Wave Motion and Sound

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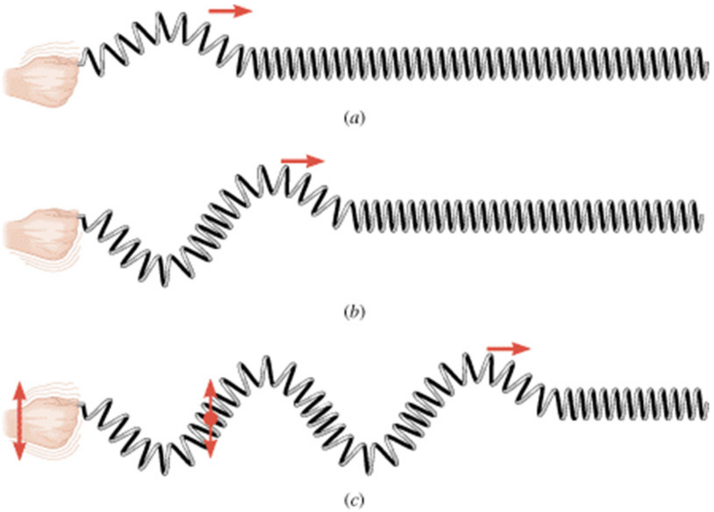
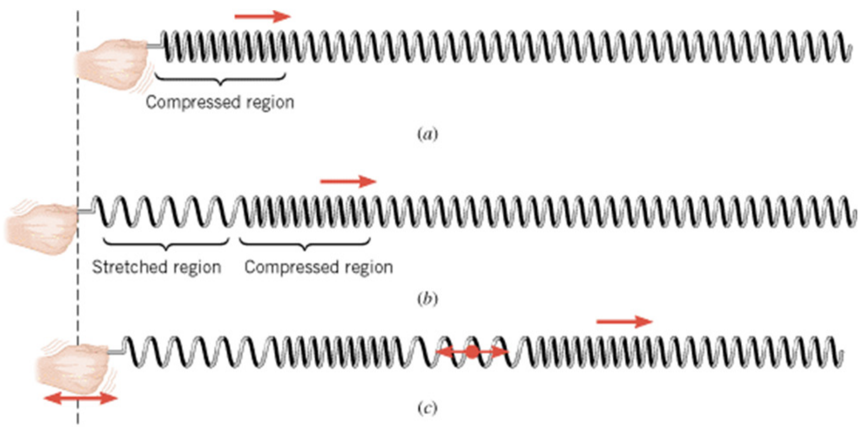
What is a wave?

- a wave is a disturbance that travels through a medium from one location to another.
- a wave is the motion of a disturbance

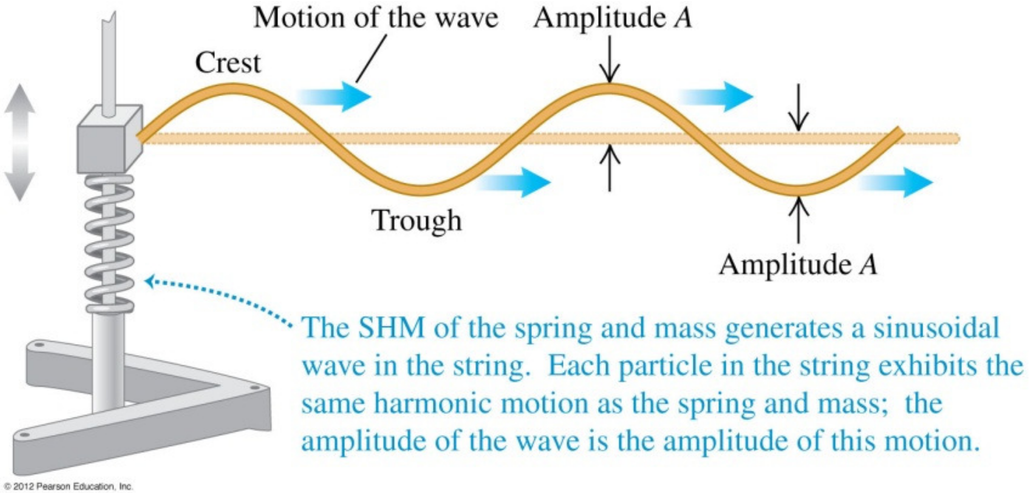
Slinky Wave

- Let's use a slinky wave as an example.
- When the slinky is stretched from end to end and is held at rest, it assumes a natural position known as the **equilibrium or rest position.**
- To introduce a wave here we must first create a disturbance (source).
- We must move a particle away from its rest position.

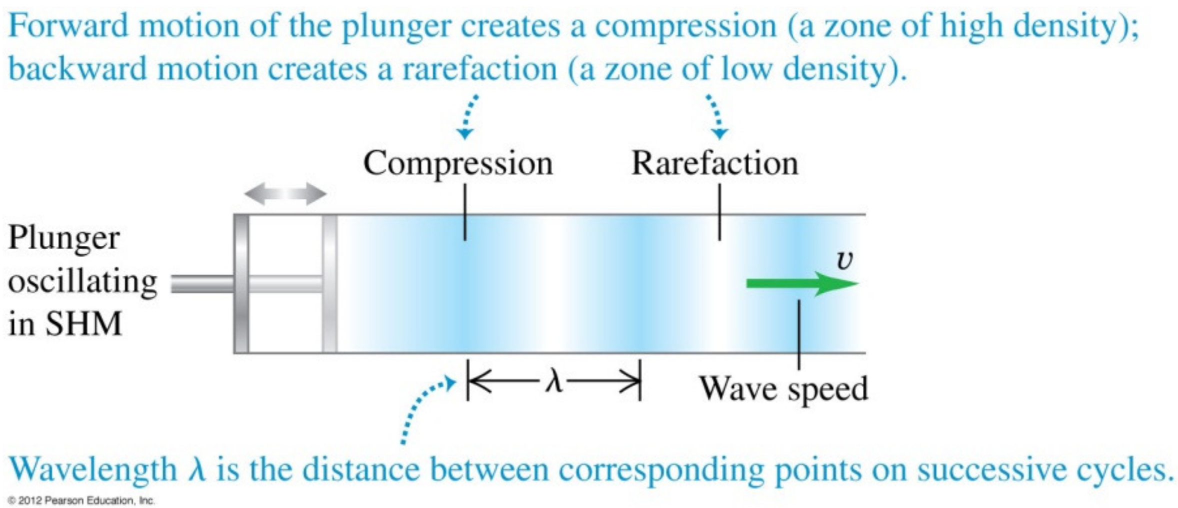




Two types of wave



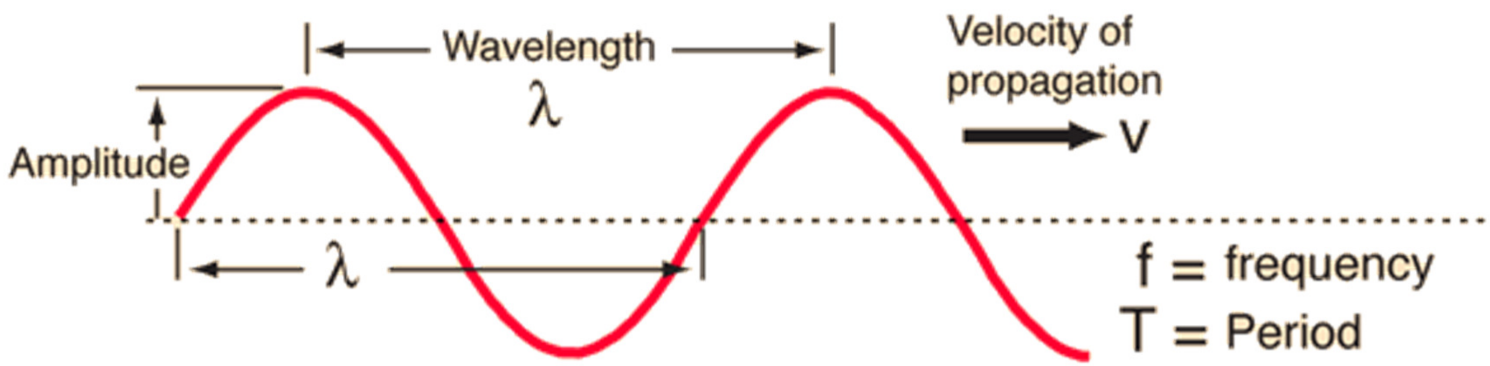
transverse,
e.g. wave in a string, water wave, EM wave



longitudinal,
e.g. sound wave

Wave length, frequency and period

- Wavelength is also measured in metres (m) - it is a length after all.
- The frequency, f, of a wave is the number of waves passing a point in **one second** (SI unit: hertz (Hz=1/s))
- period, $T = \frac{1}{f}$ is the time for a particle on a medium to make one complete vibrational cycle



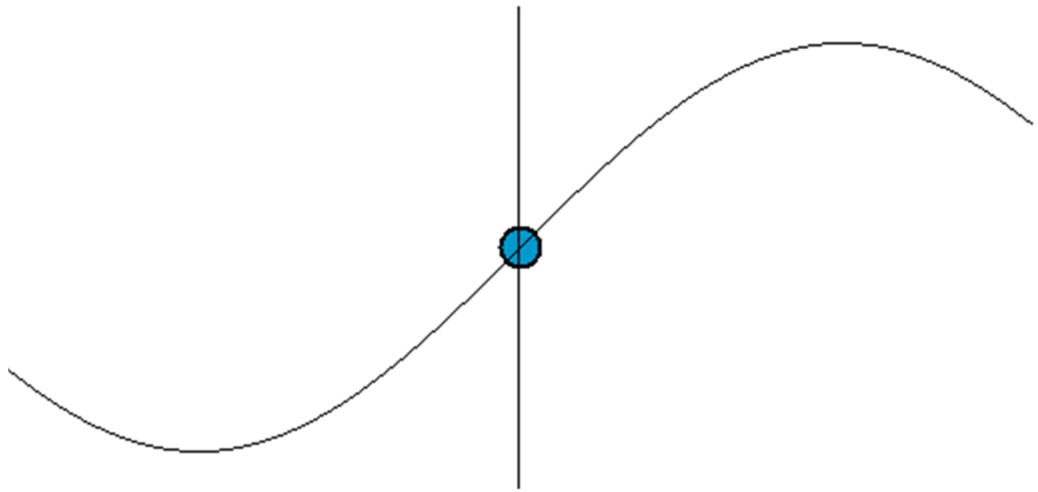
$$v = \lambda f$$

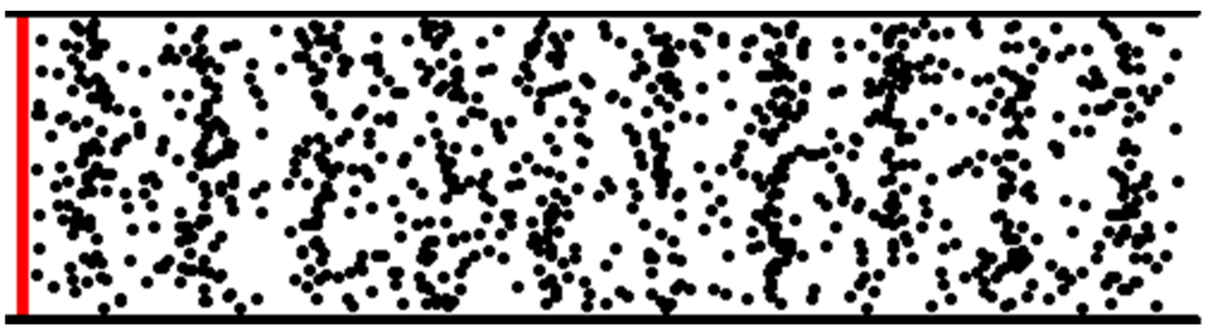
wave speed

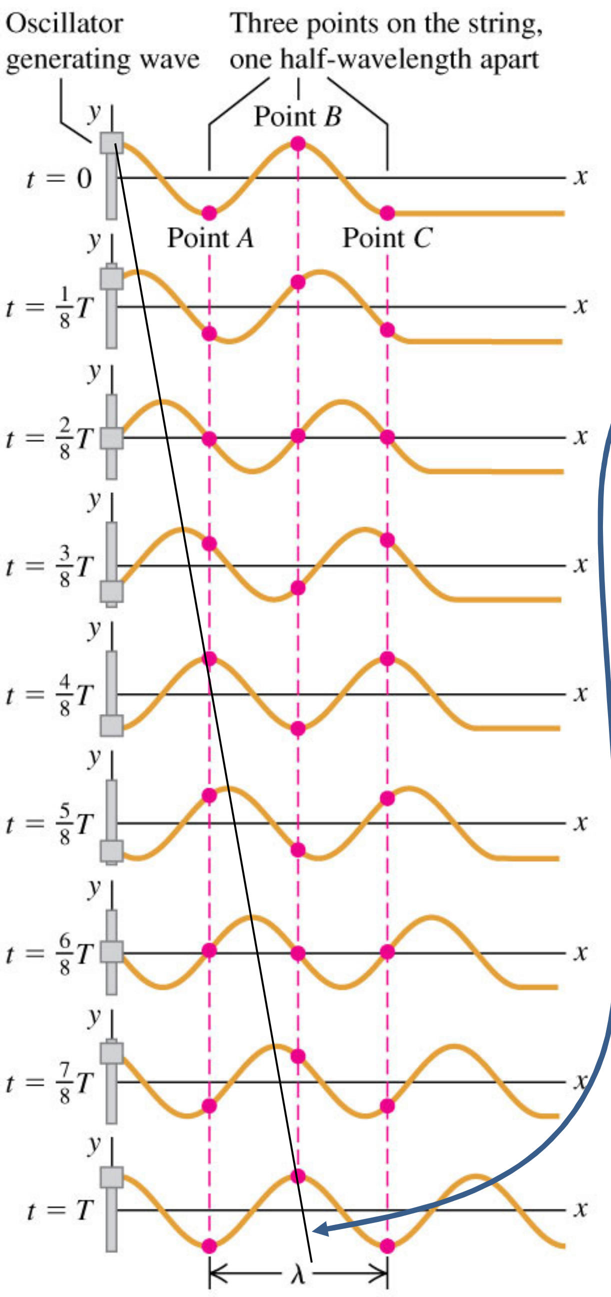
wavelength

frequency

period $T = \frac{1}{f} = \frac{2\pi}{\omega}$







Sinusoidal wave on a string as an example:

Wave motion represented by a **wavefunction** $y(x, t)$

Assume a sinusoidal generator $y(0, t) = A\cos\omega t$

Follow the time evolution of an arbitrary point, it propagates with speed v, called **phase velocity**

At time t, same phase angle as at earlier time $t - x/v$

i.e. $$y(x, t) = y\left(0, t - \frac{x}{v}\right) = A\cos\omega\left(t - \frac{x}{v}\right)$$

Define **wave number** (⚠ not a pure number, but has dimension 1/length)

$$\boxed{k = \frac{2\pi}{\lambda}} = \frac{2\pi f}{\lambda f} = \frac{\omega}{v}$$

Wavefunction becomes

$$y(x, t) = A\cos(kx - \omega t)$$

If wave traveling to the left, $v \rightarrow -v$

$$y(x,t) = A\cos\omega\left(t+\frac{x}{v}\right) = A\ \cos(kx+\omega t)$$

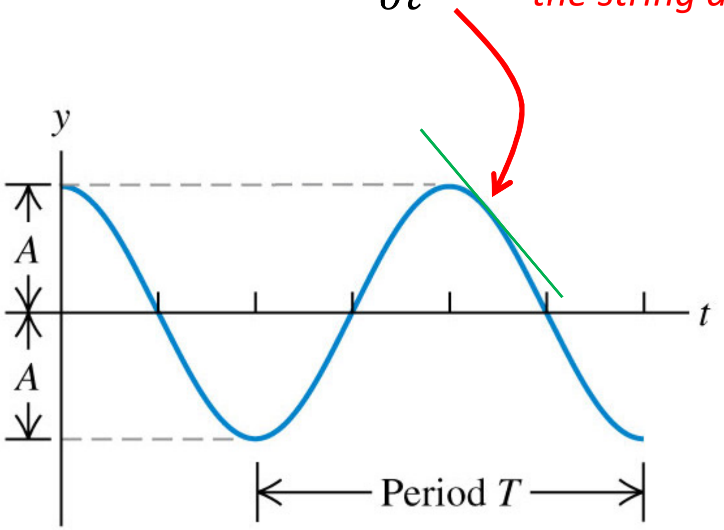
⚠ v is the magnitude, i.e., $v > 0$. The direction is shown in the phase angle $(kx \pm \omega t)$

