1.

(a) $Mg\sin\theta_1 + mg\sin\theta_2 - \mu mg\cos\theta_1 - \mu mg\cos\theta_2 = 2Ma$ [1 Mark]

$$g(\sin\theta_1 + \sin\theta_2) - \mu g(\cos\theta_1 + \cos\theta_2) = 2a$$

$$g\left[2\sin\left(\frac{\theta_1}{2}+\frac{\theta_2}{2}\right)\cos\left(\frac{\theta_1}{2}-\frac{\theta_2}{2}\right)\right] - \mu g\left[2\cos\left(\frac{\theta_1}{2}-\frac{\theta_2}{2}\right)\cos\left(\frac{\theta_1}{2}+\frac{\theta_2}{2}\right)\right]$$

$$= 2a$$

[0.5 Mark for sin(x)+sin(y)=2sin(x/2+y/2)cos(x/2-y/2)]

[0.5 Mark for cos(x)+cos(y)=2cos(x/2-y/2)cos(x/2+y/2)]

$a \geq 0$

[1 Mark]

$$\Rightarrow \quad 2\sin\left(\frac{\theta_1}{2}+\frac{\theta_2}{2}\right)\cos\left(\frac{\theta_1}{2}-\frac{\theta_2}{2}\right) - 2\mu\cos\left(\frac{\theta_1}{2}-\frac{\theta_2}{2}\right)\cos\left(\frac{\theta_1}{2}+\frac{\theta_2}{2}\right) > 0$$

$$\sin\left(\frac{\theta_1}{2}+\frac{\theta_2}{2}\right) > \mu\cos\left(\frac{\theta_1}{2}+\frac{\theta_2}{2}\right)$$

$$\tan\left(\frac{\theta_1}{2}+\frac{\theta_2}{2}\right) > \mu$$

$$\theta_1 + \theta_2 > 2\tan^{-1}(\mu)$$ [1 Mark]

1.

(b) Lower Block:

$$mg\sin\theta_1 - \mu mg\cos\theta_1 - k\Delta l = ma$$ [1 Mark]

Upper Block:

$$mg\sin\theta_2 + k\Delta l - \mu mg\cos\theta_2 = ma$$ [1 Mark]

Same Acceleration,

[0.5 Mark]

$$\Rightarrow mg\sin\theta_1 - \mu mg\cos\theta_1 - k\Delta l = mg\sin\theta_2 + k\Delta l - \mu mg\cos\theta_2$$

$$2k\Delta l = mg\sin\theta_1 - \mu mg\cos\theta_1 - mg\sin\theta_2 + \mu mg\cos\theta_2$$

$$= mg(\sin\theta_1 - \sin\theta_2) - \mu mg(\cos\theta_1 - \cos\theta_2)$$

[0.5 Mark for sin(x)-sin(y)=2sin(x/2-y/2)cos(x/2+y/2)]

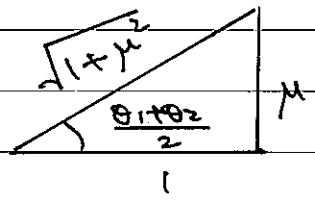
$$= 2mg\sin\left(\frac{\theta_1}{2} - \frac{\theta_2}{2}\right)\cos\left(\frac{\theta_1}{2} + \frac{\theta_2}{2}\right)$$

$$+ 2\mu mg\sin\left(\frac{\theta_1}{2} - \frac{\theta_2}{2}\right)\sin\left(\frac{\theta_1}{2} + \frac{\theta_2}{2}\right)$$

[0.5 Mark for cos(x)-cos(y)=2sin(x/2-y/2)sin(x/2+y/2)]

$$\cancel{k\Delta l = mg\sin\left(\frac{\theta_1}{2} + \frac{\theta_2}{2}\right)}$$

$$k\Delta l = mg\sin\left(\frac{\theta_1}{2} - \frac{\theta_2}{2}\right)\left[\cos\left(\frac{\theta_1}{2} + \frac{\theta_2}{2}\right) + \mu\sin\left(\frac{\theta_1}{2} + \frac{\theta_2}{2}\right)\right]$$



$$= mg\left(\frac{1}{2}\right)\left[\frac{1}{\sqrt{1+\mu^2}} + \mu\left(\frac{\mu}{\sqrt{1+\mu^2}}\right)\right]$$

$$= \frac{mg}{2}\left[\frac{1+\mu^2}{\sqrt{1+\mu^2}}\right]$$

[0.5 Mark for sin(x/2-y/2)= 1/2]

[0.5 Mark for cos(x/2+y/2)= f(u)]

[0.5 Mark for sin(x/2+y/2)= f(u)]

