1. Photons: Light Waves Behaving as Particles

Looking forward at …

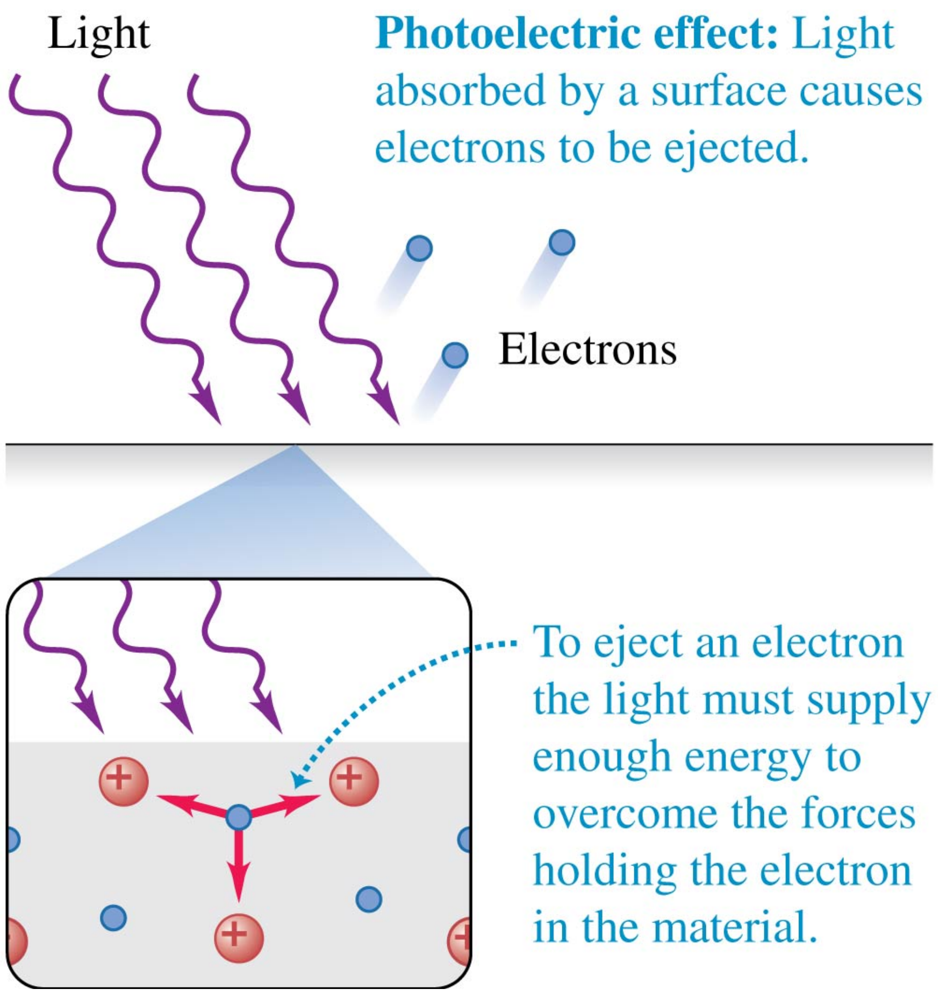
- how Einstein's photon picture of light explains the photoelectric effect.
- how experiments with x-ray production provided evidence that light is emitted in the form of photons.
- how the scattering of gamma rays helped confirm the photon picture of light.
- how the Heisenberg uncertainty principle imposes fundamental limits on what can be measured.

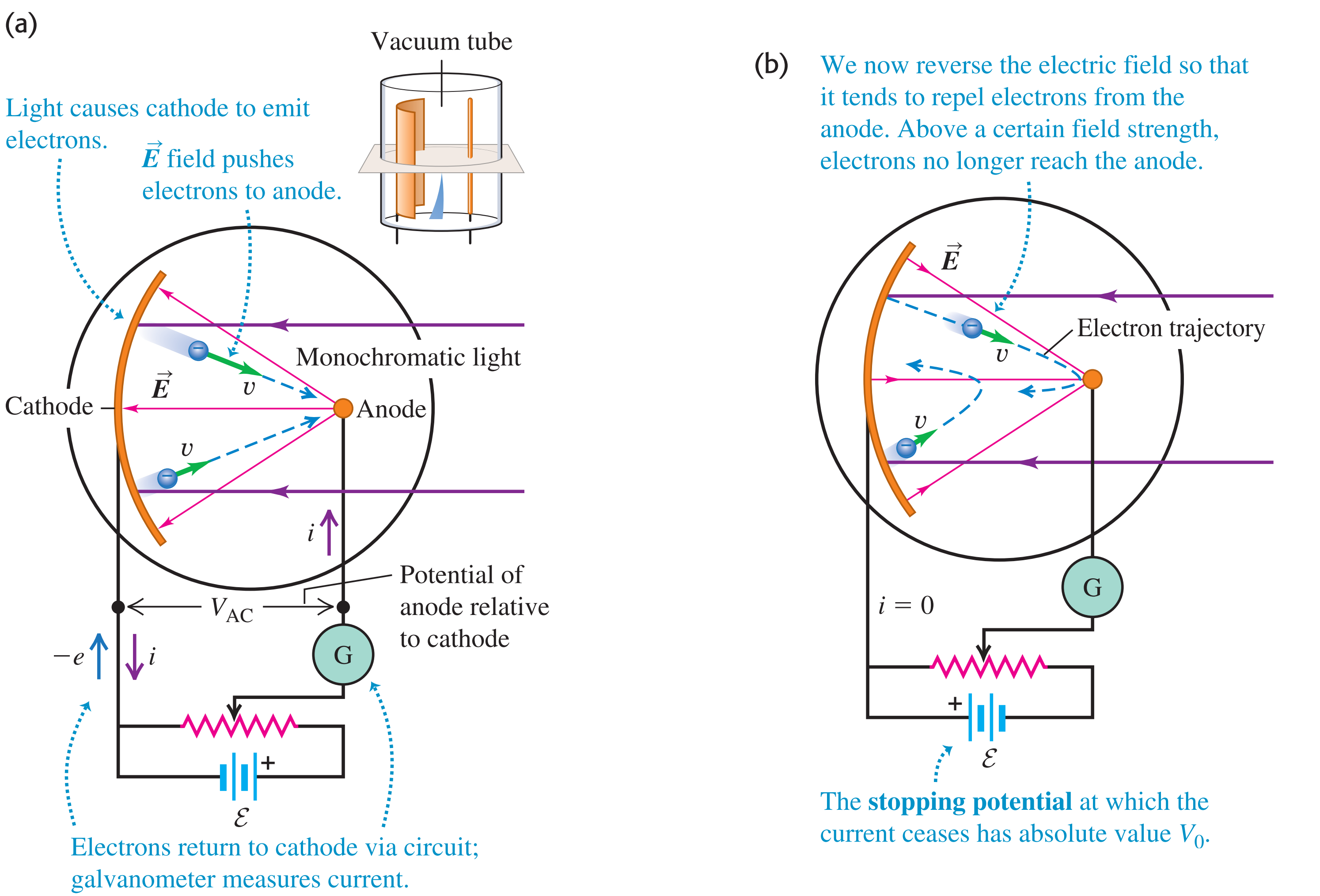
Introduction

- This plastic surgeon is using two light sources: a headlamp that emits a beam of visible light and a handheld laser that emits infrared light.
- The light from both sources is emitted in the form of packets of energy called photons.
- The individual photons in the infrared laser are actually less energetic than the photons in the visible light.

The photoelectric effect

- To escape from the surface, an electron must absorb enough energy from the incident light to overcome the attraction of positive ions in the material.
- These attractions constitute a potential-energy barrier; the light supplies the "kick" that enables the electron to escape.
- The ejected electrons form what is called a **photocurrent**.





$$W_{tot} = -eV_0 = \Delta K = 0 - K_{max}$$

$$K_{max} = \frac{1}{2}mv_{max}^{\;2} = eV_0$$

(maximum kinetic energy of photoelectrons)

Wave-Model Prediction 1: We saw in Section 32.4 that the intensity of an electromagnetic wave depends on its amplitude but not on its frequency. So the photoelectric effect should occur for light of any frequency, and *the magnitude of the photocurrent should not depend on the frequency of the light*.

Wave-Model Prediction 2: It takes a certain minimum amount of energy, called the **work function,** to eject a single electron from a particular surface (see Fig. 38.1). If the light falling on the surface is very faint, some time may elapse before the total energy absorbed by the surface equals the work function. Hence, for faint illumination, *we expect a time delay* between when we switch on the light and when photoelectrons appear.

Wave-Model Prediction 3: Because the energy delivered to the cathode surface depends on the intensity of illumination, *we expect the stopping potential to increase with increasing light intensity*. Since intensity does not depend on frequency, we further expect that *the stopping potential should not depend on the frequency of the light*.

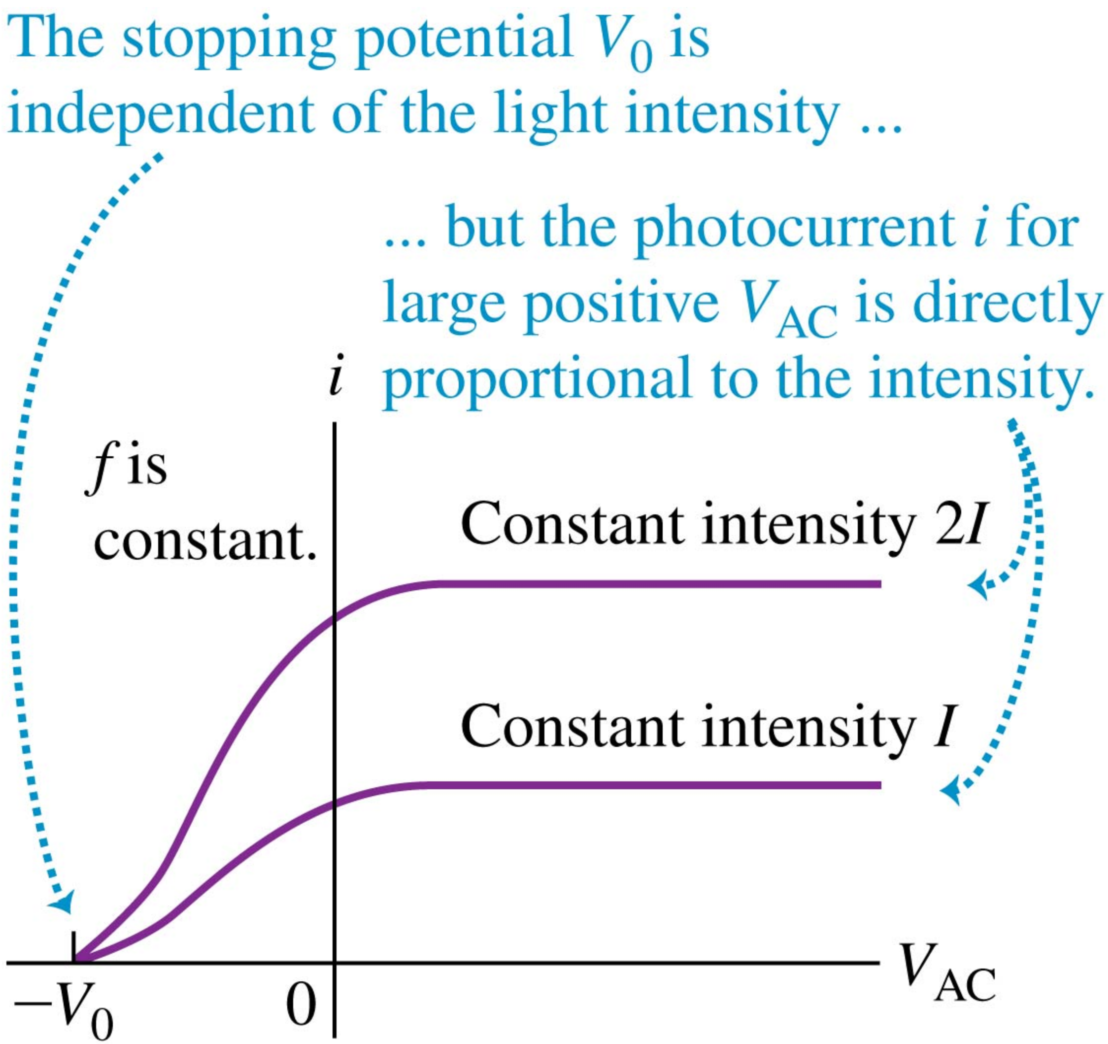
Experimental Result 1: *The photocurrent depends on the light frequency*. For a given material, monochromatic light with a frequency below a minimum **threshold frequency** produces *no* photocurrent, regardless of intensity. For most metals the threshold frequency is in the ultraviolet (corresponding to wavelengths λ between 200 and 300 nm), but for other materials like potassium oxide and cesium oxide it is in the visible spectrum (λ between 380 and 750 nm).

Experimental Result 2: There is *no measurable time delay* between when the light is turned on and when the cathode emits photoelectrons (assuming the frequency of the light exceeds the threshold frequency). This is true no matter how faint the light is.

Experimental Result 3: *The stopping potential does not depend on intensity, but does depend on frequency*. Figure 38.4 shows graphs of photocurrent as a function of potential difference V_{AC} for light of a given frequency and two different intensities. The reverse potential difference $-V_0$ needed to reduce the current to zero is the same for both intensities. The only effect of increasing the intensity is to increase the number of electrons per second and hence the photocurrent i. (The curves level off when V_{AC} is large and positive because at that point all the emitted electrons are being collected by the anode.) If the intensity is held constant but the frequency is increased, the stopping potential also increases. In other words, the greater the light frequency, the higher the energy of the ejected photoelectrons.

Photocurrent in the photoelectric effect

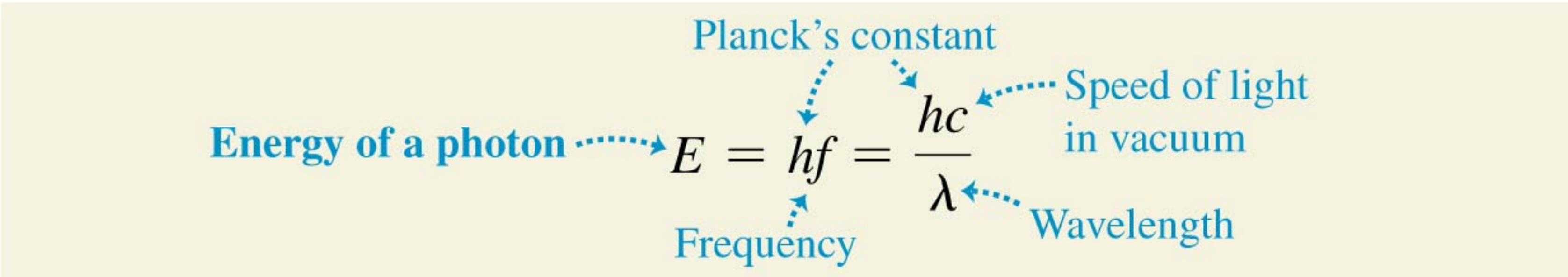
- Shown are graphs of photocurrent as a function of potential difference V_{AC} for light of a given frequency and two different intensities.
- The reverse potential difference $-V_0$ needed to reduce the current to zero is the same for both intensities.



Einstein's photon explanation

- Einstein made the radical postulate that a beam of light consists of small packages of energy called **photons** or quanta.

- The energy of an individual photon is:



- Here **Planck's constant** is $h = 6.626 \times 10^{-34}$ J · s

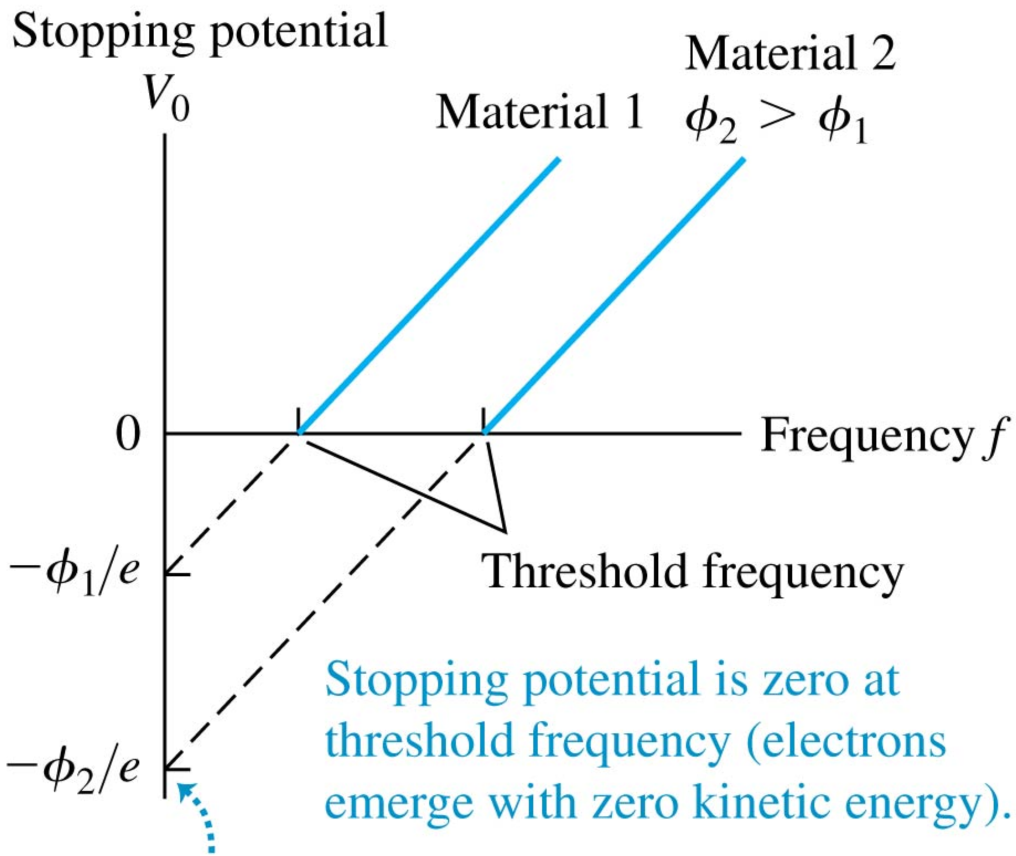
- An individual photon arriving at a surface is absorbed by a single electron.

- The electron can escape from the surface only if the energy it acquires is greater than the work function ϕ.

Einstein's explanation of the photoelectric effect

- This explains how the energy of an emitted electron in the photoelectric effect depends on the frequency of light used.

- The greater the work function of a particular material, the higher the minimum frequency needed to emit photoelectrons.



