Rules and Regulations

1. Answer all the questions in the answer book provided.
2. Full mark of this written selection test is 100 Marks.
3. The selection test is a 3-hour written test.

Useful Constants

Unless specified otherwise, the following symbols and constants will be used in this exam paper.

Astronomical Unit, 1 AU = 1.496×10^{8} km
Earth-Moon Distance, d = 384,400 km
Mass of the Sun, $M_S = 1.99\times10^{30}$ kg
Mass of the Earth, $M_e = 5.97\times10^{24}$ kg
Mass of the Moon, $M_m = 7.35\times10^{22}$ kg
Radius of the Sun, R_S = 696300 km
Radius of the Earth, R_E = 6370 km
Radius of the Moon, R_M = 1738 km
Gravitational Constant $G = 6.67\times10^{-11}$ m^3 kg^{-1} s^{-2}
Acceleration due to Gravity, g = 9.8 ms^{-2}

Air density at the sea level = 1.2 kg m^{-3}
Gas Constant = 8.31 J/(mol·K)
Velocity of Light in Vacuum, $c = 3\times10^{8}$ ms^{-1}
Specific Heat of Water, C_W = 4200 J/(kg·K)
Planck Constant, $h = 6.63\times10^{-34}$ Js
Charge of Electron, $e = 1.6\times10^{-19}$ C
Mass of Electron, $m_e = 9.1\times10^{-31}$ kg
Mass of Neutron, $m_n = 1.68\times10^{-27}$ kg
Coulomb Constant, $k_e = 8.988\times10^{9}$ N m^2/C^2

Trigonometric Identities:

$$\sin(x+y)=\sin(x)\cos(y)+\cos(x)\sin(y)$$

$$\cos(x+y)=\cos(x)\cos(y)-\sin(x)\sin(y)$$

$$\sin(2x)=2\sin(x)\cos(x)$$

$$\cos(2x)=\cos^2(x)-\sin^2(x)$$

$$\sin(x)\cos(y)=\frac{1}{2}[\sin(x+y)+\sin(x-y)]$$

$$\cos(x)\cos(y)=\frac{1}{2}[\cos(x+y)+\cos(x-y)]$$

$$\sin(x)\sin(y)=\frac{1}{2}[\cos(x-y)-\cos(x+y)]$$

Tayler Series:

$$\sin(x)\approx x-\frac{x^3}{6}+\frac{x^5}{120}-\cdots$$

$$\cos(x)\approx 1-\frac{x^2}{2}+\frac{x^4}{24}-\cdots$$

$$\tan(x)\approx x+\frac{x^3}{3}+\frac{2x^5}{15}+\cdots$$

Series Summation:

$$\sum_{k=1}^{m}k=\frac{m(m+1)}{2}$$

$$\sum_{k=1}^{m}k^2=\frac{m(m+1)(2m+1)}{6}$$

$$\sum_{k=1}^{m}k^3=\left[\frac{m(m+1)}{2}\right]^2$$

Hyperbolic functions:

$$\frac{d}{dx}(\sinh x) = \cosh x$$

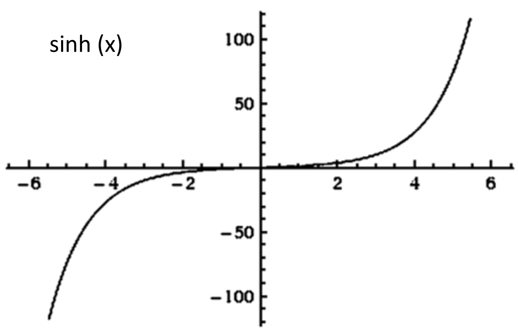
$$\frac{d}{dx}(\cosh x) = \sinh x$$

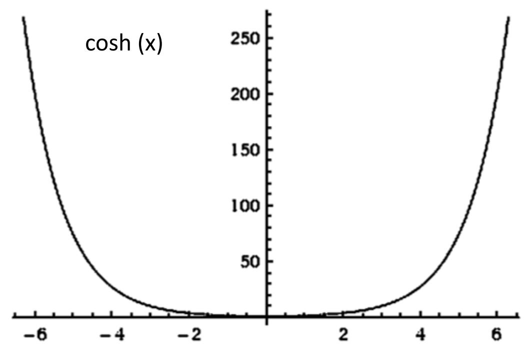
$$\frac{d}{dx}(\tanh x) = \frac{1}{\cosh^2 x}$$

