Rules and Regulations

1. Answer all the questions in the answer book provided.
2. Full mark of this written selection test is 100 Marks.
3. The selection test is a 3-hour written test.

Useful Constants

Unless specified otherwise, the following symbols and constants will be used in this exam paper.

Astronomical Unit, 1 AU = 1.496×10^8 km
Earth-Moon Distance, d = 384,400 km
Mass of the Sun, $M_S = 1.99\times10^{30}$ kg
Mass of the Earth, $M_e = 5.97\times10^{24}$ kg
Mass of the Moon, $M_m = 7.35\times10^{22}$ kg
Radius of the Sun, R_S = 696300 km
Radius of the Earth, R_E = 6370 km
Radius of the Moon, R_M = 1738 km
Gravitational Constant $G = 6.67\times10^{-11}$ m^3 kg^{-1} s^{-2}
Acceleration due to Gravity, g = 9.8 ms^{-2}

Air density at the sea level = 1.2 kg m^{-3}
Gas Constant = 8.31 J/(mol·K)
Velocity of Light in Vacuum, $c = 3\times10^8$ ms^{-1}
Specific Heat of Water, C_W = 4200 J/(kg·K)
Planck Constant, $h = 6.63\times10^{-34}$ Js
Charge of Electron, $e = 1.6\times10^{-19}$ C
Mass of Electron, $m_e = 9.1\times10^{-31}$ kg
Mass of Neutron, $m_n = 1.68\times10^{-27}$ kg
Coulomb Constant, $k_e = 8.988\times10^9$ N m^2/C^2

Trigonometric Identities:

$$\sin(x+y)=\sin(x)\cos(y)+\cos(x)\sin(y)$$

$$\cos(x+y)=\cos(x)\cos(y)-\sin(x)\sin(y)$$

$$\sin(2x)=2\sin(x)\cos(x)$$

$$\cos(2x)=\cos^2(x)-\sin^2(x)$$

$$\sin(x)\cos(y)=\frac{1}{2}[\sin(x+y)+\sin(x-y)]$$

$$\cos(x)\cos(y)=\frac{1}{2}[\cos(x+y)+\cos(x-y)]$$

$$\sin(x)\sin(y)=\frac{1}{2}[\cos(x-y)-\cos(x+y)]$$

Tayler Series:

$$\sin(x)\approx x-\frac{x^3}{6}+\frac{x^5}{120}-\cdots$$

$$\cos(x)\approx 1-\frac{x^2}{2}+\frac{x^4}{24}-\cdots$$

$$\tan(x)\approx x+\frac{x^3}{3}+\frac{2x^5}{15}+\cdots$$

Series Summation:

$$\sum_{k=1}^{m}k=\frac{m(m+1)}{2}$$

$$\sum_{k=1}^{m}k^2=\frac{m(m+1)(2m+1)}{6}$$

$$\sum_{k=1}^{m}k^3=\left[\frac{m(m+1)}{2}\right]^2$$

Hyperbolic functions:

$$\frac{d}{dx}(\sinh x) = \cosh x$$

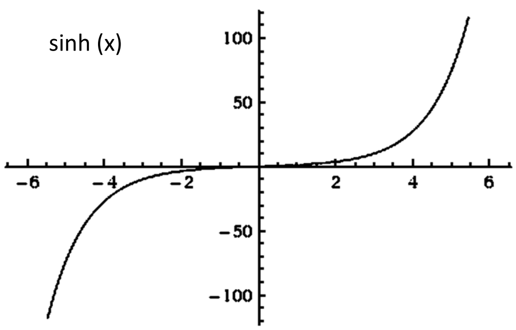
$$\frac{d}{dx}(\cosh x) = \sinh x$$

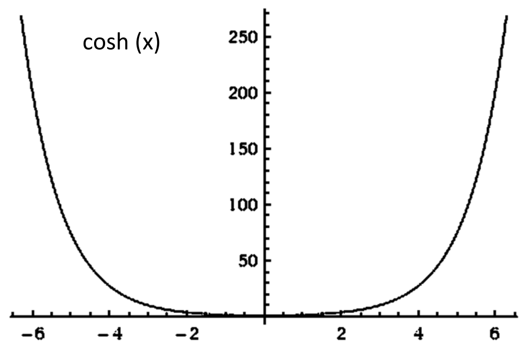
$$\frac{d}{dx}(\tanh x) = \frac{1}{\cosh^2 x}$$