f(u) = function of u

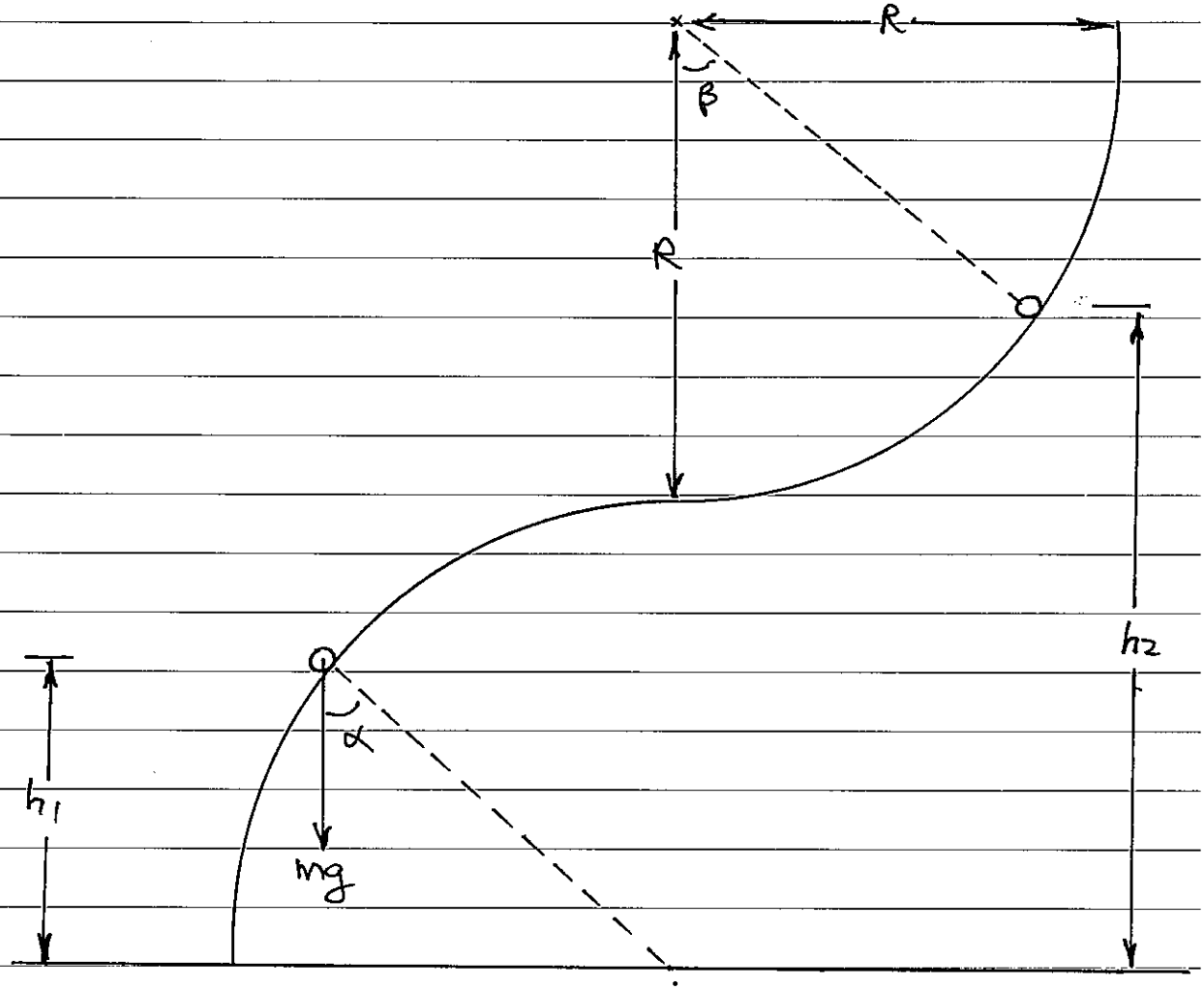
From (a),

$$\tan\left(\frac{\theta_1}{2} + \frac{\theta_2}{2}\right) = \mu$$

$$= \frac{mg\sqrt{1+\mu^2}}{2}$$

[1 Mark for Final Answer]

2.



2.

(a) Reaction force becomes zero when the object detaches.

$$mg\cos\alpha = \frac{mv^2}{R}$$ [1 Mark]

$$g\left(\frac{h_1}{R}\right) = \frac{v^2}{R} \qquad \cos\alpha = \frac{h_1}{R}$$ [0.5 Mark]

$$v^2 = gh_1 \quad \text{——} \quad (1)$$ [1 Mark]

Energy Conservation :

$$mg\left[R(1-\cos\alpha) + h_2 - R\right] = \frac{1}{2}mv^2$$ [2 Marks]

$$v^2 = 2g\left[R - R\cos\alpha + h_2 - R\right]$$

$$= 2g\left(h_2 - R\frac{h_1}{R}\right)$$

$$= 2g(h_2 - h_1) \quad \text{——} \quad (2)$$ [1 Mark]

(1) and (2) $\Rightarrow$ $gh_1 = 2g(h_2 - h_1)$ [0.5 Mark]

$$\frac{h_1}{h_2} = \frac{2}{3}$$ [0.5 Mark]

2.

(b) $S = \underset{\text{Concave}}{S_1} + \underset{\text{Convex}}{S_2}$

$\cos\beta = \dfrac{2R - h_2}{R}$

[0.5 Mark for beta]

$S_1 = R\beta = R\cos^{-1}\left(\dfrac{4R - 3h_1}{2R}\right)$

$= \dfrac{2R - \frac{3}{2}h_1}{R}$

$S_2 = R\alpha = R\cos^{-1}\left(\dfrac{h_1}{R}\right)$ [1 Mark]

$= \dfrac{4R - 3h_1}{2R}$

[0.5 Mark for alpha]

$$S = R\left[\cos^{-1}\left(\frac{4R - 3h_1}{2R}\right) + \cos^{-1}\left(\frac{h_1}{R}\right)\right]$$ [0.5 Mark]

$$= R\left[\cos^{-1}\left(\frac{4R - 3\left(\frac{2}{3}\right)h_2}{2R}\right) + \cos^{-1}\left(\frac{2h_2}{3R}\right)\right]$$

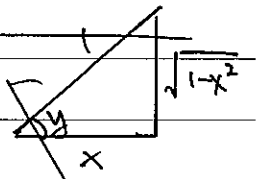
$$= R\left[\cos^{-1}\left(\frac{2R - h_2}{R}\right) + \cos^{-1}\left(\frac{2h_2}{3R}\right)\right]$$ [1 Mark for final answer]

