Rules and Regulations

1. Answer all the questions in the answer book provided.
2. Full mark of this written selection test is 100 Marks.
3. The selection test is a 3-hour written test.

Useful Constants

Unless specified otherwise, the following symbols and constants will be used in this exam paper.

Astronomical Unit, 1 AU = 1.496×10^{8} km
Earth-Moon Distance, d = 384,400 km
Mass of the Sun, $M_S = 1.99\times10^{30}$ kg
Mass of the Earth, $M_e = 5.97\times10^{24}$ kg
Mass of the Moon, $M_m = 7.35\times10^{22}$ kg
Radius of the Sun, R_S = 696300 km
Radius of the Earth, R_E = 6370 km
Radius of the Moon, R_M = 1738 km
Gravitational Constant $G = 6.67\times10^{-11}$ m^3 kg^{-1} s^{-2}
Acceleration due to Gravity, g = 9.8 ms^{-2}

Air density at the sea level = 1.2 kg m^{-3}
Gas Constant = 8.31 J/(mol·K)
Velocity of Light in Vacuum, $c = 3\times10^{8}$ ms^{-1}
Specific Heat of Water, C_W = 4200 J/(kg·K)
Planck Constant, $h = 6.63\times10^{-34}$ Js
Charge of Electron, $e = 1.6\times10^{-19}$ C
Mass of Electron, $m_e = 9.1\times10^{-31}$ kg
Mass of Neutron, $m_n = 1.68\times10^{-27}$ kg
Coulomb Constant, $k_e = 8.988\times10^{9}$ N m^2/C^2

Trigonometric Identities:

$$\sin(x+y)=\sin(x)\cos(y)+\cos(x)\sin(y)$$

$$\cos(x+y)=\cos(x)\cos(y)-\sin(x)\sin(y)$$

$$\sin(2x)=2\sin(x)\cos(x)$$

$$\cos(2x)=\cos^2(x)-\sin^2(x)$$

$$\sin(x)\cos(y)=\frac{1}{2}[\sin(x+y)+\sin(x-y)]$$

$$\cos(x)\cos(y)=\frac{1}{2}[\cos(x+y)+\cos(x-y)]$$

$$\sin(x)\sin(y)=\frac{1}{2}[\cos(x-y)-\cos(x+y)]$$

Tayler Series:

$$\sin(x)\approx x-\frac{x^3}{6}+\frac{x^5}{120}-\cdots$$

$$\cos(x)\approx 1-\frac{x^2}{2}+\frac{x^4}{24}-\cdots$$

$$\tan(x)\approx x+\frac{x^3}{3}+\frac{2x^5}{15}+\cdots$$

Series Summation:

$$\sum_{k=1}^{m}k=\frac{m(m+1)}{2}$$

$$\sum_{k=1}^{m}k^2=\frac{m(m+1)(2m+1)}{6}$$

$$\sum_{k=1}^{m}k^3=\left[\frac{m(m+1)}{2}\right]^2$$

Hyperbolic functions:

$$\frac{d}{dx}\left(\sinh ax\right)=a\cosh x$$

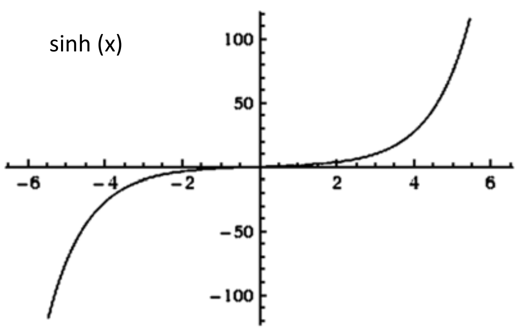
$$\frac{d}{dx}\left(\cosh ax\right)=a\sinh ax$$

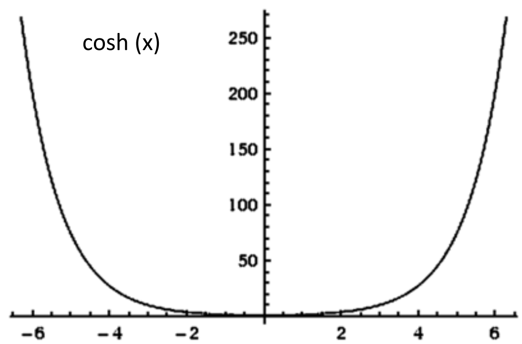
$$\frac{d}{dx}\left(\tanh ax\right)=\frac{a}{\cosh^2 ax}$$