For each material,

$$eV_0 = hf - \phi \quad \text{or} \quad V_0 = \frac{hf}{e} - \frac{\phi}{e}$$

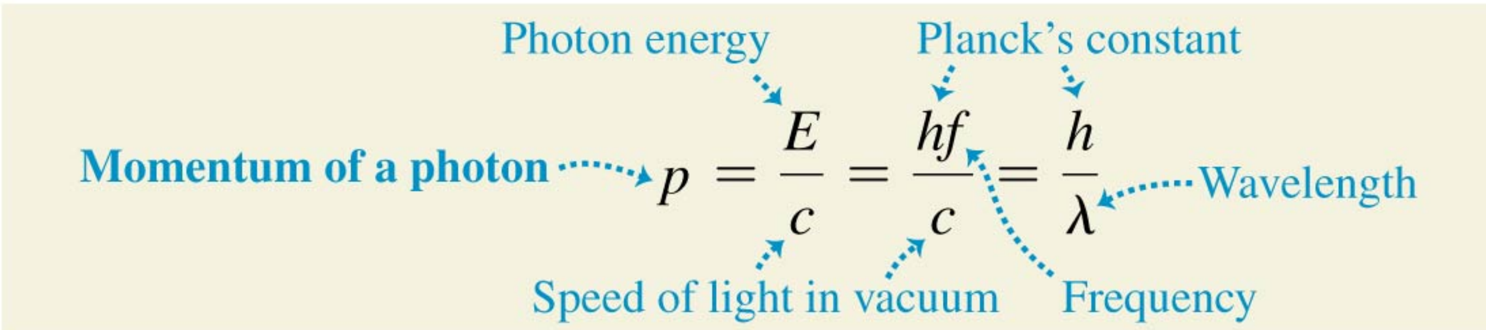
so the plots have same slope h/e but different intercepts $-\phi/e$ on the vertical axis.

Table 38.1: Work functions of several elements

Element	Work Function (eV)
Aluminum	4.3
Carbon	5.0
Copper	4.7
Gold	5.1
Nickel	5.1
Silicon	4.8
Silver	4.3
Sodium	2.7

Photon momentum

- Every particle that has energy must have momentum.
- Photons have zero rest mass, and a particle with zero rest mass and energy E has momentum with magnitude p given by $E = pc$.
- Thus the magnitude p of the momentum of a photon is:



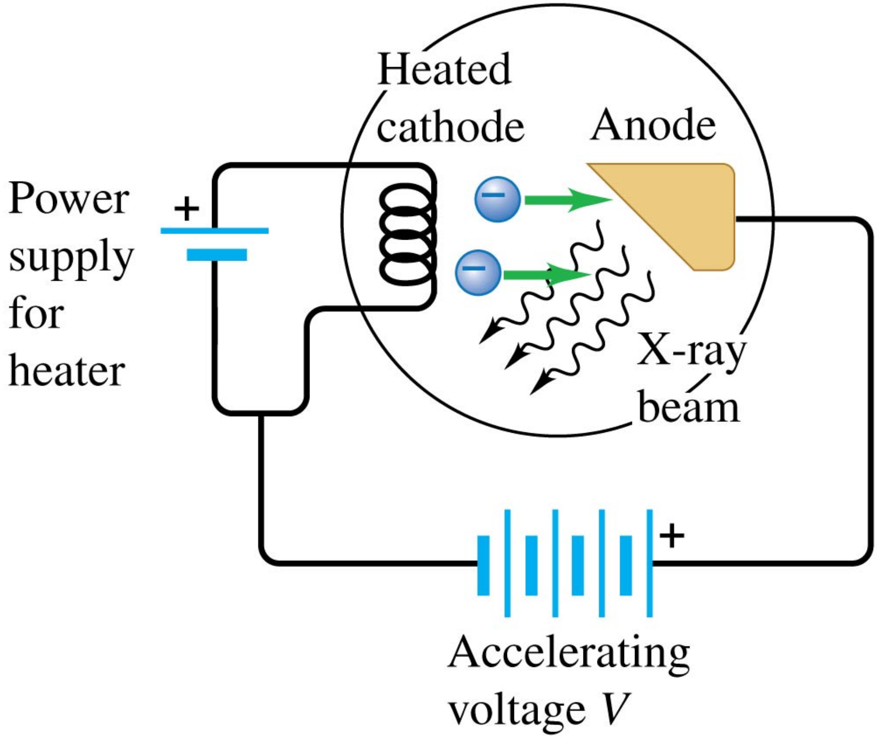
$$p = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda}$$

- The direction of the photon's momentum is simply the direction in which the electromagnetic wave is moving.

X-ray production

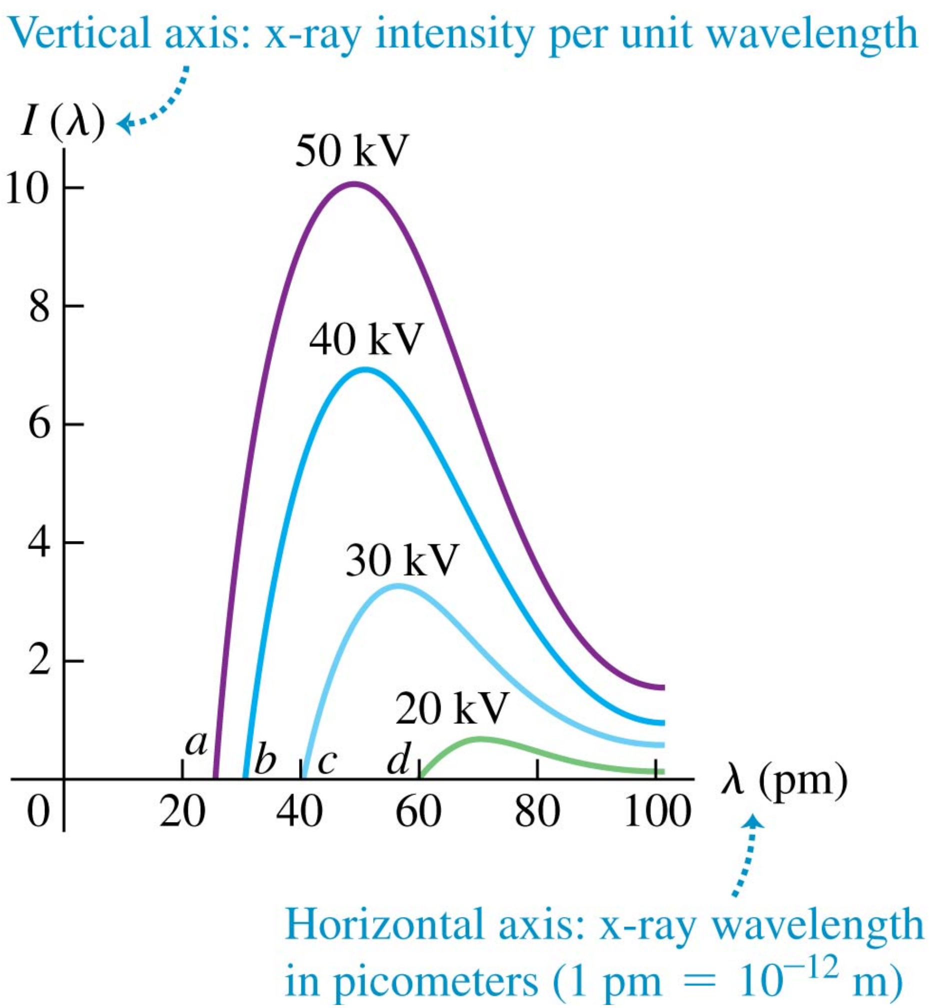
- Inverse of photoelectric effect
- Shown is an experimental arrangement for making x rays.
- The next slide shows the resulting x-ray spectrum.

Electrons are emitted thermionically from the heated cathode and are accelerated toward the anode; when they strike it, x rays are produced.



X-ray production

- The greater the kinetic energy of the electrons that strike the anode, the shorter the minimum wavelength of the x rays emitted by the anode.
- The photon model explains this behavior.
- Higher-energy electrons can convert their energy into higher-energy photons, which have a shorter wavelength.



$$eV_{AC} = hf_{max} = \frac{hc}{\lambda_{min}}$$

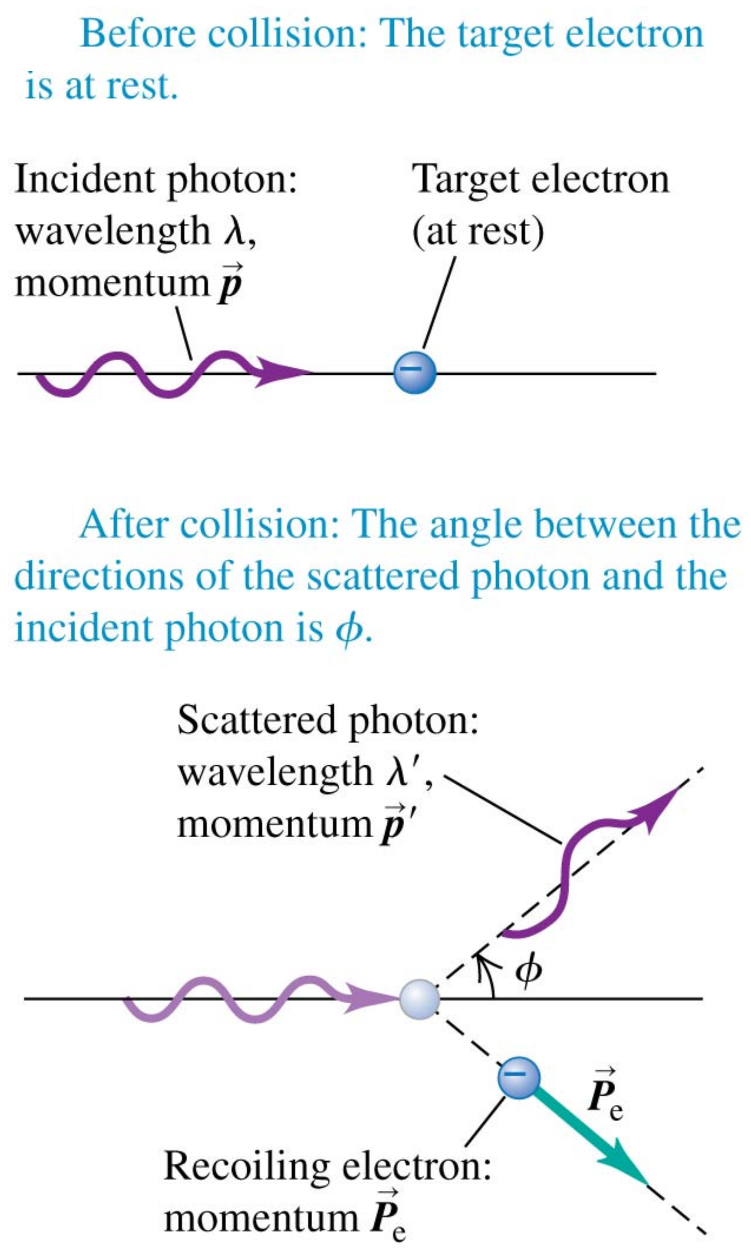
X-ray absorption and medical imaging

- Atomic electrons can absorb x rays.
- Hence materials with many electrons per atom tend to be better x-ray absorbers than materials with few electrons. (Absorption rate ~ Z^3)

- Bones contain large amounts of elements such as phosphorus and calcium, with 15 and 20 electrons per atom, respectively.
- In soft tissue, the predominant elements are hydrogen, carbon, and oxygen, with only 1, 6, and 8 electrons per atom, respectively.
- Hence x rays are absorbed by bone but pass relatively easily through soft tissue.

X-ray scattering: The Compton experiment

- In the Compton experiment, x rays are scattered from electrons.
- The scattered x rays have a longer wavelength than the incident x rays, and the scattered wavelength depends on the scattering angle ϕ.



Wave-Model Prediction: In the wave description, scattering would be a process of absorption and re-radiation. Part of the energy of the light wave would be absorbed by the electron, which would oscillate in response to the oscillating electric field of the wave. The oscillating electron would act like a miniature antenna (see Section 32.1), re-radiating its acquired energy as *scattered* waves in a variety of directions. The frequency at which the electron oscillates would be the same as the frequency of the incident light, and the re-radiated light would have the same frequency as the oscillations of the electron. So, *in the wave model, the scattered light and incident light have the same frequency and same wavelength*.

Photon-Model Prediction: In the photon model we imagine the scattering process as a collision of two *particles,* the incident photon and an electron that is initially at rest (Fig. 38.10a). The incident photon would give up part of its energy and momentum to the electron, which recoils as a result of this impact. The scattered photon that remains can fly off at a variety of angles ϕ with respect to the incident direction, but it has less energy and less momentum than the incident photon (Fig. 38.10b). The energy and momentum of a photon are given by $E = hf = hc/\lambda$ (Eq. 38.2) and $p = hf/c = h/\lambda$ (Eq. 38.5). Therefore, *in the photon model, the scattered light has a lower frequency f and longer wavelength λ than the incident light.*

Compton scattering

- In **Compton scattering**, an incident photon collides with an electron that is initially at rest.
- The photon gives up part of its energy and momentum to the electron, which recoils as a result of this impact.
- The scattered photon flies off at an angle ϕ with respect to the incident direction, but it has less energy and less momentum than the incident photon.
- Therefore, **the wavelength of the scattered photon λ' is longer than the wavelength λ of the incident photon**.

Compton scattering:

$$\lambda' - \lambda = \frac{h}{mc}(1 - \cos\phi)$$

- λ': Wavelength of scattered radiation
- λ: Wavelength of incident radiation
- h: Planck's constant
- ϕ: Scattering angle
- m: Electron rest mass
- c: Speed of light in vacuum

energy of electron

$$pc + mc^2 = p'c + E_e$$

Conservation of energy

$$E_e^2 = mc^2 + (P_e c)^2$$

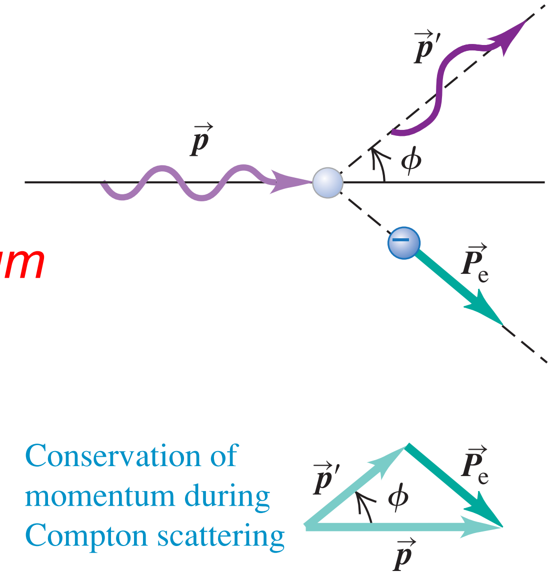
$$\vec{P}_e = \vec{p} - \vec{p}\,' \Rightarrow P_e^2 = p^2 + p'^2 + 2pp'\cos\phi$$

Conservation of momentum

Eliminate P_e, we get

$$\lambda - \lambda' = \frac{h}{mc}(1 - \cos\phi)$$

38.12 Vector diagram showing conservation of momentum in Compton scattering.

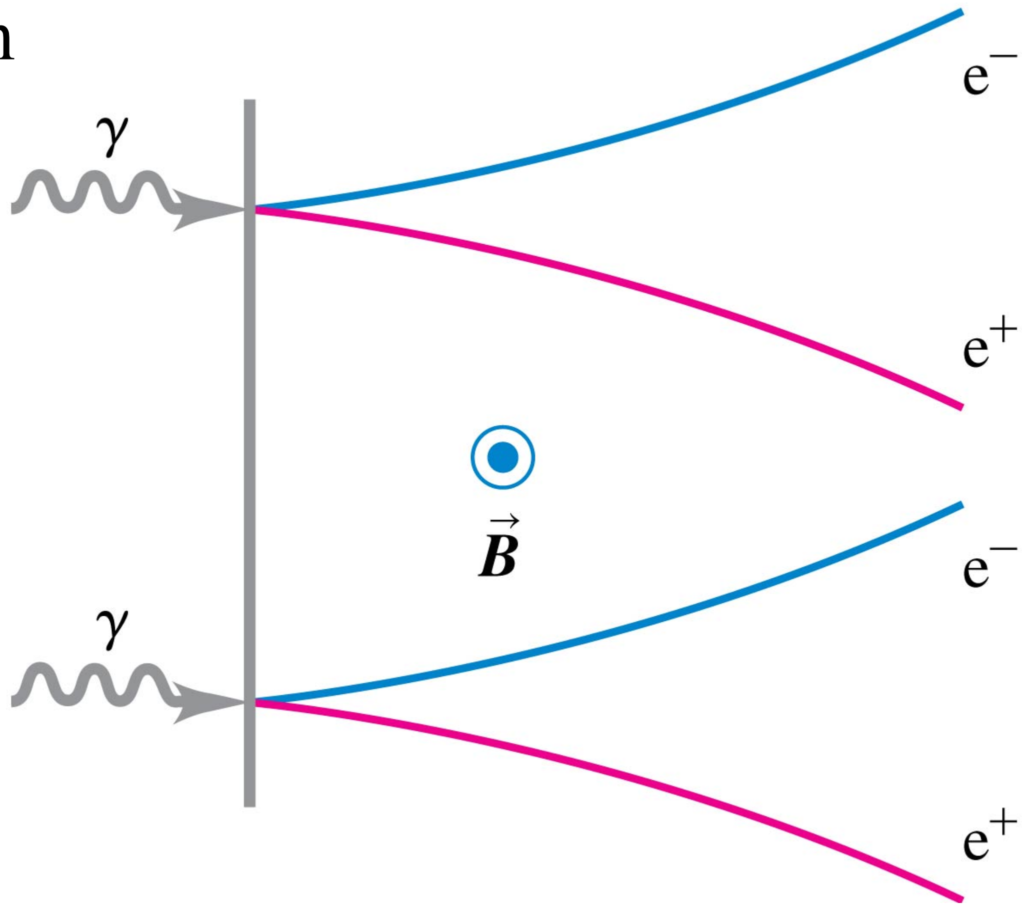


Pair production

- When gamma rays of sufficiently short wavelength are fired into a metal plate, they can convert into an electron and a **positron**, each of mass m and rest energy mc^2.