(c) Remarks: $\dfrac{d}{dx}\cos^{-1}(x) = \dfrac{-1}{\sqrt{1-x^2}}$

$\cos y = x$

$-\sin y \, dy = dx$

$\dfrac{dy}{dx} = -\dfrac{1}{\sin y} = \dfrac{-1}{\sqrt{1-x^2}}$



$$\frac{dS}{dh_2} = R\left[-\frac{1}{\sqrt{1-\left(\frac{2R-h_2}{R}\right)^2}} - \frac{1}{\sqrt{1-\left(\frac{2h_2}{3R}\right)^2}}\right] = 0$$

$$\frac{-1}{\sqrt{1-\left(\frac{2R-h_2}{R}\right)^2}} = \frac{1}{\sqrt{1-\left(\frac{2h_2}{3R}\right)^2}}$$

$$1 - \left(\frac{2R - h_2}{R}\right)^2 = 1 - \left(\frac{2h_2}{3R}\right)^2$$

$$6R - 3h_2 = 2h_2$$

$$h_2 = \frac{6}{5}R$$

$$S = R\left[\cos^{-1}\left(\frac{2R - \frac{6}{5}R}{R}\right) + \cos^{-1}\left(\frac{2 \times \frac{6}{5}R}{3R}\right)\right]$$

$$= R\left[\cos^{-1}\left(\frac{4}{5}\right) + \cos^{-1}\left(\frac{4}{5}\right)\right] = 1.287R$$

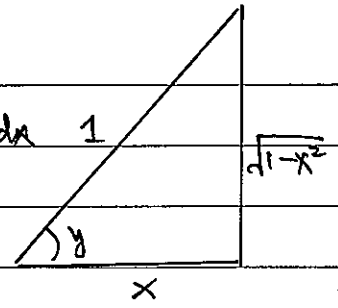
2.

(c) Remarks : $\frac{d}{dx}\cos^{-1}(x) = \frac{-1}{\sqrt{1-x^2}}$

$\cos y = x$

$-\sin y \, dy = dx$

$\frac{dy}{dx} = \frac{-1}{\sin y}$



$= \frac{-1}{\sqrt{1-x^2}}$

$$\frac{dS}{dh_2} = R\left[\frac{\frac{1}{R}}{\sqrt{1-\left(\frac{2R-h_2}{R}\right)^2}} - \frac{\frac{2}{3R}}{\sqrt{1-\left(\frac{2h_2}{3R}\right)^2}}\right] = 0$$ [2 Marks]

$$\frac{1}{\sqrt{1-\left(\frac{2R-h_2}{R}\right)^2}} = \frac{2}{3}\,\frac{1}{\sqrt{1-\left(\frac{2h_2}{3R}\right)^2}}$$

$$1 - \left(\frac{2R-h_2}{R}\right)^2 = \frac{9}{4}\left[1-\left(\frac{2h_2}{3R}\right)^2\right]$$

$$-\frac{4R^2+h_2^2-4Rh_2}{R^2} = \frac{5}{4} - \frac{9}{4}\,\frac{4h_2^2}{9R^2}$$

$$\frac{4h_2}{R} = \frac{21}{4}$$

$$h_2 = \frac{21}{16}R$$ [1 Mark]

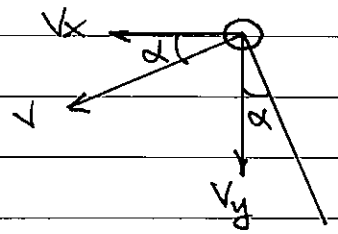
$$S = R\left[\cos^{-1}\left(\frac{2R-\frac{21}{16}R}{R}\right) + \cos^{-1}\left(\frac{2\times\frac{21}{16}R}{3R}\right)\right]$$

$$= R\left[\cos^{-1}\left(\frac{11}{16}\right) + \cos^{-1}\left(\frac{7}{8}\right)\right]$$

$$= 1.318\,R$$ [1 Mark]

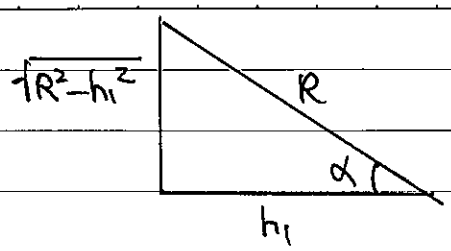
2.

(d)



$V_x = V\cos\alpha$

$V_y = V\sin\alpha$



$\sin\alpha = \dfrac{\sqrt{R^2-h_1^2}}{h_1}$

$S_G = ut + \frac{1}{2}gt^2$

$h_1 = V_y t + \frac{1}{2}gt^2$

$h_1 = V\sin\alpha\, t + \frac{1}{2}gt^2$

1/2 mv2 = mg(h2-h1) [1 Mark]

v = sqrt (gR)

$= \sqrt{gh_1}\,\dfrac{\sqrt{R^2-h_1^2}}{h_1}\,t + \frac{1}{2}gt^2$

$= \sqrt{\dfrac{g(R^2-h_1^2)}{h_1}}\,t + \frac{1}{2}gt^2$ ——— (3)

Detach at the common tangent $\Rightarrow$ $h_1 = R$

h2 = 3/2 R [0.5 Mark]

(3) $\Rightarrow$ $h_1 = \frac{1}{2}gt^2$ [0.5 Mark]

$t = \sqrt{\dfrac{2h_1}{g}}$ [0.5 Mark]

$S_G = V_x \times t$

$= V\cos\alpha\sqrt{\dfrac{2h_1}{g}}$

$= \sqrt{gh_1}\left(\dfrac{h_1}{R}\right)\sqrt{\dfrac{2h_1}{g}}$

$= \dfrac{h_1^2\sqrt{2}}{R}$

$= \sqrt{2}R$ [1 Mark]

Alternative Method:

$\begin{cases} V_y = 0 \\ V_x = V \end{cases}$

$h_1 = \frac{1}{2}gt^2$ [0.5 Mark]

$t = \sqrt{\dfrac{2h_1}{g}}$ [0.5 Mark]

$S_G = \sqrt{2}R$ [1 Mark]

3.

