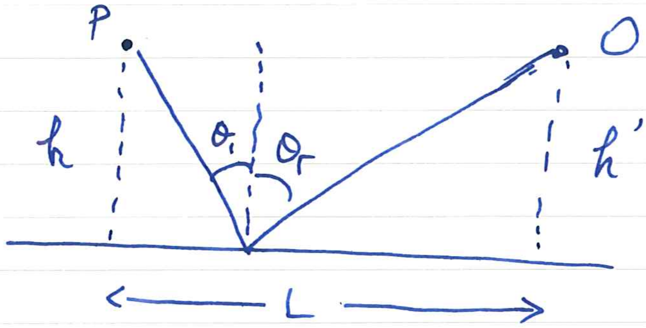
$\theta_r = \theta_r(\theta_i)$.

1) Reflection.



$$h\tan\theta_i + h'\tan\theta_r = L \quad -①$$

$$T = \left(\frac{h}{\cos\theta_i} + \frac{h'}{\cos\theta_r}\right)\frac{1}{C}. \quad -②$$

$\to T = T(\theta_i)$

From ①,

$$h\sec^2\theta_i + h'\sec^2\theta_r \frac{d\theta_r}{d\theta_i} = 0. \quad \Rightarrow \quad \frac{d\theta_r}{d\theta_i} = -\frac{h\cos^2\theta_r}{h'\cos^2\theta_i}. \quad - ③$$

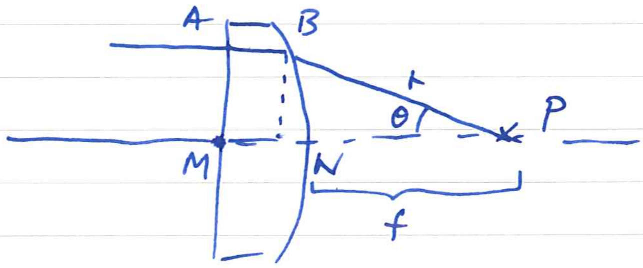
$$\frac{dT}{d\theta_i} = 0 \Rightarrow h\sec\theta_i\tan\theta_i + h'\sec\theta_r\tan\theta_r\left(\frac{d\theta_r}{d\theta_i}\right) = 0. \quad - ④$$

$$\Rightarrow h\sec\theta_i\tan\theta_i = +h'\sec\theta_r\tan\theta_r\left(\frac{h}{h'}\right)\frac{\cos^2\theta_r}{\cos^2\theta_i}$$

$$\Rightarrow h\cos\theta_i\tan\theta_i = h\,\cancel{\sec\theta_r}\,\cos\theta_r\tan\theta_r.$$

$$\Rightarrow \boxed{\theta_i = \theta_r.} \quad \text{(law of reflection).}$$

2. Planoconvex lens.



1. Along the ~~axes~~ optical axis,

light path $= n\overline{MN} + NP$
$= n\overline{MN} + f$.

2. Along ABP,
light path $= n\overline{AB} + r$

Equating the two light path, we have.

$$n(r\cos\theta - f) + f = r$$

$$\Rightarrow \quad r = \frac{(n-1)f}{n\cos\theta - 1}.$$

(N.B. (r,θ) is the polar coordinate from the origin at P.)

It is a ~~parabola!!~~ hyperbola!

$r = \dfrac{c}{\cos\theta - a} \Rightarrow r\cos\theta - ra = c$

$\Rightarrow x - c = a\sqrt{x^2+y^2}.$

$\Rightarrow x^2 - 2cx + c^2 = a^2x^2 + a^2y^2$

$\cancel{\text{FY}}\ \cancel{(x^2+x^2}$

$\Rightarrow (1-a^2)x^2 - 2cx - a^2y^2 + c^2 = 0.$

$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

$\Delta = B^2 - 4AC$

$= 0 - 4(1-a^2)(-a^2) > 0$ is hyperbola !!

Spherical mirror (concave mirror).

$\tan\alpha = \frac{h}{s-\delta}$, $\tan\beta = \frac{h}{s'-\delta}$, $\tan\phi = \frac{h}{R-\delta}$.

· If α is small $\Rightarrow \beta$, and ϕ are also small.
$\Rightarrow s, s', R \gg \delta$.

$\Rightarrow \alpha = \frac{h}{s}$, $\beta = \frac{h}{s'}$, $\phi = \frac{h}{R}$.