⚠ $y(x,t)$ is a function of two variables:

displacement of a particular point on the string – x is fixed

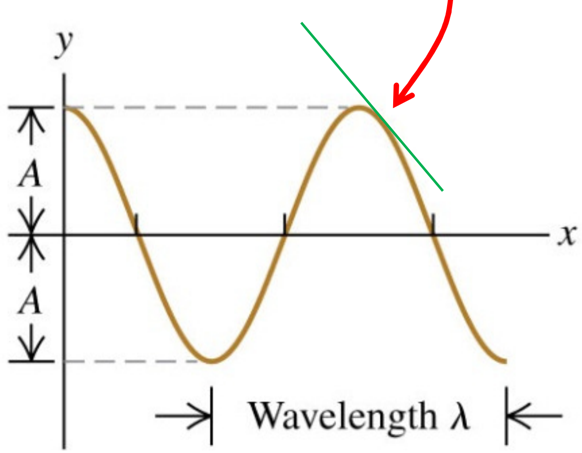
slope is $\frac{\partial y}{\partial t}$

vertical velocity of the string at this point



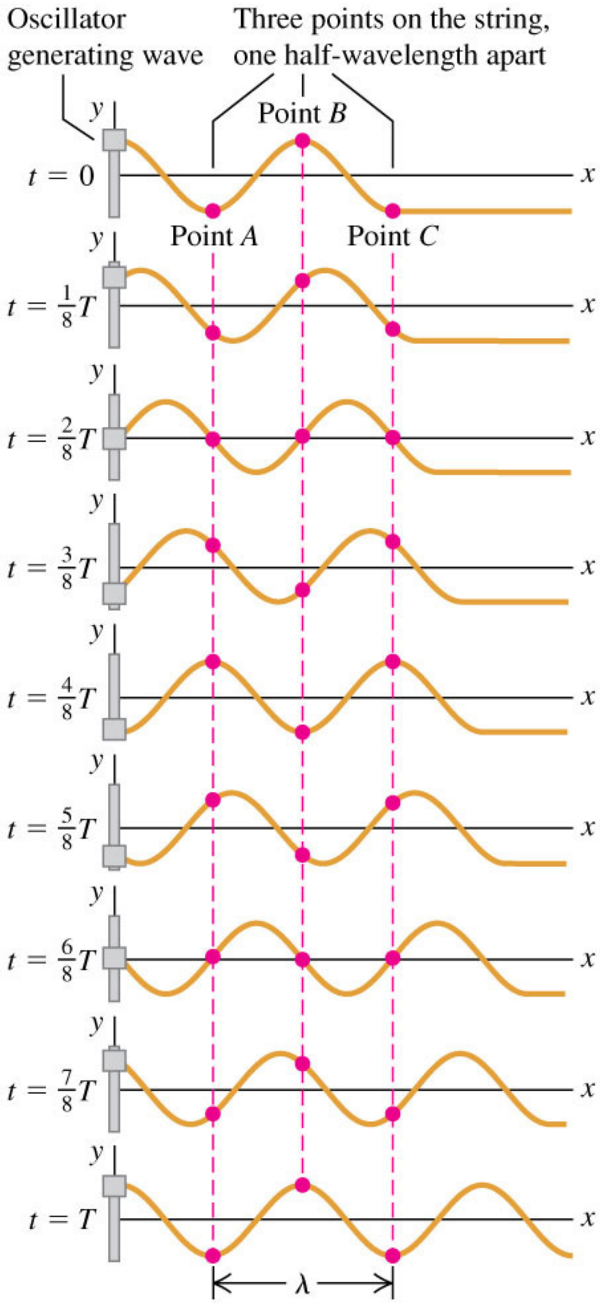
a snapshot (or photo) of the wave motion – t is fixed

slope is $\frac{\partial y}{\partial x}$

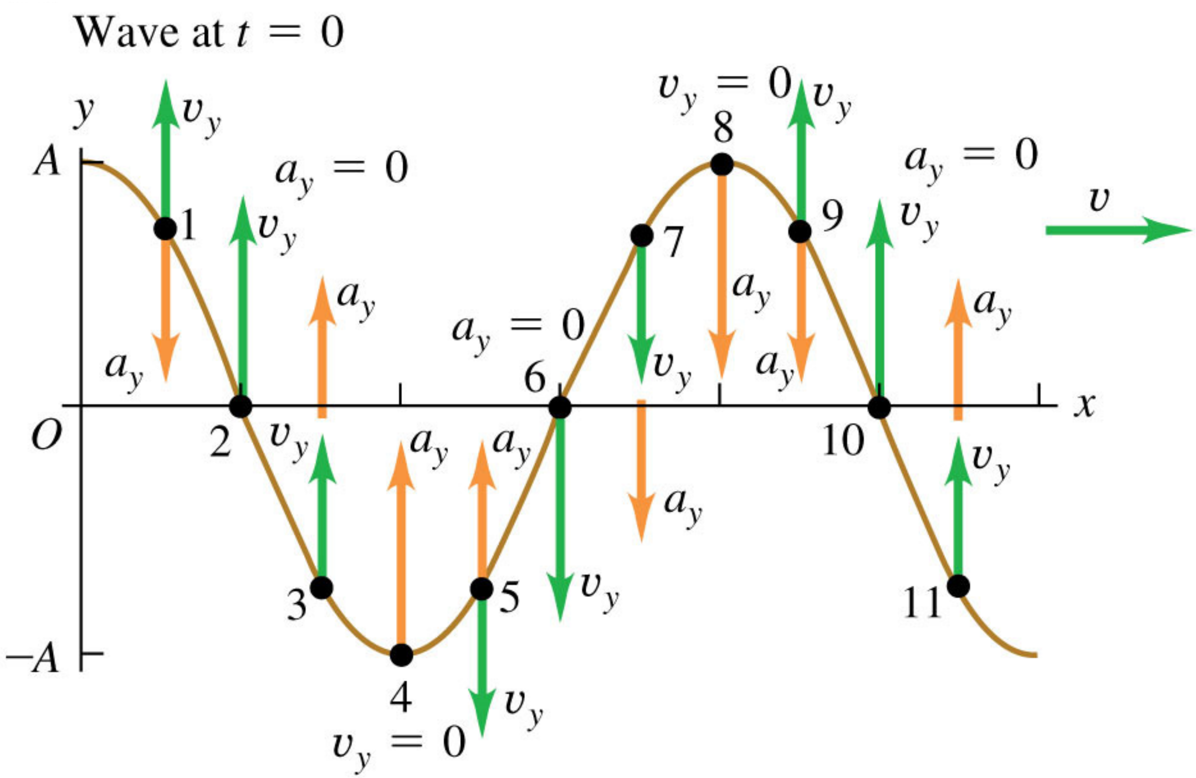


Question

In the diagram that shows a traveling wave at $t = T/8, 2T/8, \ldots T$, at which time will the point A have (a) maximum upward speed, (b) greatest upward acceleration, (c) downward acceleration but an upward velocity?

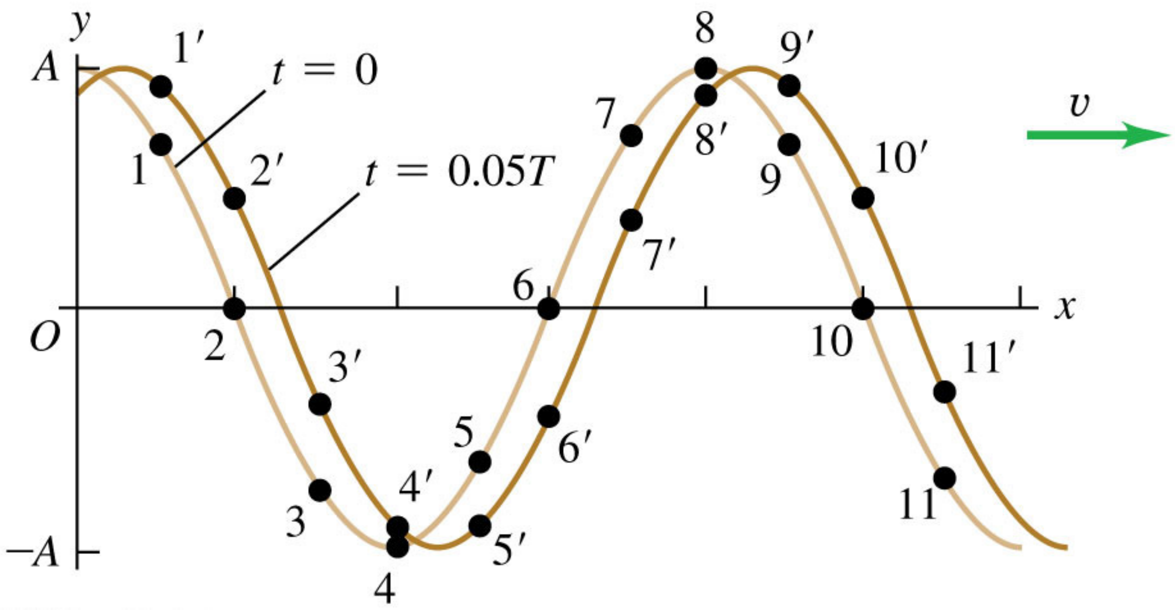


Particle velocity and acceleration in a sinusoidal wave



Particle velocity and acceleration in a sinusoidal wave

The same wave at $t = 0$ and $t = 0.05T$



Q15.2

Which of the following wave functions describe a wave that moves in the –x-direction?

A. $y(x,t) = A \sin(-kx - \omega t)$

B. $y(x,t) = A \sin(kx + \omega t)$

C. $y(x,t) = A \cos(kx + \omega t)$

D. both B. and C.

E. all of A., B., and C.

A15.2

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C. $y(x,t) = A \cos(kx + \omega t)$

D. both B. and C.

all of A., B., and C.

Example

A transverse wave travelling along a string is described by $y(x, t) = 0.00327\sin(72.1x - 2.72t)$, in which the numerical constants are in SI units.

(a) What is the amplitude of this wave?

(b) What are the wavelength, period, and frequency of this wave?

(c) What is the velocity of this wave?

(d) What is the displacement y at $x = 22.5$ cm and $t = 18.9$ s?

(e) What is the transverse velocity u of this element of the string, at that place and at that time?

(f) What is the transverse acceleration a_y at that position and at that time?

(a) $y_m = 0.00327 = 3.27\ \text{mm}$ (ans)

(b) $\lambda = \dfrac{2\pi}{k} = \dfrac{2\pi}{72.1} = 0.0871 = 8.71\,\text{cm}$ (ans)

$T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{2.72} = 2.31\,\text{s}$ (ans)

$f = \dfrac{1}{T} = \dfrac{1}{2.31} = 0.433\,\text{Hz}$ (ans)

(c) $v = \dfrac{\omega}{k} = \dfrac{2.72}{72.1} = 0.0377 = 3.77\,\text{cms}^{-1}$ (ans)

(d) $y(x,t) = 0.00327\sin(72.1\times 0.225 - 2.72\times 18.9)$

$= 0.00192 = 1.92\,\text{mm}$

(e) $u = \dfrac{\partial y}{\partial t} = -\omega y_m \cos(kx - \omega t)$

$= -(2.72)(0.00327)\sin(72.1\times 0.225 - 2.72\times 18.9)$

$= 7.20\,\text{mms}^{-1}$ (ans)

(f) $a_y = \dfrac{\partial u}{\partial t} = -\omega^2 y_m \sin(kx - \omega t) = -\omega^2 y$

$= -(2.72)^2(0.00192) = -0.0142 = -14.2\,\text{mms}^{-2}$ (ans)

Wave equation

F – equilibrium tension of the string

μ – mass per unit length of the string

$$\frac{F_{1y}}{F} = -(\text{slope at } x) = -\left(\frac{\partial y}{\partial x}\right)_x ,$$

$$\frac{F_{2y}}{F} = \left(\frac{\partial y}{\partial x}\right)_{x+\Delta x}$$

Newton's 2nd law

$$F_y = F_{1y} + F_{2y} = F\left[\left(\frac{\partial y}{\partial x}\right)_{x+\Delta x} - \left(\frac{\partial y}{\partial x}\right)_x\right] = ma_y$$

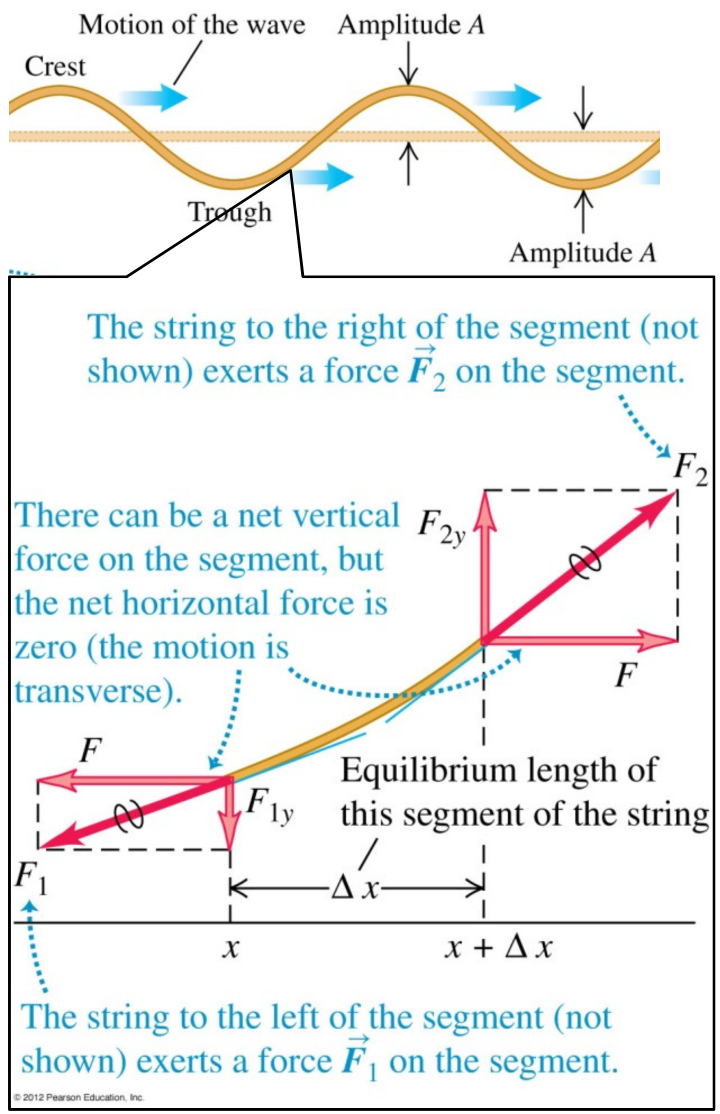
$$= (\mu \Delta x)\frac{\partial^2 y}{\partial t^2}$$

$$\Rightarrow \frac{1}{\Delta x}\left[\left(\frac{\partial y}{\partial x}\right)_{x+\Delta x} - \left(\frac{\partial y}{\partial x}\right)_x\right] = \frac{\partial^2 y}{\partial x^2} = \frac{\mu}{F}\frac{\partial^2 y}{\partial t^2}$$

From wavefunction $y(x,t) = A\cos(kx - \omega t)$

$$\left.\begin{aligned}\frac{\partial^2 y}{\partial t^2} &= -\omega^2 A\cos(kx-\omega t)\\ \frac{\partial^2 y}{\partial x^2} &= -k^2 A\cos(kx-\omega t)\end{aligned}\right\} \Rightarrow \frac{\mu}{F}\omega^2 = k^2$$

$$vk = \omega$$



$$\Rightarrow v = \sqrt{\frac{F}{\mu}}$$

wave speed on a string

∴ wavefunctions are solutions of the wave equation

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

wave equation (D'Alembert's equation)

hold for different kinds of waves

Different kinds of waves have different speed. For mechanical waves

$$v = \sqrt{\frac{\text{restoring force returning the system to equilibrium}}{\text{inertia resisting the return to equilibrium}}}$$

$$v = \sqrt{\frac{F}{\mu}}$$

v does not depend on λ nor f

Q15.1

If you double the wavelength λ of a wave on a string, what happens to the wave speed v and the wave frequency f?