(a) $P_1 V_0 = n_1 R T_1$ ——— (1) [0.5 Mark]

$P_2 V_0 = n_2 R T_2$ ——— (2) [0.5 Mark]

$P_2 = P_1 + \frac{mg}{A}$ ——— (3) [1 Mark]

[1 Mark for attempt to solve P1, P2, and Vo]

$\frac{(1)}{(2)} \Rightarrow \frac{P_1}{P_2} = \frac{n_1 T_1}{n_2 T_2}$ ——— (4)

(3) and (4) $\Rightarrow P_2 = \frac{n_1 T_1}{n_2 T_2} P_2 + \frac{mg}{A}$

$$P_2 \left(1 - \frac{n_1 T_1}{n_2 T_2} \right) = \frac{mg}{A}$$

$$P_2 \left(\frac{n_2 T_2 - n_1 T_1}{n_2 T_2} \right) = \frac{mg}{A}$$

$$P_2 = \frac{mg}{A} \left(\frac{n_2 T_2}{n_2 T_2 - n_1 T_1} \right)$$ [1 Mark]

$$P_1 = P_2 - \frac{mg}{A}$$

$$= \frac{mg}{A} \frac{n_2 T_2}{n_2 T_2 - n_1 T_1} - \frac{mg}{A}$$

$$= \frac{mg}{A} \left(\frac{n_1 T_1}{n_2 T_2 - n_1 T_1} \right)$$ [1 Mark]

$$V_0 = \frac{1}{P_1} n_1 R T_1$$

$$= \frac{A}{mg} \frac{n_2 T_2 - n_1 T_1}{n_1 T_1} \cdot n_1 R T_1$$

$$= \frac{AR}{mg} \left(n_2 T_2 - n_1 T_1 \right)$$ [1 Mark]

3.

(b) $$P_1 = \frac{mg}{A}\left(\frac{n_1 T_1}{n_2 T_2 - n_1 T_1}\right)$$ [0.5 mark for correct substitution of numerical values]

$$= \frac{(0.2)(9.8)}{0.01}\left[\frac{(0.05)(183)}{(0.03)(323) - (0.05)(183)}\right]$$

$$= 3321 \text{ Pa}$$ [0.5 Mark]

$$P_2 = \frac{mg}{A}\left(\frac{n_2 T_2}{n_2 T_2 - n_1 T_1}\right)$$ [0.5 mark for correct substitution of numerical values]

$$= \frac{(0.2)(9.8)}{0.01}\left[\frac{(0.03)(323)}{(0.03)(323) - (0.05)(183)}\right]$$

$$= 3517 \text{ Pa}$$ [0.5 Mark]

$$V_0 = \frac{AR}{mg}\left(n_2 T_2 - n_1 T_1\right)$$ [0.5 mark for correct substitution of numerical values]

$$= \frac{(0.01)(8.31)}{(0.2)(9.8)}\left[(0.03)(323) - (0.05)(183)\right]$$

$$= 22.9 \times 10^{-3} \text{ m}^3 \quad (\text{or } 22.9 \text{ dm}^3)$$

[0.5 Mark]

(c) Noble Gas, Degree of freedom = 3 [0.5 Mark]

Air, Degree of freedom = 5 [0.5 Mark]

3.

(d) Energy Conservation :

[2 Marks]

$$\frac{3}{2} n_1 R (T - T_1) + \frac{5}{2} n_2 R (T - T_2) + mc(T - T_0) + mgx = 0$$

[0.5 Mark for correct substitution of numerical values]

$$\frac{3}{2}(0.05)(8.31)(T - 183) + \frac{5}{2}(0.03)(8.31)(T - 323)$$

$$+ (0.2)(200)(T - 373) + (0.2)(9.8)x = 0$$

[1 Mark for reducing the equation to contain variables T and x]

$$41.25\,T - 15235 + 1.96\,x = 0$$

$$T = \frac{15235 - 1.96\,x}{41.25} \qquad (5)$$

Initial and Final States :

$$\frac{P_1 V_0}{T_1} = \frac{P(V_0 + xA)}{T} \qquad (6)$$

[0.5 Mark]

$$\frac{P_2 V_0}{T_2} = \frac{\left(P + \frac{mg}{A}\right)(V_0 - xA)}{T} \qquad (7)$$

[0.5 Mark]

$$(6) \Rightarrow \quad P = \frac{P_1 V_0}{T_1} \frac{T}{V_0 + xA}$$

[1 Mark]

Sub. into (7),

$$\frac{P_2 V_0}{T_2} = \left(\frac{P_1 V_0}{T_1} \frac{T}{V_0 + xA} + \frac{mg}{A}\right)\left(\frac{V_0 - xA}{T}\right)$$

[0.5 Mark]

$$\frac{P_2 V_0}{T_2} = \left(\frac{P_1 V_0}{T_1} \frac{T}{V_0 + xA}\right)\left(\frac{V_0 - xA}{T}\right) + \frac{mg}{A}\left(\frac{V_0 - xA}{T}\right)$$

$$\frac{P_2 V_0}{T_2}(V_0 + xA) = \left(\frac{P_1 V_0}{T_1}\right)(V_0 - xA) + \frac{mg}{AT}(V_0 - xA)(V_0 + xA)$$

$$mgA\,x^2 + \left(\frac{P_2}{T_2} + \frac{P_1}{T_1}\right) V_0 A T x + V_0^2 T \left(\frac{P_2}{T_2} - \frac{P_1}{T_1}\right) - \frac{mg V_0^2}{A} = 0$$

3.