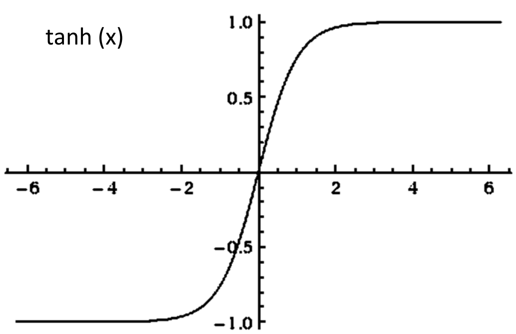
$$\sinh(x+y) = \sinh(x)\cosh(y) + \cosh(x)\sinh(y)$$

$$\cosh(x+y) = \cosh(x)\cosh(y) + \sinh(x)\sinh(y)$$

Sketch of hyperbolic functions







1. [17 Marks] The following figure (Fig. Q1) illustrates an optical phenomenon of visible light diffraction. This optical configuration can be used as a spectrometer. A collimated broadband light beam (wavelengths 400 – 760 nm) is directed to pass through a transmission grating (1000 lines/mm), which is used to separate the wavelengths of light spatially, and the diffracted light is displaced on the screen. The figure only illustrates the +1st order diffracted light.

(a) Where is the +1st order diffracted visible light on the screen, i) from x_0 to x_1, ii) from x_1 to x_2, or iii) from x_0 to x_2?

(b) Where do the red and blue colors appear on the screen? Give your answers in terms of x_0, x_1, and/or x_2.

(c) Over what range of angles and wavelengths do the diffracted lights appear on the screen?

(d) Determine the maximum order that the diffracted light appears on the screen.

(e) CCD (Charge-Coupled Device) is a device for image sensing. In replace of the screen, a linear CCD (1024 × 16 pixels, active area 3.0 cm × 2 mm) is used to detect the 1st-order diffracted light. Find Z (the distance between the transmission grating and the screen) such that all the 1st-order diffracted light can just be detected by the CCD sensor.

(f) Assume that the CCD pixel size limits the ultimate spectral resolution. Determine the mean wavelength resolution over the visible range.

(g) A HeNe laser (wavelength 632.8 nm) manufacture claims that the laser line width is 0.0005 nm. If this laser is analyzed by this optical configuration (transmission grating and CCD sensor), what will the spectral line width be?

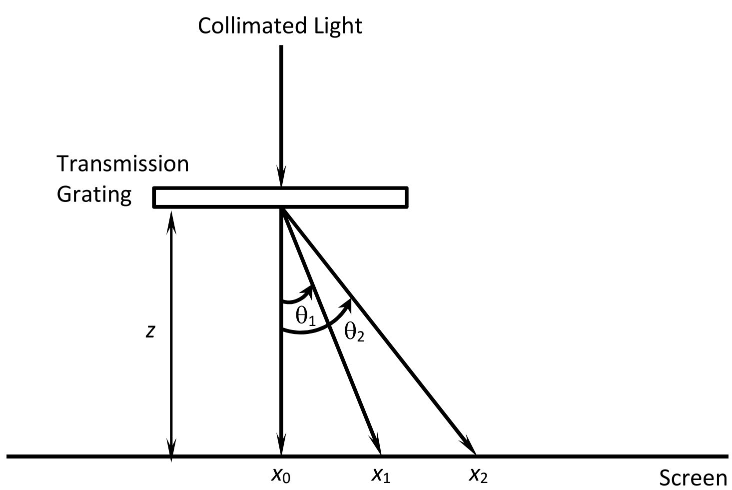


Fig Q1. Light diffraction.

2. [5 Marks] A scientist works in a laboratory where the magnetic field is constantly kept at 2 T. The scientist wears a silver necklace (mass 20 g, total resistance 0.01 Ω) enclosing an area of 0.015 m^2. Because of power failure, the magnetic field decays to 50 μT in 1 ms.

(a) Estimate the maximum current that would be generated in the necklace.

(b) If the specific heat capacity of silver is 240 J/(kg·K), estimate the temperature rises in the necklace. You may assume 100% energy conversion efficiency and heat loss to the surrounding is negligible.

3. [27 Marks] This question investigate quantitatively the importance of power factor in an AC circuit. A household electric fan is operated by the main power (220 Vac, 50 Hz). The electric fan can be equivalent to a resistive (50 Ω) and an inductive (150 mH) load.

(a) Sketch the equivalent circuit to include the power source, resistive and inductance load.

(b) Calculate the total impedance (Z_{Fan}) of the electric fan. Express your answer in complex form.

(c) Find the AC (or root-mean-square) current (I_{AC}) delivered to the electric fan.

(d) True power (P) is the power dissipated by the resistive load. Reactive power (Q) is the power stored, which returns to the source in each AC (Alternating Current) cycle. Apparent power (S) is the total power in an AC circuit, including both the dissipated and stored/returned power. Find P, Q, and S.

