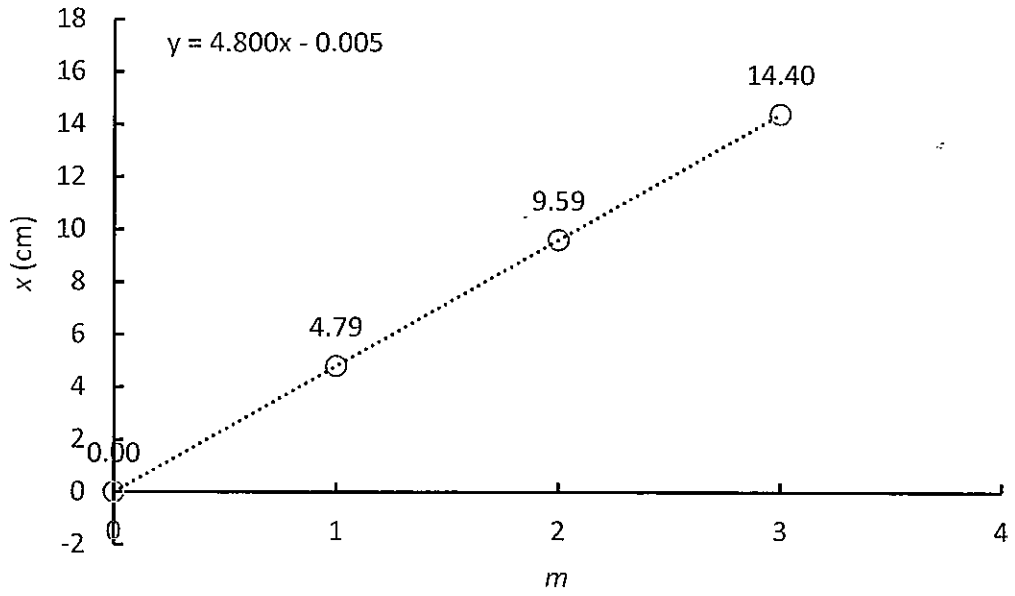
1.

(a) $d \sin\theta_m = m\lambda$ [1 Mark]

(b) $x \approx \frac{m\lambda D}{d}$ [1 Mark]

(c)



Plotting x against m: 2 Marks
Both Axis Labels: 1 Mark
Both Axis Titles + Unit(s): 1 Mark
Data Labels: 1 Mark
Data Points: 2 Marks
Linear Line: 1 Mark
> 70% Graph Area: 1 Mark; > 50% Graph Area: 0.5 Mark; Zero otherwise

-0.5 Mark for each of the missing/incorrect item above,
but minimum is zero mark.

Subtotal: 9 Marks

4.7 <= Slope <= 4.9 : 2 Marks
4.6 <= Slope <= 5.0 : 1 Mark
Zero otherwise

Slope = 4.8 cm

$\Rightarrow \frac{\lambda D}{d} = 4.8\text{ cm}$

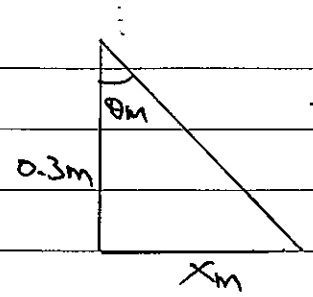
$\frac{(532\times10^{-9})(0.3)}{4.8\text{ cm}} = d$

$d = 3.325\ \mu m$ [1 Mark]

Number of Grooves
$= 300\ L/mm$ [1 Mark]

1.

(d) $d \sin\theta_m = m\lambda$



$\tan\theta_m = \frac{x}{0.3}$

$m = 0$, $\theta_m = 0°$, $x_m = 0\,cm$ [1 Mark]

$m = \pm 1$, $\theta_m = 11°$, $x_m = 5.8\,cm$ [1 Mark]

$m = \pm 2$, $\theta_m = 22.3°$, $x_m = 12.3\,cm$ [1 Mark]

$m = \pm 3$, $\theta_m = 34.8°$, $x_m = 20.9\,cm$ [1 Mark]

$m = \pm 4$, $\theta_m = 49.5°$, $x_m = 35.1\,cm$ [1 Mark]

$m = \pm 5$, $\theta_m = 71.9°$, $x_m = 91.8\,cm$ [1 Mark]

-0.5 Mark for each of the incorrect or missing answers.