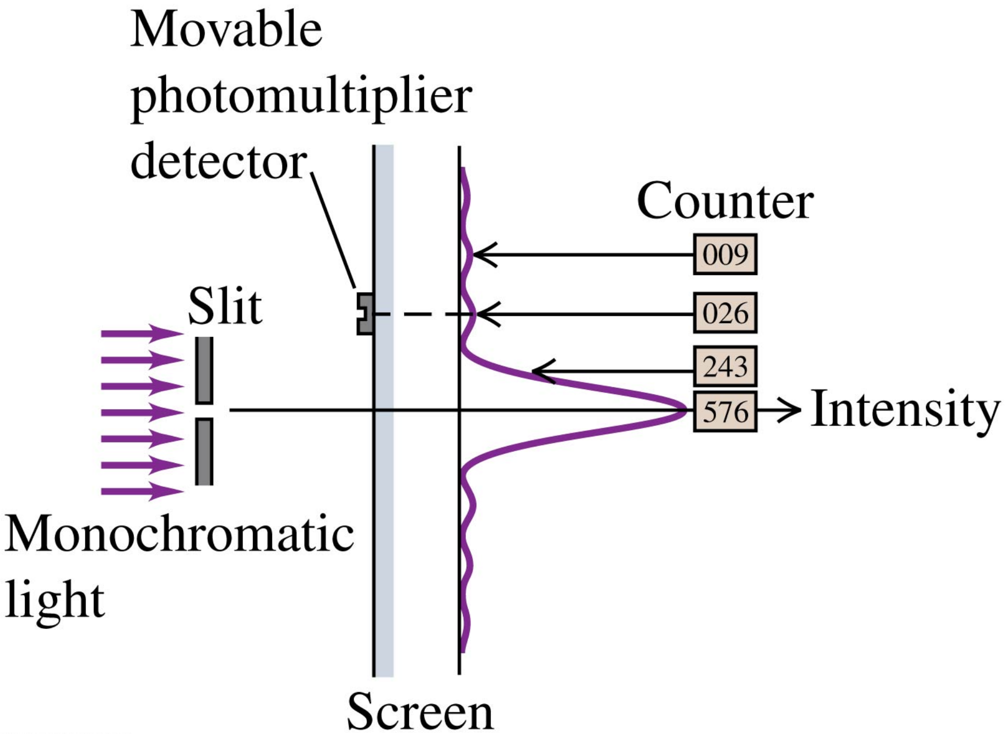
- The photon model explains this: The photon wavelength must be so short that the photon energy is at least $2mc^2$.

$$\lambda_{\text{max}} = \frac{hc}{E_{\text{min}}} = 1.213 \times 10^{-3}\, nm$$



Diffraction and uncertainty

- When a photon passes through a narrow slit, its momentum becomes uncertain and the photon can deflect to either side.



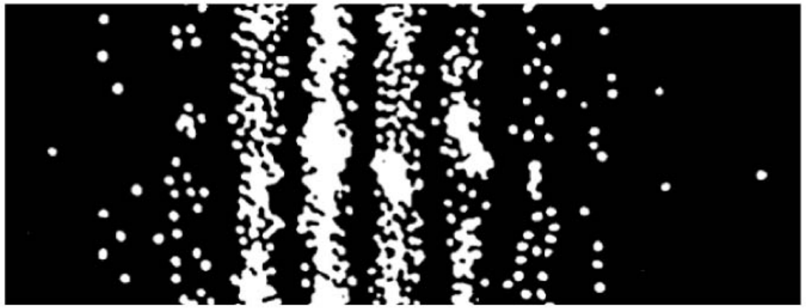
- A diffraction pattern is the result of many photons hitting the screen.
- The pattern appears even if only one photon is present at a time in the experiment (i.e. extremely low intensity).
- The pattern is the probability of any individual photon will land at a given spot

Diffraction and uncertainty

- These images record the positions where individual photons in a two-slit interference experiment strike the screen.
- As more photons reach the screen, a recognizable interference pattern appears.

After 21 photons reach the screen

After 1000 photons reach the screen



After 10,000 photons reach the screen

Uncertainty in Diffraction

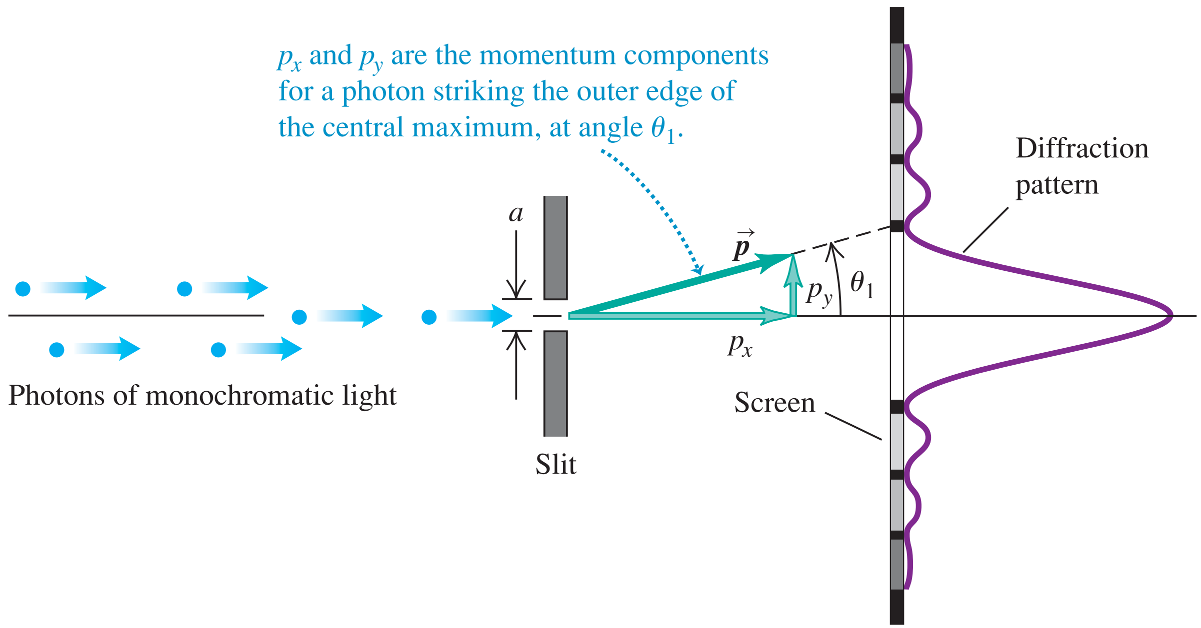
In the theory of diffraction, the angle between the central maximum and the first minimum is:

$$\sin\vartheta = \frac{\lambda}{a}, \text{ we have } \frac{p_y}{p_x} = \tan\vartheta \approx \vartheta = \frac{\lambda}{a} \Rightarrow p_y = p_x\frac{\lambda}{a}$$

i.e. the y-compoment momentum is spread out over a range between $-p_x\frac{\lambda}{a}$ and $p_x\frac{\lambda}{a}$.

There will be an uncertainty Δp_y at least eqaul to $p_x\frac{\lambda}{a}$.

$$\Delta p_y \geq p_x\frac{\lambda}{a} \Rightarrow \Delta p_y \geq \frac{h}{\lambda}\frac{\lambda}{a} = \frac{h}{a} \Rightarrow \Delta p_y a \geq h \Rightarrow \boxed{\Delta p_y \Delta y \geq h}$$



The Heisenberg uncertainty principle

- You cannot simultaneously know the position and momentum of a photon, or any other particle, with arbitrarily great precision.

- The better you know the value of one quantity, the less well you know the value of the other.

Heisenberg uncertainty principle for position and momentum:

Uncertainty in coordinate x

Planck's constant divided by 2π

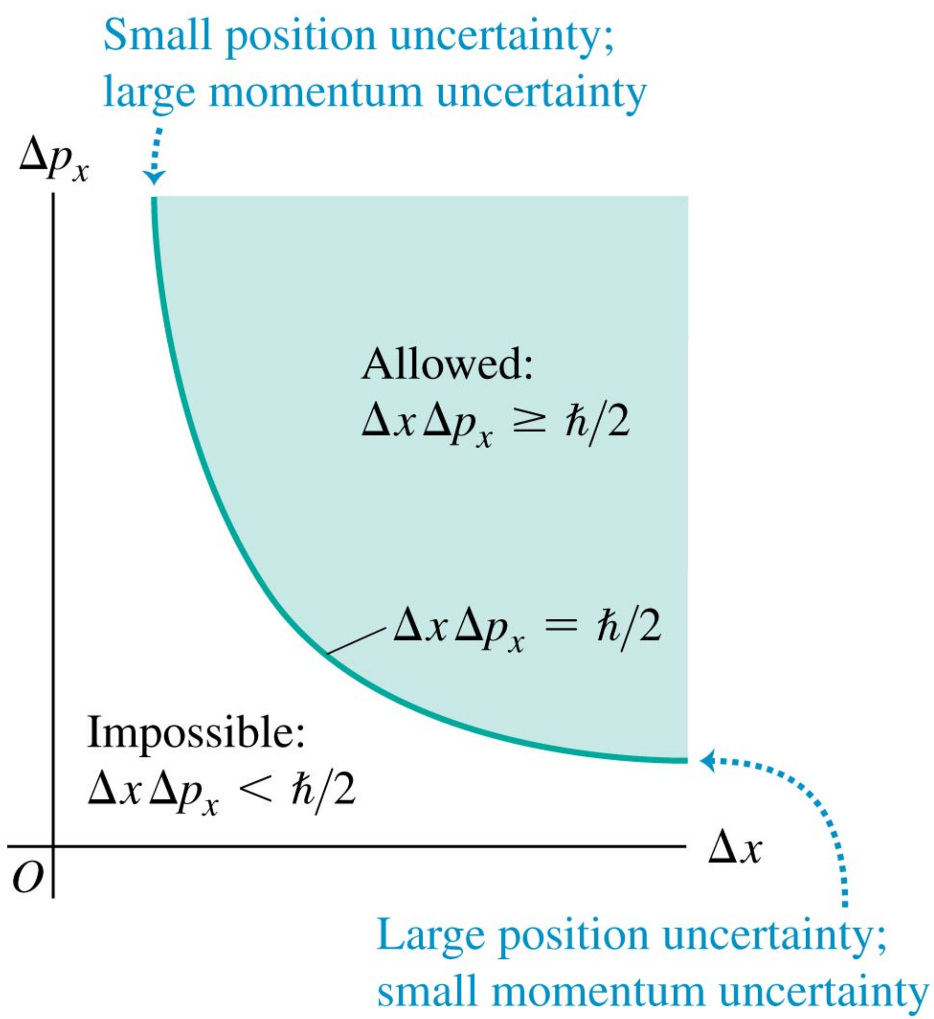
$$\Delta x \, \Delta p_x \geq \hbar/2$$

Uncertainty in corresponding momentum component p_x

- There is a similar uncertainty relationship for the *y*- and *z*-coordinate axes and their corresponding momentum components.

The Heisenberg uncertainty principle

- Shown is a graphical representation of the Heisenberg uncertainty principle.
- A measurement with uncertainties whose product puts them to the left of or below the blue line is not possible to make.



Uncertainty in energy

- There is also an uncertainty principle that involves energy and time.

- The better we know a photon’s energy, the less certain we are of when we will observe the photon:

Heisenberg uncertainty principle for energy and time:

Time uncertainty of a phenomenon

Planck’s constant divided by 2π

$$\Delta t \Delta E \geq \hbar/2$$

Energy uncertainty of same phenomenon

- This relation holds true for other kinds of particles as well.

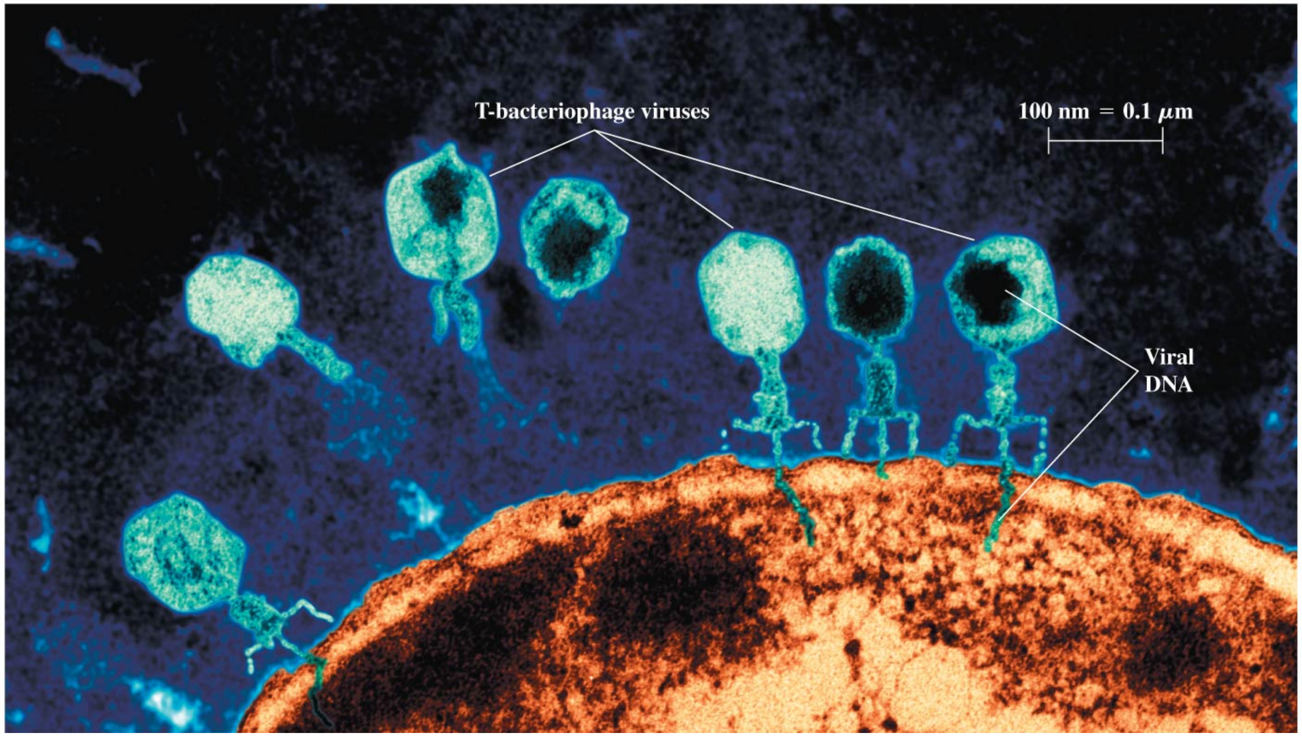
2. Particles Behaving as Waves

Looking forward at …

- de Broglie's proposal that electrons and other particles can behave like waves.
- how physicists discovered the atomic nucleus.
- how Bohr's model of electron orbits explained the spectra of hydrogen and hydrogenlike atoms.
- how a laser operates.
- how the idea of electron energy levels, coupled with the photon model of light, explains the spectrum of light emitted by a hot, opaque object.

Introduction

- Viruses (shown in blue) have landed on an *E. coli* bacterium and injected their DNA, converting the bacterium into a virus factory.
- This false-color image was made by using a beam of electrons rather than a light beam.



de Broglie waves

- In 1924 a French physicist, Louis de Broglie (pronounced “de broy”), proposed that particles may, in some situations, behave like waves.

- A free particle with rest mass *m*, moving with non-relativistic speed *v*, should have a wavelength related to its momentum:

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

De Broglie wavelength of a particle (λ); Planck’s constant (h); Particle’s momentum (p); Particle’s mass (m); Particle’s speed (v)

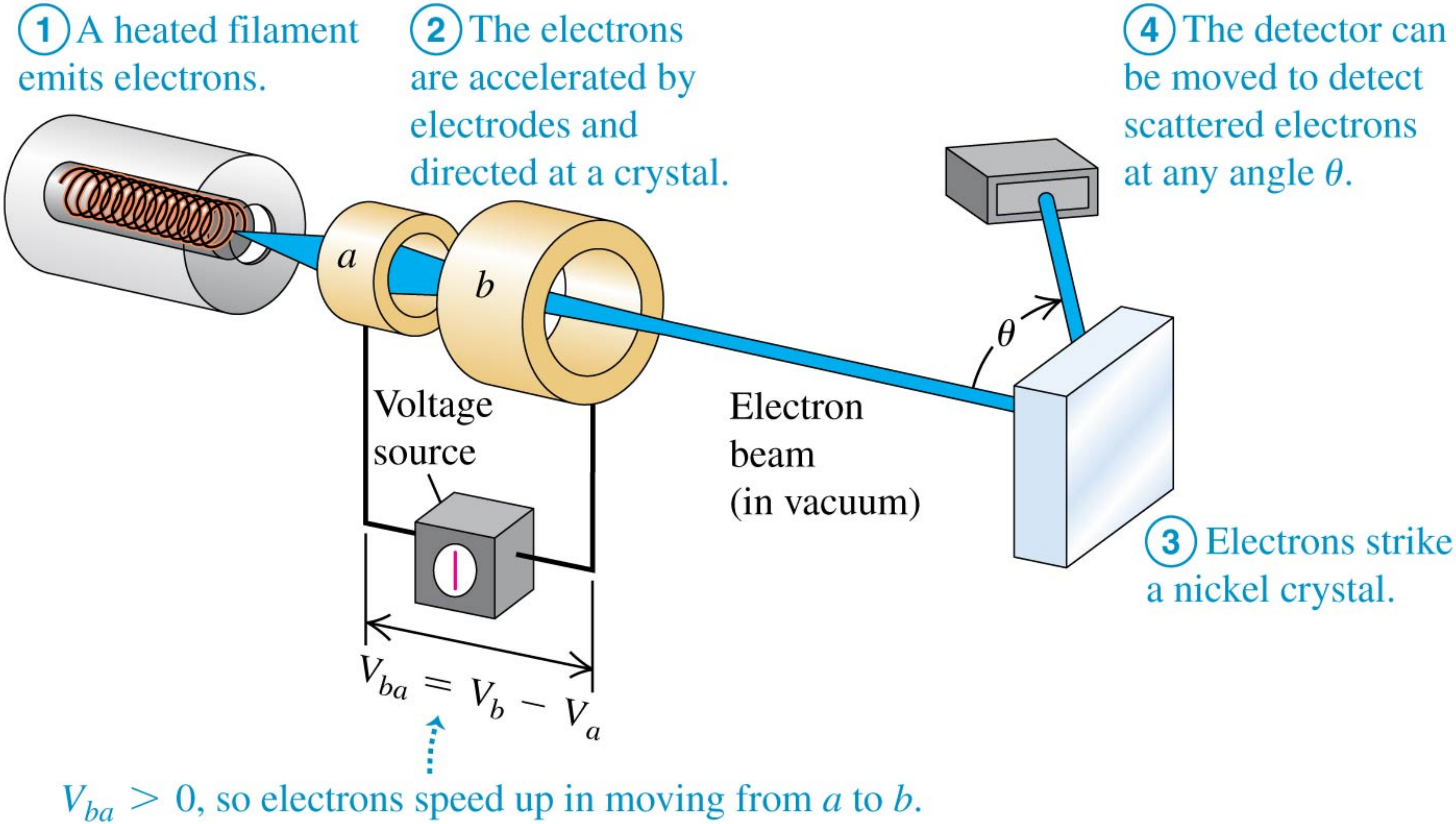
- A particle’s frequency is related to its energy in the same way as for a photon:

$$E = hf$$

Energy of a particle (E); Planck’s constant (h); Frequency (f)