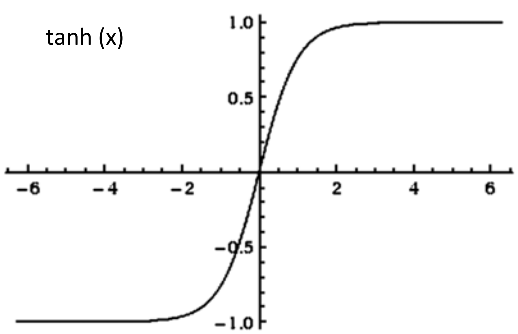
$$\sinh(x+y)=\sinh(x)\cosh(y)+\cosh(x)\sinh(y)$$

$$\cosh(x+y)=\cosh(x)\cosh(y)+\sinh(x)\sinh(y)$$

Sketch of hyperbolic functions







1. [37 Marks] Transmission grating for spectroscopy (光譜儀)

As shown in Fig. Q1a, a laser beam is directed perpendicularly to a transmission grating, and the diffracted pattern is observed on a screen. The screen is oriented parallel to the grating, which is of a distance *D* from the grating.

(a) Write down the grating equation when light is normally incident on a grating. Express your answer in terms of *d* (slit separation), θ_m (angle between the diffracted light and the grating's normal vector), *m* (the *m*th order of the diffracted light), and wavelength λ.

(b) If $D >> x$, find the relationship between *x* and λ. Express your answer in terms of *m*, λ, *d*, and *D*.

(c) In order to characterize the grating parameter, a laser (wavelength 532 nm) is used to produce the diffraction pattern, and the first 4 diffracted laser points (m = 0, 1, 2, and 3) are recorded on a graph paper, as illustrated in Fig. Q1b. Suppose D = 30 cm, by plotting a suitable *x*-*y* graph (data points of diffraction pattern, data point labels, linear regression line, linear slope), determine the number of lines (or grooves) per millimeter (L/mm) of the diffraction grating.

(d) Identify all the diffraction on the screen if another laser of wavelength 633 nm is used. You may assume that the positions and the orientations of the diffraction grating, screen, and laser remain unchanged. Your answers should include the order of diffraction, the diffracted angles, distances between all the diffracted points.

(e) A white LED consists of continuous wavelengths from 400 nm to 700 nm, which is collimated and directed perpendicularly into the diffraction grating. Sketch the diffraction patterns on separate graph paper. Your sketch should include continuous spectrum (or spectra), minimum and maximum wavelengths of the spectrum (or spectra), the order of diffraction, distances and dimensions of the spectrum (or spectra).

(f) The transmission grating can be used for spectroscopy. The requirements of the diffracted light are: i) to spatially separate light from 400 nm to 700 nm, and ii) to separate light spatially without overlapping with other order of diffracted light. Which order of the diffracted light is the best choice for the application of spectroscopy? That is, what is the best choice of *m*?

(g) Instead of using the screen to observe the diffracted light, a rectangular CCD (charge-coupled device, 感光耦合元件) is employed to detect the diffracted light. The CCD consists of 1024 pixels × 512 pixels, and its dimension is 4 cm × 2 cm. The best choice of *m* in Part (e) is used, and the CCD is oriented to optimize the spectral resolution. In order to cover the full wavelength range of light (400 nm to 700 nm), what should the distance between the transmission grating and the CCD be?

(h) Following Part (g), calculate the mean spectral resolution over the wavelength range 400 nm to 700 nm.
Hint: The number of pixels determines the spectral resolution.

(i) This experimental configuration (Fig. Q1a installed with CCD detector) is used to study the solar spectrum. Two Fraunhofer lines (Fe, absorption at 430.790 nm; and Ca, absorption at 430.774 nm) are investigated. Can the spectrometer resolve the Fraunhofer lines?

