Rules and Regulations

1. Answer all the questions in the answer book provided.
2. Full mark of this written selection test is 100 Marks.
3. The selection test is a 3-hour written test.

Useful Constants

Unless specified otherwise, the following symbols and constants will be used in this exam paper.

Astronomical Unit, 1 AU = 1.496×10^{8} km
Earth-Moon Distance, d = 384,400 km
Mass of the Sun, $M_S = 1.99\times10^{30}$ kg
Mass of the Earth, $M_e = 5.97\times10^{24}$ kg
Mass of the Moon, $M_m = 7.35\times10^{22}$ kg
Radius of the Sun, R_S = 696300 km
Radius of the Earth, R_E = 6370 km
Radius of the Moon, R_M = 1738 km
Gravitational Constant $G = 6.67\times10^{-11}$ m^3 kg^{-1} s^{-2}
Acceleration due to Gravity, g = 9.8 ms^{-2}

Air density at the sea level = 1.2 kg m^{-3}
Gas Constant = 8.31 J/(mol·K)
Velocity of Light in Vacuum, $c = 3\times10^{8}$ ms^{-1}
Specific Heat of Water, C_W = 4200 J/(kg·K)
Planck Constant, $h = 6.63\times10^{-34}$ Js
Charge of Electron, $e = 1.6\times10^{-19}$ C
Mass of Electron, $m_e = 9.1\times10^{-31}$ kg
Mass of Neutron, $m_n = 1.68\times10^{-27}$ kg
Coulomb Constant, $k_e = 8.988\times10^{9}$ N m^2/C^2

Trigonometric Identities:

$$\sin(x+y)=\sin(x)\cos(y)+\cos(x)\sin(y)$$

$$\cos(x+y)=\cos(x)\cos(y)-\sin(x)\sin(y)$$

$$\sin(2x)=2\sin(x)\cos(x)$$

$$\cos(2x)=\cos^2(x)-\sin^2(x)$$

$$\sin(x)\cos(y)=\frac{1}{2}[\sin(x+y)+\sin(x-y)]$$

$$\cos(x)\cos(y)=\frac{1}{2}[\cos(x+y)+\cos(x-y)]$$

$$\sin(x)\sin(y)=\frac{1}{2}[\cos(x-y)-\cos(x+y)]$$

Tayler Series:

$$\sin(x)\approx x-\frac{x^3}{6}+\frac{x^5}{120}-\cdots$$

$$\cos(x)\approx 1-\frac{x^2}{2}+\frac{x^4}{24}-\cdots$$

$$\tan(x)\approx x+\frac{x^3}{3}+\frac{2x^5}{15}+\cdots$$

Series Summation:

$$\sum_{k=1}^{m}k=\frac{m(m+1)}{2}$$

$$\sum_{k=1}^{m}k^2=\frac{m(m+1)(2m+1)}{6}$$

$$\sum_{k=1}^{m}k^3=\left[\frac{m(m+1)}{2}\right]^2$$

Hyperbolic functions:

$$\frac{d}{dx}\left(\sinh ax\right) = a\cosh x$$

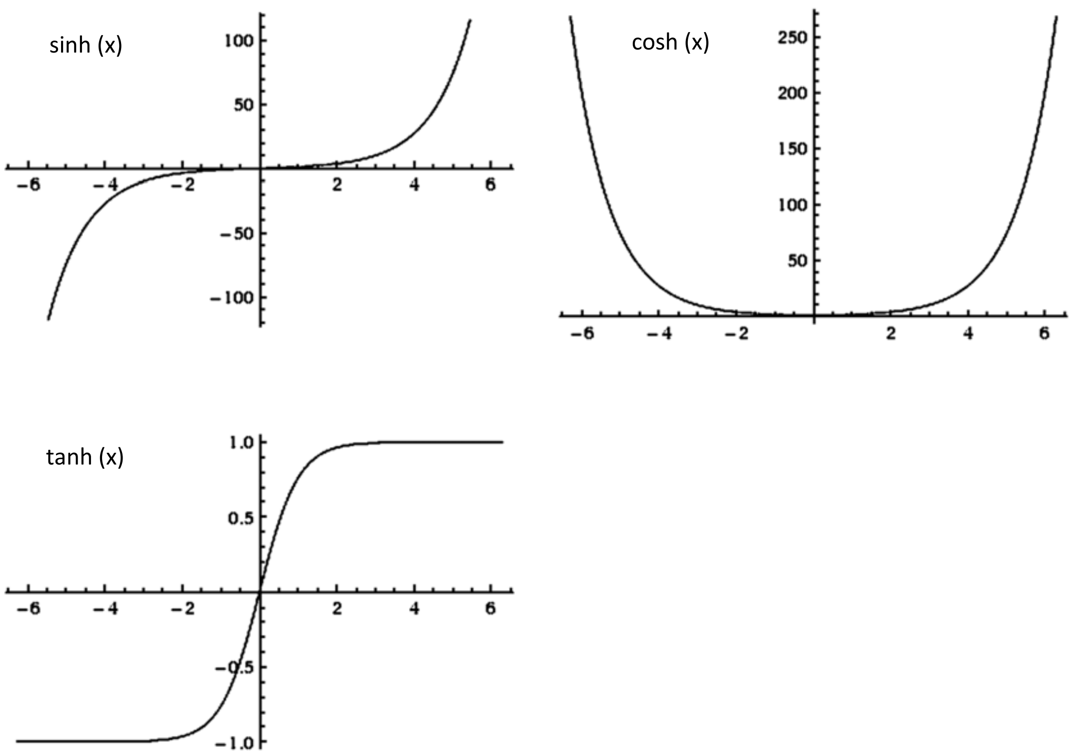
$$\frac{d}{dx}\left(\cosh ax\right) = a\sinh ax$$

$$\frac{d}{dx}\left(\tanh ax\right) = \frac{a}{\cosh^2 ax}$$

$$\sinh(x+y) = \sinh(x)\cosh(y) + \cosh(x)\sinh(y)$$

$$\cosh(x+y) = \cosh(x)\cosh(y) + \sinh(x)\sinh(y)$$

Sketch of hyperbolic functions



1. [39 Marks] Maximum ranges and maximum length of trajectory (軌道) of various projectile motions.

(a) A ball is projected at a velocity v_o at an angle θ on a horizontal ground. Find the horizontal range. Also find the maximum range, and at what projection angle the horizontal range is maximum?

(b) A ball is projected at a velocity v_o at an angle θ on a slope (angle α). Find the range on the slope. Also find the maximum range on the slope, and at what projection angle the horizontal range is maximum?

(c) A ball is projected at a velocity v_o at an angle θ above a horizontal ground (height h). Find the horizontal range. Also find the maximum range, and at what projection angle the horizontal range is maximum?

(d) A ball is projected at a velocity v_o at an angle θ on a horizontal ground. Find the length of trajectory. Also find the maximum length of trajectory, and at what projection angle the length of trajectory is maximum?

Hints:

i. Express your answers in terms of θ, α, h, and v_o.

i. All the angles are measured from the horizontal.

ii. Neglect the effect of air resistance.

iii. $\int \sqrt{1+x^2}\,dx = \dfrac{x\sqrt{x^2+1}+\ln\left(x+\sqrt{x^2+1}\right)}{2} + \text{Constant}$

iv. The following graph (Fig. Q1) may help.