Davisson and Germer experiment

- Shown is an apparatus used to study electron diffraction.



electron diffraction

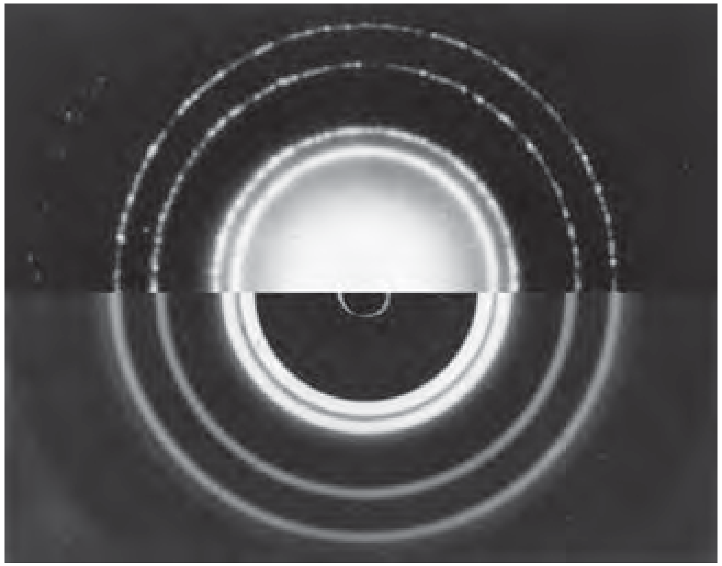
$$eV = \frac{p^2}{2m} \Rightarrow p = \sqrt{2meV} \Rightarrow \lambda = \frac{h}{p} = \frac{h}{\sqrt{2meV}}$$

The constructive interference occur when

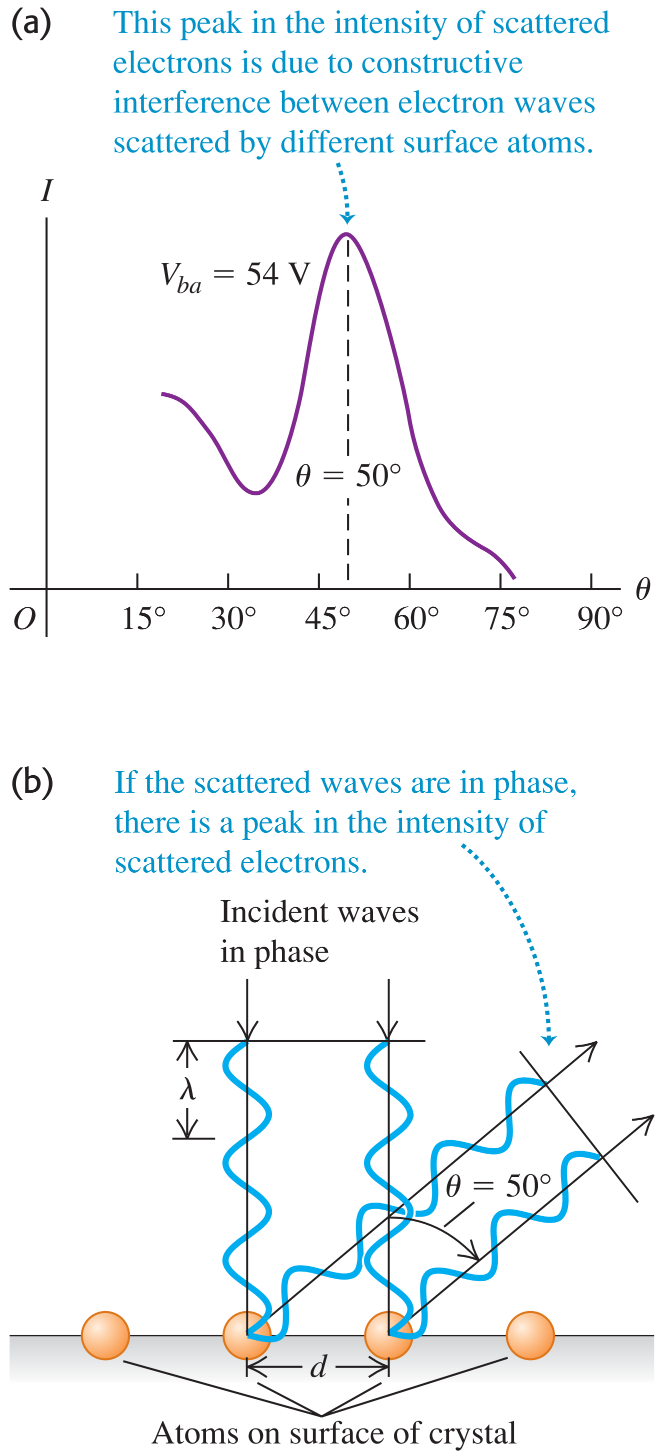
$$d \sin\vartheta = n\lambda$$

where d is the distance between rows in the crystal

Top: x-ray diffraction

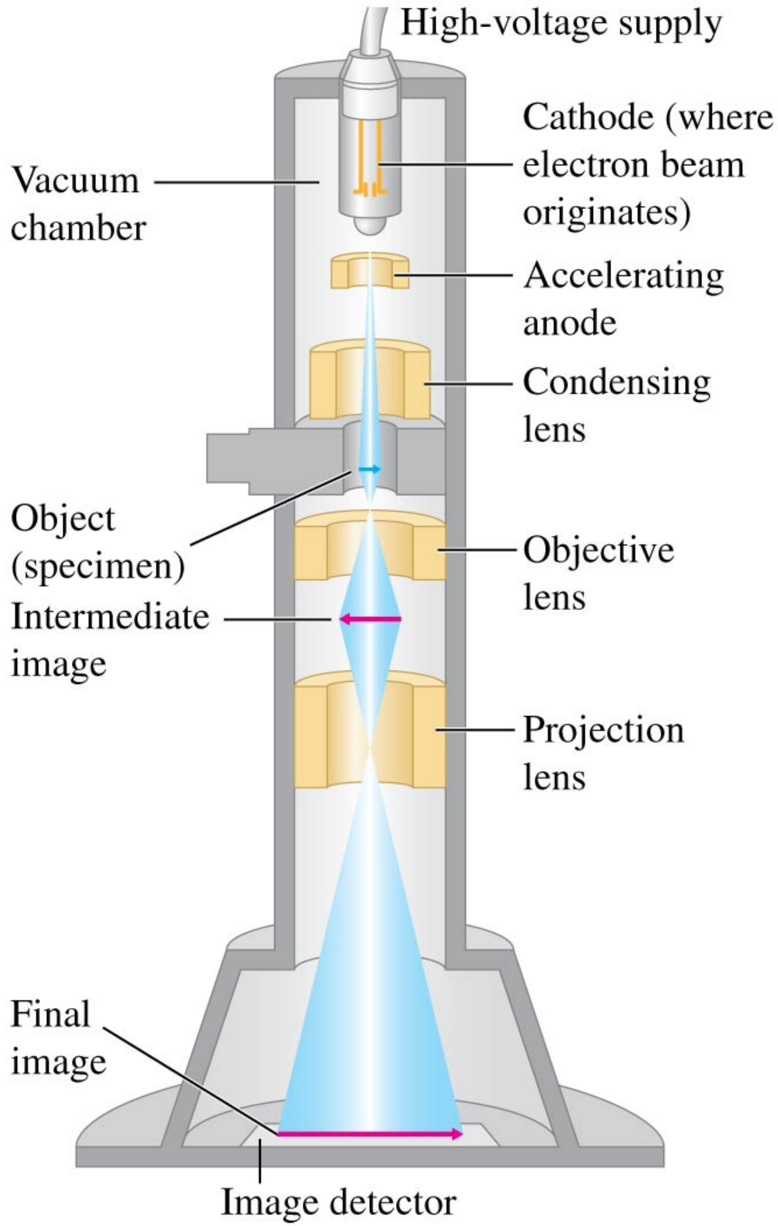


Bottom: electron diffraction

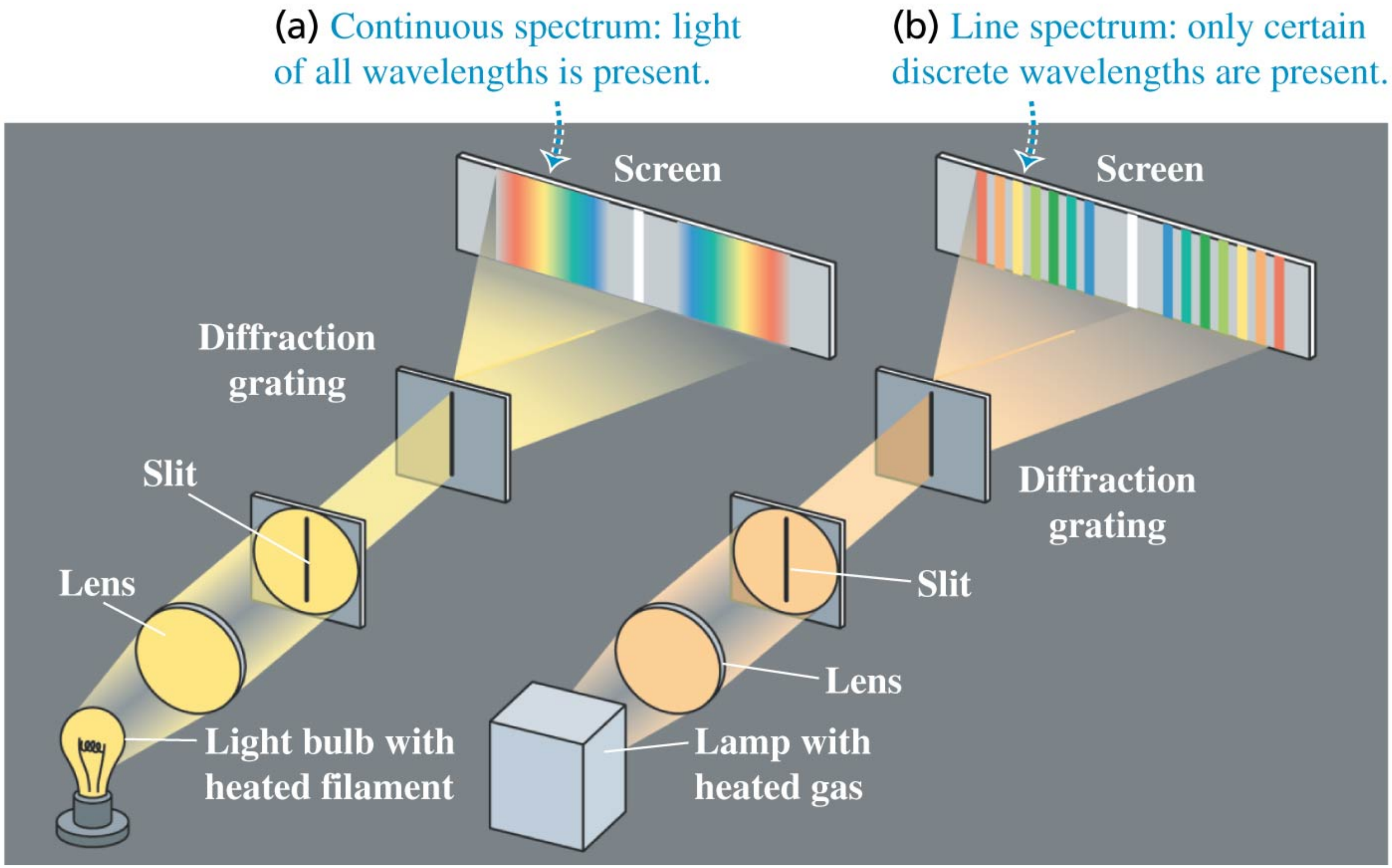


Electron microscopy

- The wave aspect of electrons means that they can be used to form images, just as light waves can.
- This is the basic idea of the transmission electron microscope (TEM), shown.
- The “lenses” are actually coils that use magnetic fields to focus the electrons.
- The resolution is limited by diffraction effect (depend on wavelength) $\lambda_e \sim 0.01nm$



Atomic line spectra



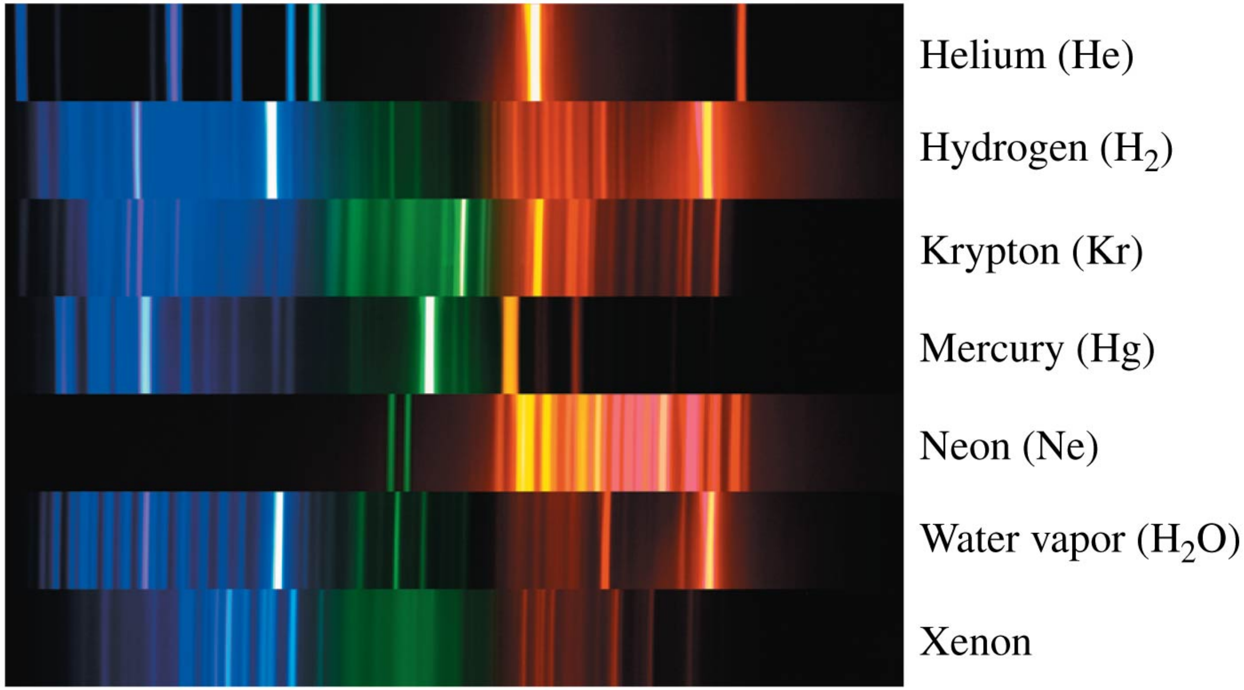
- The light emitted by atoms in a sample of heated gas includes only certain discrete wavelengths. Nineteenth-century physics does not explain this.

Atomic line spectra

- Shown are the emission line spectra of several kinds of atoms and molecules.

- No two are alike.

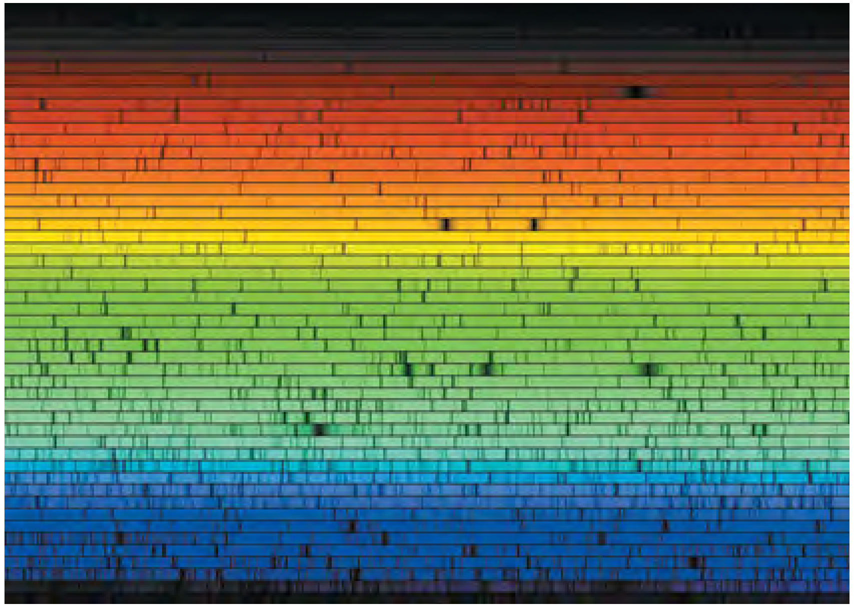
- Note that the spectrum of water vapor (H_2O) is similar to that of hydrogen (H_2), but there are important differences that make it straightforward to distinguish these two spectra.