A. v is doubled and f is doubled.

B. v is doubled and f is unchanged.

C. v is unchanged and f is halved.

D. v is unchanged and f is doubled.

E. v is halved and f is unchanged.

A15.1

If you double the wavelength λ of a wave on a string, what happens to the wave speed v and the wave frequency f?

A. v is doubled and f is doubled.

B. v is doubled and f is unchanged.

v is unchanged and f is halved.

D. v is unchanged and f is doubled.

E. v is halved and f is unchanged.

Q15.8

The four strings of a musical instrument are all made of the same material and are under the same tension, but have different thicknesses. Waves travel

A. fastest on the thickest string.

B. fastest on the thinnest string.

C. at the same speed on all strings.

D. not enough information given to decide

A15.8

The four strings of a musical instrument are all made of the same material and are under the same tension, but have different thicknesses. Waves travel

A. fastest on the thickest string.

fastest on the thinnest string.

C. at the same speed on all strings.

D. not enough information given to decide

Kinetic energy of the wave

Consider a wave of the form:

$$y(x,t) = A\cos(kx - \omega t)$$

Consider a string element of mass dm.

The kinetic energy is:

$$dK = \frac{1}{2} dm u^2 \text{ and } u = \frac{\partial y}{\partial t} = A\omega \sin(kx - \omega t)$$

And

$$dm = \mu dx$$

We get

$$dK = \frac{1}{2}\mu dx A^2 \omega^2 \sin^2(kx - \omega t)$$

Rate of kinetic energy transmission:

$$\begin{aligned} \frac{dK}{dt} &= \frac{1}{2}\mu A^2 \omega^2 \sin^2(kx - \omega t)\frac{dx}{dt} \\ &= \frac{1}{2}\mu v A^2 \omega^2 \sin^2(kx - \omega t) \end{aligned}$$

Kinetic energy is maximum at the y=0 position

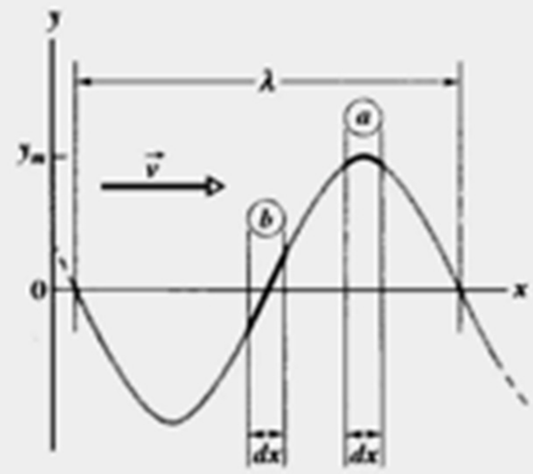


FIG. 16-12 A snapshot of a traveling wave on a string at time $t = 0$. String element a is at displacement $y = y_m$, and string element b is at displacement $y = 0$. The kinetic energy of the string element at each position depends on the transverse velocity of the element. The potential energy depends on the amount by which the string element is stretched as the wave passes through it.

(elastic) Potential energy of the wave

- Potential energy is carried in the string when it is stretched.
- Stretching is largest when the displacement has the largest gradient. Hence, the potential energy is also maximum at the $y = 0$ position. This is different from the harmonic oscillator, in which case energy is conserved.

Consider the extension Δs of a string element.

$$\Delta s = \sqrt{(dx)^2 + [y(x+dx,t) - y(x,t)]^2} - dx$$
$$\approx \sqrt{(dx)^2 + \left(\frac{\partial y}{\partial x}dx\right)^2} - dx = \left[\sqrt{1 + \left(\frac{\partial y}{\partial x}\right)^2} - 1\right]dx.$$

Using power series expansion,

$$\Delta s \approx \left[1 + \frac{1}{2}\left(\frac{\partial y}{\partial x}\right)^2 - 1\right]dx = \frac{1}{2}\left(\frac{\partial y}{\partial x}\right)^2 dx.$$

The potential energy is given by work done in extending the string element

$$dU = F\Delta s \approx \frac{F}{2}\left(\frac{\partial y}{\partial x}\right)^2 dx = \frac{F}{2}k^2A^2\sin^2(kx - \omega t)dx$$

Rate of potential energy transmission:

$$\frac{dU}{dt} = \frac{1}{2}\mu v A^2\omega^2\sin^2(kx - \omega t) = \frac{dK}{dt}$$

Power of transmission:

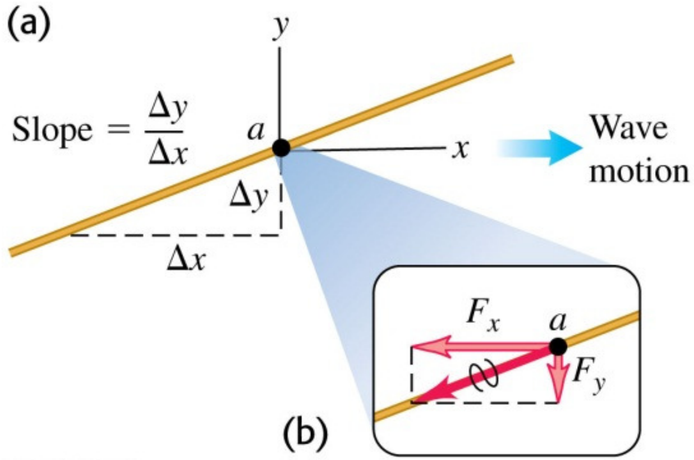
$$P = \frac{dK}{dt} + \frac{dU}{dt} = \mu v A^2 \omega^2 \sin^2(kx - \omega t)$$

time-averaged Power of transmission:

$$\boxed{\langle P \rangle = \frac{1}{2}\mu v A^2 \omega^2}$$

Another way to look at power propagation in wave motion

Consider a vibrating string



F_y *y*-component of force acting on point *a*

as point *a* moves, F_y does work. The power is

$$P(x,t) = F_y v_y = \left(-F\frac{\partial y}{\partial x}\right)\frac{\partial y}{\partial t}$$

$$= [-FkA\sin(kx-\omega t)][-\omega A\sin(kx$$

$$= Fk\omega A^2\sin^2(kx-\omega t)$$

$$k = \frac{\omega}{v} = \omega\sqrt{\frac{\mu}{F}}$$

$$P(x,t) = \sqrt{\mu F}\,\omega^2 A^2 \sin^2(kx-\omega t)$$

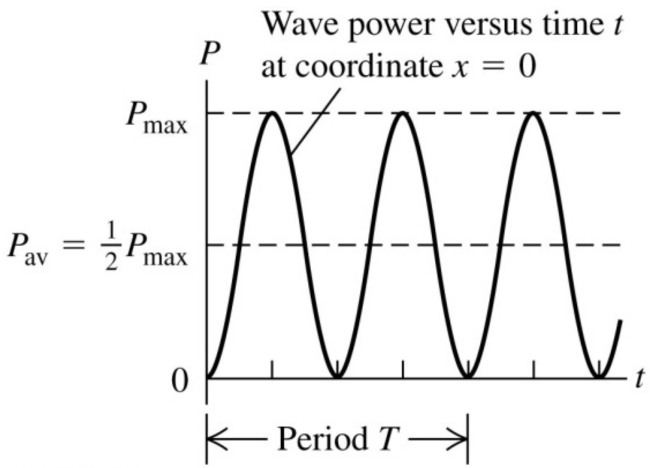
propagating, like a wave

$$P_{\max} = \sqrt{\mu F}\,\omega^2 A^2$$

$$P_{av} = \frac{1}{T}\int_0^T P(x,t)dt$$

$$= \sqrt{\mu F}\,\omega^2 A^2\left(\frac{\omega}{2\pi}\right)\underbrace{\int_0^{\frac{2\pi}{\omega}} \sin^2(kx-\omega t)\,dt}_{\pi/\omega}$$

$$P_{av} = \frac{1}{2}\sqrt{\mu F}\,\omega^2 A^2 = \frac{1}{2}P_{\max}$$



Q15.7

Two identical strings are each under the same tension. Each string has a sinusoidal wave with the same average power P_{av}.

If the wave on string #2 has twice the amplitude of the wave on string #1, the *wavelength* of the wave on string #2 must be

A. 4 times the wavelength of the wave on string #1.

B. twice the wavelength of the wave on string #1.

C. the same as the wavelength of the wave on string #1.

D. 1/2 of the wavelength of the wave on string #1.

E. 1/4 of the wavelength of the wave on string #1.

A15.7

Two identical strings are each under the same tension. Each string has a sinusoidal wave with the same average power P_{av}.

If the wave on string #2 has twice the amplitude of the wave on string #1, the *wavelength* of the wave on string #2 must be

A. 4 times the wavelength of the wave on string #1.

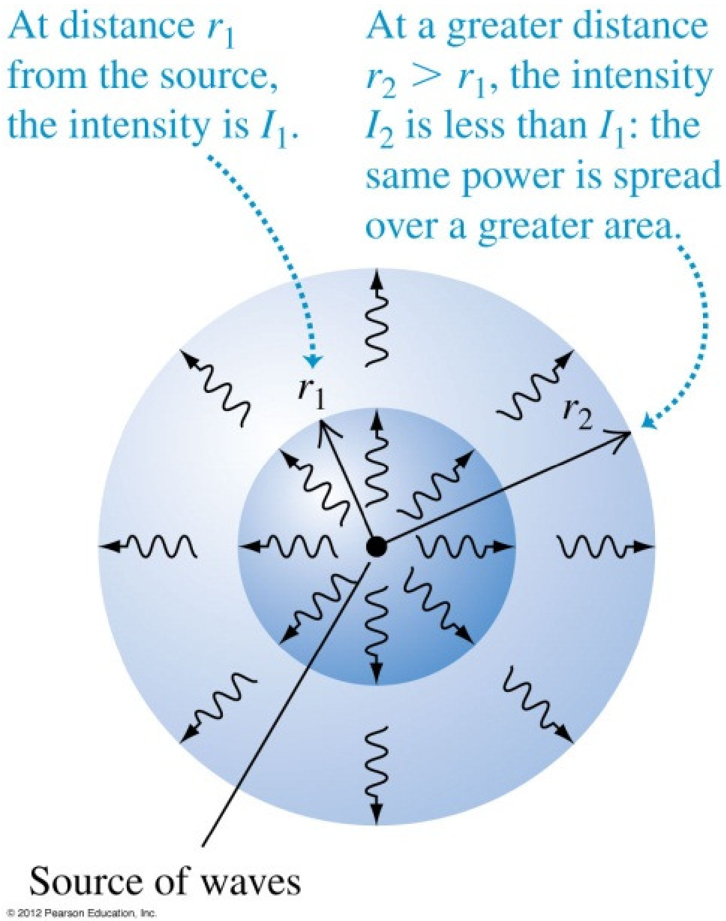
twice the wavelength of the wave on string #1.

C. the same as the wavelength of the wave on string #1.

D. 1/2 of the wavelength of the wave on string #1.

E. 1/4 of the wavelength of the wave on string #1.

For wave in 3D, define **intensity** = average power per unit area, SI unit: W/m^2



Suppose power of source is P

intensity at distance r

$$I = \frac{P}{4\pi r^2}$$

an *inverse square law!* Just like the Newton's law of gravitation and the Coulomb's law, although in a different context

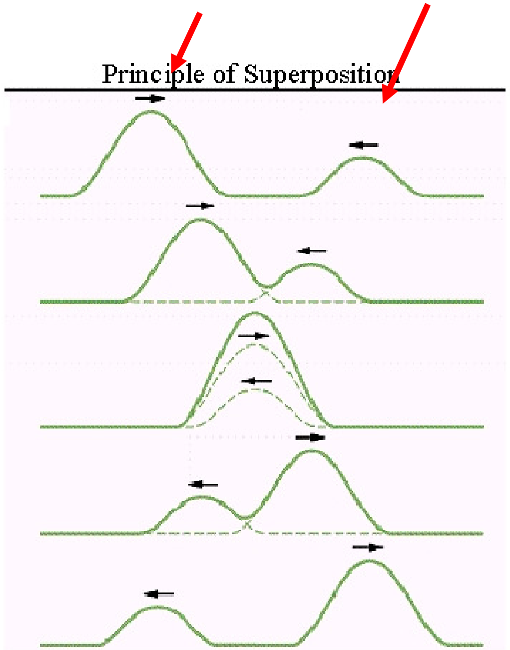
⚠ In the case of intensity it is clear that the inverse square law results from the surface area of a sphere, i.e., the dimensionality of space. The Newton's law of gravitation and Coulomb's law can also be formulated in a similar way to show that the inverse square laws are results of the dimensionality of space. This more general formulation is known as the Gauss Law.

Spherical wave

Principle of superposition of wave

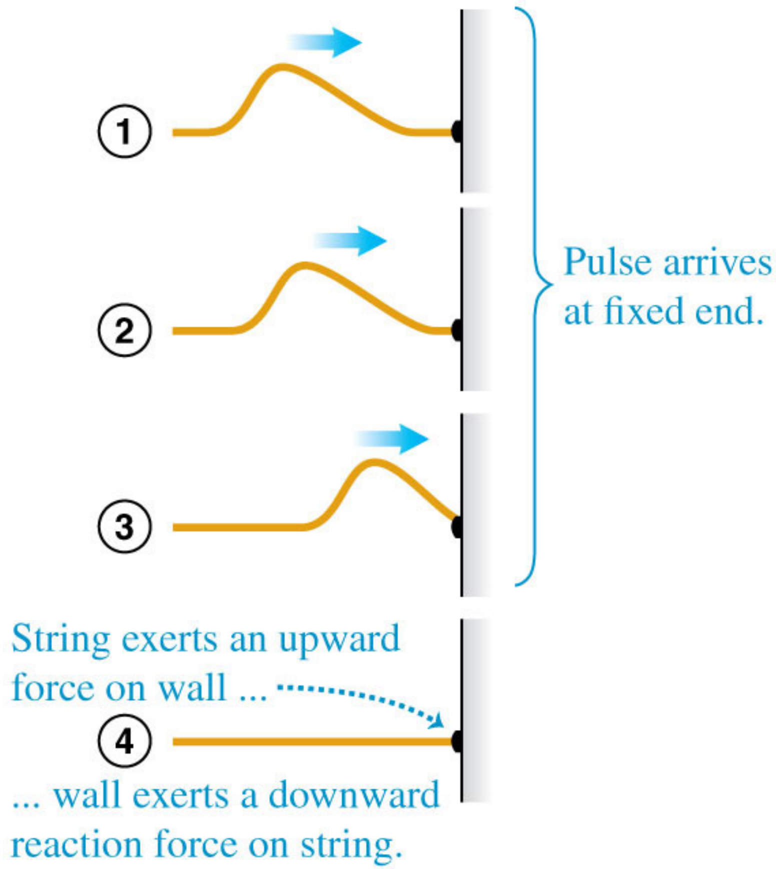
Whenever two (or more) **waves** travelling through the same medium at the same time. The **waves** pass through each other without being disturbed. The net displacement of the medium at any point in space or time, is simply the sum of the individual **wave** displacements.

$$y(x,t) = y_1(x,t) + y_2(x,t)$$



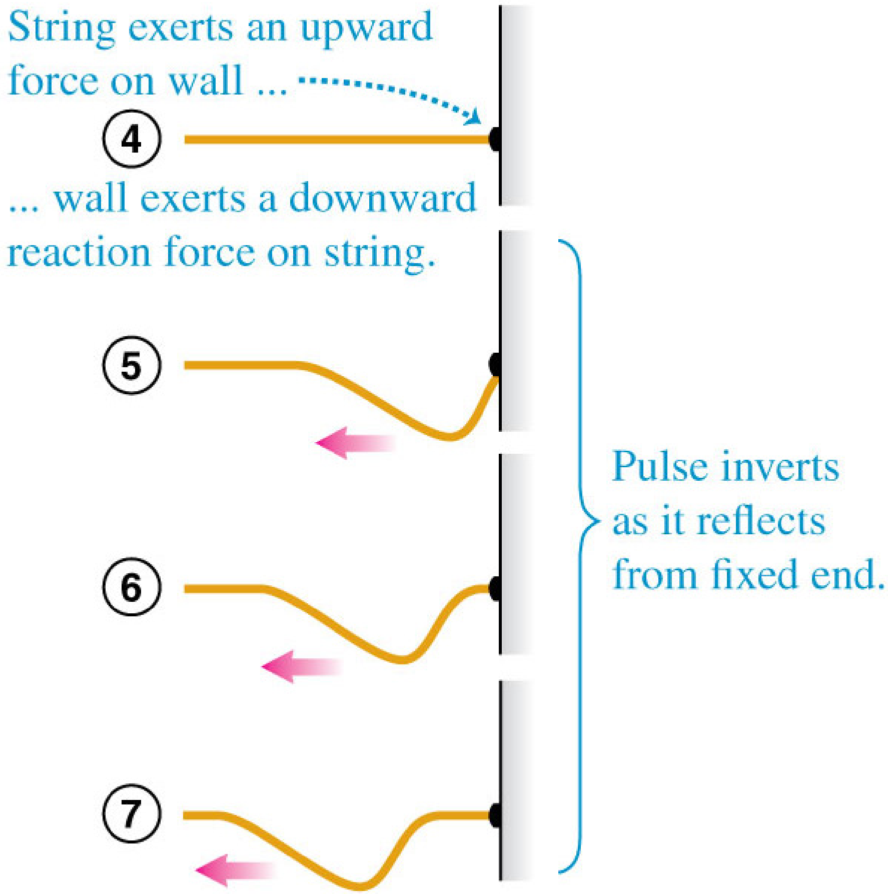
Reflection of a wave pulse at a fixed end of a string

- What happens when a wave pulse or a sinusoidal wave arrives at the end of the string?
- If the end is fastened to a rigid support, it is a *fixed* end that cannot move.
- The arriving wave exerts a force on the support (drawing 4).



