radius of curvature of the cornea, and rays from an object at infinity are focused in front of the retina.





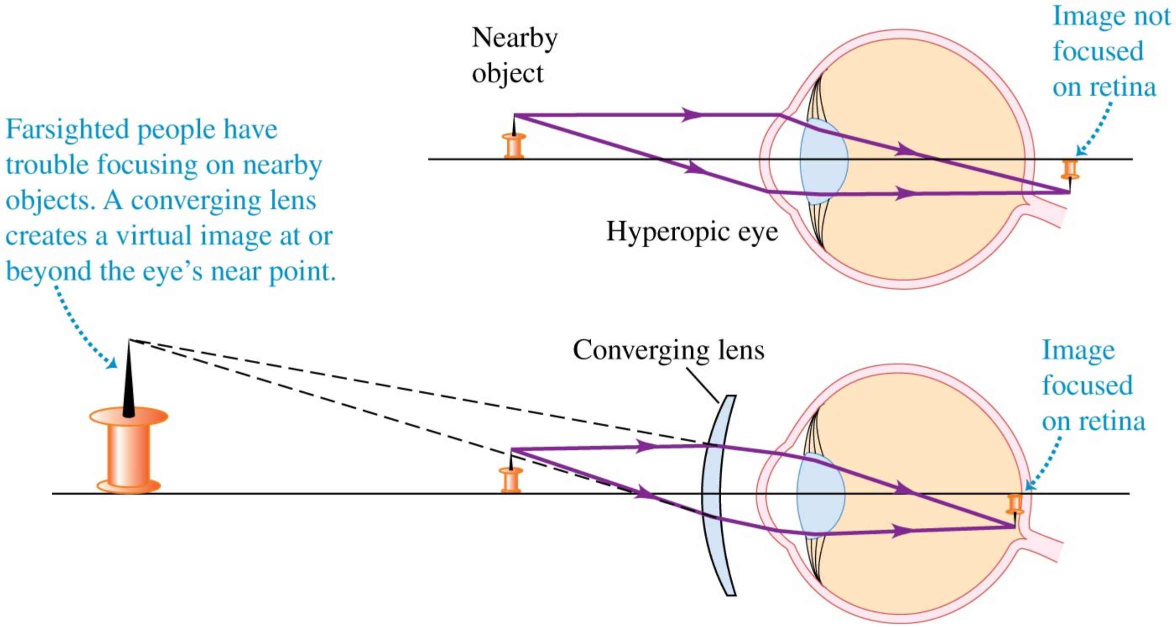
Nearsighted correction

- The far point of a certain myopic eye is 50 cm in front of the eye.
- When a diverging lens of focal length $f = -48$ cm is worn 2 cm in front of the eye, it creates a virtual image at 50 cm that permits the wearer to see clearly.



Farsighted correction

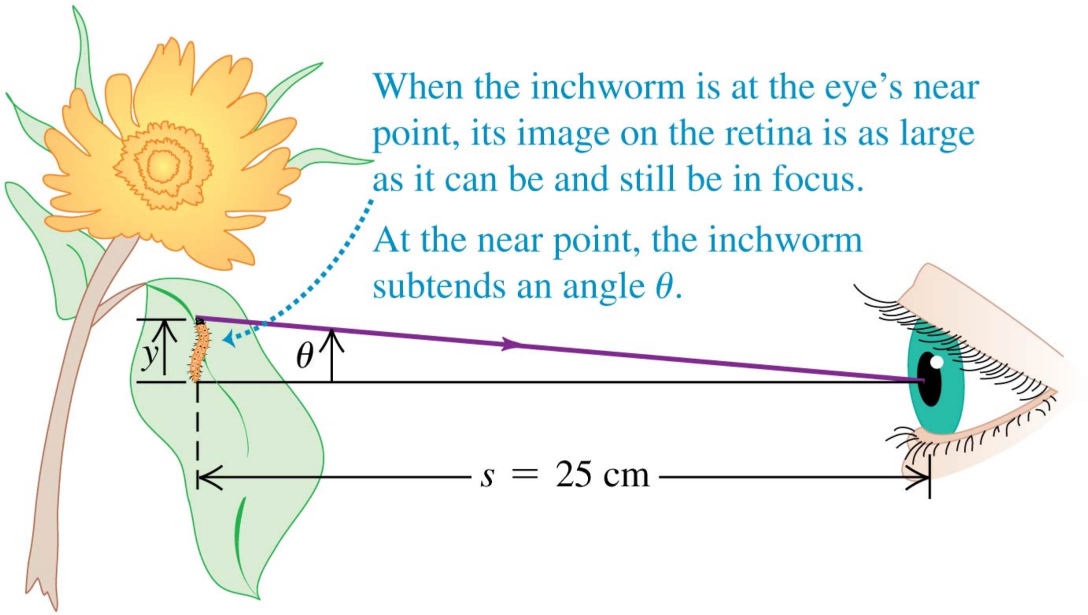
- A converging lens can be used to create an image far enough away from the hyperopic eye at a point where the wearer can see it clearly.