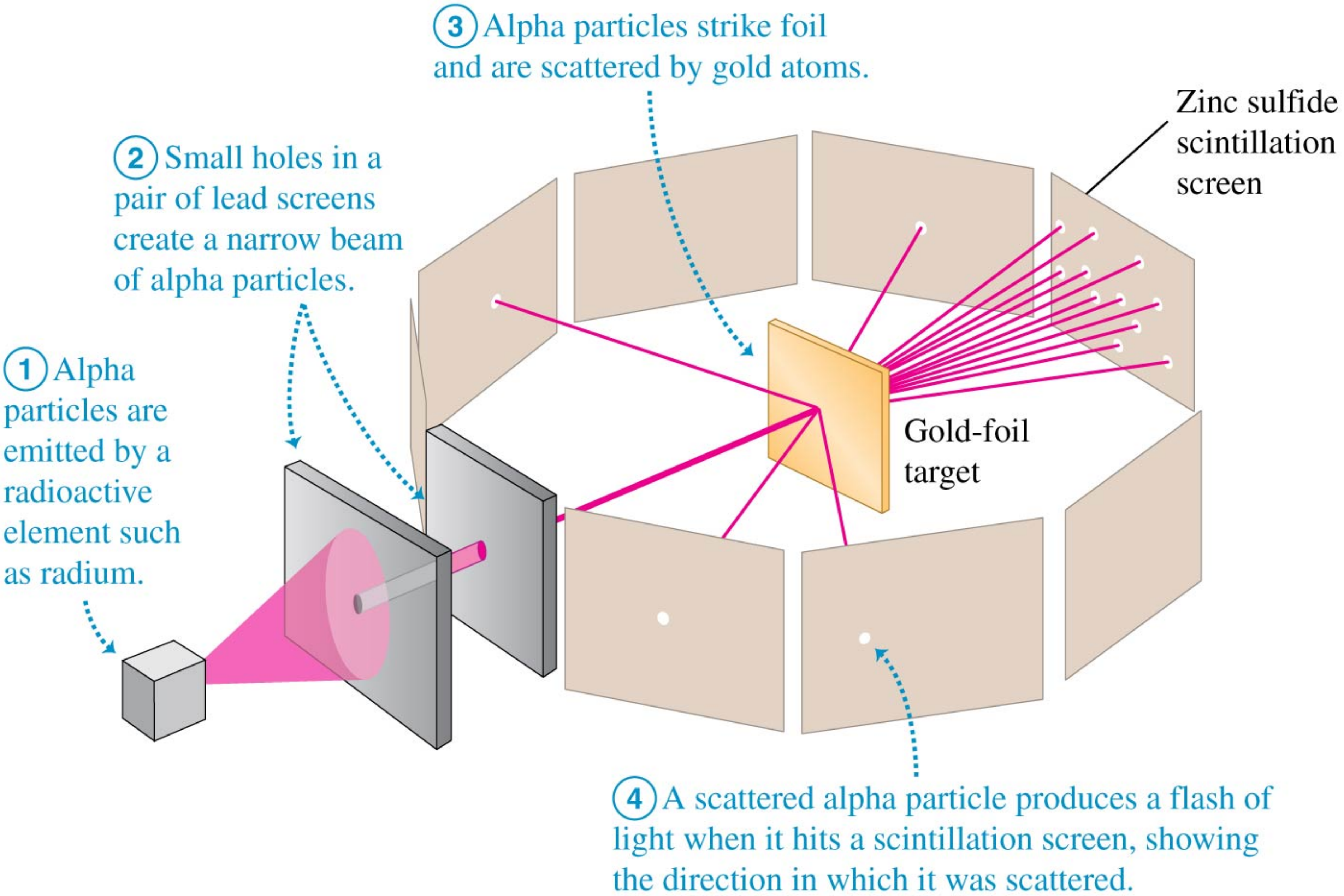
Absorption line spectrum

- If white light pass through the gas and look at the transmitted light with a spectrometer.
- The dark lines correspond to the wavelengths that have been absorbed by the gas.

39.9 The absorption line spectrum of the sun. (The spectrum "lines" read from left to right and from top to bottom, like text on a page.) The spectrum is produced by the sun's relatively cool atmosphere, which absorbs photons from deeper, hotter layers. The absorption lines thus indicate what kinds of atoms are present in the solar atmosphere.



The Rutherford scattering experiment (1911)

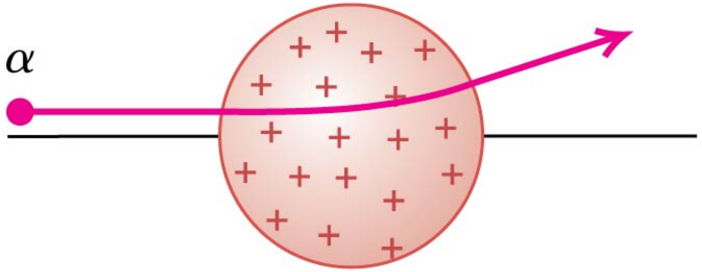


The nuclear atom

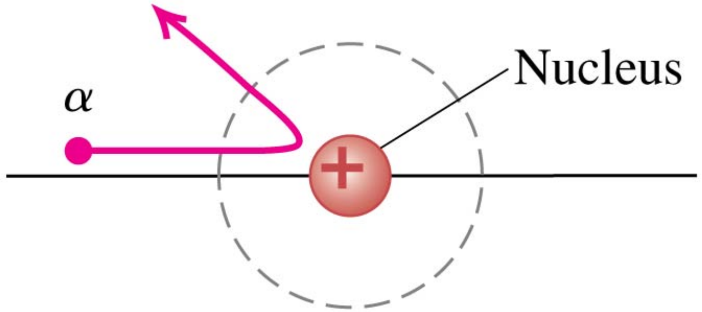
- Rutherford probed the structure of the atom by sending alpha particles at a thin gold foil.

- Some alpha particles were scattering by **large** angles, leading him to conclude that the atom's positive charge is concentrated in a nucleus at its center.

(a) Thomson's model of the atom: An alpha particle is scattered through only a small angle.

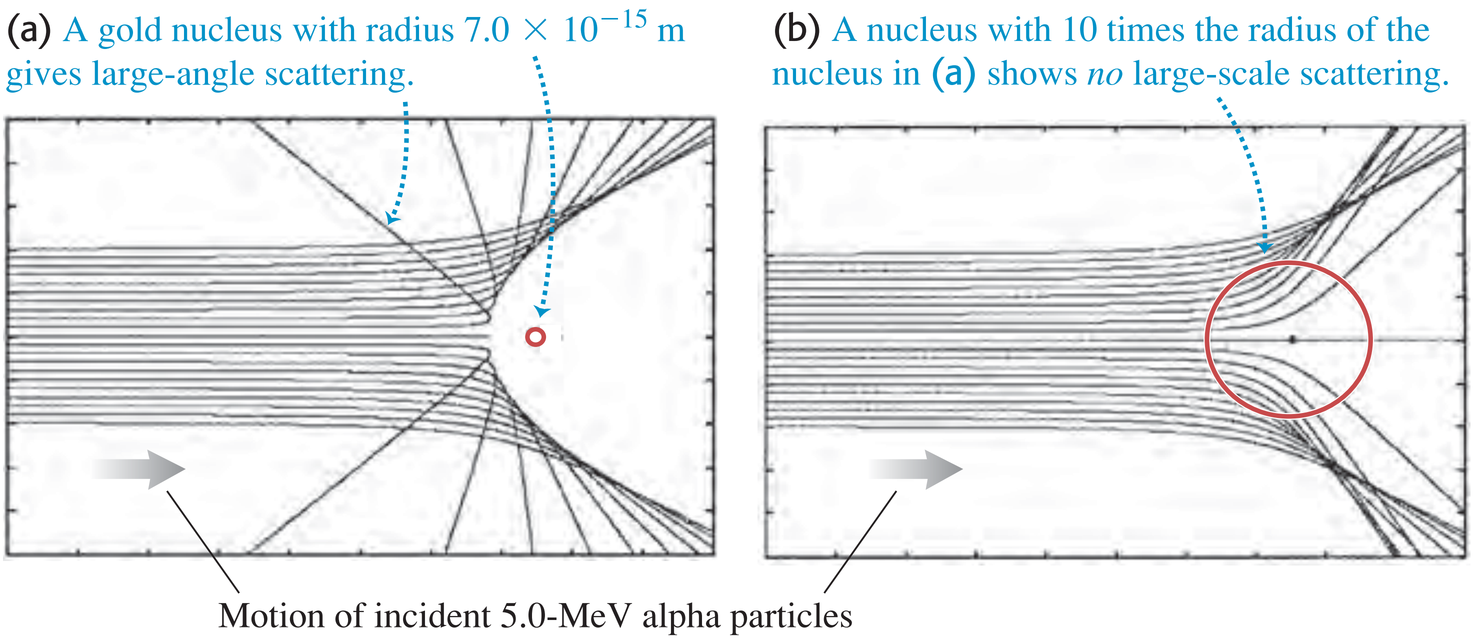


(b) Rutherford's model of the atom: An alpha particle can be scattered through a large angle by the compact, positively charged nucleus (not drawn to scale).



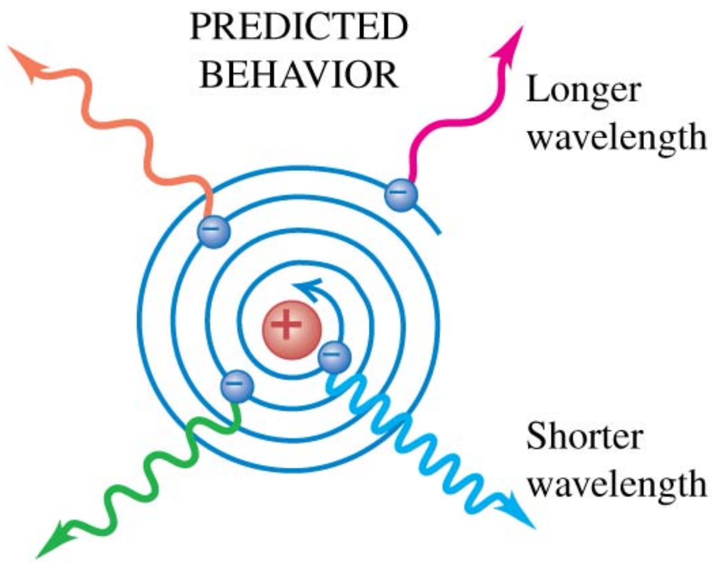
The Rutherford scattering experiment

39.13 Computer simulation of scattering of 5.0-MeV alpha particles from a gold nucleus. Each curve shows a possible alpha-particle trajectory. **(a)** The scattering curves match Rutherford's experimental data if a radius of 7.0×10^{-15} m is assumed for a gold nucleus. **(b)** A model with a much larger radius for the gold nucleus does not match the data.



The failure of classical physics

ACCORDING TO CLASSICAL PHYSICS



- An orbiting electron is accelerating, so it should radiate electromagnetic waves.
- The electron's angular speed would increase as its orbit shrank (Kepler's 2nd law), so the frequency of the radiated waves should increase.
- The waves would carry away energy, so the electron should lose energy and spiral inward.
- Thus, classical physics says that atoms should collapse within a fraction of a second and should emit light with a continuous spectrum as they do so.

The Bohr model of hydrogen

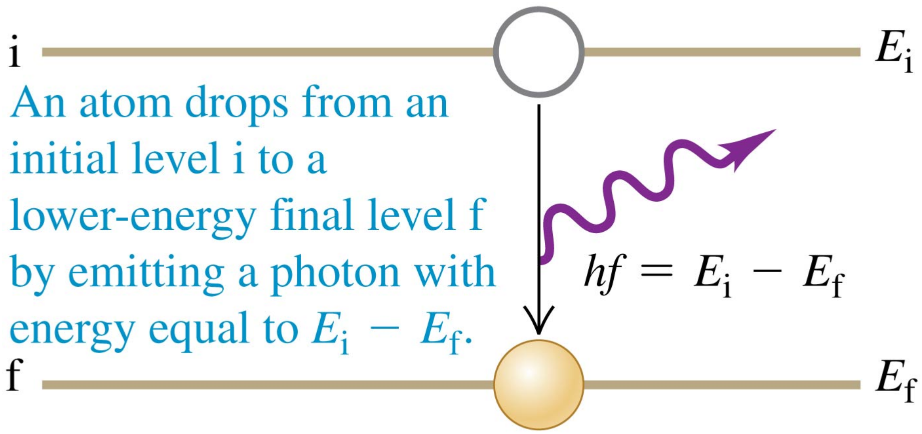
- Niels Bohr (1885–1962) postulated that each energy level of a hydrogen atom corresponds to a **specific stable circular orbit** of the electron around the nucleus, where the corresponding energy can have only certain particular values **(energy level).**

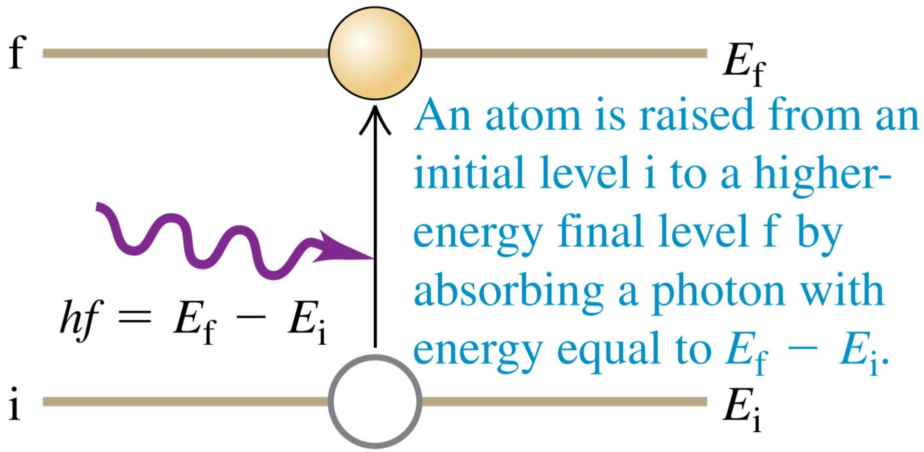
- In the Bohr model, an atom radiates energy only when an electron makes a transition from an orbit of energy E_i to a different orbit with lower energy E_f , emitting a photon of energy $hf = E_\mathrm{i} - E_\mathrm{f}$ in the process.

- Bohr won the 1922 Nobel Prize in physics for these ideas.

Atomic energy levels

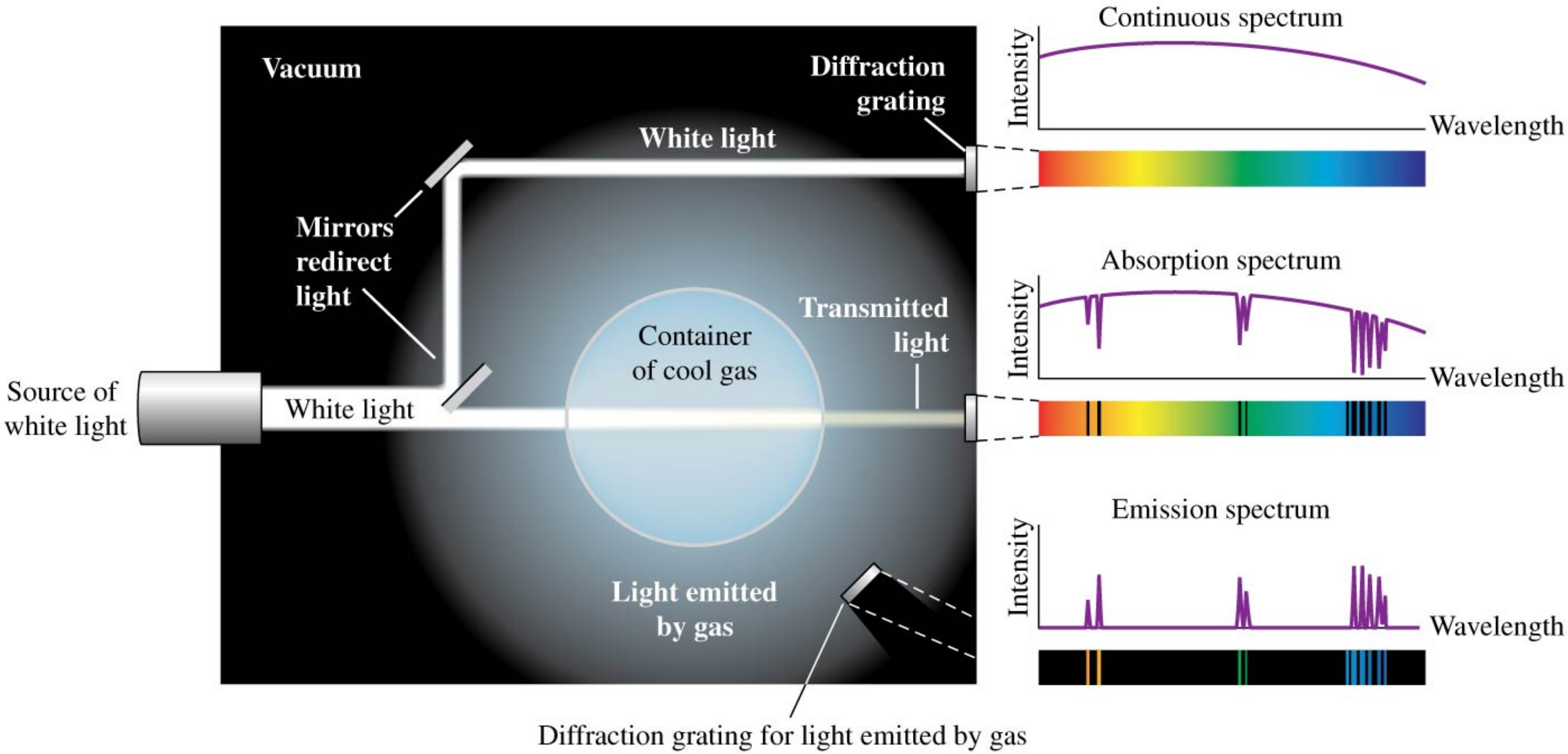
- When an atom makes a transition from one energy level to a lower level, it **emits** a photon whose energy equals that lost by the atom.
- An atom can also **absorb** a photon, provided the photon energy equals the difference between two energy levels.
- The lowest energy level is called ground state, levels with energies greater than the ground level are called excited levels.





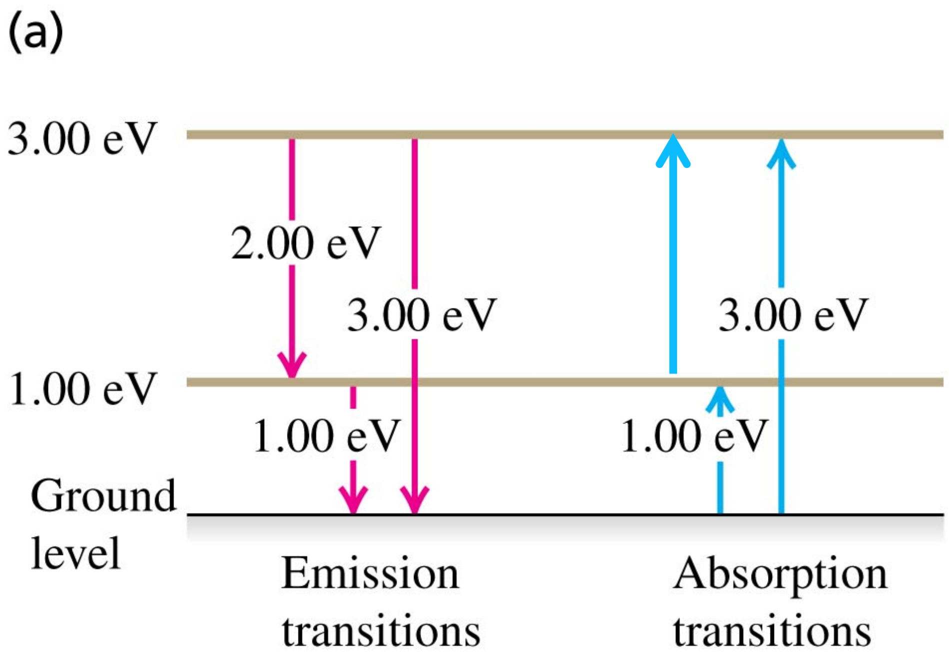
Atomic energy levels

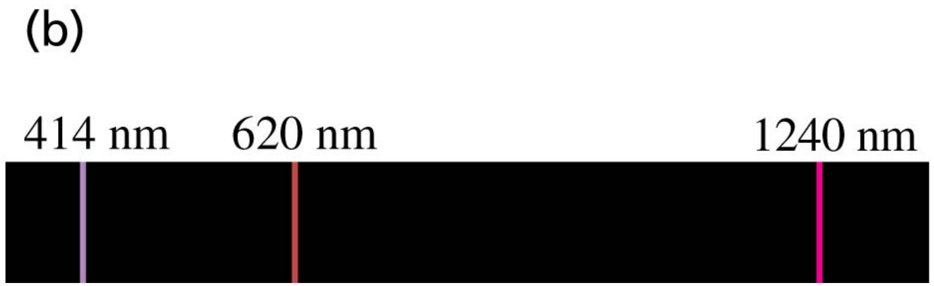
- A cool gas that's illuminated by white light to make an absorption line spectrum also produces an emission line spectrum when viewed from the side.