(e) Power factor is defined as the ratio between true power and apparent power. Find the power factor and the impedance phase angle.

(f) Power factor is an important aspect in household appliances. Power factor is always equal to or less than 1. Any power factor less than 1 implies that the electric company costs you more money than the necessary power dissipated by household appliances. Power factor can be corrected by adding another load of capacitive reactance, which is in parallel with the power source. Calculate the theoretical capacitance such that the power factor is equal to one.
Hint: Correction of power factor is to bring the circuit total impedance equal to its total resistance.

(g) Suppose you have the following choices of capacitors, ignoring the capacitor breakdown voltage, which one is the best choice to correct the power factor?

1 pF	2 pF	3 pF	5 pF	10 pF	20 pF	30 pF	50 pF	100 pF	200 pF	300 pF	500 pF
1 nF	2 nF	3 nF	5 nF	10 nF	20 nF	30 nF	50 nF	100 nF	200 nF	300 nF	500 nF
1 μF	2 μF	3 μF	5 μF	10 μF	20 μF	30 μF	50 μF	100 μF	200 μF	300 μF	500 μF
1 mF	2 mF	3 mF	5 mF	10 mF	20 mF	30 mF	50 mF	100 mF	200 mF	300 mF	500 mF
1 F	2 F	3 F	5 F	10 F	20 F	30 F	50 F	100 F	200 F	300 F	500 F

(h) With the best choice of the correction capacitor, find the new total impedance (Z_{total}, express your answer in complex form), true power (P'), apparent power (S'), and power factor.

(i) The electric price is HKD1/kWh. By comparison of energy consumptions of the electric fan with and without the correction capacitor, calculate the amount of money saved if the electric fan is operated for 8 hours × 30 days.
Hint: The electric company costs the consumption of apparent energy only.

4. [8.5 Marks] A soap bubble (spherical shell, radius a, wall thickness t) is charged to a potential V. The bubble bursts and becomes a spherical drop.

(a) Assume that the mass of bubble drop remains unchanged after the bubble burst, find the radius of the bubble drop. Express your answer in terms of a and t.

(b) In terms of ε_o (the permittivity of free space), a, t, and V, find the capacitances and the charge of the bubble shell and bubble drop. For the bubble drop, you may assume that all the charge resides on the surface of the bubble drop.

(c) Find the potential of the bubble drop. Express your answer in terms of a and t.

5. [3.5 Marks] Two identical masses (mass m) with equal charges $+Q$ are suspended by 2 identical massless strings (length L) from the same point. The strings hang at θ to the vertical. Find Q in terms of ε_o (the permittivity of free space), L, m, g (acceleration due to gravity), and θ.

6. [11 Marks] A light ray is incident at $\theta_1 = 30°$ on a glass plate (width h, refractive index n), and emerges from the other side of the glass, as shown in Fig. Q6a.

(a) Find the linear displacement (x) of the light ray caused by refraction. Express your answer in terms of h and n only.

(b) What is the minimum refractive index such that the linear displacement is ~~positive~~ possible?

(c) Figure Q6b illustrates a light ray passing through another block. At the interface of the light ray incidence, the refractive index is 1.0. The refractive index inside the block is monotonic linearly increasing until at the other end, the refractive index is 2.0. Calculate the linear displacement (x) of the light ray in terms of h.

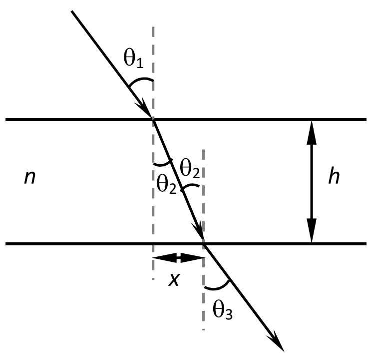


Fig Q6a. Light beam passing through a glass block.

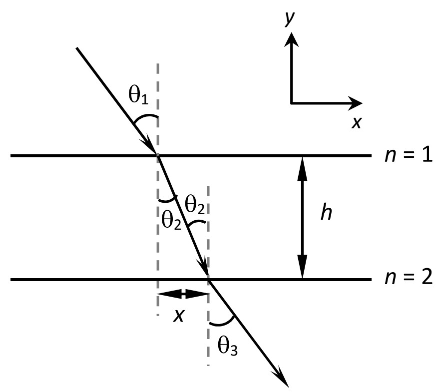


Fig Q6b. Light beam passing through a block where the refractive index is monotonic linearly increasing with the width.