Angular size

- The maximum **angular** size of an object viewed at a comfortable distance is the angle it subtends at a distance of 25 cm.

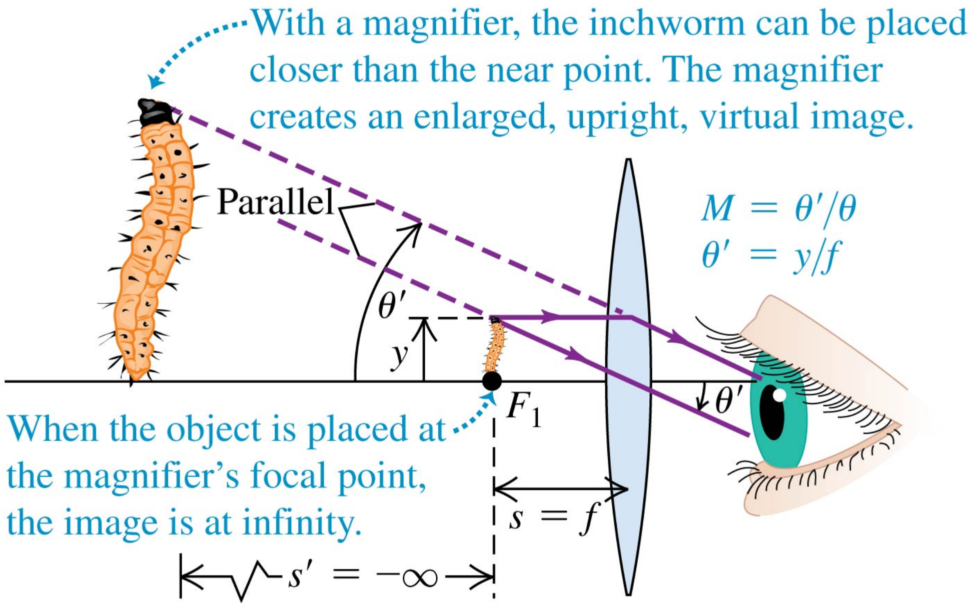
$$\theta \approx \tan\theta = \frac{y}{25cm}$$