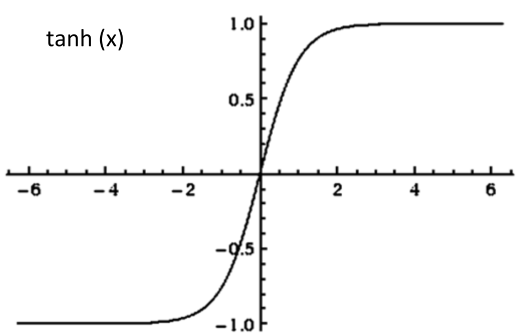
$$\sinh(x+y) = \sinh(x)\cosh(y) + \cosh(x)\sinh(y)$$

$$\cosh(x+y) = \cosh(x)\cosh(y) + \sinh(x)\sinh(y)$$

Sketch of hyperbolic functions



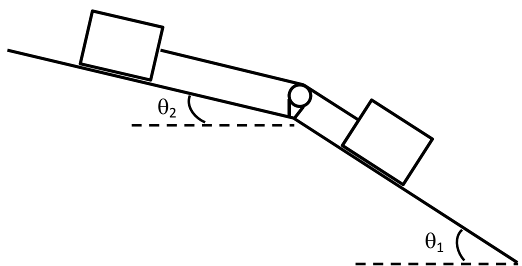




1. [10 Marks] Two identical blocks, each of mass *m*, are connected by a rubber band (massless, spring constant *k*). They are placed onto inclined planes of angles θ_1 and θ_2, as shown in the following figure. A frictionless pulley is mounted between the 2 inclined planes such that the rubber band is parallel to the inclined planes. The coefficient of friction between the blocks and the inclined planes is μ.

(a) What is the condition that the blocks will just move down the inclined planes when they are released from rest? Express your answer in terms of μ, θ_1, and θ_2.

(b) Suppose the angles of the inclined planes are adjusted such that i) the blocks just move down the inclined planes and ii) $\theta_1 - \theta_2 = 60°$ (assume $\theta_1 > \theta_2$). After a while, the 2 blocks move with the same acceleration (assume the 2 blocks remain on their original inclined planes). Find the extension of the rubber band. Express your answer in terms of *m*, μ, *g* (the acceleration due to gravity), and *k*.



2. [17.5 Marks] As illustrated in the following figure, an object (mass *m*) is released from rest at height h_2 (Position A) on a curved slope, which consists of a concave and a convex circular arc (both radii are *R*). The arcs join smoothly with a horizontal common tangent and have a right angle at the center. The object detaches from the slope at height h_1 (Position B).

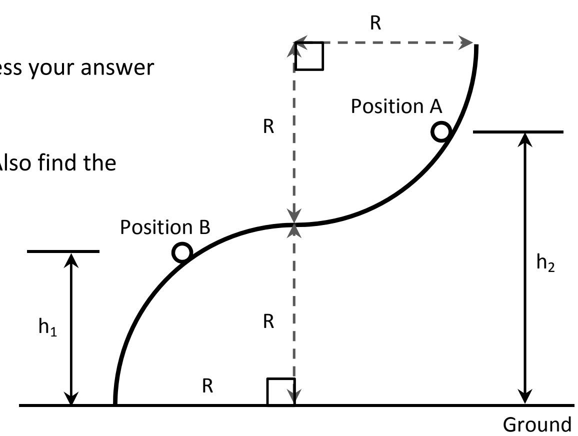
(a) Find h_2 in terms of h_1 and/or R.

(b) Find the distance covered by the object on the slope. Express your answer in terms of h_2 and R.

(c) Find h_2 such that the distance in Part (b) is the maximum. Also find the maximum distance.

(d) If the object detaches at the common tangent, find the horizontal distance from the detachment point when it reaches the ground.

Hint: You may assume that friction and air resistance is negligible.