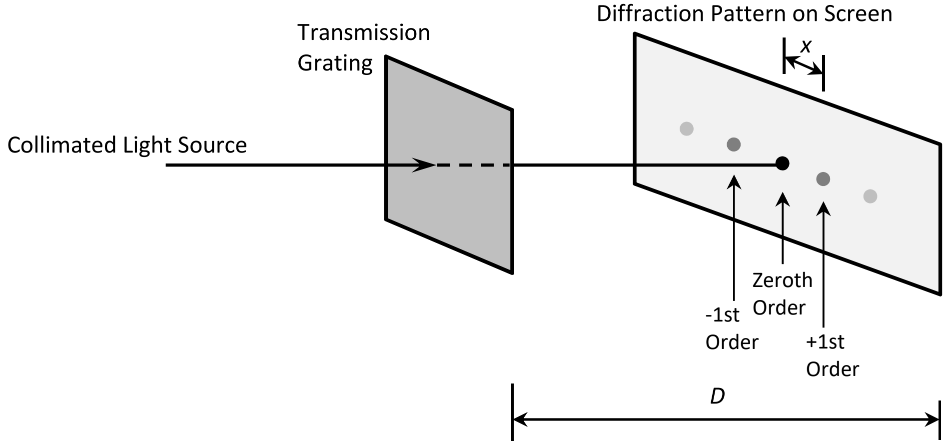


Fig. Q1a

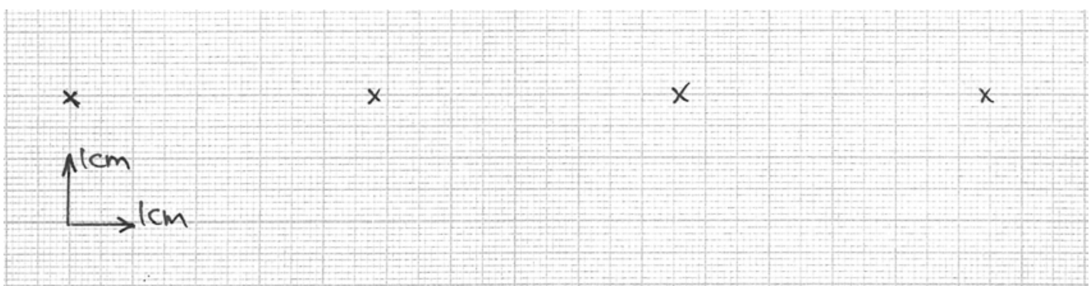


Fig. Q1b

2. [41 Marks] Maximum energy conversion efficiency of solar cell

Solar cell is a device to convert sunlight into electrical energy. Most of the commercial solar cells are fabricated by the raw material of silicon. Silicon is a semiconductor with bandgap energy 1.14 eV (electron volt) at room temperature. Light of photon energy larger than the bandgap energy will be absorbed by the semiconductor. Therefore, for a particular light source, some of the photons (with photon energy larger than the bandgap energy) can be absorbed for electrical energy generation, but some of the photons (with photon energy smaller than the bandgap energy) cannot be absorbed. We call this is the spectral mismatch. Spectral mismatch is one of the important limiting factors that a solar cell cannot achieve 100% energy conversion efficiency.

(a) What is the wavelength region of light that can be absorbed by a single crystal silicon solar cell?

Energy conversion efficiency can be studied under laboratory environment. A circular solar cell (diameter 10 cm) is put under a tungsten lamp (temperature at 2500 K). The distance between the solar cell and tungsten lamp is 37.3 cm. You may consider the tungsten lamp as a point source with black body characteristics.

(b) What is the peak emission wavelength of the tungsten lamp?

(c) Find the total irradiance (W/m^2) of the tungsten lamp.

(d) The fraction of total emission from a blackbody between the wavelength interval 0 and λ, F(0, λ), can be written as

$$F(0,\lambda) = \frac{\int_a^b L(\lambda)\,\mathrm{d}\lambda}{\int_c^d L(\lambda)\,\mathrm{d}\lambda}, \qquad \text{(Eqn. Q2)}$$

where $L(\lambda)$ is the Planck's law spectral radiance at the wavelength λ. What are the values a, b, c, and d?

(e) In Equation Q2, the integration variable is dλ. Rewrite Equation Q2 such that the integration variable is d(λ*T*). Express your answer in terms of $L(\lambda)$, λ, *T*, and σ (Stefan's constant).

(f) The following table lists the fraction of total emission from a blackbody between the wavelength interval 0 and λ as a function of λT. Sketch the data points of F(0,λ) against λT on a *x*-*y* graph.

λT (μm·K)	2000	3000	4000	5000	6000	7000	8000
F(0,λ)	0.066728	0.273232	0.480877	0.633747	0.737818	0.808109	0.856288

λT (μm·K)	9000	10000	11000	12000	13000	14000	15000
F(0,λ)	0.890029	0.914199	0.931890	0.945098	0.955139	0.962898	0.969981

(g) By considering the spectral mismatch only, calculate the maximum efficiency of the solar cell if it is illuminated by the tungsten lamp at 2500 K.

(h) Find the spectral power illuminating on the solar cell.

Hint: The spectral power means that the spectral power that can be absorbed by the solar cell.