Note that x_m is measured from the zero order diffraction beam.

With m = -1, -2, ..., -5: 1 Mark

1.

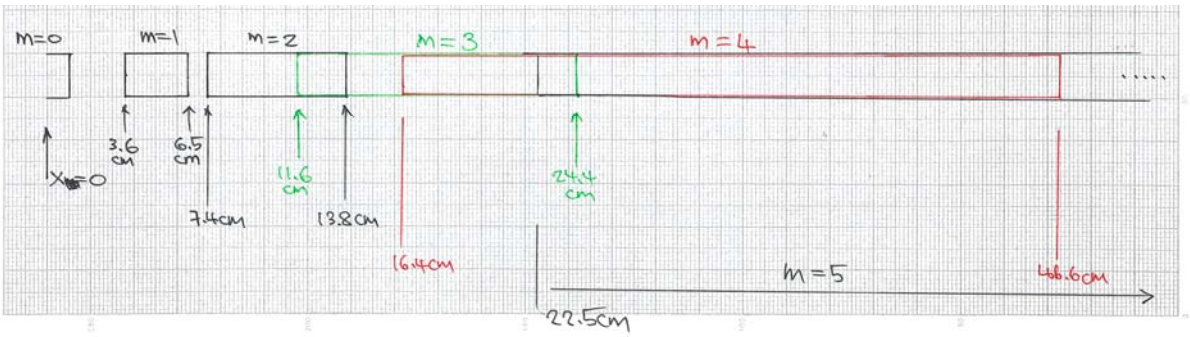
(e)

Diffraction Order	Wavelength	θ_m	x_m	
0	All Wavelengths	0	0	[1 Mark]
±1	400 nm	±6.9°	±3.6 cm	[1 Mark]
	700 nm	±12.1°	±6.5 cm	
±2	400 nm	±13.9°	±7.4 cm	[1 Mark]
	700 nm	±24.7°	±13.8 cm	
±3	400 nm	±21.1°	±11.6 cm	[1 Mark]
	700 nm	±39.1°	±24.4 cm	
±4	400 nm	±28.7°	±16.4 cm	[1 Mark]
	700 nm	±57.2°	±46.6 cm	
±5	400 nm	±36.9°	±22.5 cm	[1 Mark]
	700 nm	/	/	

With m = -1, -2, ..., -5: 1 Mark

Note: There is no 5th order diffraction for λ=700nm, but there exists a maximum wavelength of light (666 nm, $x_m \rightarrow \infty$) with the 5th order of diffraction.

Continuous diffraction: 1 Mark



Diffraction order from zeroth to 5th: 1 Mark

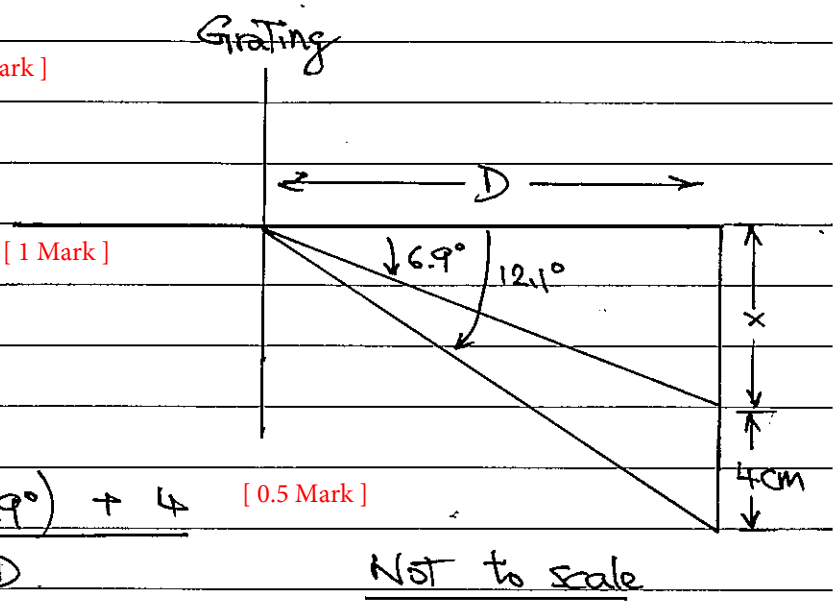
Note: m = -1, …., -5 diffraction pattern is not shown for clarity. The size of zero order diffraction depends on the dimension of the incident light.