3. [20.5 Marks] The following figure illustrates an isolated cylinder (negligible heat capacity). A piston (mass m, cross section area A, specific heat c) can move freely (frictionless) in vertical direction. The initial conditions are:

i. piston temperature is T_o;

ii. the volumes, V_o, of the gases are equal;

iii. an amount of n_1 mole noble gas at temperature T_1 and pressure P_1 above the piston; and

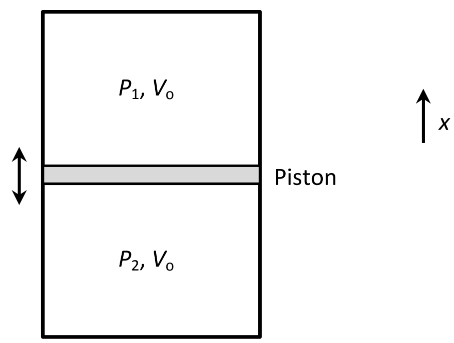
iv. an amount of n_2 mole air at temperature T_2 and pressure P_2 below the piston.

(a) Find the initial pressures P_1, P_2, and initial volume V_o. Express your answers in terms of m, g (acceleration due to gravity), A, R (gas constant), n_1, n_2, T_1, and T_2.

(b) Suppose m = 0.2 kg, c = 200 J/(kg·K), n_1 = 0.05 mole, n_2 = 0.03 mole, T_0 = 100°C, T_1 = -90°C, T_2 = 50°C, and A = 100 cm^2, evaluate the numerical values of P_1, P_2, and V_0.

(c) How many degrees of freedom do the gases have?

(d) Determine the final temperature and the displacement of the piston.



4. [8 Marks] Gravity waves are waves generated at the interface between two media when the force of gravity tries to restore the parcel toward equilibrium. An example is the interface between the atmosphere and the ocean. The phase velocity v_p of gravity waves in ocean of depth h can be described by the following equation:

$$v_p^{\ 2} = \frac{g}{k}\tanh(kh)$$

where g is the acceleration due to gravity, k is the wavenumber, and $\tanh(\ )$ is the hyperbolic tangent.

(a) Find v_g/v_p in terms of k and h, where v_g is the group velocity of the gravity wave.

(b) Find the minimum and maximum values of v_g/v_p.

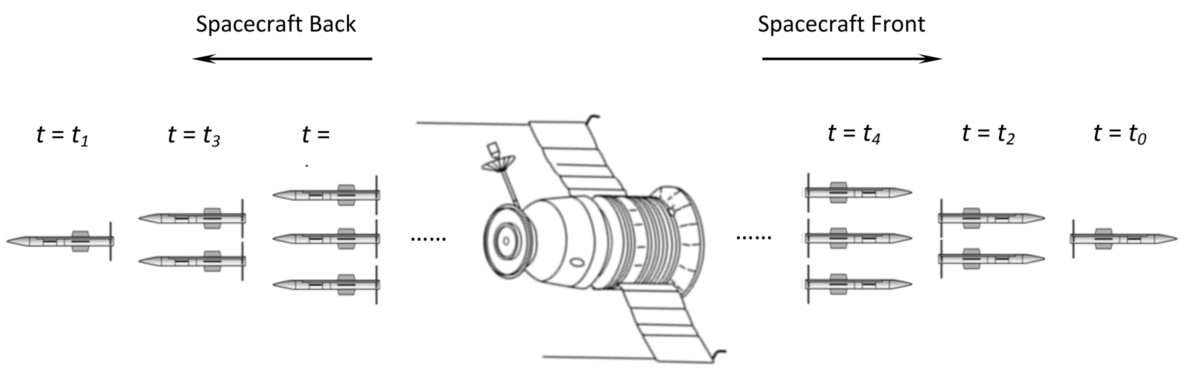
5. [16 Marks] A stationary spacecraft (mass *M*) carries *n* missiles (mass *m* each) in space. The spacecraft launches a series of missiles (mass *m* each) at time $t = t_i$ (i = 0, 1 ,2 …, *n*) where $t_0 < t_1 < t_2 < t_3 \ldots\ldots < t_n$. For all launches of even *i* , the directions of launches are in front of the spacecraft; whereas for all the odd number launches, the directions of launches are opposite to that of the even number launches. After each of the complete front and back launches, an additional missile will be added in the next launching cycle, that is, at $t = t_0$ and t_1, there is 1 missile launched, respectively; at $t = t_2$ and t_3, there are 2 missiles launched, respectively; whereas at $t = t_4$ and t_5, there are 3 missiles launched, respectively. The launching procedure continues until all the missiles have been used up.