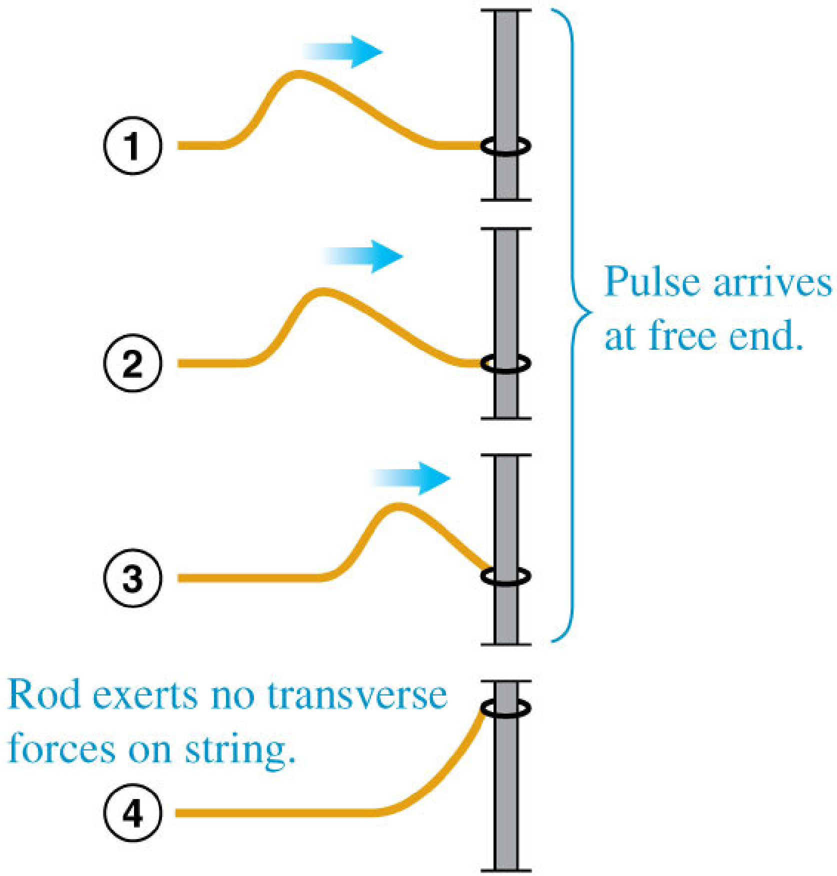
Reflection of a wave pulse at a fixed end of a string

- The reaction to the force of drawing 4, exerted by the support on the string, “kicks back” on the string and sets up a reflected pulse or wave traveling in the reverse direction.



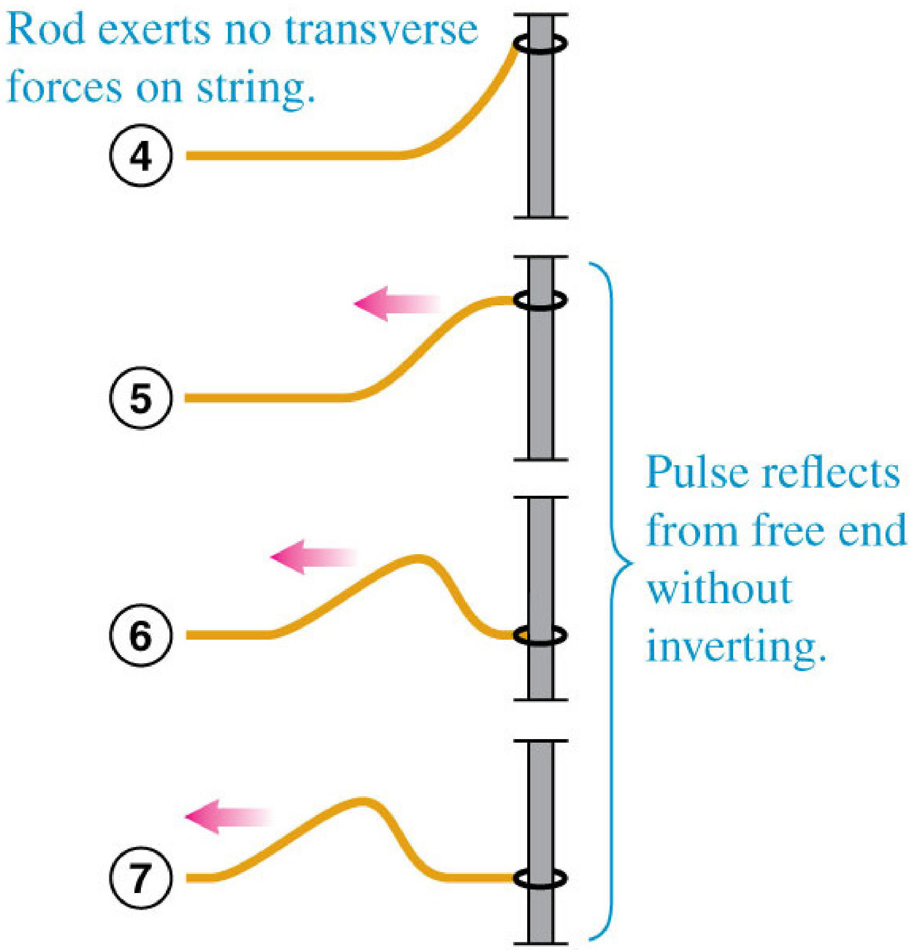
Reflection of a wave pulse at a free end of a string

- A **free end** is one that is perfectly free to move in the direction perpendicular to the length of the string.
- When a wave arrives at this free end, the ring slides along the rod, reaching a maximum displacement, coming momentarily to rest (drawing 4).



Reflection of a wave pulse at a free end of a string

- In drawing 4, the string is now stretched, giving increased tension, so the free end of the string is pulled back down, and again a reflected pulse is produced.



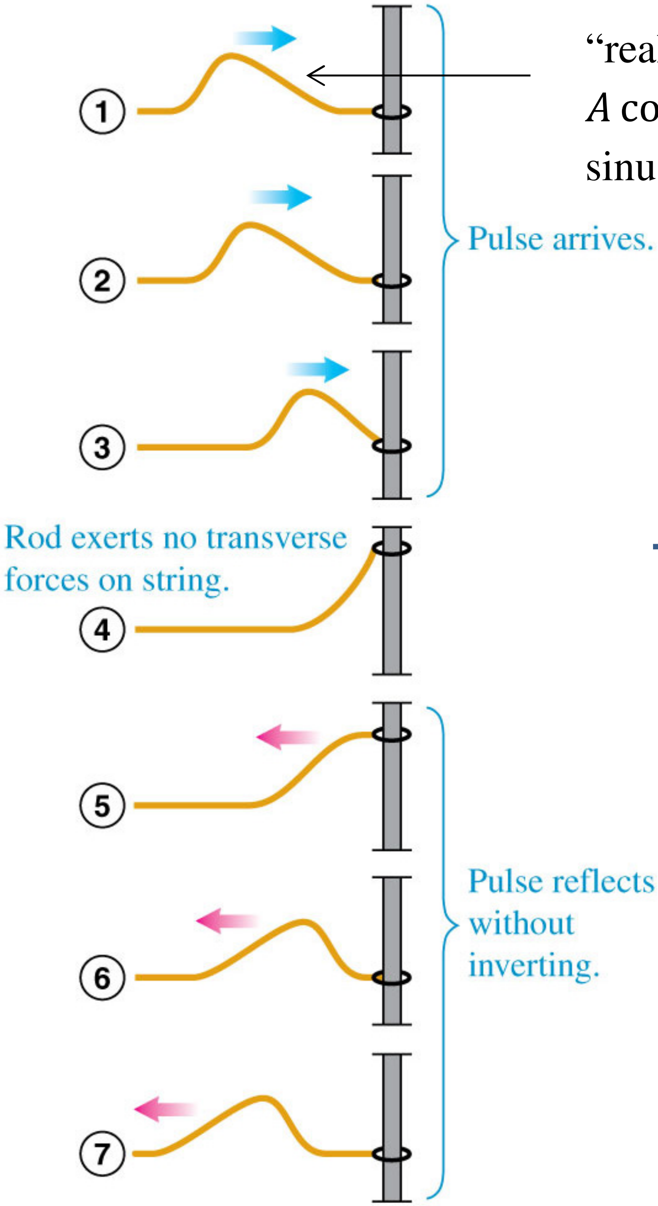
Wave reflection

https://www.youtube.com/watch?v=DbtQj8lNGFY

Image method –proof involves solving the d'Alembert equation
Mirror image of the wave coming in opposite direction, reflected wave results from the superposition of the "real" wave and its "image".

⚠ must observe the **boundary condition**

open boundary condition – free to move at one end

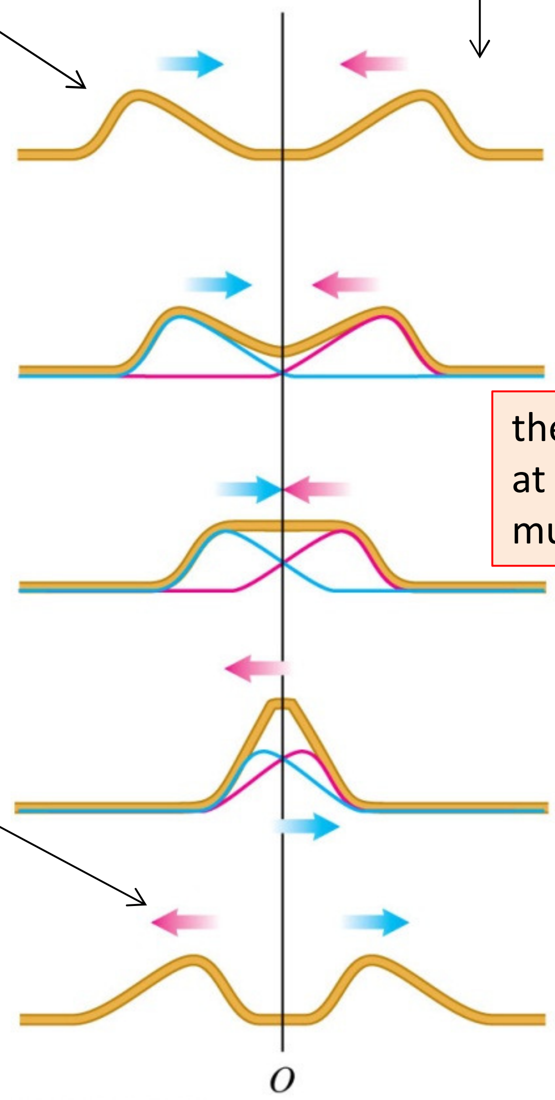


"real" incoming wave, $A\cos(kx-\omega t)$ if sinusoidal

mirror image, going to the left, $A\cos(kx+\omega t)$ if sinusoidal

image method

"image" becomes "real" wave, i.e., reflected wave is $A\cos(kx+\omega t)$, no phase change

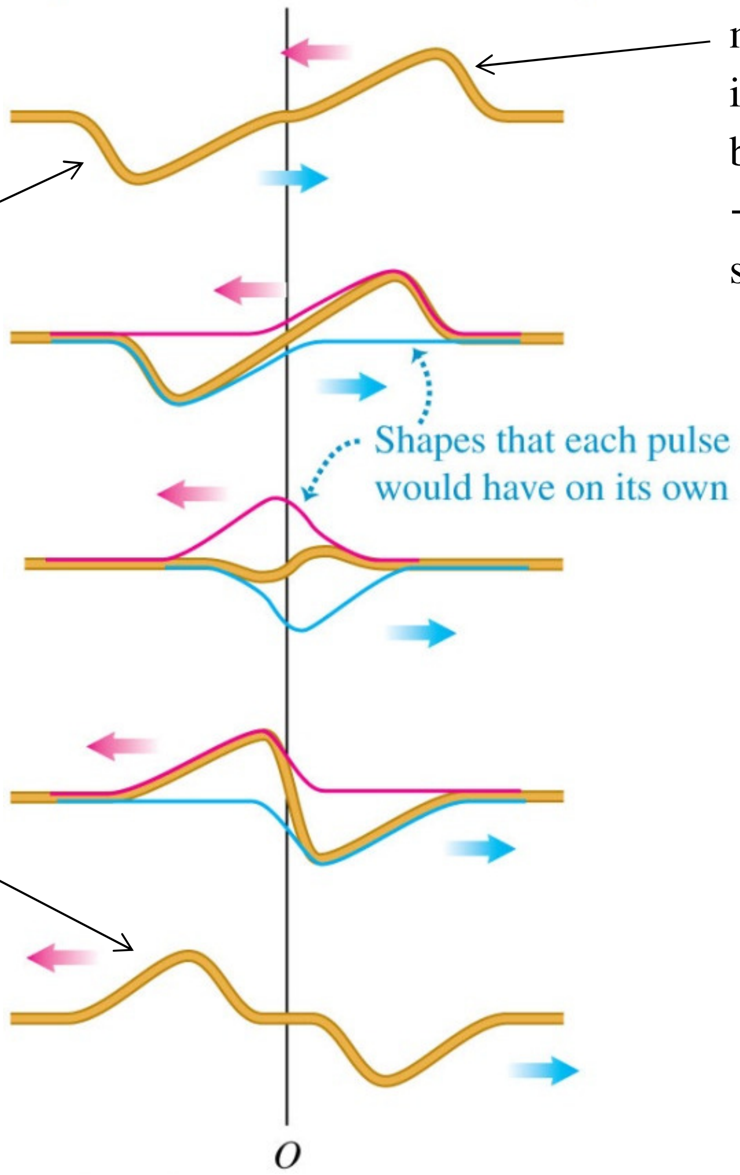


the slope at the boundary must be zero

fixed boundary condition – string clamped at one end

"real" in coming wave, $A\cos(kx - \omega t)$ if sinusoidal

mirror image – must be inverted to preserve boundary condition, i.e. $-A\cos(kx + \omega t)$ if sinusoidal



"image" becomes "real" wave, i.e., reflected wave is $-A\cos(kx + \omega t)$

$= A\ \cos(kx + \omega t + \pi)$

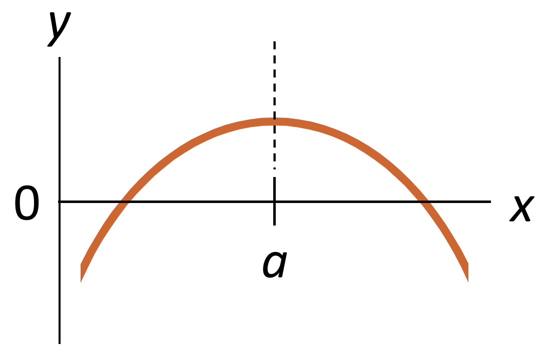
means a phase change of π

the wave at the boundary must be zero

Q15.3

A wave on a string is moving to the right. This graph of *y*(*x, t*) versus coordinate *x* for a specific time *t* shows the shape of part of the string at that time.