(f) m = 1 [2 Marks]

1.

(g) $\tan 6.9^\circ = \dfrac{x}{D}$ [1 Mark]

$\tan 12.1^\circ = \dfrac{x + 4\text{cm}}{D}$ [1 Mark]



$\Rightarrow \tan 12.1^\circ = \dfrac{(D \tan 6.9^\circ) + 4}{D}$ [0.5 Mark]

$D \tan 12.1^\circ = D \tan 6.9^\circ + 4$

$\Rightarrow D = 42.8\text{ cm}$ [1 Mark]

(h) Mean Spectral Resolution $= \dfrac{300\text{ nm}}{\cancel{102}\ 1024\text{ pixels}}$ [1 Mark]

$= 0.29\text{ nm/pixel}$ [0.5 Mark]

(i) The spectrometer cannot resolve the Fraunhofer lines. [1 Mark]

2.

(a) $E_g = 1.14\ eV$

$\lambda_g = \dfrac{1.24}{1.14} = 1.09\ \mu m$ [1 Mark]

$\Rightarrow \lambda < 1.09\ \mu m$, light can be absorbed. [1 Mark]

(b) $\lambda_{max} = \dfrac{2898}{\cancel{3000}\ 2500} = \cancel{0.966\ \mu m}\ 1.16\ um$ [0.5 Mark] [1 Mark]

(c) $E = \sigma T^4$ [1 Mark]

$= (5.67 \times 10^{-8})(2500)^4$ [0.5 Mark]

$= 2.21 \times 10^6\ W/M^2$ [1 Mark]

(d) $a = 0$ [1 Mark] $c = 0$ [1 Mark]

$b = 1.09\ \mu m$ [1 Mark] $d \to \infty$ [1 Mark]

(e) Fraction of Total Emission :

$$F(0, \lambda) = \frac{\int_0^{\lambda} L(\lambda)\, d\lambda}{\int_0^{\infty} L(\lambda)\, d\lambda}$$ [1 Mark]

$$= \frac{\int_0^{\lambda} L(\lambda)\, d\lambda}{\sigma T^4}$$ [1 Mark]

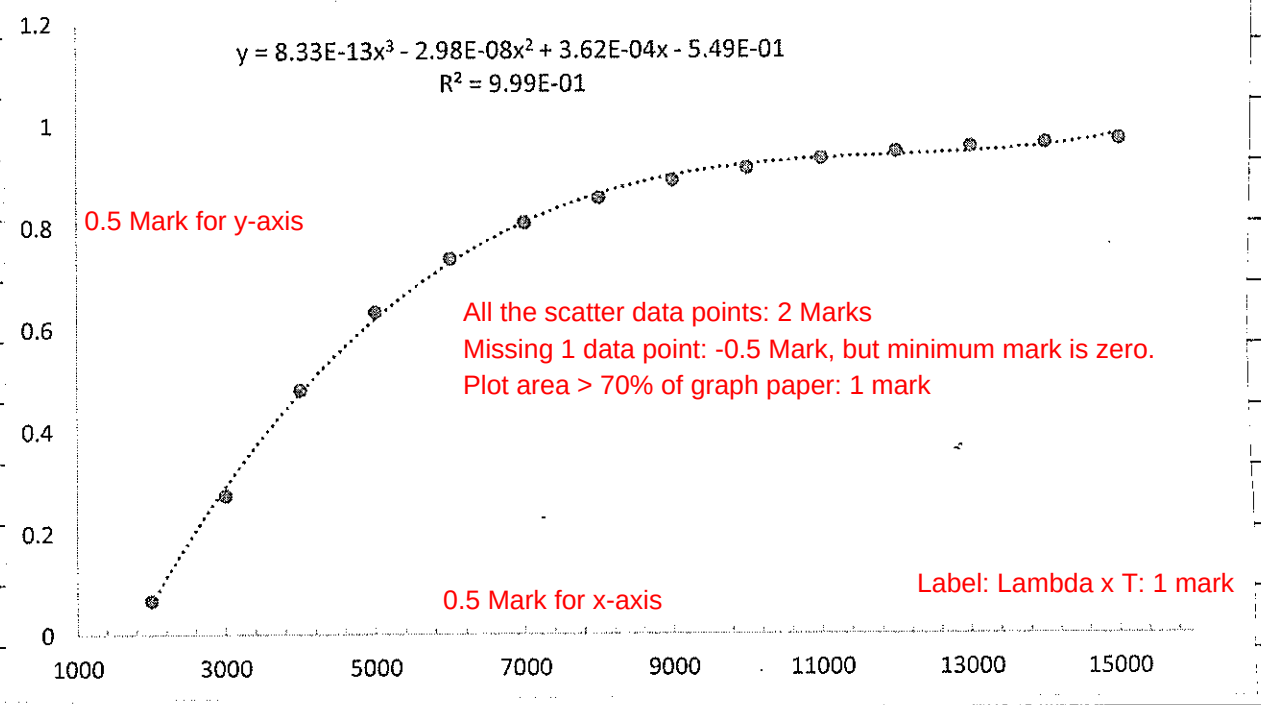
$$= \int_0^{\lambda T} \frac{L(\lambda)}{\sigma T^5}\, d(\lambda T)$$ [1 Mark]