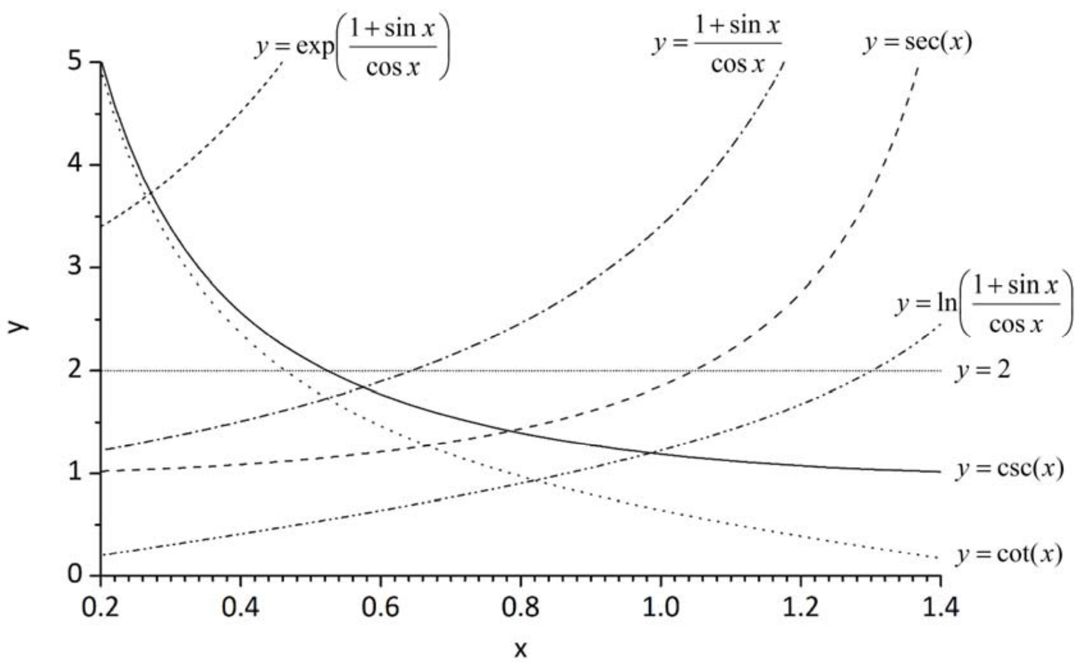


Fig. Q1

2. [11 Marks] A uniform rod (mass m, length $2L$) is held so that one end is supported by a vertical wall, and the other end rests on a horizontal floor. Initially, the rod makes an angle θ_o with the floor. When the rod is released from rest, because of the influence of gravity, the rod slides down. Ignore all frictions.

(a) Find the moment of inertia of the rod about its end. Express your answer in terms of m and L.

(b) Find $\dot{\theta}$ when the rod reaches a new angle θ. Express your answer in terms of θ, θ_o, L, and g (acceleration due to gravity).

(c) Find the angle θ when the upper end of the rod leave the wall.

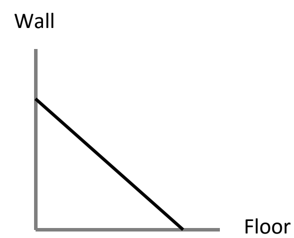


Fig. Q2

3. [10 Marks] The vapor pressure of an unknown solid can be approximated by a second-order polynomial equation:

$$P = 4.33\times10^{-4}\ T^2 + 4.25\times10^{-2}\ T - 24.2,$$

whereas the vapor pressure of the unknown substance in liquid form is approximated by another second-order polynomial equation:

$$P = -0.27\ T^2 + 154.3\ T - 20841,$$

where P is the vapor pressure in atm, and T is the absolute temperature in K.

(a) Find the triple point.

(b) Find the latent heat of vaporization at the triple point (specific volume change of the phase transition = 72.6 m^3).

(c) Find the latent heat of melting at the triple point, if the latent heat of sublimation at the triple point is 715 kJ/kg.

4. [23 Marks] A family consists of 4 members. Each member takes a 5-min shower every day. The average water flow rate through the shower head is 12 litre per minute. The warm water (40°C) from the shower head is a mixture of cold water (20°C) and hot water (55°C) at a T-elbow (Fig. Q4), and the hot water is heated by an electric water heater.