(i) An optical filter is used to filter the light from the tungsten lamp. The optical filter has the following characteristics:

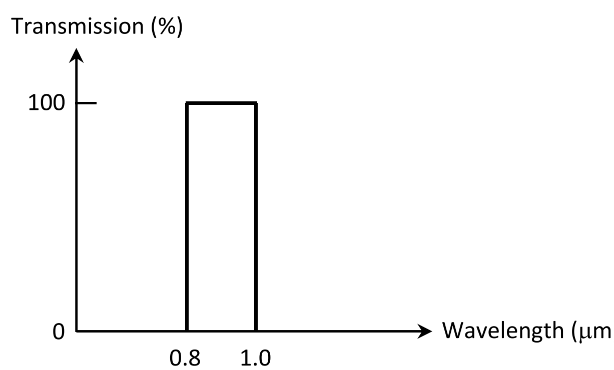


Fig. Q2

What is the optical filter?

Hints: Short wavelength pass filter, long wavelength pass filter, band pass filter, notch filter, interference filter, polarizer, neutral density filter, or variable neutral density filter?

(j) With the optical filter, find the spectral power illuminating on the solar cell.

Hint: The spectral power means that the spectral power that can be absorbed by the solar cell.

3. [22 Marks] One Dimensional Collisions

(a) Warm-up question: In a one-dimension collision, find the relative velocity of two participles (masses m and M; initial velocities v_i and V_i; final velocities v_f and V_f, respectively) after collision. Express your answer in terms of relative velocity before collision. You may want to use the symbols Δv_i and Δv_f as the relative velocities before and after collisions.

(b) A tennis ball (mass m) sits on top of a basketball (mass M), where mass M is much larger than mass m (Fig. Q3a). The bottom of the basketball is of height h above the ground, and the bottom of the tennis ball is of (d + h) above the ground. The balls are dropped from rest. Find the height (measured from the ground) that the tennis ball bounces.

Hints: (i) All collisions are elastic, and (ii) collisions are limited to one dimension only.

Consider n balls (Ball 1, Ball 2, ….., Ball n) having masses m_1, m_2, ……, m_n where $m_1 >> m_2 >> \ldots\ldots >> m_n$ (the symbol '>>' represents 'much larger than'), as illustrated in Fig. Q3b. The bottom of Ball 1 is of height h above the ground, the bottom of ball 2 is of (d_1 + h) above, and so on. All balls are dropped from rest.

(c) Find the upward speed of the nth ball (v_n) after bouncing off the ball below it. Express your answer in terms of n and v, where $v = \sqrt{2gh}$.

(d) Suppose ($d_1 + d_2 + \ldots\ldots + d_n$ = D), find the height (measured from the ground) that the nth ball bounces. Express your answer in terms of n, D, and h.

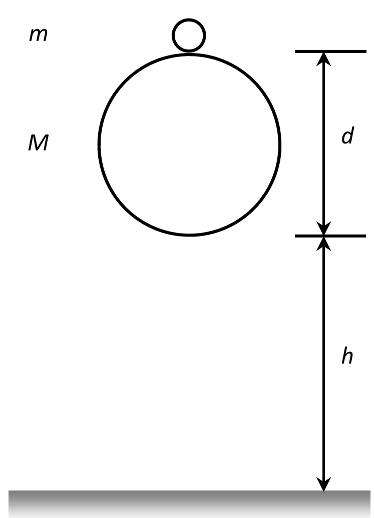


Fig. Q3a

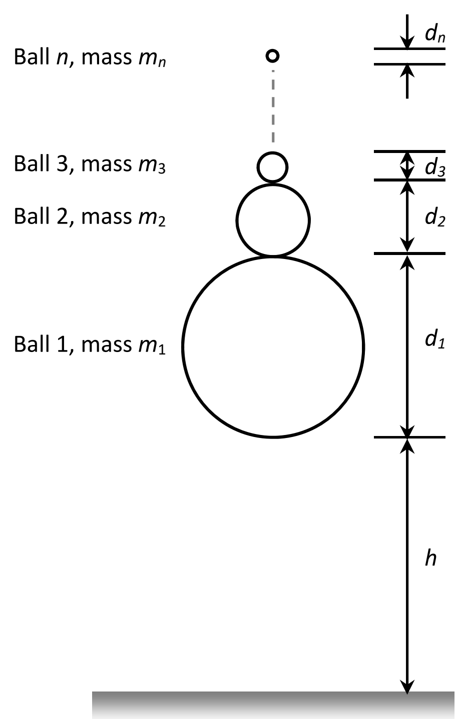


Fig. Q3b

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