Also, we have. $\left.\begin{aligned}\alpha+\theta &= \phi\\ \phi+\theta &= \beta\end{aligned}\right\}$ $\phi-\alpha = \beta-\phi$

$\Rightarrow \alpha+\beta = 2\phi$

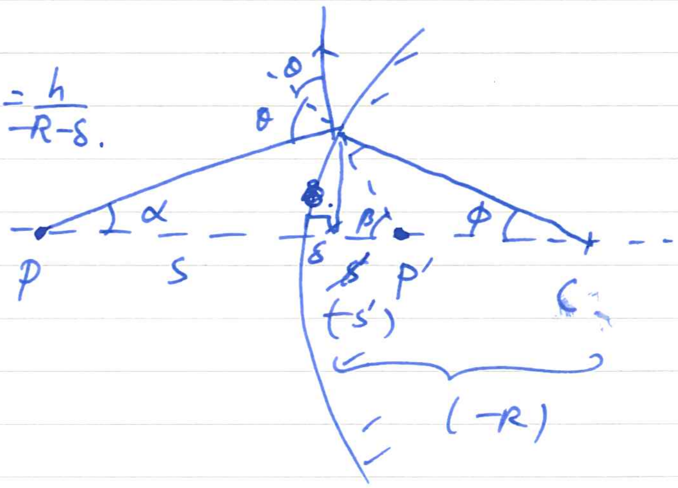
$\Rightarrow \frac{1}{s} + \frac{1}{s'} = \frac{2}{R}$.

* Define $f = \frac{2}{R}$, we have.

$$\boxed{\frac{f}{s} + \frac{f}{s'} = 1.}$$

· (convex mirror).

$\tan\alpha = \frac{h}{s+\delta}$, $\tan\beta = \frac{h}{-s'-\delta}$, $\tan\phi = \frac{h}{-R-\delta}$.



$\left.\begin{aligned}\alpha+\phi &= \theta\\ \phi+\cancel{\phi}\,\theta &= \beta\end{aligned}\right\}$ $\alpha+\phi = \beta-\phi$
$\alpha-\beta = -2\phi$.

$\Rightarrow \frac{1}{s} + \frac{1}{s'} = +\frac{2}{R}$.

~~Now, we use the convention~~
$\Rightarrow$ We get the same formulae for convex and concave mirror under the sign convention.

Image of a point object at a spherical refracting surface

$\theta_a = \alpha + \phi$ — ① , $\theta_b + \beta = \phi$ — ②

Snell's law: $n_a \sin\theta_a = n_b \sin\theta_b$.

$\tan\alpha = \frac{h}{s+\delta}$, $\tan\beta = \frac{h}{s'-\delta}$, $\tan\phi = \frac{h}{R-\delta}$.

Under small angle approximation, $n_a\theta_a = n_b\theta_b$.

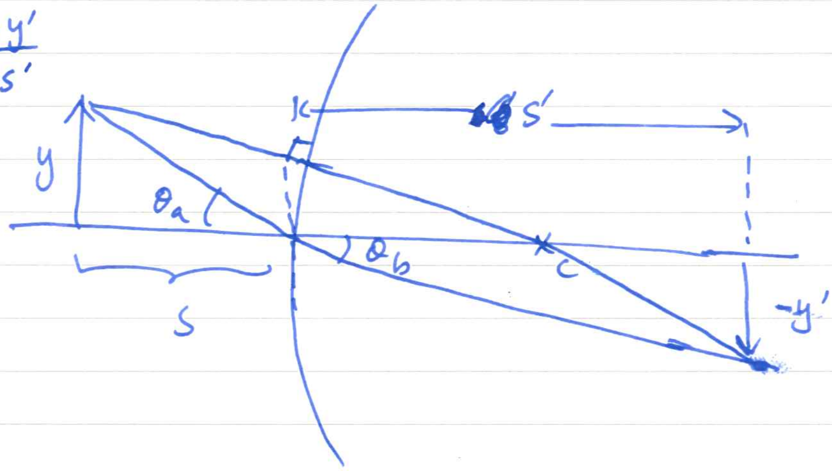
① $\Rightarrow$ $\theta_b = \frac{n_a}{n_b}(\alpha+\phi)$

② $\Rightarrow$ $n_a\alpha + n_b\beta = (n_b - n_a)\phi$. —(*).