Emission spectrum of a hypothetical atom

- Consider a hypothetical atom that has energy levels at 0.00 eV, 1.00 eV, and 3.00 eV.
- (a) shows the energy-level diagram for the hypothetical atom.
- (b) shows the emission spectrum of this hypothetical atom.





The Bohr model of hydrogen

- Bohr found that the magnitude of the electron's angular momentum is **quantized**; that is, this magnitude must be an integral multiple of $h/2\pi$.
- Let's number the orbits by the **principal quantum number** n, where $n = 1, 2, 3, \ldots$, and call the radius of orbit n, r_n, and the speed of the electron in that orbit v_n.
- The magnitude of the angular momentum of an electron of mass m in such an orbit is:

Quantization of angular momentum:

$$L_n = mv_n r_n = n\frac{h}{2\pi}$$

Orbital angular momentum (L_n)

Principal quantum number ($n = 1, 2, 3, \ldots$)

Planck's constant (h)

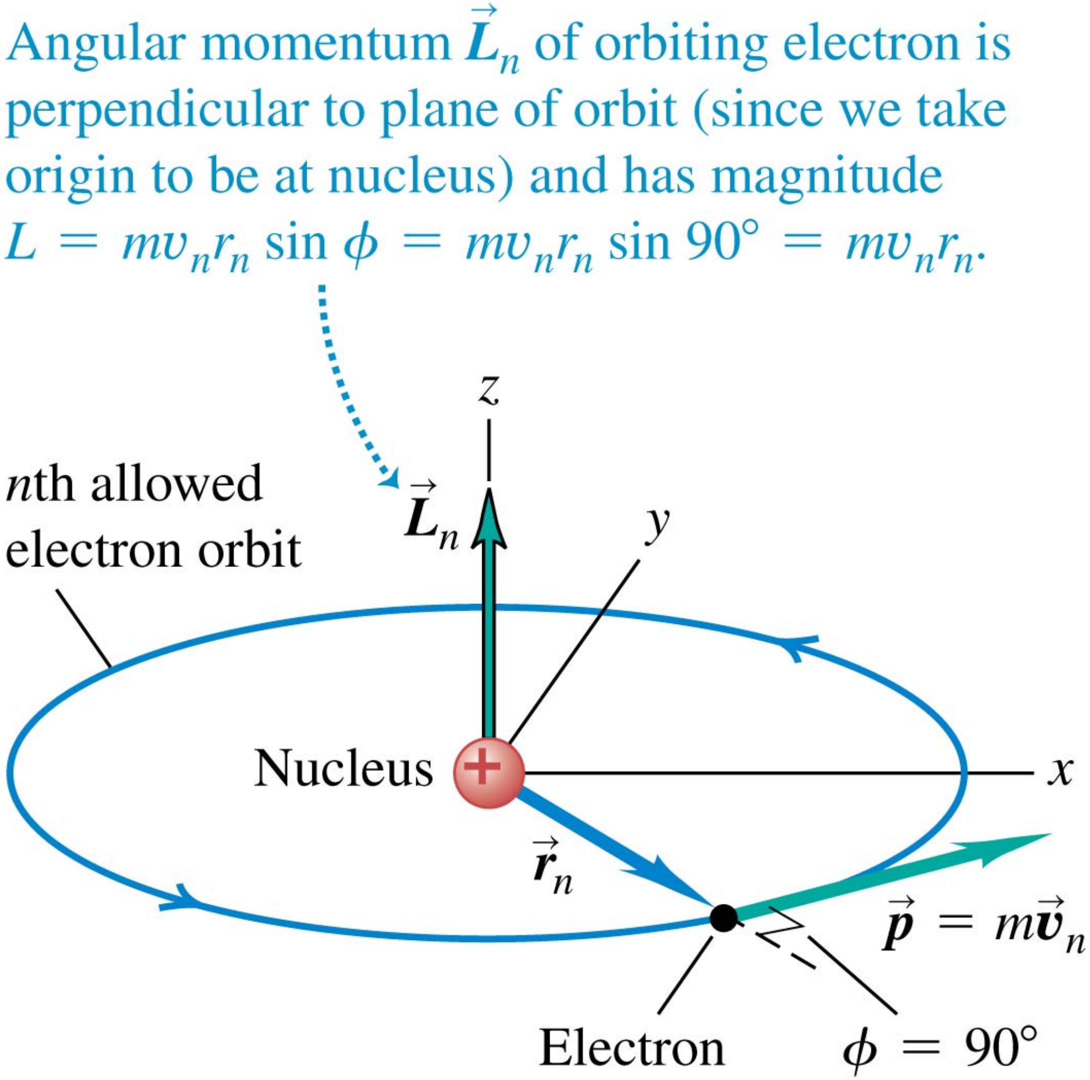
Electron mass (m)

Electron speed (v_n)

Electron orbital radius (r_n)

The Bohr model of hydrogen

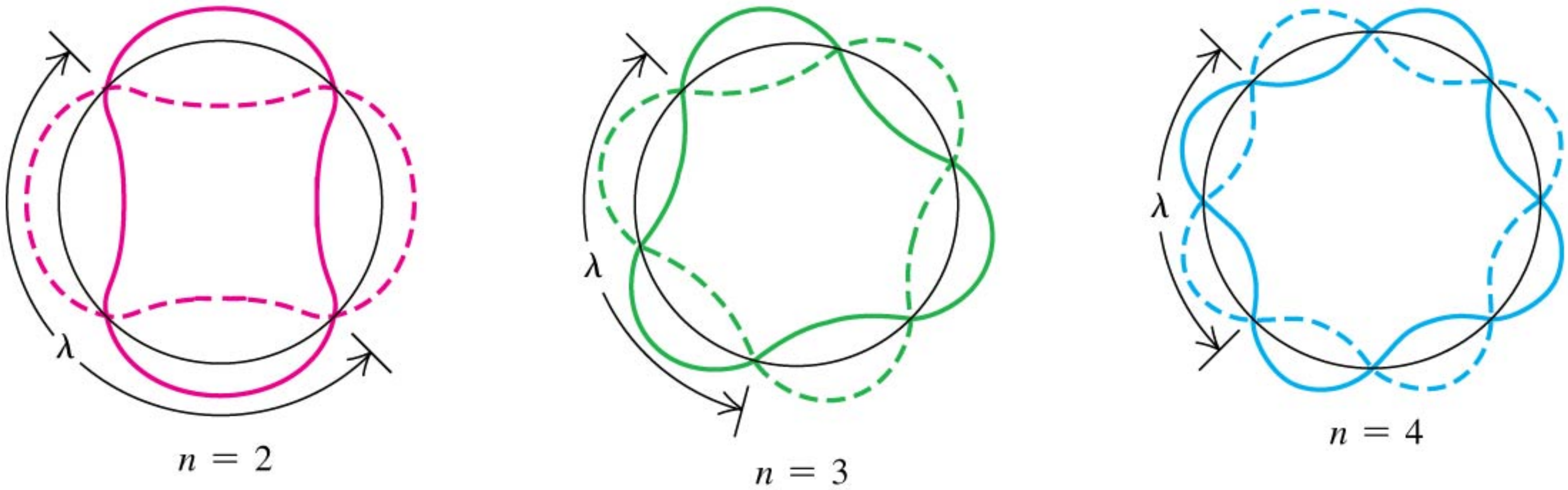
- Shown is the angular momentum of an electron in a circular orbit around an atomic nucleus.



The Bohr model of hydrogen

- A standing wave on a string transmits no energy, and **electrons in Bohr's orbits radiate no energy.**

- For the wave to "come out even" and join onto itself smoothly, the circumference of this circle must include some whole number of wavelengths.

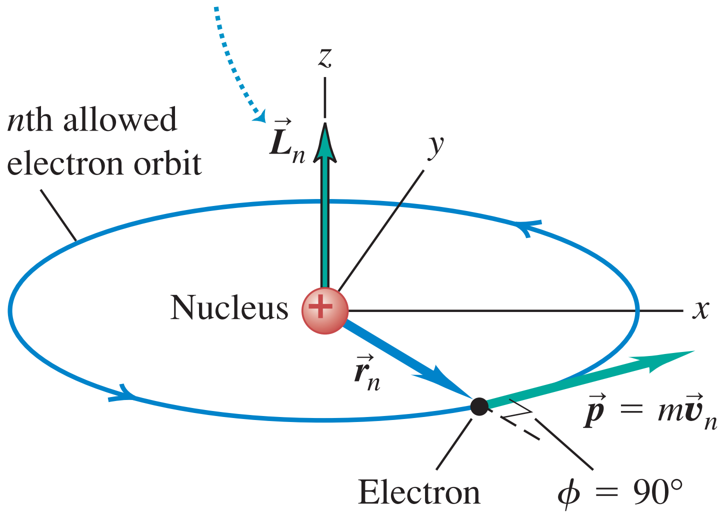


$$2\pi r_n = n\lambda \Rightarrow r_n = \frac{n\lambda}{2\pi} = \frac{nh}{2\pi p_n} \Rightarrow L_n = r_n p_n = n\hbar$$

The Bohr model of hydrogen

$$\begin{cases} L_n = mv_n r_n = n\dfrac{h}{2\pi} \\ \dfrac{e^2}{4\pi\varepsilon_0 r_n^2} = \dfrac{mv_n^2}{r_n} \end{cases}$$

$$\Rightarrow r_n = \frac{\varepsilon_0 h^2}{\pi m e^2} n^2 = a_0 n^2 \quad \text{and} \quad v_n = \frac{1}{\varepsilon_0}\frac{e^2}{2nh}$$



Total mechanical energy of the electron:

$$E_n = K_n + U_n = \frac{1}{2}mv_n^2 - \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_n} = -\frac{1}{\varepsilon_0^2}\frac{me^4}{8n^2h^2} = -\frac{hcR}{n^2}$$

where $R = \dfrac{me^4}{8\varepsilon_0^2 h^3 c}$ is th Rydberg constant

The Bohr model of hydrogen

- The orbital speed of the electron in Bohr's model of a hydrogen atom is:

$$v_n = \frac{1}{\epsilon_0}\frac{e^2}{2nh}$$

Orbital speed in nth orbit in the Bohr model (v_n); Magnitude of electron charge (e); Planck's constant (h); Electric constant (ϵ_0); Principal quantum number ($n = 1, 2, 3, \ldots$)

- The radius of this orbit is:

$$r_n = n^2 a_0$$

Radius of nth orbit in the Bohr model (r_n); Bohr radius (a_0); Principal quantum number ($n = 1, 2, 3, \ldots$)

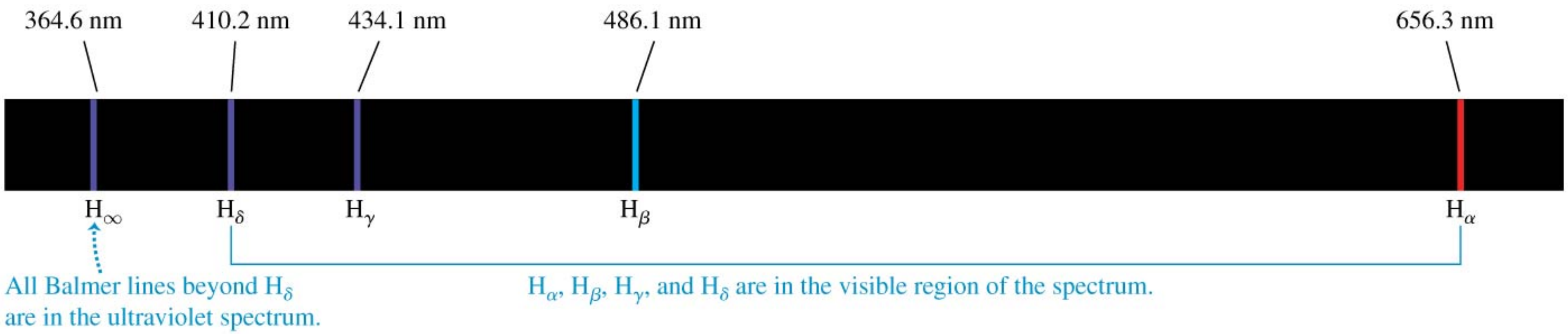
where the Bohr radius is $a_0 = 5.29 \times 10^{-11}$ m.

The Bohr model of hydrogen

- The Bohr model predicts the observable energy levels of the hydrogen atom, which give rise to the hydrogen spectrum, below.

$$E_n = -\frac{hcR}{n^2}, \quad \text{where} \quad R = \frac{me^4}{8\epsilon_0^2 h^3 c}$$

Total energy for *n*th orbit in the Bohr model (E_n)
Planck's constant (h)
Speed of light in vacuum (c)
Principal quantum number ($n = 1, 2, 3, \ldots$)
Rydberg constant (R)
Electron mass (m)
Magnitude of electron charge (e)
Electric constant (ϵ_0)



Hydrogen spectrum in more detail

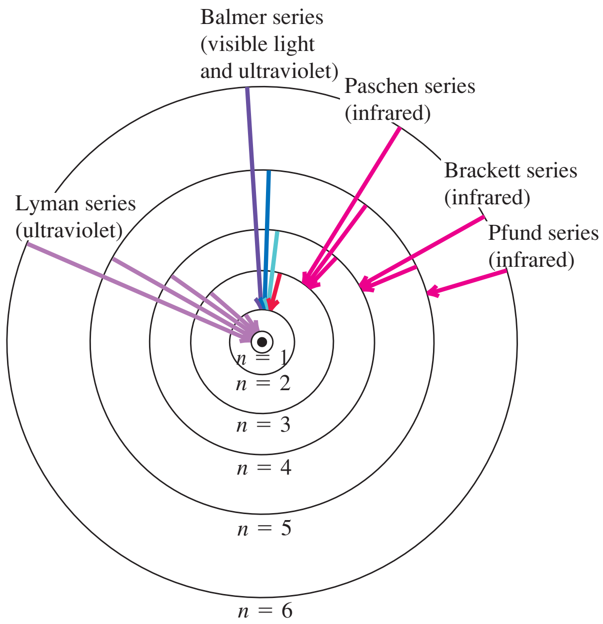
- The Balmer series is not the entire spectrum of hydrogen; it's just the visible-light portion.

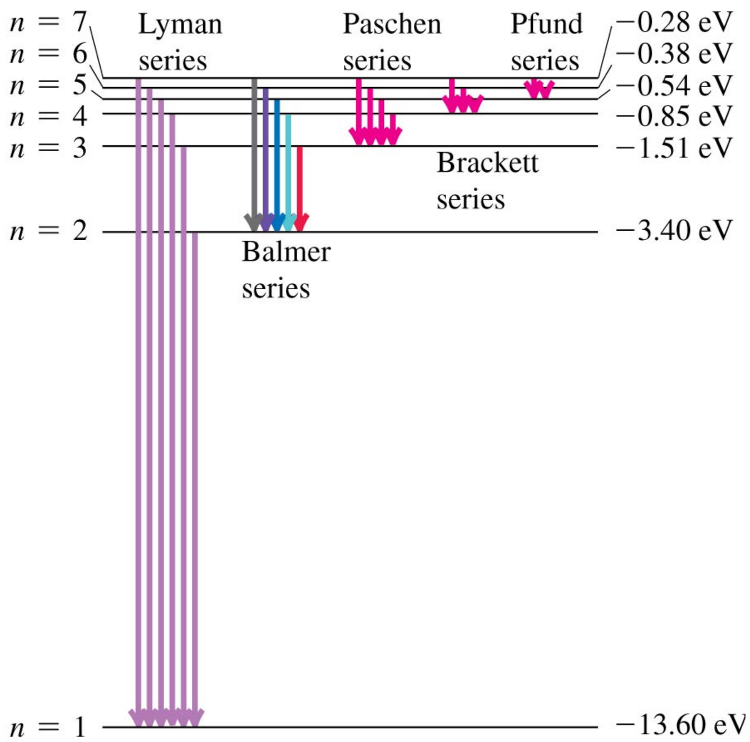
- Hydrogen also has a series of spectral lines in the ultraviolet (Lymann), and several series of spectral lines in the infrared.

$$\frac{hc}{\lambda} = E_n - E_m = hcR\left(\frac{1}{n^2} - \frac{1}{m^2}\right)$$

$$\frac{1}{\lambda} = R\left(\frac{1}{n^2} - \frac{1}{m^2}\right)$$

(a) Permitted orbits of an electron in the Bohr model of a hydrogen atom (not to scale). Arrows indicate the transitions responsible for some of the lines of various series.