(a) Suppose the total mass of the missiles is equal to half of the mass of the spacecraft, how many complete cycles (*k*) that the spacecraft can launch all the missiles. Express your answer in terms of *M* and/or *m*.

(b) Determine *n* in terms of *M* and/or *m*.

(c) The missile launching does not rely on fuel burning, but it is operated similar to the principle of cannon by giving an impulse to the missile in a small duration of time. The spacecraft has an initial velocity V_0. Supposing all the missiles in front of the spacecraft are launched at a speed of 2*v*, while all the missiles behind the spacecraft are launched at a speed of *v*, and the entire launching process costs $\gamma - 1$ of the total initial kinetic energy, find the change in velocity of the spacecraft after the entire launching process in terms of *v* and γ . Determine the possible value(s) of γ to make this missile launching possible. Deduce whether or not the spacecraft will move backward.

Hints: You may ignore all gravitational effects.



6. [6.5 Marks] The pressure, *P*, on walls for a given *N* identical gas molecules (mass *m* each and root-mean-square speed v_{rms}) confined in a cubic volume *V* is given by

$$P = \frac{Nm{v_{rms}}^2}{3V}.$$

(a) Express v_{rms} in terms of absolute temperature *T*, Boltzmann constant *k*, and molecular mass *m*.

(b) Calculate the root-mean-square speed of air at 300 K. You may assume air contains 80% nitrogen (molar mass 28 g) and 20% oxygen (molar mass 32 g).

(c) An unknown extrasolar planet (0.2 Earth Mass and 25 Earth Radius) has an atmospheric composition (80% nitrogen and 20% oxygen). Estimate the maximum surface temperature of the extrasolar planet so that the planet's atmosphere still exists.

7. [5.5 Marks] A wave (for example, sound) propagating in the *x*-direction can be described as

$$\Psi(x,t) = A\sin\left(2\pi\upsilon t - \frac{2\pi x}{\lambda}\right),$$

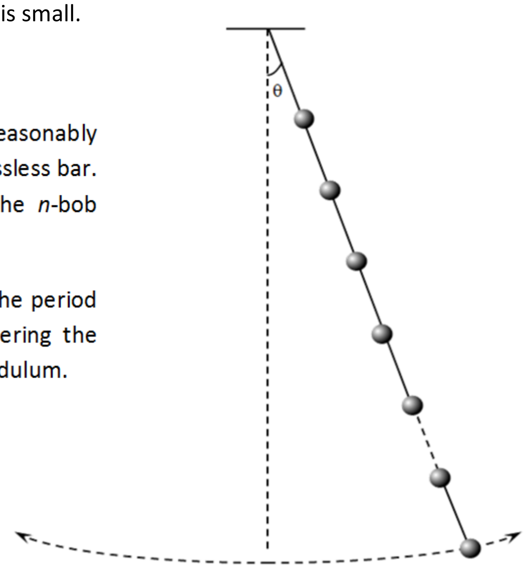
where *A* is the amplitude, λ is the wavelength, and υ is the wave frequency.

(a) Suppose the wave frequency υ = 1000 Hz and phase velocity υ_Φ = 500 ms^{-1}, determine the wavelength.

(b) Find the distance between 2 points at a given time where the phase difference is π / 9.

(c) Determine the phase change at a fixed point over a time interval 10 ms.

8. [16 Marks] As shown in the following figure, a simple pendulum with *n* identical small bobs (mass *m* each) on a massless rigid bar (length *L*) makes an angle θ with the vertical. All the bobs are uniformly distributed along the bar. The oscillation is restricted to the plane of the paper, and the pendulum oscillation θ is small.



(a) Find the period of the pendulum.

(b) A uniform rod (mass *M*) can be approximated to consist of reasonably large number of uniformly distributed small bobs along a massless bar. For small oscillation, determine the oscillation period of the *n*-bob pendulum.

(c) The oscillation period of the *n*-bob pendulum is identical to the period of a physical pendulum (that is, rod pendulum). By considering the moment of inertia of the rod, derive the period of the rod pendulum.

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