(a) Find the power input to the electric heater.

(b) Determine the amount of entropy generation per year by the family.

In order to conserve energy, drain warm water (35°C) is used to preheat the cold water (20°C). The effectiveness of the preheat procedure is 0.5. The effectiveness of 0.5 means that only half of the energy from the drain water is transferred to the cold water.

(c) Determine the electric power required to produce 40°C warm water from the shower head, if the preheat procedure is adopted.

(d) Determine the reduction in the amount of entropy generation per year.

Hints:
i. You may assume that the efficiency of the heater is 100%.
ii. Heat loss is negligible, and water is incompressible.
iii. Specific heat (c) is constant over the temperature range, c = 4180 J/(kg·K).

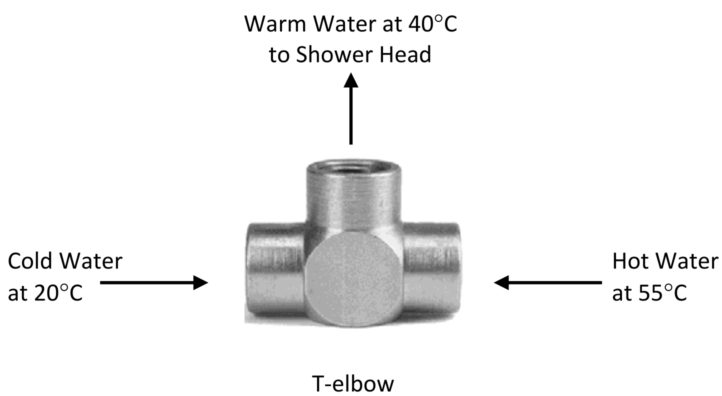


Fig. Q4

5. [17 Marks] A chain with uniform mass density (ρ) per unit length hangs between 2 points on 2 vertical walls, and the shape of the chain is represented in an *x*-*y* coordinate system (Fig. Q5). On any subpart of the chain, the tension of the chain can be resolved into two components, the horizontal component (T_x) and the vertical (T_y) component of the tension.

(a) Given that one of the component of the tension is constant (C_1) throughout the chain, but the other component of the tension is proportional to the slope of the chain with the constant of proportionality C_2, write down the equations of T_x and T_y.

(b) Considering the vertical component of the tension and the weight in a small piece of the chain, find the expression of the differential equation dT_y/dx. Express your answer in terms of ρ, g (acceleration due to gravity), and dy/dx.

(c) Given that when x = 0, $y = \frac{C_2}{\rho g}$, and $\left.\frac{dy}{dx}\right|_{x=0} = 0$, find the equation of the chain, that is, y = *f*(x). Express your answer in terms of ρ, g, and C_2.

Hints:

$$\int \frac{dx}{\sqrt{1+x^2}} = \sinh^{-1} x + \text{Constant}$$

$$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + \text{Constant}$$

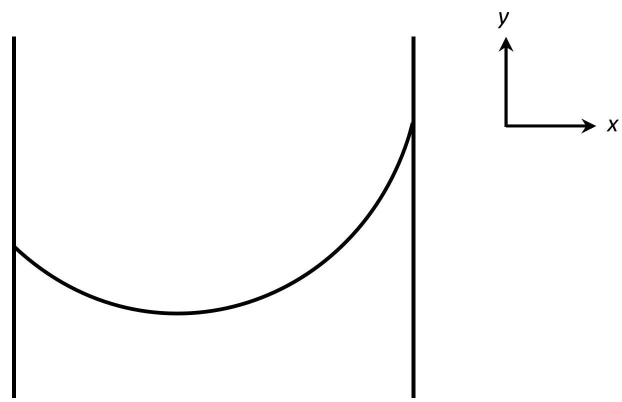


Fig. Q5

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