2.

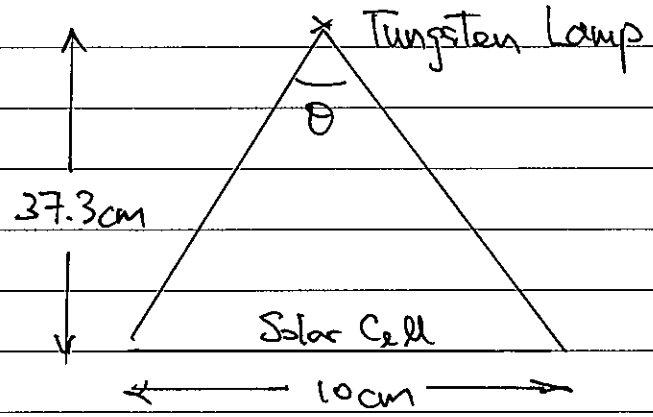
Full mark in Part (g): 6 marks

Label: F(0, lambda): 1 Mark

(f)



(g) $\theta = 30^\circ \left(\text{or } \frac{\pi}{6}\right)$ [0.5 Mark]



λ : Integrate from 0 to 1.09 μm.

[0.5 Mark] [0.5 Mark] [0.5 Mark]

$$E = \int_0^{1.09}\int_0^{2\pi}\int_0^{\frac{\pi}{6}} L(\lambda)\cos\theta\sin\theta\, d\theta\, d\phi\, d\lambda$$ [1 Mark]

$$= \int_0^{1.09} L(\lambda)\left(\int_0^{2\pi}\int_0^{\frac{\pi}{6}} \cos\theta\sin\theta\, d\theta\, d\phi\right) d\lambda$$

$$= \int_0^{1.09} L(\lambda)\left[2\pi\,\frac{\sin^2\theta}{2}\right]_0^{\frac{\pi}{6}} d\lambda$$ [1 Mark]

$$= \frac{1}{4}\int_0^{1.09} \pi L(\lambda)\, d\lambda$$ [1 Mark]

$\lambda_1 T = (0)(2500) = 0$ [1 Mark]

$\lambda_2 T = (1.09\,\mu m)(2500\,K) = 2725\ \mu mK$ [1 Mark]

2.

(g) Either (i) Best solution, more marks.

Polynomial Fit (3rd order):

[3 Marks]

$$y = 8.33\times10^{-13}x^3 - 2.98\times10^{-8}x^2 + 3.62\times10^{-4}x - 5.49\times10^{-1}$$

Since $\lambda T = 2725\ \mu m\,K$,

$\Rightarrow F(0, 1.09) = 0.231$ [1 Mark]

$\Rightarrow$ Maximum Efficiency $= 23.1\%$ [1 Mark]

OR (ii)