At this time, what is the *velocity* of a particle of the string at *x* = *a*?



A. The velocity is upward.

B. The velocity is downward.

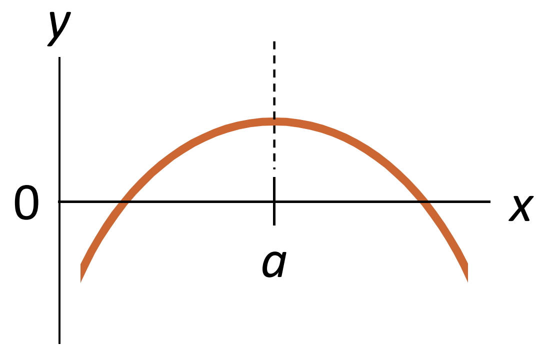
C. The velocity is zero.

D. not enough information given to decide

A15.3

A wave on a string is moving to the right. This graph of *y*(*x, t*) versus coordinate *x* for a specific time *t* shows the shape of part of the string at that time.

At this time, what is the *velocity* of a particle of the string at $x = a$?



A. The velocity is upward.

B. The velocity is downward.

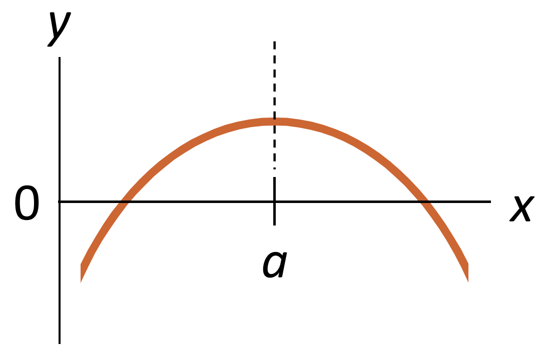
C. The velocity is zero.

D. not enough information given to decide

Q15.4

A wave on a string is moving to the right. This graph of $y(x, t)$ versus coordinate x for a specific time t shows the shape of part of the string at that time.

At this time, what is the *acceleration* of a particle of the string at $x = a$?



A. The acceleration is upward.

B. The acceleration is downward.

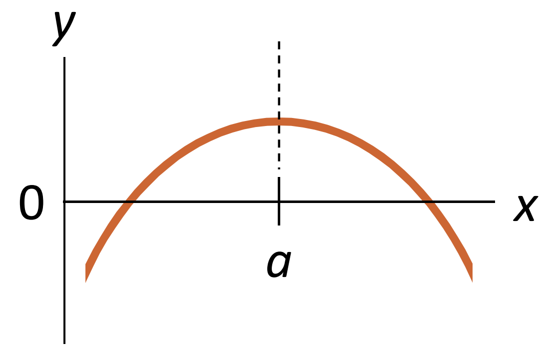
C. The acceleration is zero.

D. not enough information given to decide

A15.4

A wave on a string is moving to the right. This graph of $y(x, t)$ versus coordinate x for a specific time t shows the shape of part of the string at that time.

At this time, what is the *acceleration* of a particle of the string at $x = a$?



A. The acceleration is upward.

B. The acceleration is downward.

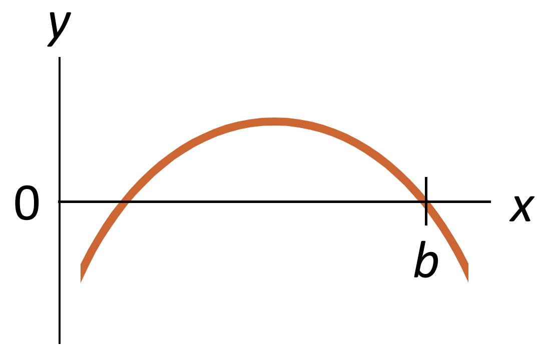
C. The acceleration is zero.

D. not enough information given to decide

Q15.5

A wave on a string is moving to the right. This graph of $y(x, t)$ versus coordinate x for a specific time t shows the shape of part of the string at that time.

At this time, what is the *velocity* of a particle of the string at $x = b$?



A. The velocity is upward.

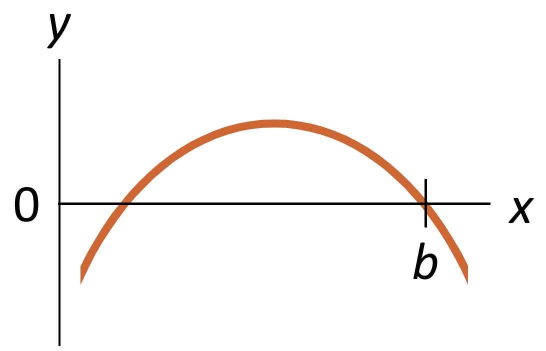
B. The velocity is downward.

C. The velocity is zero.

D. not enough information given to decide

A15.5

A wave on a string is moving to the right. This graph of *y*(*x, t*) versus coordinate *x* for a specific time *t* shows the shape of part of the string at that time.



At this time, what is the *velocity* of a particle of the string at *x* = *b*?

The velocity is upward.

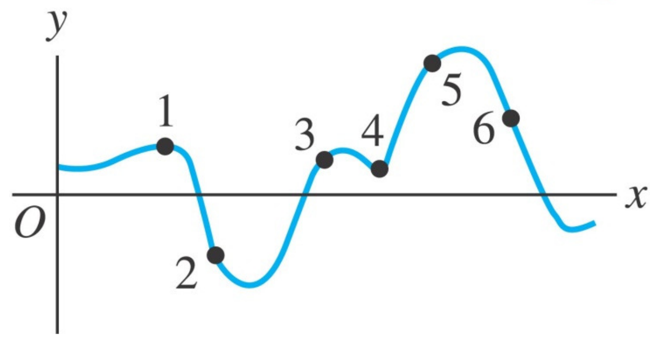
B. The velocity is downward.

C. The velocity is zero.

D. not enough information given to decide

Q15.6

A wave on a string is moving to the right. This graph of *y*(*x*, *t*) versus coordinate *x* for a specific time *t* shows the shape of part of the string at that time.



At this time, the velocity of a particle on the string is *upward* at

A. only one of points 1, 2, 3, 4, 5, and 6.

B. point 1 and point 4 only.

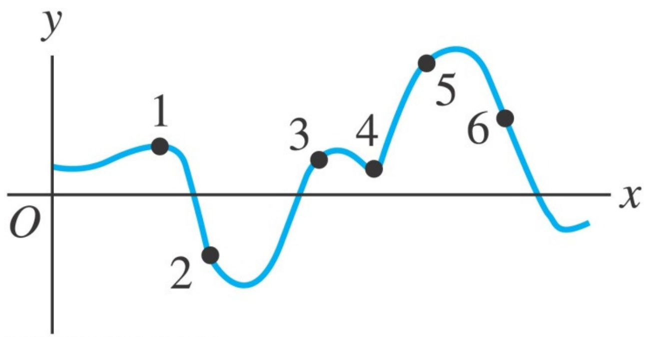
C. point 2 and point 6 only.

D. point 3 and point 5 only.

E. three or more of points 1, 2, 3, 4, 5, and 6.

A15.6

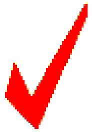
A wave on a string is moving to the right. This graph of *y*(*x, t*) versus coordinate *x* for a specific time *t* shows the shape of part of the string at that time.



At this time, the velocity of a particle on the string is *upward* at

A. only one of points 1, 2, 3, 4, 5, and 6.

B. point 1 and point 4 only.

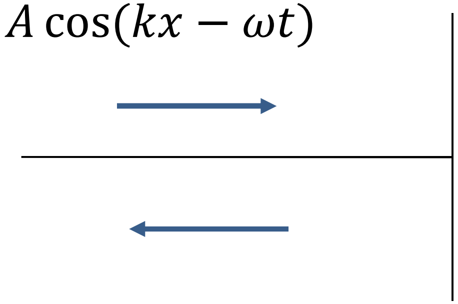
 point 2 and point 6 only.

D. point 3 and point 5 only.

E. three or more of points 1, 2, 3, 4, 5, and 6.

Standing wave – result of superposition between incident and reflected waves

continuous incident
wave train (not **pulse**)
$A\cos(kx-\omega t)$



For open boundary condition, reflected wave is
$A\cos(kx+\omega t)$
Resulting wave:

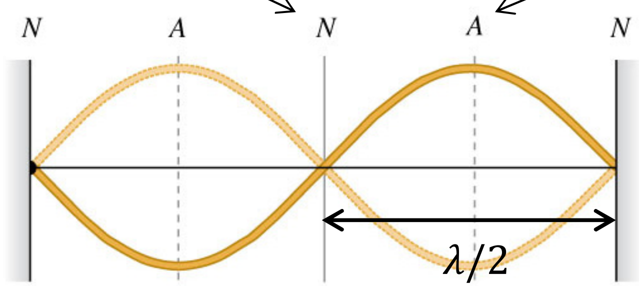
$$y(x,t) = A\cos(kx-\omega t) + A\cos(kx+\omega t)$$
$$= \underbrace{2A\cos kx}_{\text{sinusoidal amplitude}}\ \underbrace{\cos\omega t}_{\text{time variation}}$$

not propagating because no $\cos(kx-\omega t)$ term

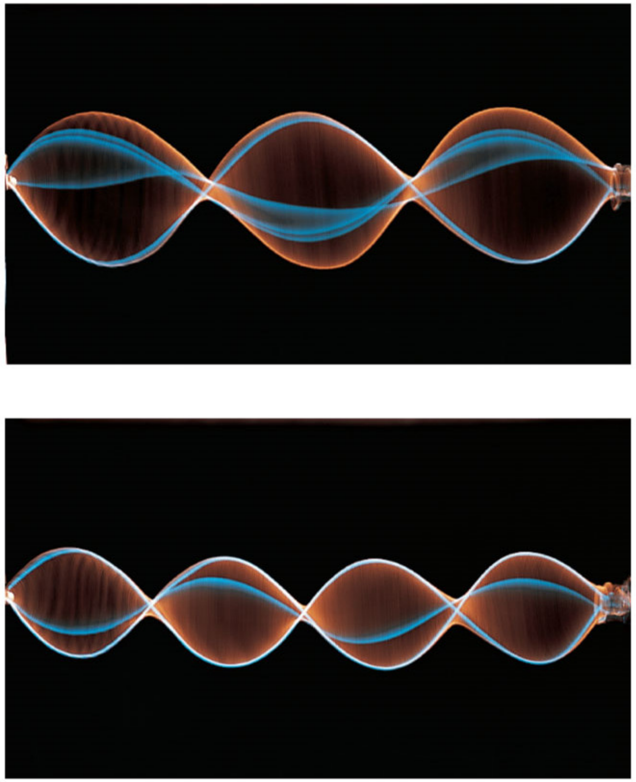
For fixed boundary condition, reflected wave is $-A\cos(kx+\omega t)$
resulting wave: $y(x,t) = A\cos(kx-\omega t) - A\cos(kx+\omega t)$
$= 2A\sin kx \sin\omega t$

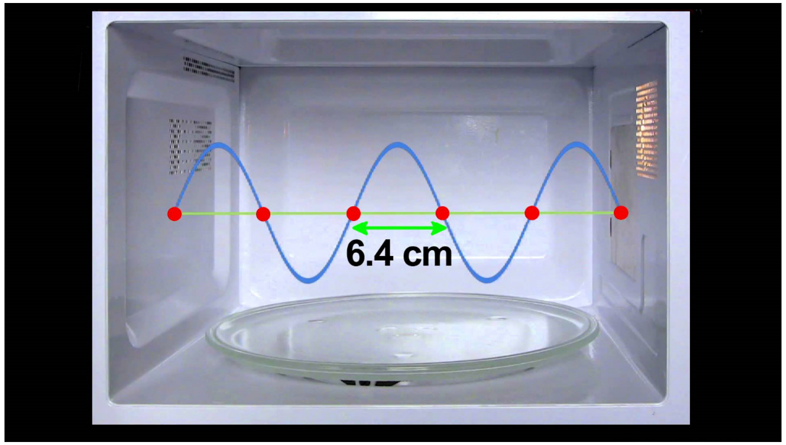
node – zero amplitude, called **destructive interference**

antinode – maximum amplitude, called **constructive interference**



Demonstration: standing wave applet
http://www.walter-fendt.de/ph14e/stwaverefl.htm





How a microwave oven works
https://www.youtube.com/watch?v=kp33ZprO0Ck

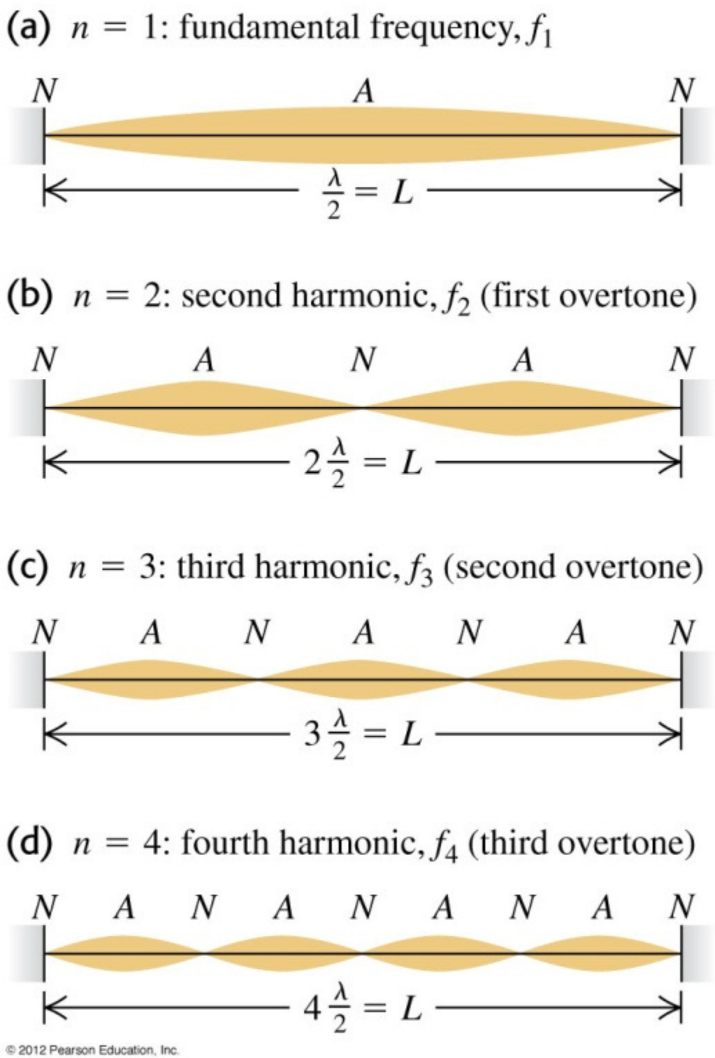
For a string of length L clamped on both ends, **normal modes** of vibration are those standing waves that can be fitted into the string

Normal mode frequencies are

$$L = n\frac{\lambda}{2} \quad \Rightarrow \quad \lambda_n = \frac{2L}{n},$$
$$n = 1, 2, \ldots$$

and frequencies are

$$f_n = n\left(\frac{v}{2L}\right) = nf_1, \qquad n = 1, 2, \ldots$$



Q15.9

While a guitar string is vibrating, you gently touch the midpoint of the string to ensure that the string does not vibrate at that point.