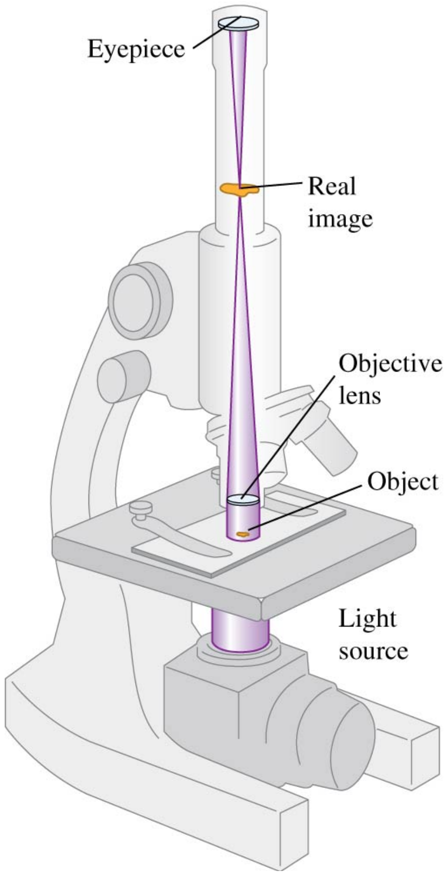
The magnifier

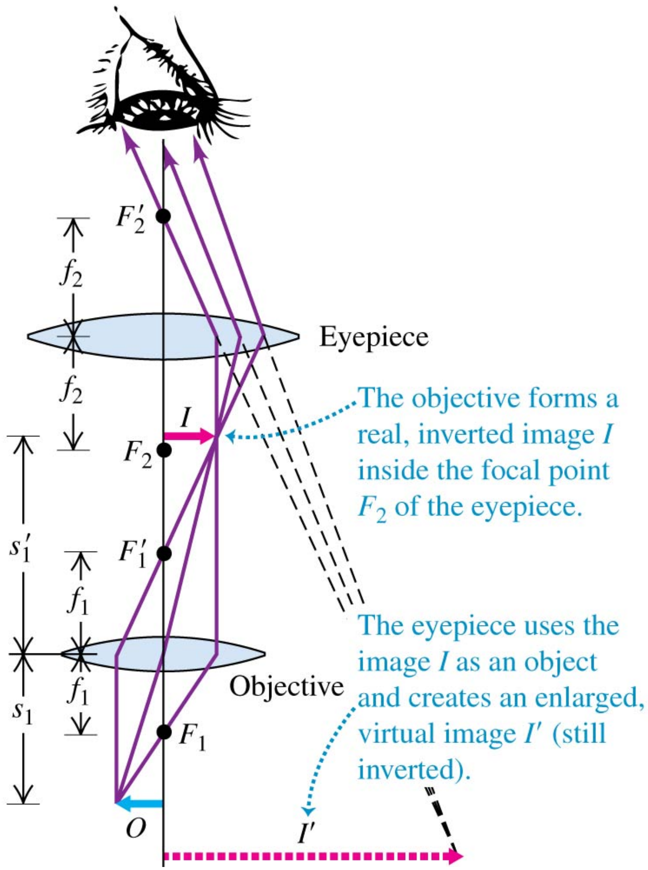
- The **angular magnification** of a simple magnifier is:

$$M \equiv \theta'/\theta = (25\text{ cm})/f.$$



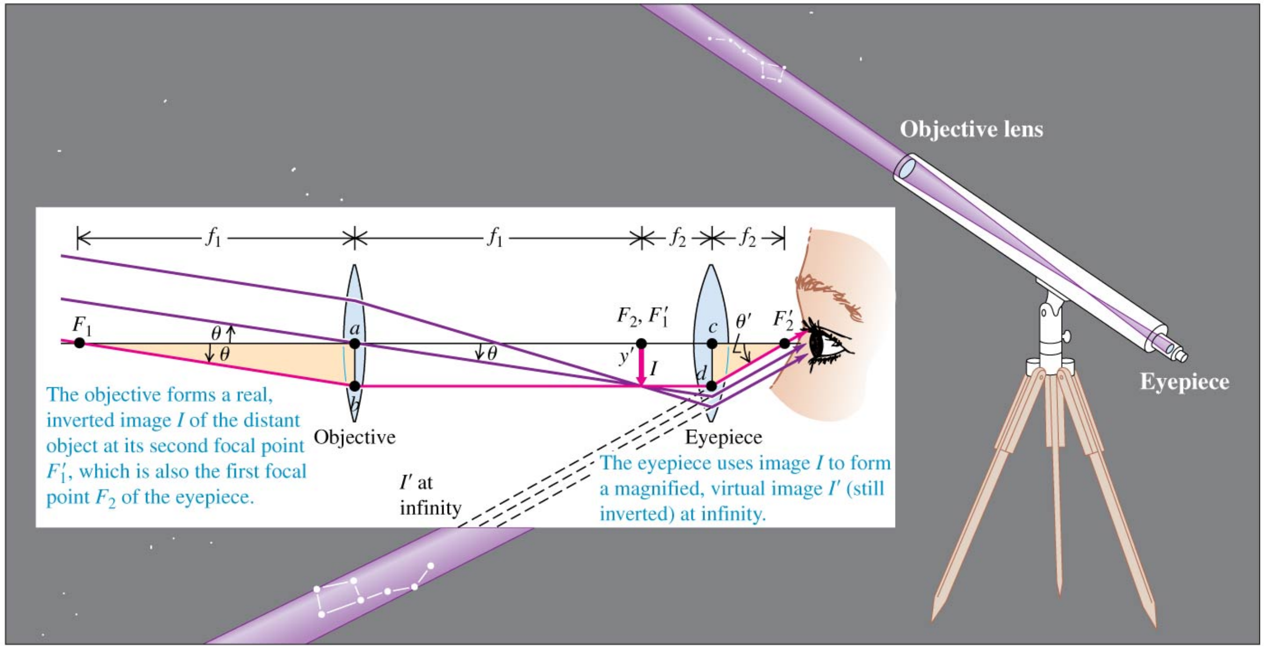
The compound microscope



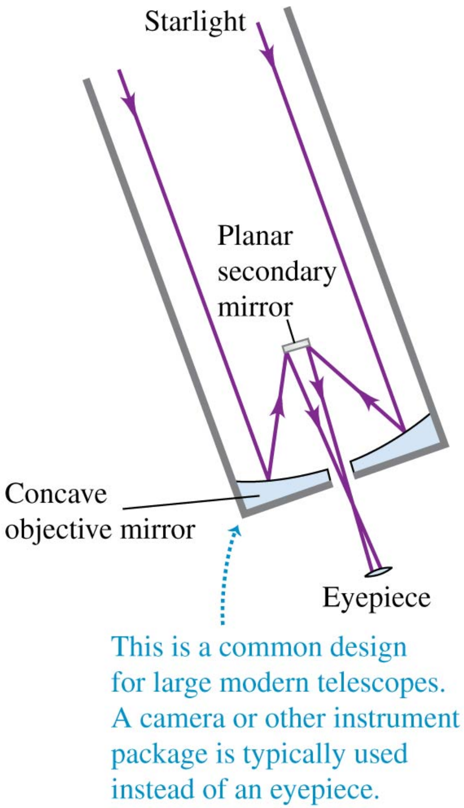


The astronomical telescope

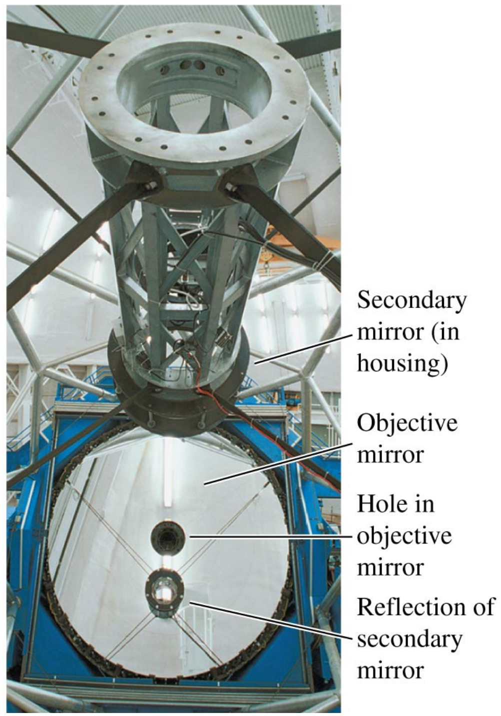
- The figure below shows the optical system of an *astronomical refracting telescope*.



The reflecting telescope



- The Gemini North telescope uses an objective mirror 8 meters in diameter.



Example: In a telescope, the objective lens forms a real, reduced image I of the object. This image is the object for the eyepiece, which forms an enlarged, virtual image of I which is at infinity (for mast comfortable viewing by a eye).

$$\vartheta = \frac{-y'}{f_1} \quad \text{and} \quad \vartheta' = \frac{y'}{f_2}$$

$$\text{angular magnification } M = \frac{\vartheta'}{\vartheta} = -\frac{f_1}{f_2} \quad \text{(-ve for the inverted final image)}$$