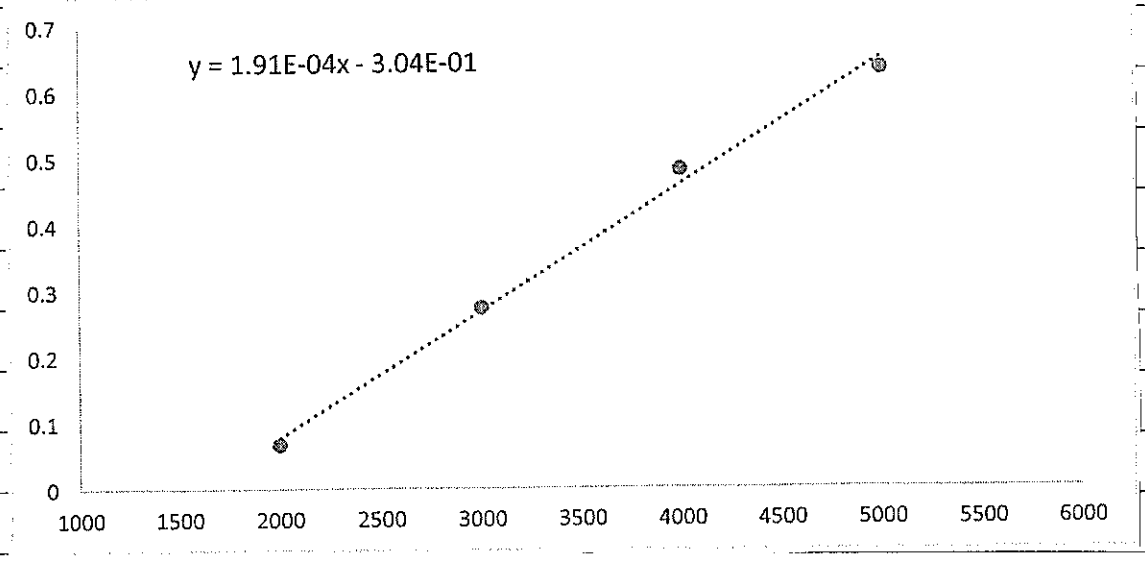
Linear Regression Fit of the first 4 data points.

$$y = 1.91\times10^{-4}x - 3.04\times10^{-1}$$ [1 Mark]

Since $\lambda T = 2725\ \mu m\,K$,

$\Rightarrow F(0, 1.09) = 0.216$ [1 Mark]

$\Rightarrow$ Maximum Efficiency $= 21.6\%$ [1 Mark]



(g) OR by observation.

$0.210 \leq F(0, 1.09) \leq 0.235$ [1 Mark]

Maximum efficiency written correctly [1 Mark]

(h) $E = (0.25)(0.231)(5.67\times10^{-8})(2500)^4 \times \pi(0.05)^2$ [1 Mark]

$= 1000\,W$

$950\,W \leq E \leq 1050\,W$ [1 Mark]

(i) Band pass filter [2 Marks]

(j) $E = \int_{0.8}^{1.0}\int_{0}^{2\pi}\int_{0}^{\frac{\pi}{6}} L(\lambda)\cos\theta\sin\theta\, d\theta\, d\phi\, d\lambda$

$= \frac{1}{4}\int_{0.8}^{1.0} \pi L(\lambda)\, d\lambda$ [1 Mark]

$\lambda_1 T = (0.8)(2500) = 2000\ \mu m\,K$ [1 Mark]

$\lambda_2 T = (1.0)(2500) = 2500\ \mu m\,K$ [1 Mark]

[1 Mark] [0.5 Mark]

$E = \frac{1}{4}\left[F(0, 1\,\mu m) - F(0, 0.8\,\mu m)\right]\sigma T^4 \pi(0.05)^2$

[0.5 Mark for substitution of numerical values correctly]

$= \frac{1}{4}(0.1617 - 0.066728)(5.67\times10^{-8})(2500)^4 \pi(0.05)^2$

$= 413\,W$ [1 Mark]

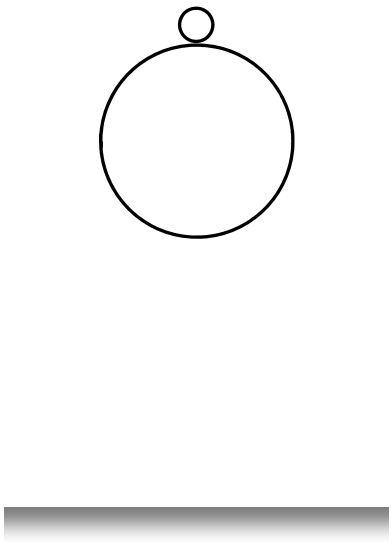
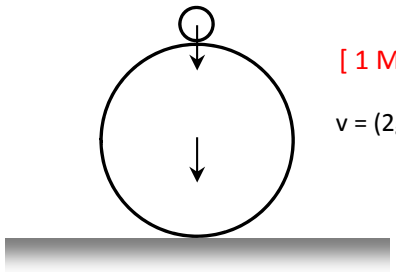
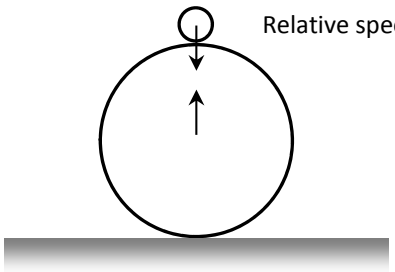
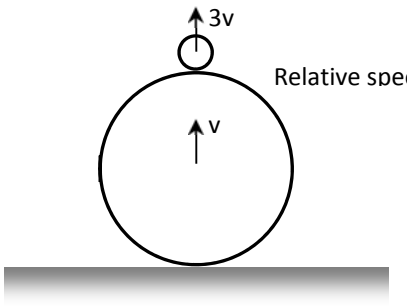
3(a)