(d) (cont.)

$$(0.2)(9.8)(0.01)x^2 + \left(\frac{3517}{323} + \frac{3321}{183}\right)(22.9\times10^{-3})(0.01)Tx$$

$$+ (22.9\times10^{-3})^2 T\left(\frac{3517}{323} - \frac{3321}{183}\right)$$

$$- \frac{(0.2)(9.8)}{6.01}(22.9\times10^{-3})^2 = 0$$

$$0.0196x^2 + 6.649\times10^{-3}Tx - 3.807\times10^{-3}T - 0.1028 = 0$$

[1 Mark for reducing the equation to contain variables T and x]

$$x^2 + 0.3392Tx - 0.1942T - 5.245 = 0 \quad \text{—— (8)}$$

Sub. (5) into (8),

[0.5 Mark]

$$x^2 + 0.3392\left(\frac{15235 - 1.96x}{41.25}\right)x - 0.1942\left(\frac{15235 - 1.96x}{41.25}\right)$$

$$- 5.245 = 0$$

$$0.9839x^2 + 125.3x - 76.97 = 0$$ [0.5 Mark]

$$x = -128\text{ m} \quad \text{(rejected)}$$ [0.5 Mark]

or $x = 0.6114$ m [1 Mark]

$T = 369$ K [1 Mark]

4.

(a)

$$V_p^2 = \frac{g}{k} \tanh(kh)$$

$$\left(\frac{w}{k}\right)^2 = \frac{g}{k} \tanh(kh)$$ [1 Marks for Vp = w / k]

$$w^2 = gk \tanh(kh) \quad \text{——} \quad (1)$$

$$2w \frac{dw}{dk} = g \tanh(kh) + \frac{gkh}{\cosh^2(kh)}$$ [1 mark for differentiation (dw/dk)]

$$\frac{V_g}{V_p} = \left(\frac{dw}{dk}\right)\left(\frac{k}{w}\right)$$ [1 Marks for Vg = dw / dk]

$$= \frac{1}{2w}\left[g \tanh(kh) + \frac{gkh}{\cosh^2(kh)}\right]\left(\frac{k}{w}\right)$$

$$= \frac{1}{2w^2}\left[gk \tanh(kh) + \frac{gk^2h}{\cosh^2(kh)}\right]$$

$$= \frac{1}{2gk \tanh(kh)}\left[gk \tanh(kh) + \frac{gk^2h}{\cosh^2(kh)}\right]$$

From (1)

$$= \frac{1}{2} + \frac{kh}{2\tanh(kh)\cosh^2(kh)}$$ [0.5 Mark]

$$= \frac{1}{2} + \frac{kh}{2\sinh(kh)\cosh(kh)}$$ $\sinh 2x = 2\sinh x \cosh x$

$$= \frac{1}{2} + \frac{kh}{\sinh(2kh)}$$ [1 Mark]

4.

(b)

L' Hospital Rule :

$$\lim \frac{kh}{\sinh(2kh)} = \lim \frac{1}{2\cosh(kh)}$$ [1 Mark]

$kh \to 0$, $\cosh(kh) = 1$

$kh \to \infty$, $\cosh(kh) \to \infty$ [0.5 Mark]

$$\Rightarrow \quad 1 \leq \cosh(kh) \leq \infty$$

$$\Rightarrow \quad 0 \leq \frac{kh}{\sinh(2kh)} \leq \frac{1}{2}$$ [1 Mark]

$$\Rightarrow \quad \frac{1}{2} \leq \frac{V_g}{V_p} \leq 1$$

[0.5 Mark] [0.5 Mark]

5.

(a) $$2(m + 2m + 3m + \cdots\cdots + km) = \frac{M}{2}$$ [1 Marks]

$$2m(1 + 2 + 3 + \cdots\cdots + k) = \frac{M}{2}$$

$$2m\left[\frac{(k+1)k}{2}\right] = \frac{M}{2}$$ [0.5 Mark for Sum of Series]

$$k^2 + k = \frac{M}{2m} \quad \text{———} \quad (1)$$

$$k = \frac{\sqrt{2M/m + 1} - 1}{2}$$ [0.5 Mark]

(b) $$n = \frac{M}{2m}$$ [0.5 Mark]

(c) $$m(2v) + 2m(2v) + 3m(2v) + \cdots + km(2v)$$

$$- mv - 2mv - 3mv - \cdots\cdots - kmv = M\bar{V}$$

$$2mv(1 + 2 + \cdots\cdots + k) - mv(1 + 2 + \cdots + k) = M\bar{V}$$

$$(k+1)\frac{k}{2}(2mv - mv) = M\bar{V}$$

$$\frac{(k+1)k\,mv}{2} = M\bar{V}$$

$$\bar{V} = \frac{k(k+1)mv}{2M}$$

$$(1) \Rightarrow \quad = \frac{M}{2m} \cdot \frac{mv}{2M}$$

$$= \frac{v}{4}$$

(c) Conservation of momentum:

$$(nm+M)V_0 = \frac{n}{2}m(-v) + \frac{n}{2}m(2v) + MV' \quad , V' : \text{spacecraft final velocity} \qquad \boxed{1}$$

$$\Rightarrow \quad \frac{3M}{2}V_0 = \frac{Mv}{4} + MV' \qquad (\because n = \frac{M}{2m}) \qquad \boxed{0.5}$$

$$\Rightarrow \quad \frac{3}{2}V_0 = \frac{v}{4} + V' \quad — ① \qquad \boxed{0.5}$$