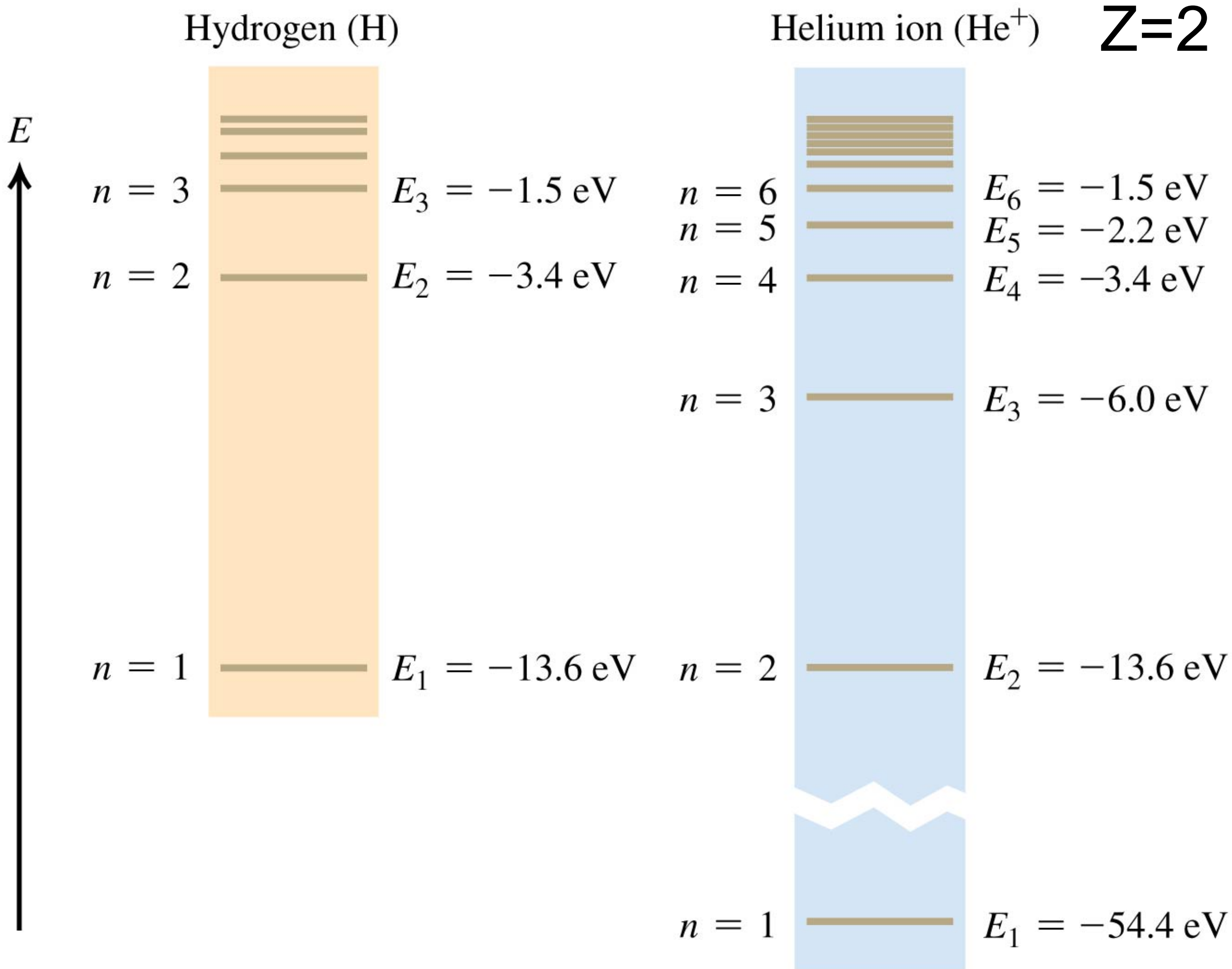




Hydrogen-like atoms

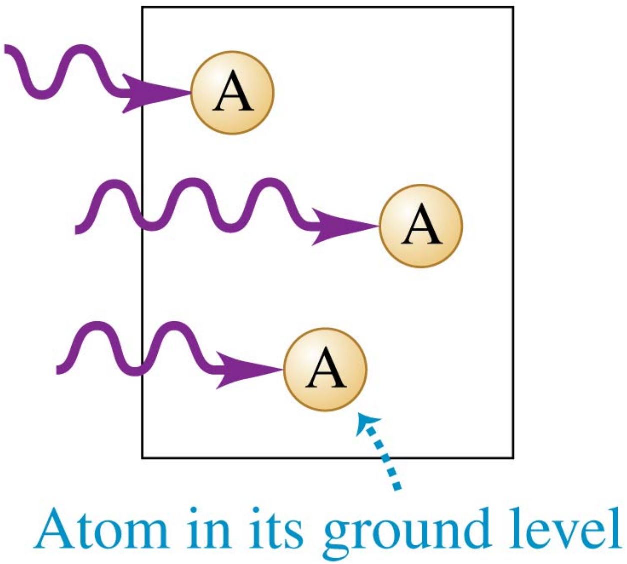
- The Bohr model can be applied to any atom with a single electron.

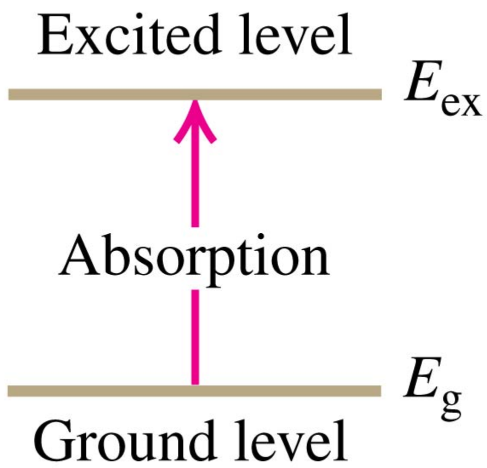
$$e^2 \rightarrow Ze^2 \quad \text{and} \quad R \rightarrow Z^2 R$$



Absorption

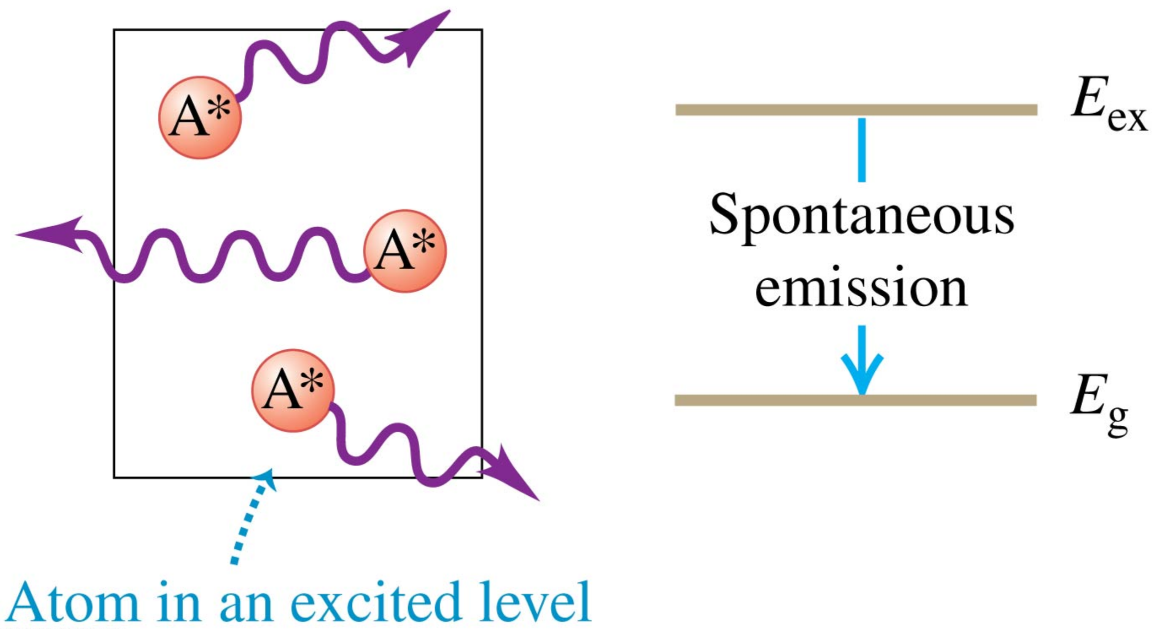
- Consider a gas of atoms in a transparent container.
- Each atom is initially in its ground level of energy E_g and also has an excited level of energy E_{ex}.
- If we shine light of frequency f on the container, an atom can **absorb** one of the photons provided the photon energy $E = hf$ equals the energy difference $E_{ex} - E_g$ between the levels.
- The figure shows this process, in which three atoms A each absorb a photon and go into the excited level.





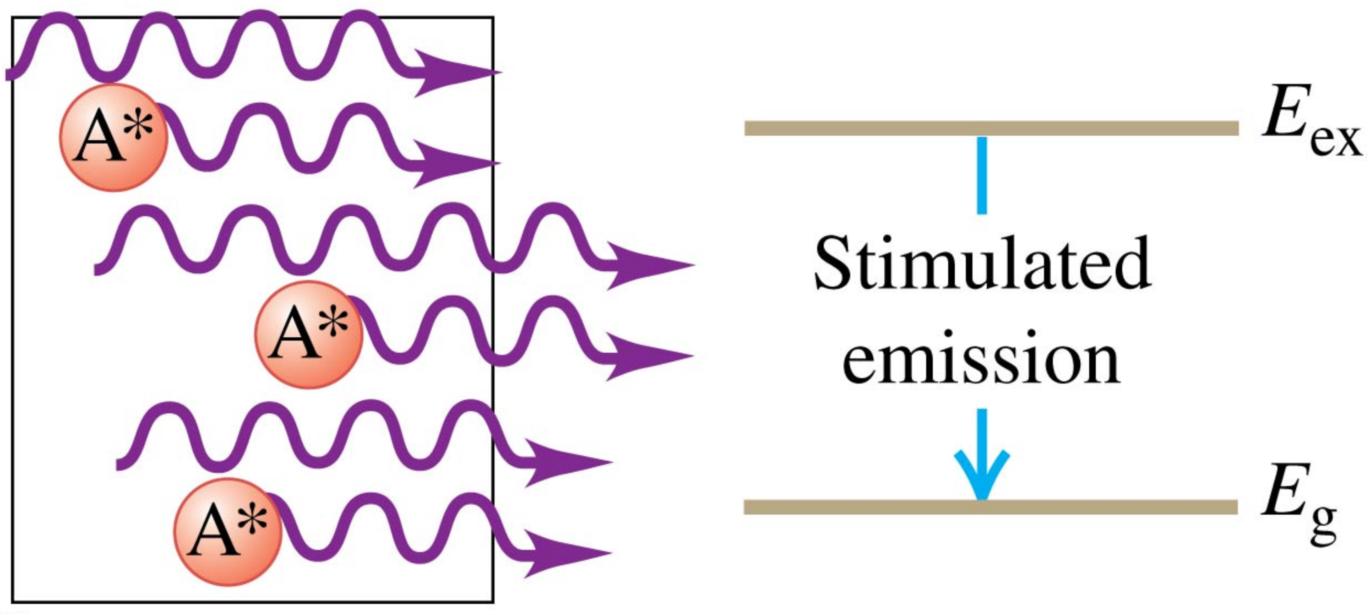
Spontaneous emission

- Excited atoms (which we denote as A*) can return to the ground level by each emitting a photon with the same frequency as the one originally absorbed.
- This process is called **spontaneous emission**.
- The direction and phase of each spontaneously emitted photon are **random**.



Stimulated emission

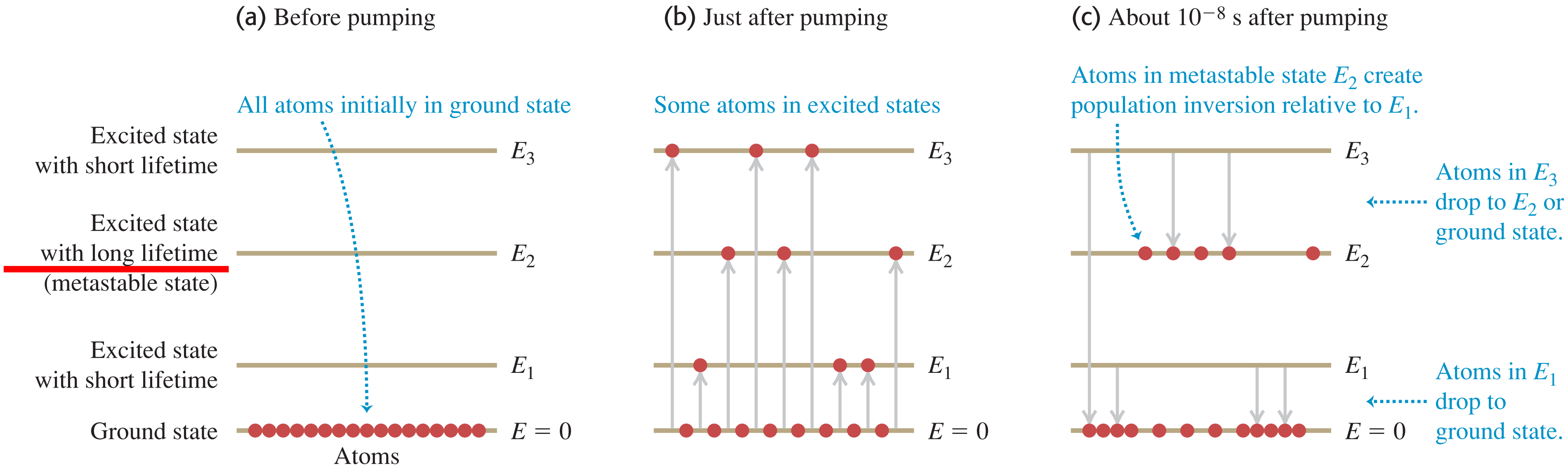
- In **stimulated emission**, each incident photon encounters a previously excited atom.
- A kind of resonance effect induces each excited atom to emit a second photon with the **same frequency, direction, phase, and polarization** as the incident photon, which is not changed by the process.



Population Inversion

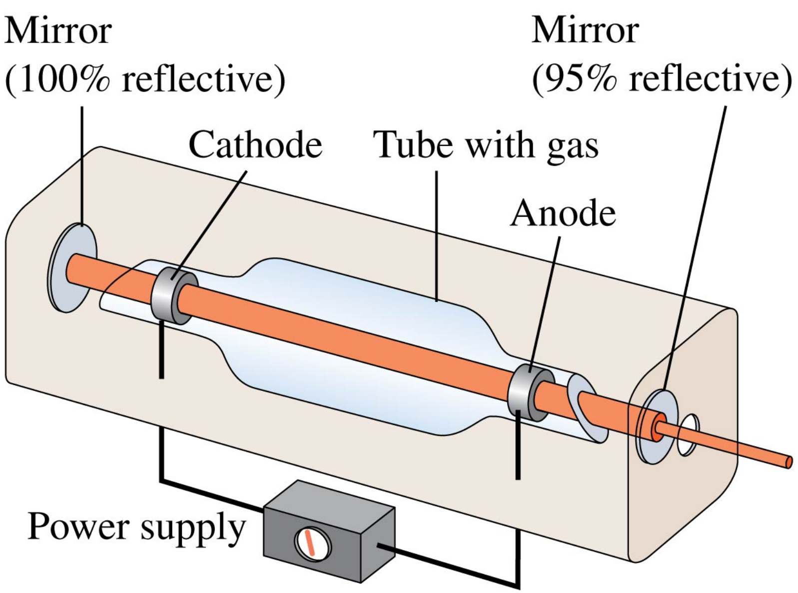
- We can pump the material to excite the atoms out of the ground state into the excited states -> enhanced stimulated emission

39.29 (a), (b), (c) Stages in the operation of a four-level laser. (d) The light emitted by atoms making spontaneous transitions from state E_2 to state E_1 is reflected between mirrors, so it continues to stimulate emission and gives rise to coherent light. One mirror is partially transmitting and allows the high-intensity light beam to escape.



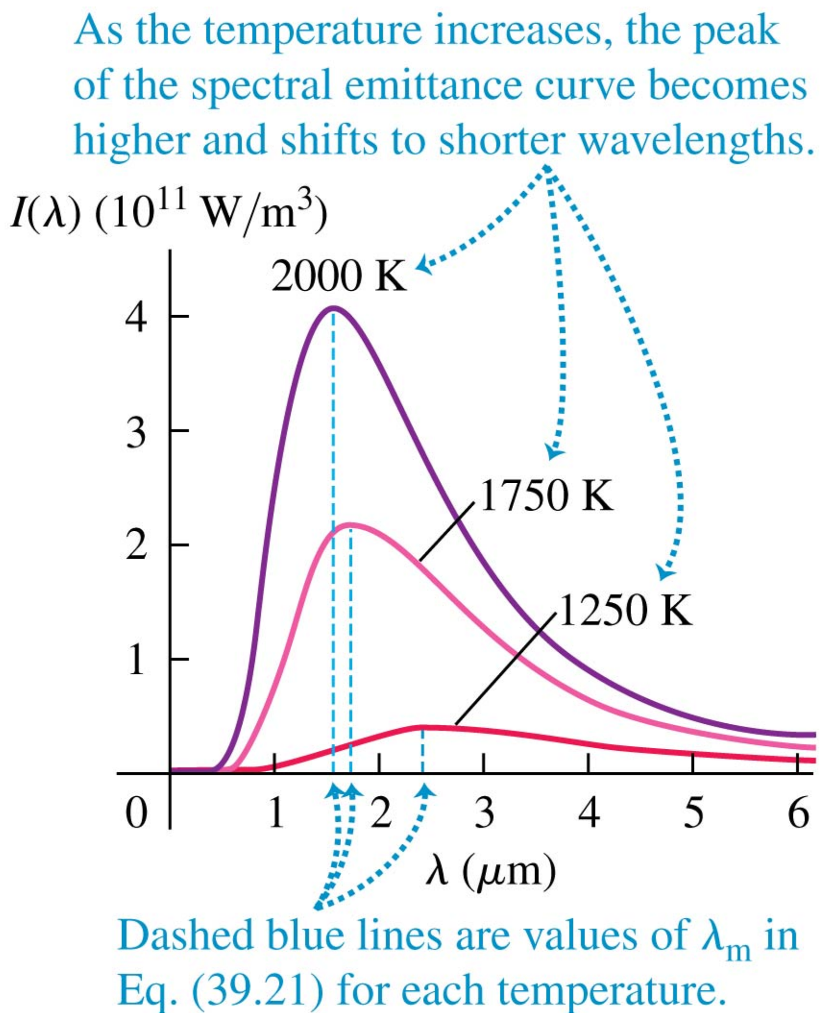
The laser

- The laser is a light source that produces a beam of highly coherent and very nearly monochromatic light as a result of cooperative emission from many atoms.
- The name “laser” is an acronym for “light amplification by stimulated emission of radiation.”



Continuous spectra and blackbody radiation

- A blackbody is an idealized case of a hot, dense object.
- The figure shows the continuous spectrum produced by a blackbody at different temperatures.
- Planck provided the first explanation of blackbody radiation by assuming that atoms in the blackbody have evenly spaced energy levels, and emit photons by jumping from one energy level down to the next one.



$$I(\lambda) = \frac{2\pi hc^2}{\lambda^5\left(e^{hc/\lambda kT} - 1\right)}$$

(Planck radiation law)

Continuous spectra and blackbody radiation

- The spectral emittance $I(\lambda)$ for radiation from a blackbody has a peak whose wavelength depends on temperature:

Wien displacement law for a blackbody:

Peak wavelength in spectral emittance curve

$$\lambda_m T = 2.90 \times 10^{-3}\ \text{m} \cdot \text{K}$$

Absolute temperature of blackbody

- We can obtain the Stefan–Boltzmann law for a blackbody by integrating $I(\lambda)$ over all wavelengths to find the total radiated intensity:

$$I = \int_0^\infty I(\lambda)\, d\lambda = \frac{2\pi^5 k^4}{15c^2h^3} T^4 = \sigma T^4$$

where $\sigma = 5.6704 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4$ is the Stefan–Boltzmann constant.