$$\begin{cases} mv_i + MV_i = mv_f + MV_f \\ \frac{1}{2}mv_i^2 + \frac{1}{2}MV_i^2 = \frac{1}{2}mv_f^2 + \frac{1}{2}MV_f^2 \end{cases}$$ [1 Mark]

$$\Rightarrow \quad \begin{cases} m(v_i - v_f) = M(v_f - v_i) \\ m(v_i^2 - v_f^2) = M(v_f^2 - v_i^2) \end{cases}$$ [1 Mark]

$$\Rightarrow \quad (v_i - V_i) = -(v_f - V_f)$$ [1 Mark]

(b)

Step (i) Balls drop from rest 	Step (ii) Right after hitting the ground, both balls move downward with speed v = $(2gh)^{1/2}$  [1 Mark] v = $(2gh)^{1/2}$
Step (iii) Right after bounce off the ground, the relative speed = 2v. [1 Mark]  Relative speed = 2v	Step (iv)  Relative speed = 2v

Step (iv)

After bounce each other, from Result (a), the relative speed is 2v. [1 Mark]

The speed of the basketball is essentially equal to v (M >> m), [1 Mark]

$$\underbrace{v_f}_{\substack{\text{speed of tennis ball}\\ \text{after collision}}} - \underbrace{v}_{\text{speed of basketball}} = \underbrace{2v}_{\text{relative speed}} \qquad \text{Eqn (Q3.1)} \qquad [\text{ 2 Marks }]$$

$$\Rightarrow \qquad v_f = 3v \qquad [\text{ 1 Mark }]$$

Conservation of energy to find the height (H') that the tennis ball bounces:

$$\frac{1}{2}m(3v)^2 = mgH' \qquad [\text{ 1 Mark }]$$

$$H' = \frac{9v^2}{2g} = \frac{9(2gh)}{2g} = 9h \qquad [\text{ 1 Mark }]$$

The height (from the ground) that the tennis ball bounces:

H = 9 h + d [1 Mark]

(c) Inductively determine the speed of each ball after bouncing off the ball below it.

From Eqn (Q3.1),

Ball 2 and Ball 1: $v_2 - v = 2v_1$, where $v = \sqrt{2gh}$ [1 Mark]

Ball n and Ball (n – 1): $v_n - v = 2v_{n-1}$ [2 Marks]

$$\Rightarrow \qquad v_n = 2v_{n-1} + v$$

$$\Rightarrow \qquad v_n = (2^n - 1)v \qquad [\text{ 3 Marks }]$$

For those students who cannot write the general form $v_n = (2^n - 1)v$, partial marks are given for v_2, v_3, and v_4:

$v_2 = 2v_1 + v = 3v$ (note: $v_1 = v$) [0.5 Mark for v_1 and v_2]

$v_3 = 2(2v_1 + v) + v = 4v_1 + 3v = 7v$ [0.5 Mark]

$v_4 = 2(4v_1 + 3v) + v = 8v_1 + 7v = 15v$ [0.5 Mark]

(d) Conservation of energy to find the height (H_n') that the nth ball bounces:

$$\frac{1}{2}m[(2^n - 1)v]^2 = mgH_n' \qquad [\text{ 1 Mark }]$$

$$H_n' = \frac{[(2^n - 1)v]^2}{2g} \qquad [\text{ 1 Mark }]$$

$$H_n = \frac{[(2^n - 1)v]^2}{2g} + D \qquad [\text{ 1 Mark }]$$