$$\Rightarrow \boxed{\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}}$$

Lateral magnification : $m = \frac{y'}{y}$

$\left\{\begin{array}{l} \tan\theta_a = \frac{y}{s}, \quad \tan\theta_b = \frac{-y'}{s'} \\ n_a \sin\theta_a = n_b \sin\theta_b \end{array}\right.$



$\frac{n_a y}{s} = -\frac{n_b y'}{s'}$

$$\Rightarrow \boxed{m = \frac{y'}{y} = -\frac{n_a s'}{n_b s}}$$

Lensmaker's Equation

- We apply the single-surface equation twice.

$$\frac{1}{s_1} + \frac{n}{s_1'} = \frac{n-1}{R_1}. \qquad (1^{st} \text{ surface})$$

$$\frac{n}{s_2} + \frac{1}{s_2'} = \frac{1-n}{R_2}. \qquad (2^{nd} \text{ surface}).$$

Notice that the real image from the first surface becomes the virtual image of the second surface.

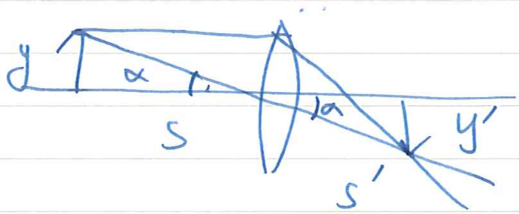
$$\Rightarrow \quad s_2 = -(s_1' - t) \simeq -s_1'$$

$$\Rightarrow \quad \left.\begin{aligned} \frac{1}{s_1} + \frac{n}{s_1'} &= \frac{n-1}{R_1} \\ -\frac{n}{s_1'} + \frac{1}{s_2'} &= \frac{1-n}{R_2} \end{aligned}\right\} \quad \frac{1}{s_1} + \frac{1}{s_2'} = \underbrace{(n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}_{f}$$

$$\Rightarrow \boxed{\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}.}$$

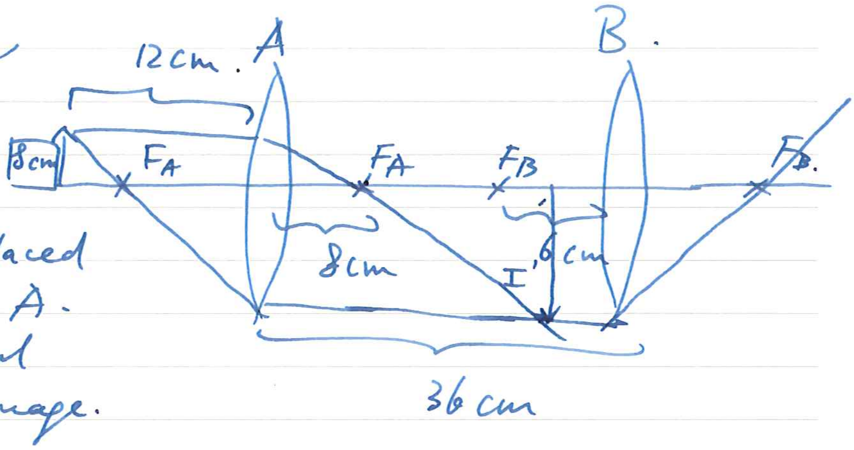
- magnification

$$m = \frac{y'}{y}. = -\frac{s'}{s}.$$



Ex Image of a Image.

2 converging lens A and B, of focal length 8cm and 6cm ~~say~~ respectively, are placed 36 cm apart.
An object 8 cm high is placed 12cm to the left of lens A.
Find the position, size, and orientation of the final image.



1). $\frac{1}{12} + \frac{1}{s_1'} = \frac{1}{8} \Rightarrow s_1' = 24\text{cm}.$

$\rightarrow$ 12 cm to the left of lens B.

$\Rightarrow m_1 = -(\frac{24}{12}) = -2$

$\Rightarrow \frac{1}{12} + \frac{1}{s_2'} = \frac{1}{6} \Rightarrow s_2' = 12\text{ cm}.$

$\Rightarrow m_2 = \frac{-12}{12} = -1$

$\therefore$ The final image is 12cm right of len B.

~~Total~~ The overall magnification $m = m_1 m_2 = 2$ ~~which~~ (same orientation).