Conservation of energy:

$$\gamma\left[\frac{1}{2}(nm+M)V_0^2\right] = \frac{1}{2}\left(\frac{n}{2}m\right)(-v)^2 + \frac{1}{2}\left(\frac{n}{2}m\right)(2v)^2 + \frac{1}{2}MV'^2 \qquad \boxed{1}$$

$$\Rightarrow \quad \frac{3}{2}\gamma V_0^2 = \frac{5}{4}v^2 + V'^2 \quad — ② \qquad \boxed{0.5}$$

$$\gamma ① - \frac{3}{2} ② \Rightarrow \gamma\left(\frac{v}{4}+V'\right)^2 - \frac{15}{8}v^2 - \frac{3}{2}V'^2 = 0 \qquad \boxed{0.5}$$

$$\Rightarrow \left(\gamma - \frac{3}{2}\right)V'^2 + \frac{\gamma}{2}vV' + \left(\frac{\gamma}{16} - \frac{15}{8}\right)v^2 = 0 \quad — (*) \qquad \boxed{0.5}$$

$$\Rightarrow \quad V' = \frac{-\frac{\gamma}{2} \pm \sqrt{\frac{\gamma^2}{4} - 4\left(\gamma - \frac{3}{2}\right)\left(\frac{\gamma}{16} - \frac{15}{8}\right)}}{2\left(\gamma - \frac{3}{2}\right)}v \qquad \boxed{0.5}$$

$$= \frac{-2\gamma \pm 3\sqrt{2(7\gamma-10)}}{4(2\gamma-3)}v \quad , \gamma \neq \frac{3}{2} \qquad \boxed{0.5}$$

$$\gamma = \frac{3}{2}, \ (*) \Rightarrow \frac{1}{2}\left(\frac{3}{2}\right)vV' + \left(\frac{1}{16}\left(\frac{3}{2}\right) - \frac{15}{8}\right)v^2 = 0$$

$$\Rightarrow \quad V' = \frac{19}{8}v \qquad \boxed{0.5}$$

~~V' should be finite an~~

launching is possible for $2(7\gamma-10) \geq 0 \Rightarrow \gamma \geq \frac{10}{7}$ $\boxed{0.5}$ $\boxed{0.5}$

V' should be finite and continuous for all finite $\gamma \geq \frac{10}{7}$,

however, $\lim_{\gamma \to \frac{3}{2}+} \frac{-2\gamma - 3\sqrt{2(7\gamma-10)}}{4(2\gamma-3)}v \to \frac{-6}{0}v \to -\infty$ is undefined (not physical) $\boxed{0.5}$

$$\lim_{\gamma \to \frac{3}{2}} \frac{-2\gamma + 3\sqrt{2(7\gamma-10)}}{4(2\gamma-3)}v = \lim_{\gamma \to \frac{3}{2}} \frac{-2 + \frac{3\sqrt{2}\cdot 7}{2\sqrt{7\gamma-10}}}{8}v \quad \text{(L'Hopital's Rule)} \qquad \boxed{0.5}$$

$$= \frac{19}{8}v$$

$$= V'(\gamma = \tfrac{3}{2}) \quad \text{is continuous} \qquad \boxed{0.5}$$

$$\therefore \quad V' = \begin{cases} \dfrac{-2\gamma + 3\sqrt{2(7\gamma-10)}}{4(2\gamma-3)} & , \gamma \geq \frac{10}{7}, \gamma \neq \frac{3}{2} \\ \frac{19}{8}v & , \gamma = \frac{3}{2} \end{cases}$$

(fc)

5.

Put into ① $\Rightarrow$ $V_0 = \frac{2}{3}\left[\frac{1}{4} + \frac{-2\gamma + 3\sqrt{2(7\gamma-10)}}{4(2\gamma-3)}\right]v$

$$= \frac{-1+\sqrt{2(7\gamma-10)}}{2(2\gamma-3)}v \quad , \gamma \neq \frac{3}{2}$$

$$V_0 = \frac{7}{4}v \quad , \gamma = \frac{3}{2}$$

$\boxed{0.5} + \boxed{0.5}$

$$\therefore \quad \Delta V = V' - V_0 = \left[\frac{-2\gamma + 3\sqrt{2(7\gamma-10)}}{4(2\gamma-3)} - \frac{-1+\sqrt{2(7\gamma-10)}}{2(2\gamma-3)}\right]v$$

$$= \frac{2(1-\gamma)+\sqrt{2(7\gamma-10)}}{4(2\gamma-3)}v \quad , \gamma \neq \frac{3}{2}$$

$$\Delta V = \frac{5}{8}v \quad , \gamma = \frac{3}{2}$$

$\boxed{0.5} + \boxed{0.5}$

$$V' < 0 \Rightarrow \frac{-2\gamma + 3\sqrt{2(7\gamma-10)}}{4(2\gamma-3)} < 0$$

$$\Rightarrow \begin{cases} -2\gamma + 3\sqrt{2(7\gamma-10)} < 0 & , \gamma > \frac{3}{2} \\ -2\gamma + 3\sqrt{2(7\gamma-10)} > 0 & , \frac{10}{7} \leq \gamma < \frac{3}{2} \end{cases}$$

$\boxed{0.5}$

$$\Rightarrow \begin{cases} \cancel{18}^{9}(7\gamma-10) < \cancel{4}^{2}\gamma^2 & , \gamma > \frac{3}{2} \\ 9(7\gamma-10) > 2\gamma^2 & , \frac{10}{7} \leq \gamma < \frac{3}{2} \end{cases}$$

$$\Rightarrow \begin{cases} 2\gamma^2 - 63\gamma + 90 > 0 & , \gamma > \frac{3}{2} \\ 2\gamma^2 - 63\gamma + 90 < 0 & , \frac{10}{7} \leq \gamma < \frac{3}{2} \end{cases}$$

$\boxed{0.5}$

$$\Rightarrow \begin{cases} (\gamma-30)(2\gamma-3) > 0 & , \gamma > \frac{3}{2} \\ (\gamma-30)(2\gamma-3) < 0 & , \frac{10}{7} \leq \gamma < \frac{3}{2} \end{cases}$$

$$\Rightarrow \left[\left(\gamma < \frac{3}{2} \text{ or } \gamma > 30\right) \text{ and } \gamma > \frac{3}{2}\right]$$

$$\text{or } \left(\frac{3}{2} < \gamma < 30 \text{ and } \frac{10}{7} \leq \gamma < \frac{3}{2}\right)$$

$\boxed{1}$

$$\Rightarrow \gamma > 30$$

$\boxed{1}$

13.5

Total - 16

6.