The lowest-frequency standing wave that could be present on the string

A. vibrates at the fundamental frequency.

B. vibrates at twice the fundamental frequency.

C. vibrates at 3 times the fundamental frequency.

D. vibrates at 4 times the fundamental frequency.

E. not enough information given to decide

A15.9

While a guitar string is vibrating, you gently touch the midpoint of the string to ensure that the string does not vibrate at that point.

The lowest-frequency standing wave that could be present on the string

A. vibrates at the fundamental frequency.

B. vibrates at twice the fundamental frequency.

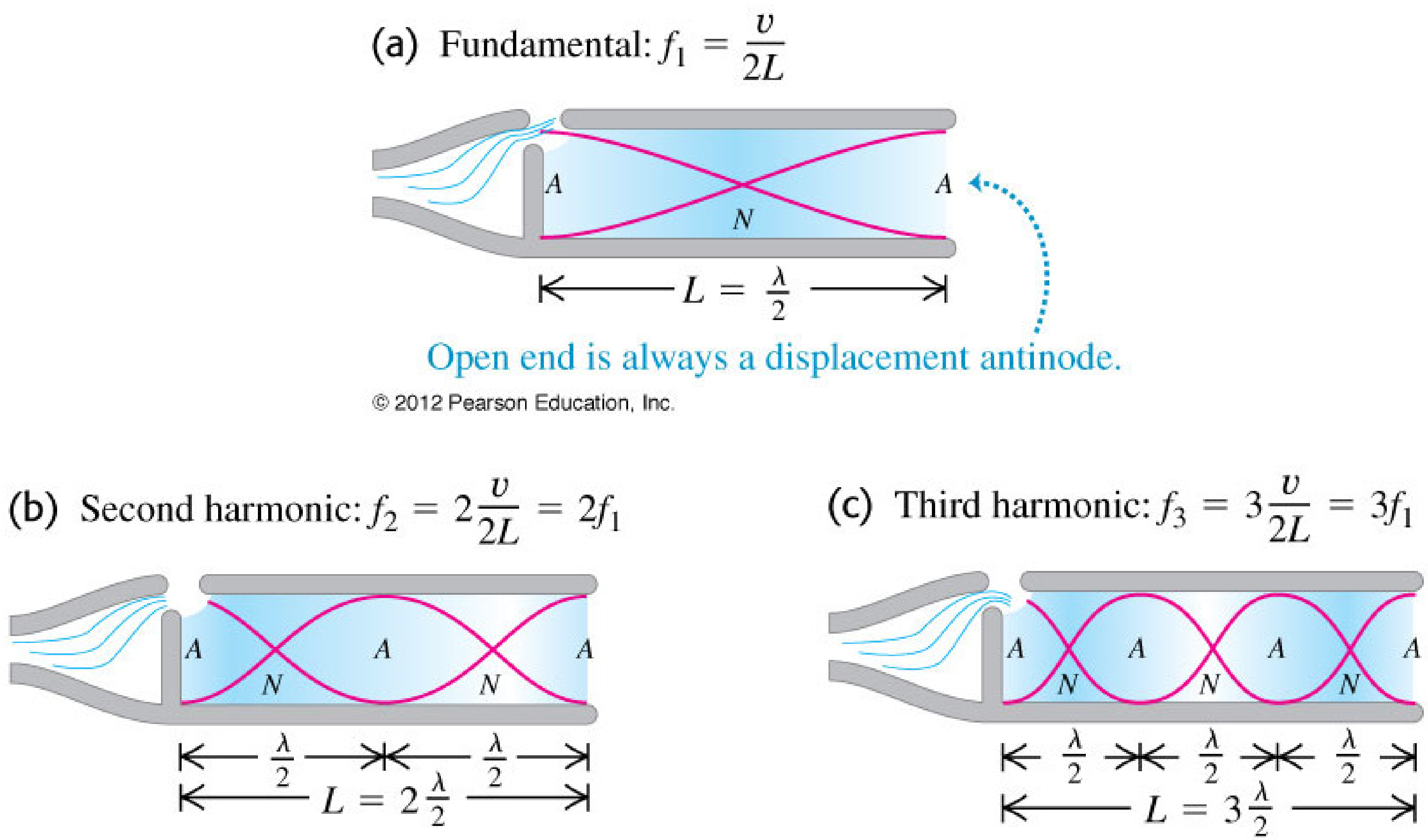
C. vibrates at 3 times the fundamental frequency.

D. vibrates at 4 times the fundamental frequency.

E. not enough information given to decide

For an open organ pipe, normal modes of sound wave are :

$$L = n\frac{\lambda}{2}, \qquad n = 1, 2, \ldots \text{ i.e. } \qquad \lambda_n = \frac{2L}{n}, \qquad f_n = n\left(\frac{v}{2L}\right) = nf_1,$$

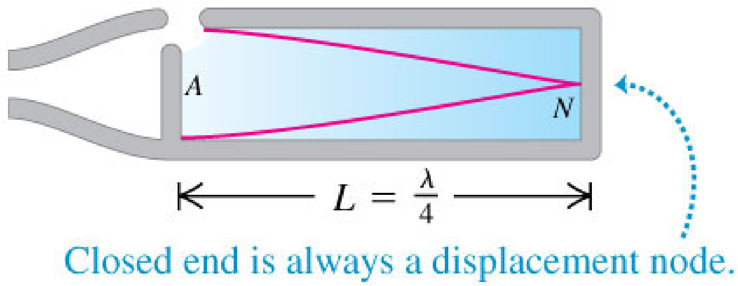


⚠ sound wave is **longitudinal**, not transverse as sketched in the diagrams

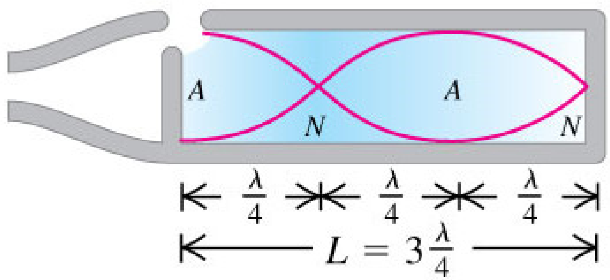
For a closed (at one end) organ pipe, normal modes of sound wave are :

$$L = n\frac{\lambda}{4}, \qquad n = 1, 3, 5, \ldots \text{ i.e.} \qquad \lambda_n = \frac{4L}{n}, \qquad f_n = n\left(\frac{v}{4L}\right) = nf_1,$$

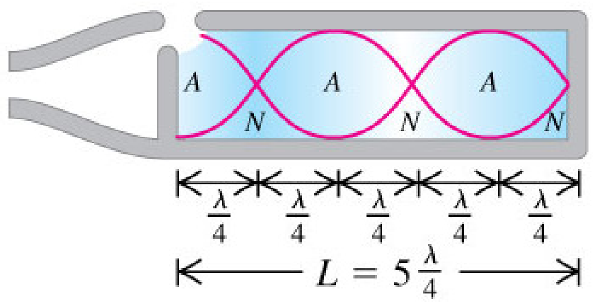
(a) Fundamental: $f_1 = \frac{v}{4L}$



(b) Third harmonic: $f_3 = 3\frac{v}{4L} = 3f_1$



(c) Fifth harmonic: $f_5 = 5\frac{v}{4L} = 5f_1$



Q16.6

When you blow air into an open organ pipe, it produces a sound with a fundamental frequency of 440 Hz.

If you close one end of this pipe, the new fundamental frequency of the sound that emerges from the pipe is

A. 110 Hz.

B. 220 Hz.

C. 440 Hz.

D. 880 Hz.

E. 1760 Hz.

A16.6

When you blow air into an open organ pipe, it produces a sound with a fundamental frequency of 440 Hz.

If you close one end of this pipe, the new fundamental frequency of the sound that emerges from the pipe is

A. 110 Hz.

✓ 220 Hz.

C. 440 Hz.

D. 880 Hz.

E. 1760 Hz.

Beats – interference of two traveling waves with *slightly* different frequencies

At a fixed spatial point $x_0 = 0$ for simplicity

$$y_a(t) = A\cos(-2\pi f_a t + \phi_a)$$
$$y_b(t) = A\cos(-2\pi f_b t + \phi_b)$$

Resulting note

$$y_a(t) + y_b(t) = 2A\cos\left(-2\pi\frac{f_a + f_b}{2}t + \frac{\phi_a + \phi_b}{2}\right)\cos\left(-2\pi\frac{f_a - f_b}{2}t + \frac{\phi_a - \phi_b}{2}\right)$$

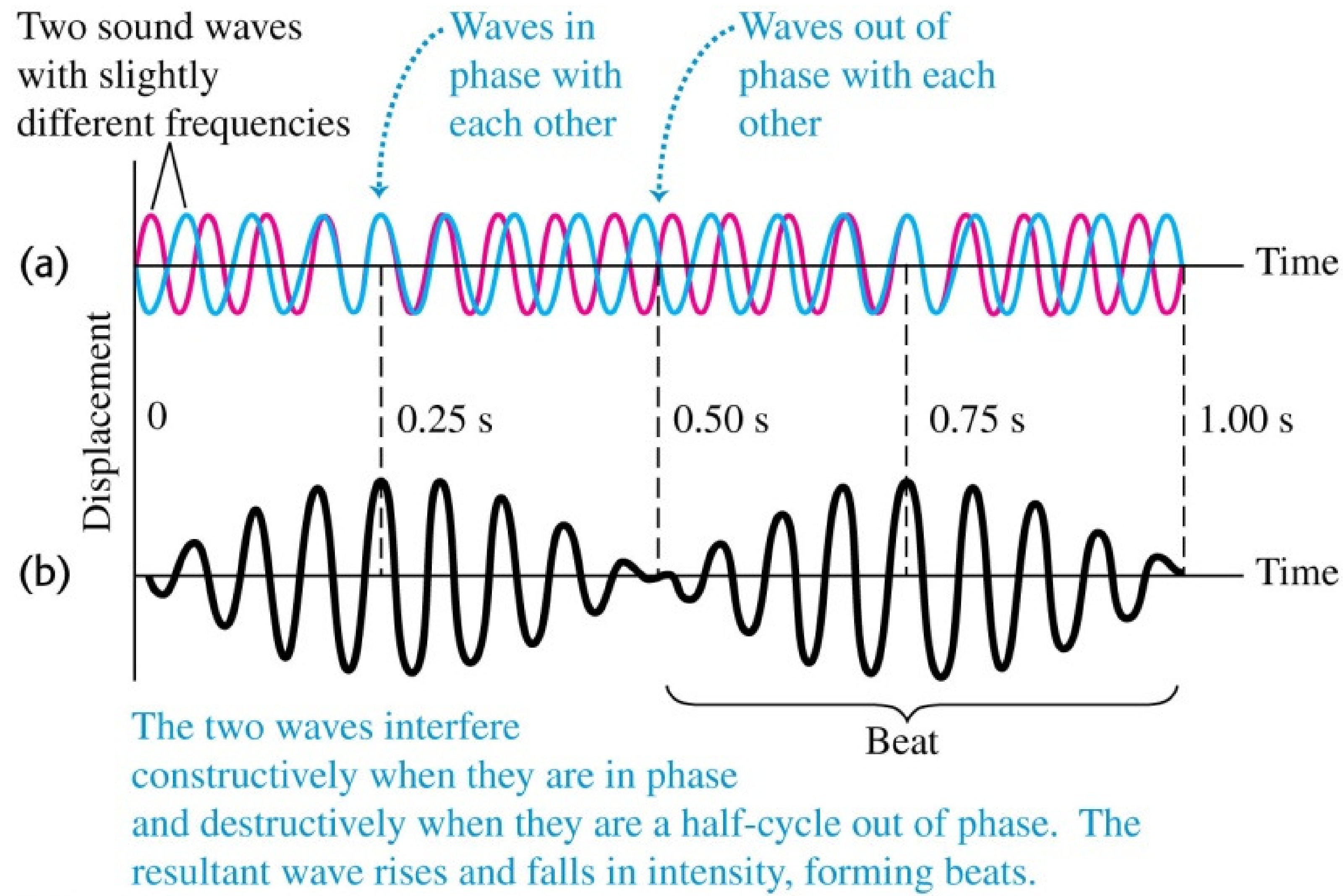
fast varying with frequency $\frac{1}{2}(f_a + f_b) \approx f_a \approx f_b$

slow varying with frequency $\frac{1}{2}|f_a - f_b|$, hear rise and fall in intensity with period

$$T = \frac{1}{2}\frac{1}{\frac{1}{2}|f_a - f_b|} = \frac{1}{|f_a - f_b|}$$

Beat frequency $\boxed{f_{\text{beat}} = |f_a - f_b|}$

A graphical illustration:



Demonstration – beats

https://www.youtube.com/watch?v=8vUuGRaTQ2E

Question

A tuning fork vibrates at 440 Hz, while a second tuning fork vibrates at an unknown frequency. They produce a tone that rises and falls in intensity three times per second. The frequency of the second tuning fork is

A. 434 Hz B. 437 Hz C. 443 Hz D. 446 Hz

E. either 434 or 446 Hz F. either 437 or 443 Hz

Q16.7

You hear a sound with a frequency of 256 Hz. The amplitude of the sound increases and decreases periodically: it takes 2 seconds for the sound to go from loud to soft and back to loud. This sound can be thought of as a sum of two waves with frequencies

A. 256 Hz and 2 Hz.

B. 254 Hz and 258 Hz.

C. 255 Hz and 257 Hz.

D. 255.5 Hz and 256.5 Hz.

E. 255.75 Hz and 256.25 Hz.

A16.7

You hear a sound with a frequency of 256 Hz. The amplitude of the sound increases and decreases periodically: it takes 2 seconds for the sound to go from loud to soft and back to loud. This sound can be thought of as a sum of two waves with frequencies

A. 256 Hz and 2 Hz.

B. 254 Hz and 258 Hz.

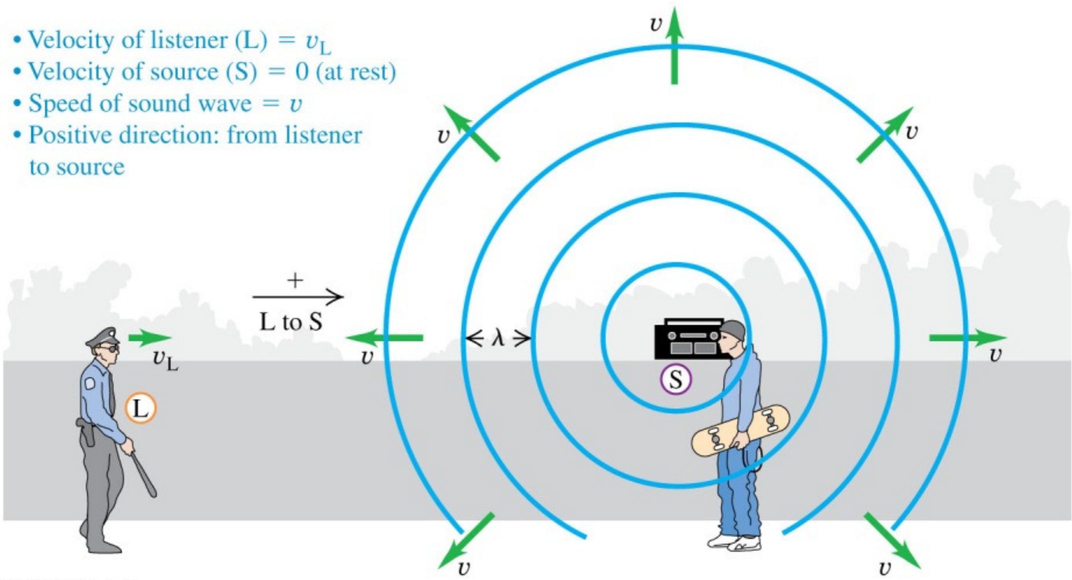
C. 255 Hz and 257 Hz.

D. 255.5 Hz and 256.5 Hz.

✓ 255.75 Hz and 256.25 Hz.