Example 39.7 Light from the sun

To a good approximation, the sun's surface is a blackbody with a surface temperature of 5800 K. (We are ignoring the absorption produced by the sun's atmosphere, shown in Fig. 39.9.) (a) At what wavelength does the sun emit most strongly? (b) What is the total radiated power per unit surface area?

SOLUTION

IDENTIFY and SET UP: Our target variables are the peak-intensity wavelength λ_m and the radiated power per area I. Hence we'll use the Wien displacement law, Eq. (39.21) (which relates λ_m to the blackbody temperature T), and the Stefan–Boltzmann law, Eq. (39.19) (which relates I to T).

EXECUTE: (a) From Eq. (39.21),

$$\lambda_m = \frac{2.90 \times 10^{-3}\ \text{m} \cdot \text{K}}{T} = \frac{2.90 \times 10^{-3}\ \text{m} \cdot \text{K}}{5800\ \text{K}}$$
$$= 0.500 \times 10^{-6}\ \text{m} = 500\ \text{nm}$$

(b) From Eq. (39.19),

$$I = \sigma T^4 = (5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4)(5800\ \text{K})^4$$
$$= 6.42 \times 10^7\ \text{W/m}^2 = 64.2\ \text{MW/m}^2$$

EVALUATE: The 500-nm wavelength found in part (a) is near the middle of the visible spectrum. This should not be a surprise: The human eye evolved to take maximum advantage of natural light.

The enormous value $I = 64.2\ \text{MW/m}^2$ found in part (b) is the intensity at the *surface* of the sun, a sphere of radius 6.96×10^8 m. When this radiated energy reaches the earth, 1.50×10^{11} m away, the intensity has decreased by the factor $[(6.96 \times 10^8\ \text{m})/(1.50 \times 10^{11}\ \text{m})]^2 = 2.15 \times 10^{-5}$ to the still-impressive $1.4\ \text{kW/m}^2$.

Example 39.8 A slice of sunlight

Find the power per unit area radiated from the sun's surface in the wavelength range 600.0 to 605.0 nm.

SOLUTION

IDENTIFY and SET UP: This question concerns the power emitted by a blackbody over a narrow range of wavelengths, and so involves the spectral emittance $I(\lambda)$ given by the Planck radiation law, Eq. (39.24). This requires that we find the area under the $I(\lambda)$ curve between 600.0 and 605.0 nm. We'll *approximate* this area as the product of the height of the curve at the median wavelength $\lambda = 602.5$ nm and the width of the interval, $\Delta\lambda = 5.0$ nm. From Example 39.7, $T = 5800$ K.

EXECUTE: To obtain the height of the $I(\lambda)$ curve at $\lambda = 602.5\ \text{nm} = 6.025 \times 10^{-7}$ m, we first evaluate the quantity $hc/\lambda kT$ in Eq. (39.24) and then substitute the result into Eq. (39.24):

$$\frac{hc}{\lambda kT} = \frac{(6.626 \times 10^{-34}\ \text{J}\cdot\text{s})(2.998 \times 10^{8}\ \text{m/s})}{(6.025 \times 10^{-7}\ \text{m})(1.381 \times 10^{-23}\ \text{J/K})(5800\ \text{K})} = 4.116$$

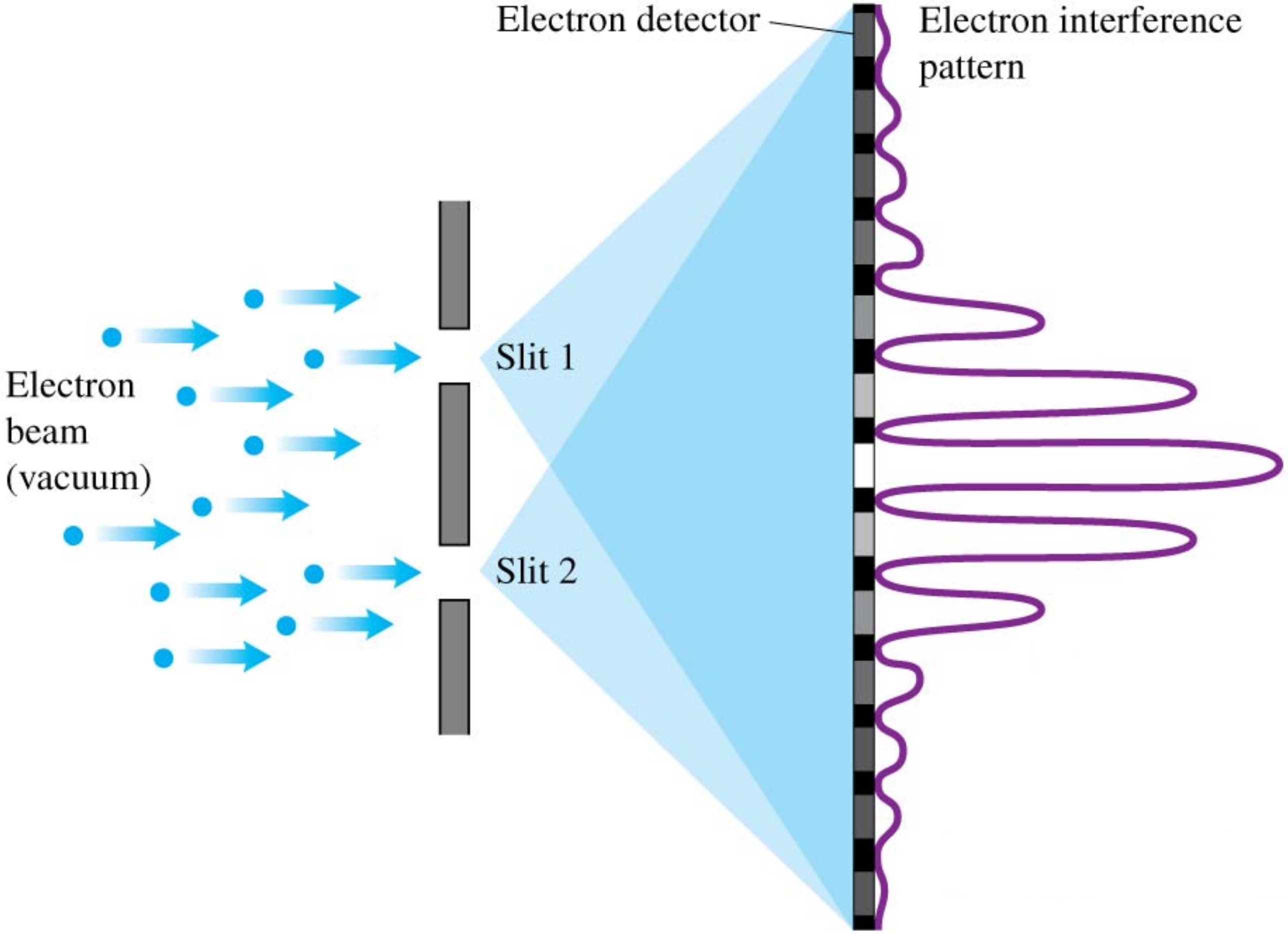
$$I(\lambda) = \frac{2\pi(6.626 \times 10^{-34}\ \text{J}\cdot\text{s})(2.998 \times 10^{8}\ \text{m/s})^2}{(6.025 \times 10^{-7}\ \text{m})^5(e^{4.116} - 1)}$$
$$= 7.81 \times 10^{13}\ \text{W/m}^3$$

The intensity in the 5.0-nm range from 600.0 to 605.0 nm is then approximately

$$I(\lambda)\Delta\lambda = (7.81 \times 10^{13}\ \text{W/m}^3)(5.0 \times 10^{-9}\ \text{m})$$
$$= 3.9 \times 10^{5}\ \text{W/m}^2 = 0.39\ \text{MW/m}^2$$

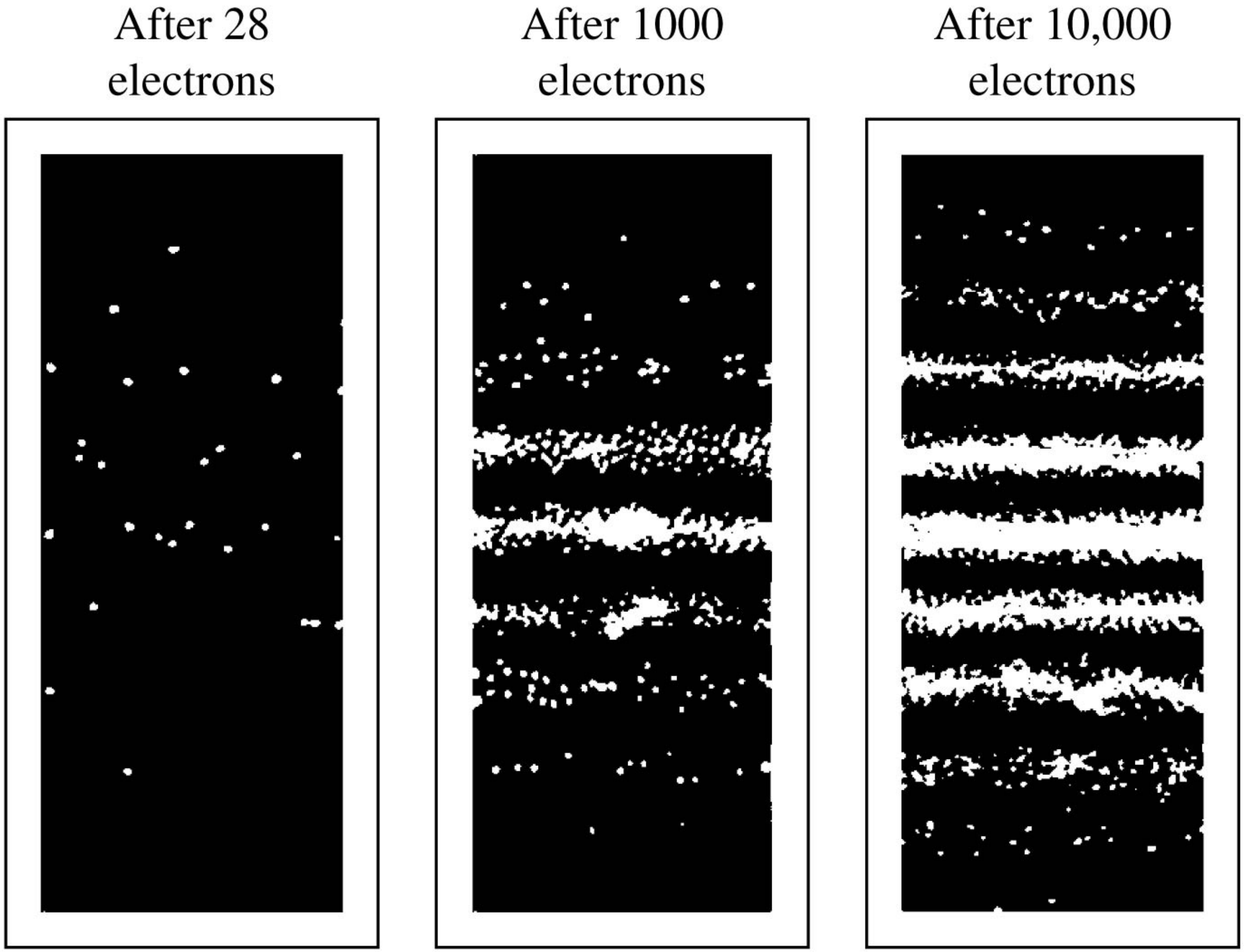
EVALUATE: In part (b) of Example 39.7, we found the power radiated per unit area by the sun at *all* wavelengths to be $I = 64.2\ \text{MW/m}^2$; here we have found that the power radiated per unit area in the wavelength range from 600 to 605 nm is $I(\lambda)\Delta\lambda = 0.39\ \text{MW/m}^2$, about 0.6% of the total.

A two-slit interference experiment for electrons



We can send the electron once a time through the slits. Each electron wave interferes with itself!!!

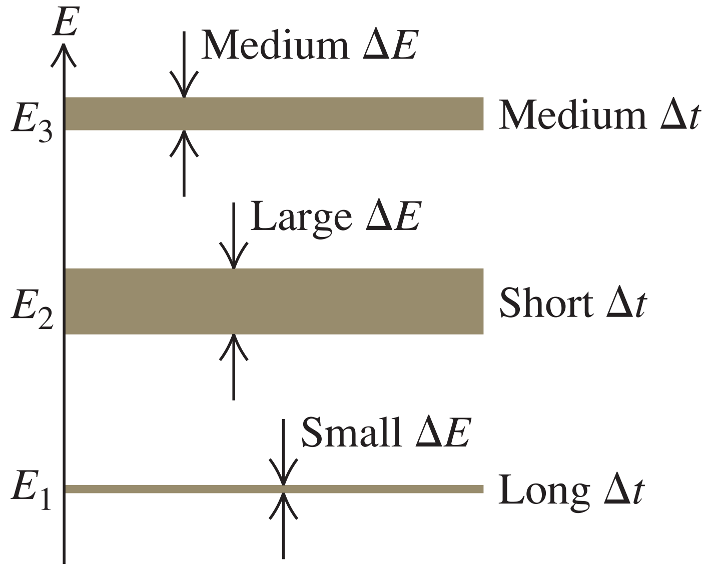
A two-slit interference experiment for electrons



The Uncertainty Principle

$$\Delta x \Delta p \geq \frac{\hbar}{2} \quad \text{and} \quad \Delta E \Delta t \geq \frac{\hbar}{2}$$

39.35 The longer the lifetime Δt of a state, the smaller is its spread in energy (shown by the width of the energy levels).



Example: Estimate the ground state energy of a particle confined in a box of size a

$$\Delta x \leq a$$

$$\Delta p \geq \frac{\hbar}{2a}$$

$$E \sim \frac{(\Delta p)^2}{2m} \geq \frac{1}{2m}\left(\frac{\hbar}{2a}\right)^2$$

Example 39.9 The uncertainty principle: position and momentum

An electron is confined within a region of width 5.000×10^{-11} m (roughly the Bohr radius). (a) Estimate the minimum uncertainty in the x-component of the electron's momentum. (b) What is the kinetic energy of an electron with this magnitude of momentum? Express your answer in both joules and electron volts.

SOLUTION

IDENTIFY and SET UP: This problem uses the Heisenberg uncertainty principle for position and momentum and the relationship between a particle's momentum and its kinetic energy. The electron could be anywhere within the region, so we take $\Delta x = 5.000 \times 10^{-11}$ m as its position uncertainty. We then find the momentum uncertainty Δp_x using Eq. (39.29) and the kinetic energy using the relationships $p = mv$ and $K = \frac{1}{2}mv^2$.

EXECUTE: (a) From Eqs. (39.29), for a given value of Δx, the uncertainty in momentum is minimum when the product $\Delta x \Delta p_x$ equals $\hbar$. Hence

$$\Delta p_x = \frac{\hbar}{2\Delta x} = \frac{1.055 \times 10^{-34}\ \text{J}\cdot\text{s}}{2(5.000 \times 10^{-11}\ \text{m})} = 1.055 \times 10^{-24}\ \text{J}\cdot\text{s/m}$$
$$= 1.055 \times 10^{-24}\ \text{kg}\cdot\text{m/s}$$

(b) We can rewrite the nonrelativistic expression for kinetic energy as

$$K = \tfrac{1}{2}mv^2 = \frac{(mv)^2}{2m} = \frac{p^2}{2m}$$

Hence an electron with a magnitude of momentum equal to Δp_x from part (a) has kinetic energy

$$K = \frac{p^2}{2m} = \frac{(1.055 \times 10^{-24}\ \text{kg}\cdot\text{m/s})^2}{2(9.11 \times 10^{-31}\ \text{kg})}$$
$$= 6.11 \times 10^{-19}\ \text{J} = 3.81\ \text{eV}$$

EVALUATE: This energy is typical of electron energies in atoms. This agreement suggests that the uncertainty principle is deeply involved in atomic structure.

A similar calculation explains why electrons in atoms do not fall into the nucleus. If an electron were confined to the interior of a nucleus, its position uncertainty would be $\Delta x \approx 10^{-14}$ m. This would give the electron a momentum uncertainty about 5000 times greater than that of the electron in this example, and a kinetic energy so great that the electron would immediately be ejected from the nucleus.

Example 39.10 The uncertainty principle: energy and time

A sodium atom in one of the states labeled "Lowest excited levels" in Fig. 39.19a remains in that state, on average, for 1.6×10^{-8} s before it makes a transition to the ground level, emitting a photon with wavelength 589.0 nm and energy 2.105 eV. What is the uncertainty in energy of that excited state? What is the wavelength spread of the corresponding spectral line?

SOLUTION

IDENTIFY and SET UP: This problem uses the Heisenberg uncertainty principle for energy and time interval and the relationship between photon energy and wavelength. The average time that the atom spends in this excited state is equal to Δt in Eq. (39.30). We find the minimum uncertainty in the energy of the excited level by replacing the $\geq$ sign in Eq. (39.30) with an equals sign and solving for ΔE.

EXECUTE: From Eq. (39.30),

$$\Delta E = \frac{\hbar}{2\Delta t} = \frac{1.055 \times 10^{-34}\ \text{J} \cdot \text{s}}{2(1.6 \times 10^{-8}\ \text{s})}$$
$$= 3.3 \times 10^{-27}\ \text{J} = 2.1 \times 10^{-8}\ \text{eV}$$

The atom remains in the ground level indefinitely, so that level has *no* associated energy uncertainty. The fractional uncertainty of the *photon* energy is therefore

$$\frac{\Delta E}{E} = \frac{2.1 \times 10^{-8}\ \text{eV}}{2.105\ \text{eV}} = 1.0 \times 10^{-8}$$

You can use some simple calculus and the relation $E = hc/\lambda$ to show that $\Delta\lambda/\lambda \approx \Delta E/E$, so that the corresponding spread in wavelength, or "width," of the spectral line is approximately

$$\Delta\lambda = \lambda\frac{\Delta E}{E} = (589.0\ \text{nm})(1.0 \times 10^{-8}) = 0.0000059\ \text{nm}$$

EVALUATE: This irreducible uncertainty $\Delta\lambda$ is called the *natural line width* of this particular spectral line. Though very small, it is within the limits of resolution of present-day spectrometers. Ordinarily, the natural line width is much smaller than the line width arising from other causes such as the Doppler effect and collisions among the rapidly moving atoms.