(a) $P = \frac{N m V_{rms}^2}{3V}$ and $PV = NkT$ [1 Mark]

$\Rightarrow P = \frac{PV}{kT} \frac{m V_{rms}^2}{3V}$

$V_{rms}^2 = \frac{3kT}{m}$ [0.5 Mark]

(b) Mean molar mass of air $= 28 \times 0.8 + 32 \times 0.2$

$= 28.8\,g$ [0.5 Mark]

$V_{rms,air}^2 = \frac{3(1.38 \times 10^{-23})(300)}{0.0288 / 6.02 \times 10^{23}}$ [0.5 Mark for mean mass of single air molecule]

$V_{rms,air} = 509.5\ ms^{-1}$ [0.5 Mark]

(c) $V_{escape} = \sqrt{\frac{2GM}{R}}$ [1 Mark]

$V_{rms} < V_{escape}$

$\sqrt{\frac{3kT}{m}} < \sqrt{\frac{2GM}{R}}$ [1 Mark]

$T < \frac{2GMm}{3kR}$ [0.5 Mark for correct substitution of numerical values]

$= \frac{(2)(6.67 \times 10^{-11})(0.2)(5.97 \times 10^{24})}{(3)(1.38 \times 10^{-23})(25 \times 6370{,}000)} \times \frac{0.0288}{6.02 \times 10^{23}}$

$= 1156\ K$ [1 Mark]

7.

(a) Phase Velocity $V_\phi = \lambda \nu$ [1 Mark]

$$\lambda = \frac{V_\phi}{\nu} = \frac{500\ ms^{-1}}{1000\ Hz} = 0.5\ m$$ [0.5 Mark]

(b) $$\psi(x,t) = A \sin\left[2\pi\left(\frac{x}{\lambda} - \nu t\right)\right]$$

$$\Delta\phi = \phi_1 - \phi_2$$

$$= 2\pi\left(\frac{x_1}{\lambda} - \nu t\right) - 2\pi\left(\frac{x_2}{\lambda} - \nu t\right)$$

$$2\pi\left(\frac{x_1}{\lambda} - \frac{x_2}{\lambda}\right) = \frac{\pi}{9}$$ [1 Mark]

$$x_1 - x_2 = \frac{0.5\ m}{18}$$

$$= 0.02778\ m$$ [1 Mark]

(c) $$\Delta\phi = \phi_1 - \phi_2$$

$$= 2\pi\left(\frac{x}{\lambda} - \nu t_1\right) - 2\pi\left(\frac{x}{\lambda} - \nu t_2\right)$$ [1 Mark]