Doppler effect – frequency changes when source and/or observer are "moving"
Consider mechanical wave (sound as an example) only, all speeds relative to the medium (air), which is assumed to be stationary.

Case I: Source not moving (relative to the medium)



assume listener *approaching* source with speed v_L

wave front approach listener with speed $v + v_L$

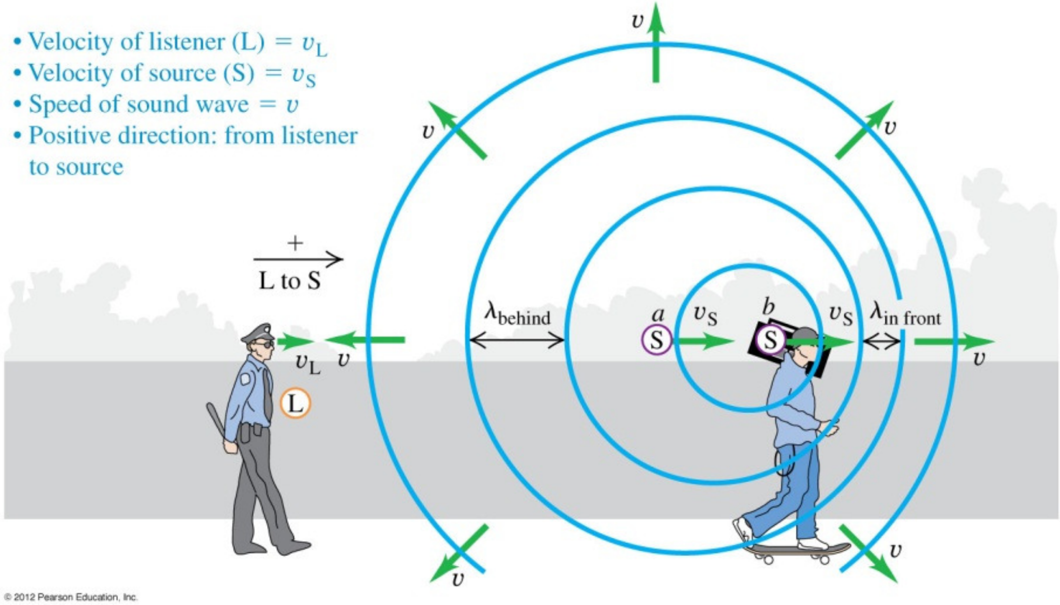
$$f_L = \frac{v + v_L}{\lambda} = \left(1 + \frac{v_L}{v}\right) f_S$$

(λ ↖ v/f_S)

If listener *approaching* source, $v_L > 0$ and $f_L > f_S$, hear a higher pitch
If listener *leaving* source, $v_L < 0$ and $f_L < f_S$, hear a lower pitch

Case II: Source moving

$$\lambda_{\text{behind}} = \frac{v + v_S}{f_S}$$



$$\lambda_{\text{in front}} = \frac{v - v_S}{f_S}$$

$$f_L = \frac{v + v_L}{\lambda_{\text{behind}}} \quad \Longrightarrow \quad \boxed{f_L = \left(\frac{v + v_L}{v + v_S}\right) f_S}$$

⚠ Sign convention – direction in which listener would approach source is taken to be +ve – check that the formula works in all possible cases

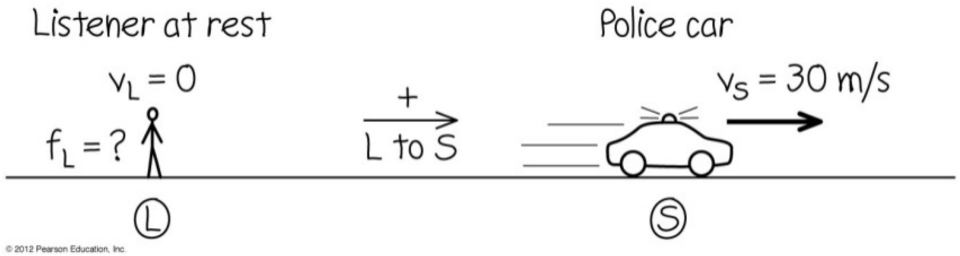
⚠ If listener at rest ($v_L = 0$), source approaching listener, then v_S(> / <) 0, and f_L (> / <) f_S

⚠ What if $v_S > v$? A condition called **supersonic**, leads to **shock wave**. Read textbook if you are interested.

Example 16.15 and 16.17

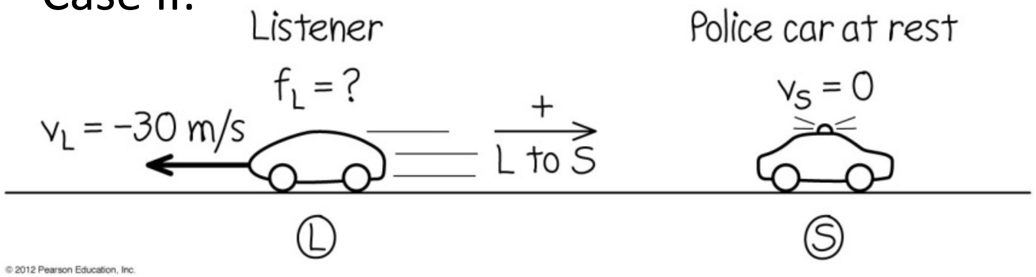
A police car's siren has frequency $f_S = 300$ Hz. Take speed of sound in still air, v, to be 340 m/s

Case I:



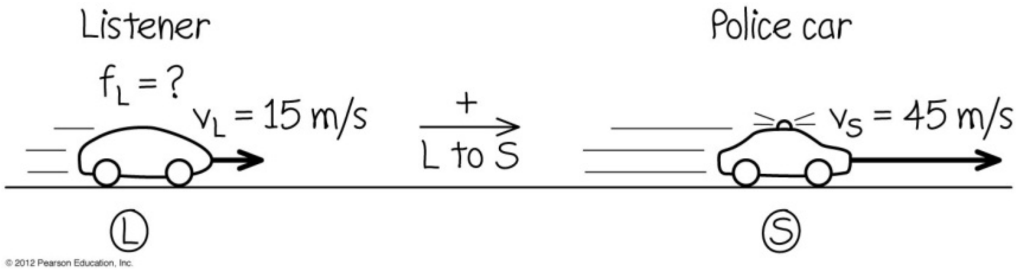
$$f_L = \frac{340\ \text{m/s}}{340\ \text{m/s} + 30\ \text{m/s}}(300\ \text{Hz})$$
$$= 276\ \text{Hz}$$

Case II:



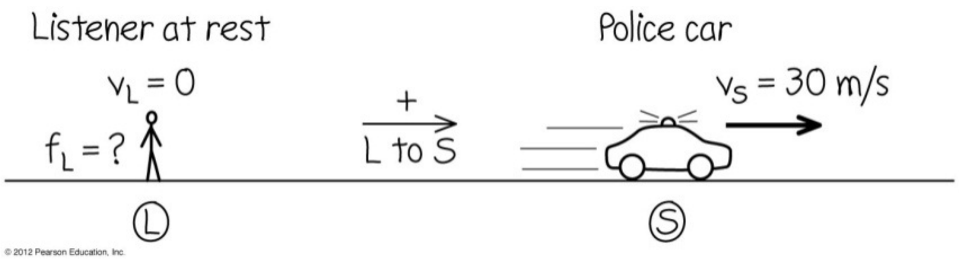
$$f_L = \frac{340\ \text{m/s} - 30\ \text{m/s}}{340\ \text{m/s}}(300\ \text{Hz})$$
$$= 274\ \text{Hz}$$

Case III:

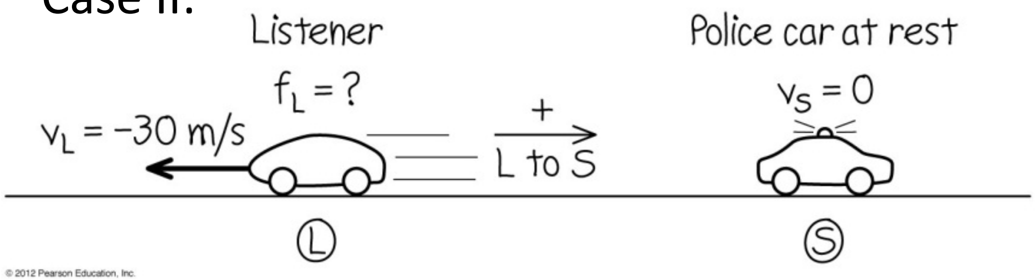


$$f_L = \frac{340\ \text{m/s} + 15\ \text{m/s}}{340\ \text{m/s} + 45\ \text{m/s}}(300\ \text{Hz})$$
$$= 277\ \text{Hz}$$

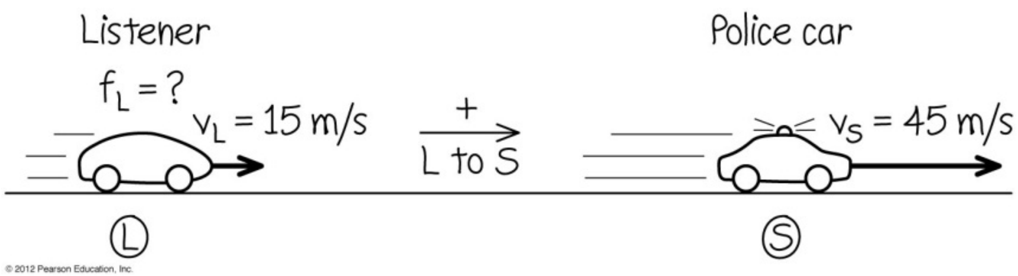
Case I:



Case II:



Case III:



In all 3 cases, the source and listener have the same relative velocity, but different f_L, i.e., cannot use either source or listener as frame of reference because there exist an absolute frame of reference – the medium.

How about waves without medium, such as light? All inertia frame of references are equivalent and Doppler effect can depend on the relative motion of the source and receiver only.

$$f_R = \sqrt{\frac{c - v}{c + v}} f_S$$

v is the relative velocity between source and receiver, +ve if moving away from each other.

Question

If remote star moving away from us, see (red / blue) shift in the light it emits.

Q16.8

On a day when there is no wind, you are moving toward a stationary source of sound waves. Compared to what you would hear if you were not moving, the sound that you hear has

A. a higher frequency and a shorter wavelength.

B. the same frequency and a shorter wavelength.

C. a higher frequency and the same wavelength.

D. the same frequency and the same wavelength.

A16.8

On a day when there is no wind, you are moving toward a stationary source of sound waves. Compared to what you would hear if you were not moving, the sound that you hear has

A. a higher frequency and a shorter wavelength.

B. the same frequency and a shorter wavelength.

✓ a higher frequency and the same wavelength.

D. the same frequency and the same wavelength.

Q16.9

On a day when there is no wind, you are at rest and a source of sound waves is moving toward you. Compared to what you would hear if the source were not moving, the sound that you hear has

A. a higher frequency and a shorter wavelength.

B. the same frequency and a shorter wavelength.

C. a higher frequency and the same wavelength.

D. the same frequency and the same wavelength.

A16.9

On a day when there is no wind, you are at rest and a source of sound waves is moving toward you. Compared to what you would hear if the source were not moving, the sound that you hear has

✓ a higher frequency and a shorter wavelength.

B. the same frequency and a shorter wavelength.

C. a higher frequency and the same wavelength.

D. the same frequency and the same wavelength.

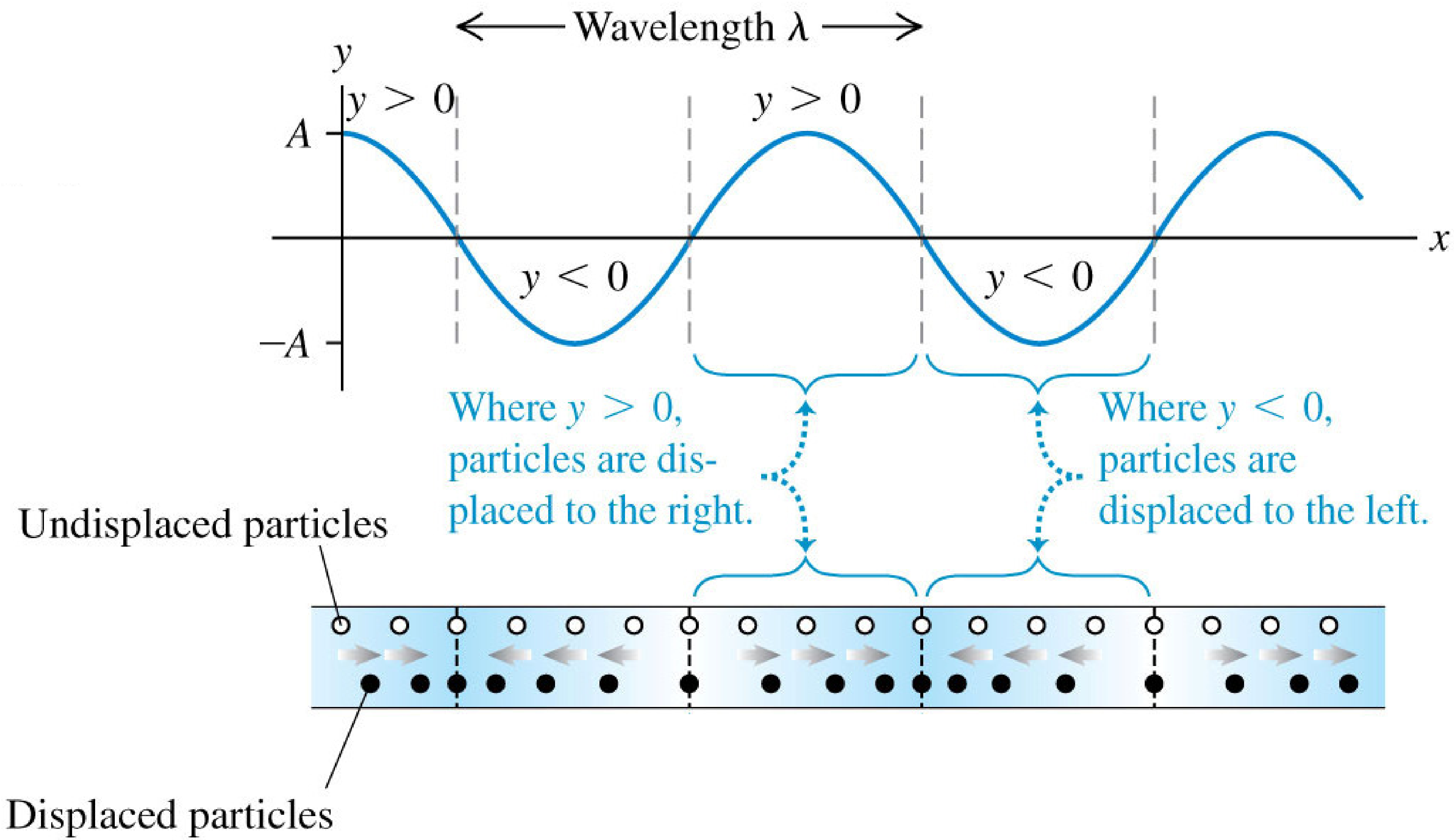
Sound waves

- **Sound** is simply any longitudinal wave in a medium.
- The **audible range** of frequency for humans is between about 20 Hz and 20,000 Hz.
- For a sinusoidal sound wave traveling in the *x*-direction, the wave function *y*(*x*, *t*) gives the instantaneous displacement *y* of a particle in the medium at position *x* and time *t*:

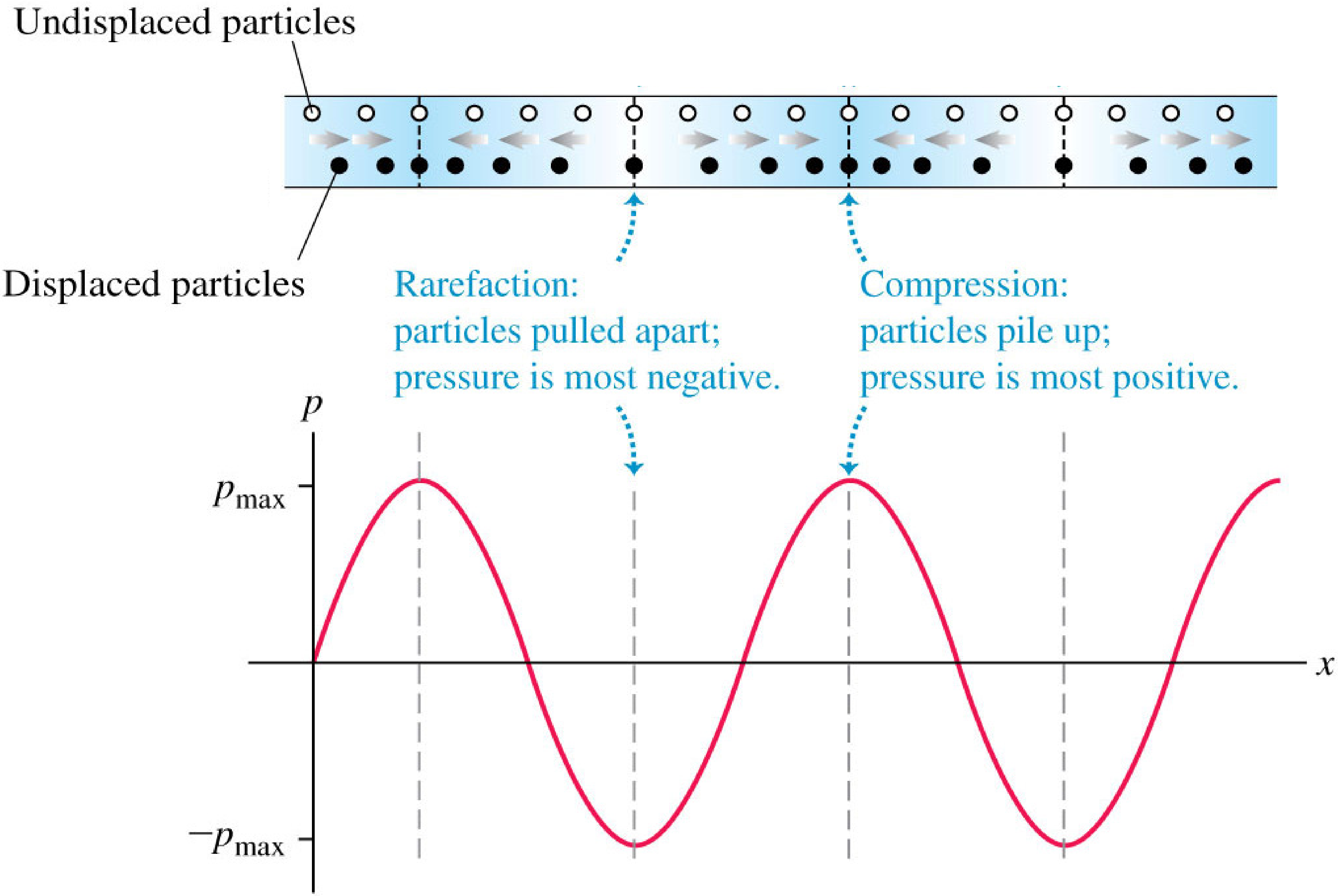
$$y(x, t) = A\cos(kx - \omega t) \quad \text{(sound wave propagating in the } +x\text{-direction)}$$

- In a longitudinal wave the displacements are *parallel* to the direction of travel of the wave, so distances *x* and *y* are measured parallel to each other, not perpendicular as in a transverse wave.

Different ways to describe a sound wave



Different ways to describe a sound wave



Different ways to describe a sound wave

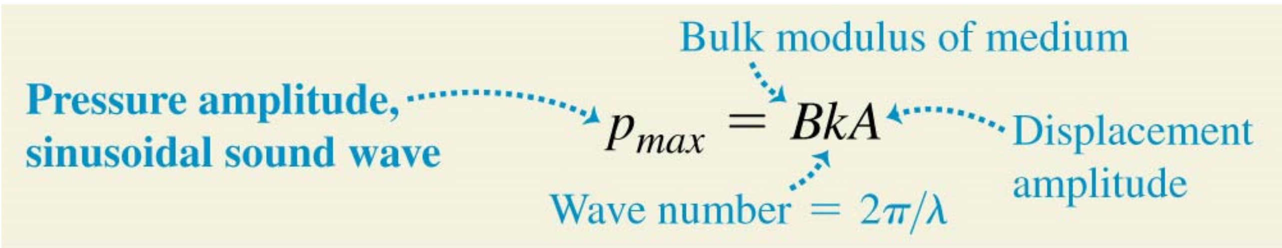
- Sound can be described mathematically as a displacement wave:

$$y(x, t) = A\cos(kx - \omega t) \quad \text{(sound wave propagating in the } +x\text{-direction)}$$

- The same sound wave can alternatively be described mathematically as a pressure wave:

$$p(x, t) = BkA\sin(kx - \omega t)$$

- The quantity *BkA* represents the maximum pressure fluctuation, and is called the **pressure amplitude**:



Speed of sound waves

- The speed of sound depends on the characteristics of the medium.
- In a fluid, such as water, the speed of sound is:

Speed of a longitudinal wave in a fluid

$$v = \sqrt{\frac{B}{\rho}}$$

B: Bulk modulus of fluid; ρ: Density of fluid

- In a solid rod or bar, the speed of sound is:

Speed of a longitudinal wave in a solid rod

$$v = \sqrt{\frac{Y}{\rho}}$$

Y: Young's modulus of rod material; ρ: Density of rod material

- In an ideal gas, such as air, the speed of sound is:

Speed of sound in an ideal gas

$$v = \sqrt{\frac{\gamma R T}{M}}$$

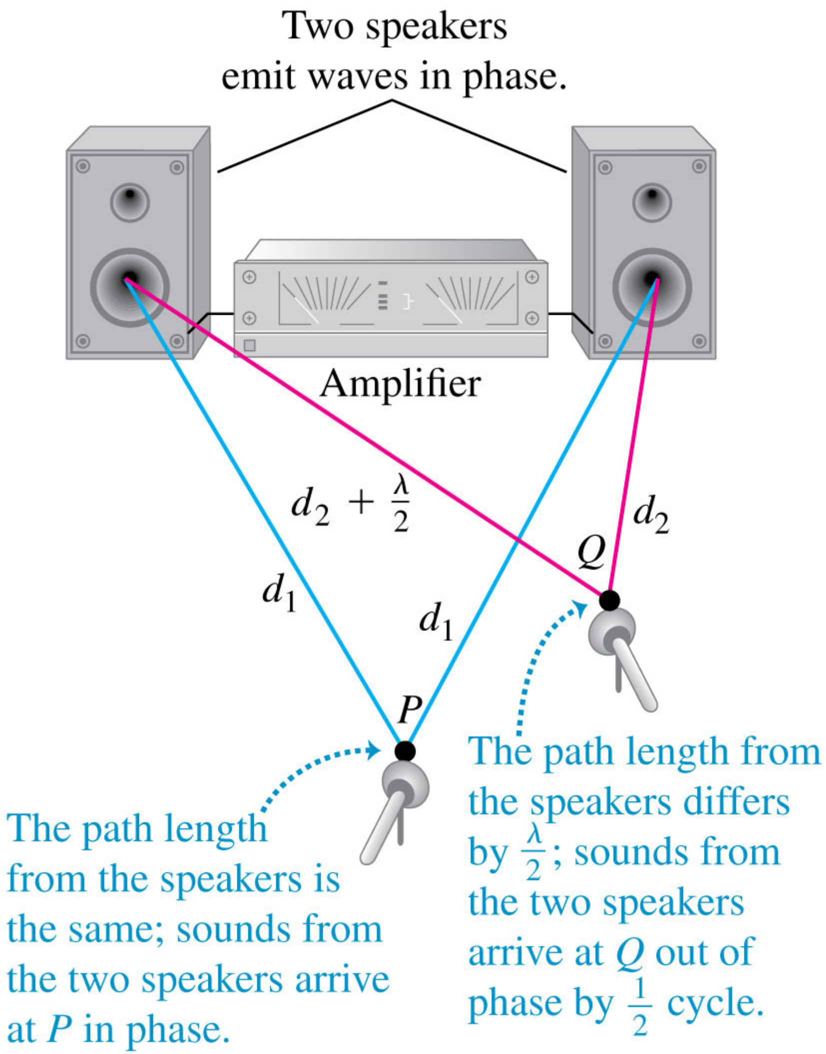
γ: Ratio of heat capacities; R: Gas constant; T: Absolute temperature; M: Molar mass

Table 16.1: Speed of sound in various bulk materials

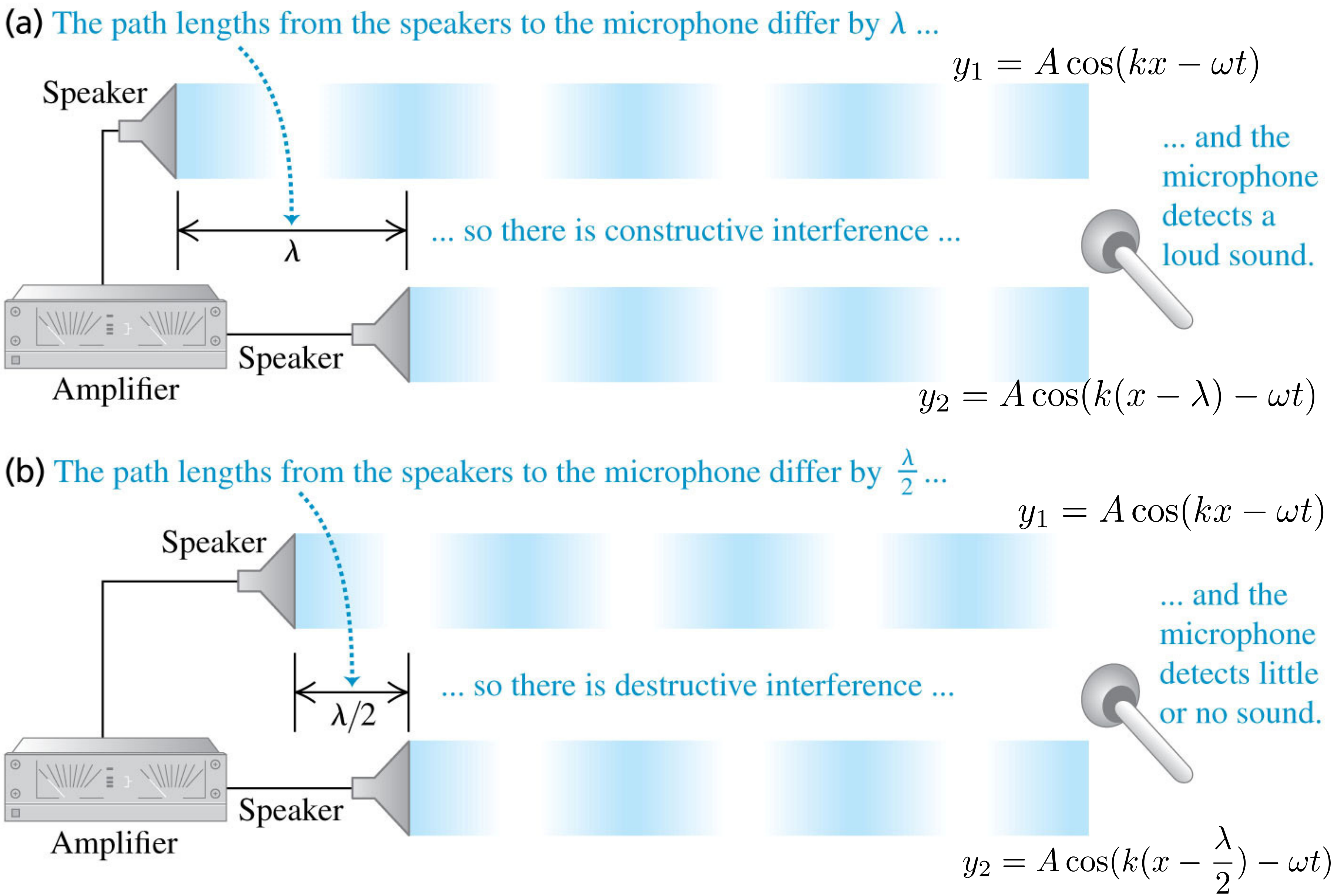
Material	Speed of Sound (m/s)
Gases	
Air (20°C)	344
Helium (20°C)	999
Hydrogen (20°C)	1330
Liquids	
Liquid helium (4 K)	211
Mercury (20°C)	1451
Water (0°C)	1402
Water (20°C)	1482
Water (100°C)	1543
Solids	
Aluminum	6420
Lead	1960
Steel	5941

Interference

- When two or more waves overlap in the same region of space they **interfere**.
- In the figure, two speakers are driven by the same amplifier.
- Constructive interference occurs at point *P*, and destructive interference occurs at point *Q*.



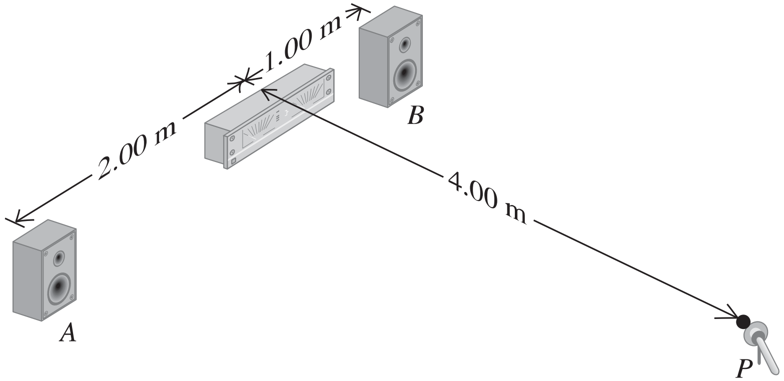
Interference



Example 16.13 Loudspeaker interference

Two small loudspeakers, A and B (Fig. 16.23), are driven by the same amplifier and emit pure sinusoidal waves in phase. (a) For what frequencies does constructive interference occur at point P? (b) For what frequencies does destructive interference occur? The speed of sound is 350 m/s.

16.23 What sort of interference occurs at P?



SOLUTION

IDENTIFY and SET UP: The nature of the interference at P depends on the difference d in path lengths from point A to P and from point B to P. We calculate the path lengths using the Pythagorean theorem. Constructive interference occurs when d equals a whole number of wavelengths, while destructive interference occurs when d is a half-integer number of wavelengths. To find the corresponding frequencies, we use $v = f\lambda$.

EXECUTE: The distance from A to P is $[(2.00\text{ m})^2 + (4.00\text{ m})^2]^{1/2} = 4.47$ m, and the distance from B to P is $[(1.00\text{ m})^2 + (4.00\text{ m})^2]^{1/2} = 4.12$ m. The path difference is $d = 4.47\text{ m} - 4.12\text{ m} = 0.35$ m.

(a) Constructive interference occurs when $d = 0, \lambda, 2\lambda, \ldots$ or $d = 0, v/f, 2v/f, \ldots = nv/f$. So the possible frequencies are

$$f_n = \frac{nv}{d} = n\frac{350\text{ m/s}}{0.35\text{ m}} \qquad (n = 1, 2, 3, \ldots)$$
$$= 1000\text{ Hz}, 2000\text{ Hz}, 3000\text{ Hz}, \ldots$$

(b) Destructive interference occurs when $d = \lambda/2, 3\lambda/2, 5\lambda/2, \ldots$ or $d = v/2f, 3v/2f, 5v/2f, \ldots$. The possible frequencies are

$$f_n = \frac{nv}{2d} = n\frac{350\text{ m/s}}{2(0.35\text{ m})} \qquad (n = 1, 3, 5, \ldots)$$
$$= 500\text{ Hz}, 1500\text{ Hz}, 2500\text{ Hz}, \ldots$$

EVALUATE: As we increase the frequency, the sound at point P alternates between large and small (near zero) amplitudes, with maxima and minima at the frequencies given above. This effect may not be strong in an ordinary room because of reflections from the walls, floor, and ceiling. It is stronger outdoors and best in an anechoic chamber, which has walls that absorb almost all sound and thereby eliminate reflections.