7. [7 Marks] A monochromatic light beam (wavelength λ) is directed to a soap film (refractive index n). The incident angle is 30°. Find the thickness of the film that gives the 1st order interference of the reflected light. Express your answer in terms of λ and n. You may assume that the film is parallel-sided slab within a small region.

8. [7 Marks] A telephoto lens (L_2, negative lens, focal length 100 mm) is placed in a camera lens (L_1, positive lens, focal length 50 mm). Find (a) the distance x if the optical system is focused at an object 100 cm in front of L_1, and (b) the magnification produced by the lens combination.

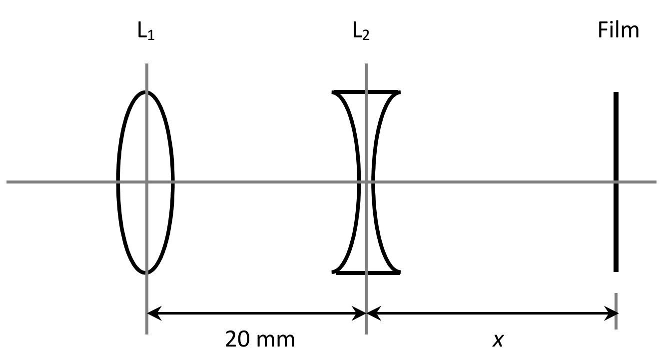


Fig Q8.

9. [14 Marks] Recently the lead-in-water crisis draws the public concern on the heath of drinking water. Lead (Pb) has four stable and naturally-existed isotopes, including ^{204}Pb (1.4%), ^{206}Pb (24.1%), ^{207}Pb (22.1%), and ^{208}Pb (52.4%). To detect lead in drinking water, one method is to analyze the mass spectrum, which can be produced by a mass spectrometer. Mass spectrometer is an instrument to measure mass distribution of charged atoms and molecules. The working principle is to separate different masses of ions spatially under the influence of a magnetic field.

The following diagram (Fig. Q9, drawing is not to scale) illustrates the layout of a typical mass spectrometer. Different speeds of Pb ions first travel in perpendicular electric and magnetic fields (Chamber A), then some of the ions further travel into Chamber B with magnetic field only. The Pb ions eventually hit a detector at the positions x_1, x_2, x_3, and x_4. The intensities in the positions x_1, x_2, x_3, and x_4 respectively represent the concentrations of ^{204}Pb, ^{206}Pb, ^{207}Pb, and ^{208}Pb ions. Both the Chambers A and B have the magnetic field strength of 0.02 T.

You may consider the Pb ions carry negative charge (but in reality it is not true).

(a) What is the function of Chamber A? Answer this question within 20 words.

(b) If x_1 = 1.020 m and x_2 = 1.030 m, determine the speed of the Pb isotopes when they just enter Chamber B.

(c) Find x_3 and x_4.

(d) Determine the magnitude of the electric field in Chamber A.

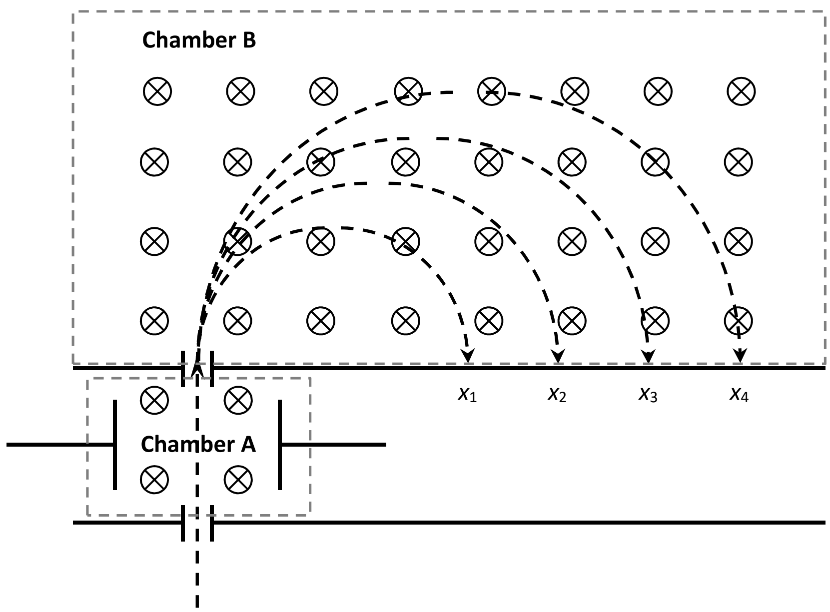


Fig Q9. Layout of a typical mass spectrometer. The symbol '⊗' represents the direction of the magnetic field.

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