$$= 2\pi\nu\,(t_2 - t_1)$$

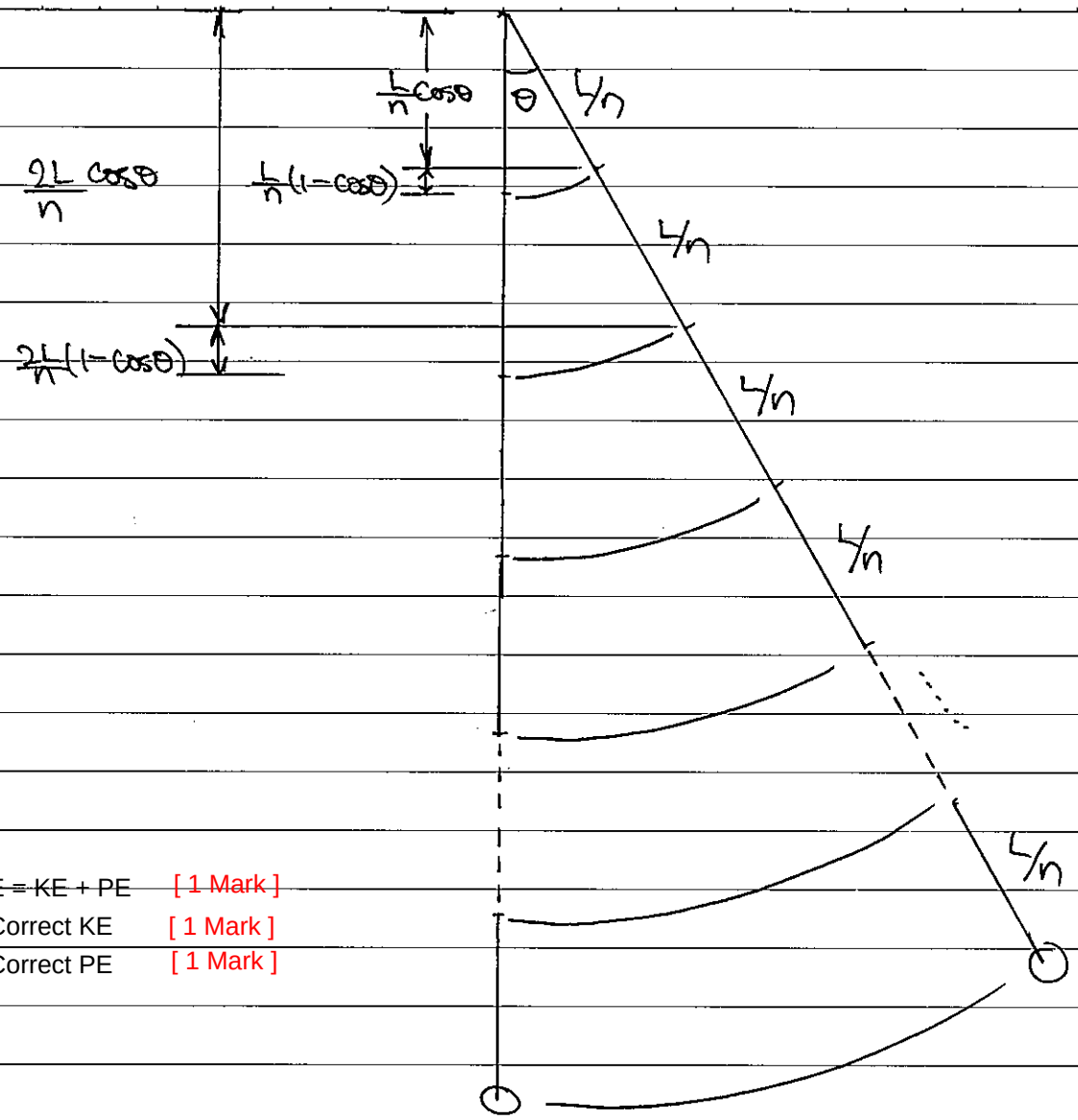
$$= 2\pi(1000)(10\ ms)$$

20 pi rad

$= \cancel{xxxxxx}$ rad. [1 Mark]

8.

(a)



E = KE + PE [1 Mark]

Correct KE [1 Mark]

Correct PE [1 Mark]

$$E = \frac{1}{2}m\left(\frac{L}{n}\frac{d\theta}{dt}\right)^2 + \frac{1}{2}m\left(\frac{2L}{n}\frac{d\theta}{dt}\right)^2 + \frac{1}{2}m\left(\frac{3L}{n}\frac{d\theta}{dt}\right)^2 + \cdots\cdots$$

$$+ mg\frac{L}{n}(1-\cos\theta) + mg\left(\frac{2L}{n}\right)(1-\cos\theta) + mg\left(\frac{3L}{n}\right)(1-\cos\theta) + \cdots\cdots$$

[1 Mark for correct differentiation]

$$\frac{dE}{dt} = \frac{1}{2}m\left(\frac{L}{n}\right)^2 2\left(\frac{d\theta}{dt}\right)\frac{d^2\theta}{dt^2} + \frac{1}{2}m\left(\frac{2L}{n}\right)^2(2)\left(\frac{d\theta}{dt}\right)\left(\frac{d^2\theta}{dt^2}\right)$$

$$+ \frac{1}{2}m\left(\frac{3L}{n}\right)^2(2)\left(\frac{d\theta}{dt}\right)\left(\frac{d^2\theta}{dt^2}\right) + \cdots\cdots$$

$$+ mg\left(\frac{L}{n}\right)\sin\theta\frac{d\theta}{dt} + mg\left(\frac{2L}{n}\right)\sin\theta\frac{d\theta}{dt} + mg\left(\frac{3L}{n}\right)\sin\theta\frac{d\theta}{dt} + \cdots\cdots$$

8.

(a) $$\left(\frac{L}{n}\right)^2 \frac{d^2\theta}{dt^2} + \left(\frac{2L}{n}\right)^2 \frac{d^2\theta}{dt^2} + \left(\frac{3L}{n}\right)^2 \frac{d^2\theta}{dt^2} + \cdots\cdots$$

$$+ \frac{gL}{n}\sin\theta + g\left(\frac{2L}{n}\right)\sin\theta + g\left(\frac{3L}{n}\right)\sin\theta + \cdots\cdots = 0$$

[1 Mark for dE/dt = 0]

[0.5 Mark for sin x ~ x]

$$\frac{L}{n^2}\frac{d^2\theta}{dt^2}\left(1^2 + 2^2 + 3^2 + \cdots + n^2\right)$$

$$+ \frac{g}{n}\theta\left(1 + 2 + 3 + \cdots + n\right) = 0$$

[0.5 Mark for Series] [0.5 Mark for Series]

$$\frac{L}{n^2}\frac{d^2\theta}{dt^2}\frac{n(n+1)(2n+1)}{6} + \frac{g\theta}{n}\frac{n(n+1)}{2} = 0$$

$$\frac{L}{n}\frac{d^2\theta}{dt^2}\frac{(2n+1)}{3} = -g\theta$$

$$\frac{d^2\theta}{dt^2} = -\frac{3ng}{(2n+1)L}\theta$$ [1 Mark]

$$\Rightarrow \omega = \sqrt{\frac{3ng}{(2n+1)L}}$$ [1 Mark]

$$T = 2\pi\sqrt{\frac{(2n+1)L}{3ng}}$$ [1 Mark]

8.

(b) $$T = 2\pi\sqrt{\frac{(2n+1)L}{3ng}}$$

[1 Mark]

$$= 2\pi\sqrt{\frac{(2+1/n)L}{3g}}$$

$n \to \infty$, [0.5 Mark]

$$T = 2\pi\sqrt{\frac{2L}{3g}}$$ [1 Mark]

Alternative Method:

$$\lim_{n\to\infty} \frac{(2n+1)L}{3ng}$$

$$= \lim_{n\to\infty} \frac{2L}{3g}$$

$$= \frac{2L}{3g}$$

(c) Rod Pendulum, or physical pendulum,

[1 Mark]

$$T_{Rod} = 2\pi\sqrt{\frac{I}{Mg\,L_{cm}}}$$ [1 Mark] where $I = \frac{1}{3}ML^2$

L_{cm} ← distance from center of mass

$$= 2\pi\sqrt{\frac{\frac{1}{3}ML^2}{Mg(L/2)}}$$ [1 Mark]

$$= 2\pi\sqrt{\frac{2L}{3g}}$$ [1 Mark]