**Rules and Regulations**

1. Answer all the questions in the answer book provided.
2. Full mark of this written selection test is 100 Marks.
3. The selection test is a 3-hour written test.

**Useful Constants**

Unless specified otherwise, the following symbols and constants will be used in this exam paper.

Astronomical Unit, 1 AU = $1.496\times10^8$ km
Earth-Moon Distance, $d$ = 384,400 km
Mass of the Sun, $M_S = 1.99\times10^{30}$ kg
Mass of the Earth, $M_e = 5.97\times10^{24}$ kg
Mass of the Moon, $M_m = 7.35\times10^{22}$ kg
Radius of the Sun, $R_S$ = 696300 km
Radius of the Earth, $R_E$ = 6370 km
Radius of the Moon, $R_M$ = 1738 km
Gravitational Constant $G = 6.67\times10^{-11}$ m$^3$ kg$^{-1}$ s$^{-2}$
Acceleration due to Gravity, $g$ = 9.8 ms$^{-2}$

Air density at the sea level = 1.2 kg m$^{-3}$
Gas Constant = 8.31 J/(mol·K)
1 mole = $6.0\times10^{22}$ atoms (molecules)
Velocity of Light in Vacuum, $c = 3\times10^8$ ms$^{-1}$
Specific Heat of Water, $C_W$ = 4200 J/(kg·K)
Planck Constant, $h = 6.63\times10^{-34}$ Js
Charge of Electron, $e = 1.6\times10^{-19}$ C
Mass of Electron, $m_e = 9.1\times10^{-31}$ kg
Mass of Neutron, $m_n = 1.68\times10^{-27}$ kg
Coulomb Constant, $k_e = 8.988\times10^9$ N m$^2$/C$^2$

**Trigonometric Identities:**

$$\sin(x+y)=\sin(x)\cos(y)+\cos(x)\sin(y)$$

$$\cos(x+y)=\cos(x)\cos(y)-\sin(x)\sin(y)$$

$$\sin(2x)=2\sin(x)\cos(x)$$

$$\cos(2x)=\cos^2(x)-\sin^2(x)$$

$$\sin(x)\cos(y)=\frac{1}{2}[\sin(x+y)+\sin(x-y)]$$

$$\cos(x)\cos(y)=\frac{1}{2}[\cos(x+y)+\cos(x-y)]$$

$$\sin(x)\sin(y)=\frac{1}{2}[\cos(x-y)-\cos(x+y)]$$

**Tayler Series:**

$$\sin(x)\approx x-\frac{x^3}{6}+\frac{x^5}{120}-\cdots$$

$$\cos(x)\approx 1-\frac{x^2}{2}+\frac{x^4}{24}-\cdots$$

$$\tan(x)\approx x+\frac{x^3}{3}+\frac{2x^5}{15}+\cdots$$

**Series Summation:**

$$\sum_{k=1}^{m}k=\frac{m(m+1)}{2}$$

$$\sum_{k=1}^{m}k^2=\frac{m(m+1)(2m+1)}{6}$$

$$\sum_{k=1}^{m}k^3=\left[\frac{m(m+1)}{2}\right]^2$$

**Hyperbolic functions:**

$$\frac{d}{dx}(\sinh ax) = a\cosh x$$

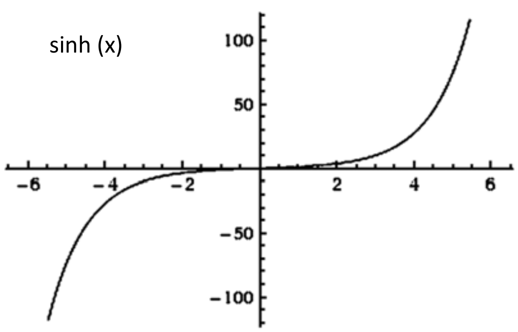
$$\sinh(x+y) = \sinh(x)\cosh(y) + \cosh(x)\sinh(y)$$

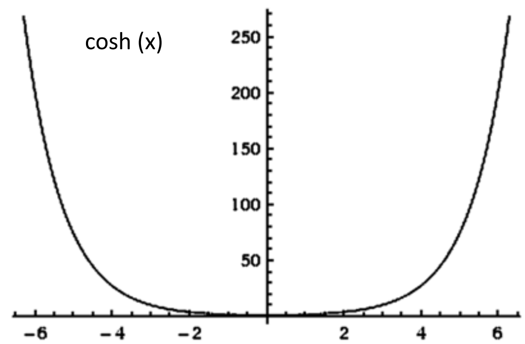
$$\frac{d}{dx}(\cosh ax) = a\sinh ax$$

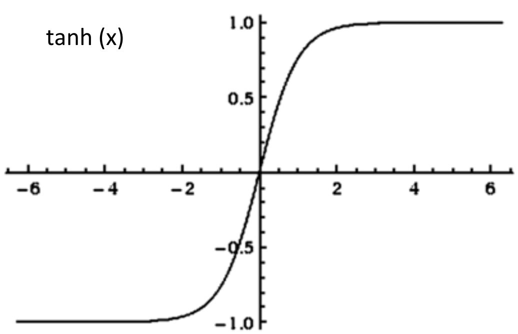
$$\cosh(x+y) = \cosh(x)\cosh(y) + \sinh(x)\sinh(y)$$

$$\frac{d}{dx}(\tanh ax) = \frac{a}{\cosh^2 ax}$$

**Sketch of hyperbolic functions**



1. [ 44 Marks] Derivation of system temperatures and temperature dependent physical properties

(a) Find the temperature of 0.1 mole of helium gas, which occupy a volume of 2 litres at atmospheric pressure (1 Atmospheric Pressure = 101,325 Pascal).

(b) A system of gas occupying energy levels exhibits the Maxwell-Boltzmann distribution. The population distribution in the energy levels is given in Table Q1a. Find the temperature of the gas system.
Hint: A suitable graph plotting with linear regression equation may help.

Table Q1a

| Energy (eV) | Population ~~(%)~~ (Arbitrary Unit) |
|---|---|
| $3.01\times10^{-2}$ | 3.1 |
| $2.15\times10^{-2}$ | 8.5 |
| $1.29\times10^{-2}$ | 23 |
| $4.30\times10^{-3}$ | 63 |
| $2.00\times10^{-4}$ | 97 |

(c) In a cryogenic (低溫) experiment, heat is transferred to a sample at a constant rate of 0.05 W. The entropy of the sample with respect with time is listed in Table Q1b. i) Plot a nonlinear curve of entropy against time. ii) With the help of the second-order polynomial curve fitting, derive the general equation of the sample temperature as a function of time t. iii) Find the temperature of the sample at time t = 500 seconds. (iv) Find the time at which the entropy of the sample becomes steady.

Table Q1b

| Time (s) | 100 | 200 | 300 | 400 | 500 | 600 | 700 | 800 |
|---|---|---|---|---|---|---|---|---|
| Entropy (J/K) | 2.3 | 2.6 | 2.85 | 3.0 | 3.11 | 3.2 | 3.25 | 3.27 |

(d) The molar specific heat of a gas is measured at constant volume and found to be $11R/2$, where $R$ is the Universal gas constant. Is the gas most likely to be monatomic, diatomic, or polyatomic?

(e) Figure Q1 illustrates 2 identical solid copper spheres. One of the copper spheres lies on a thermally insulating plate (Sphere A), whereas the other hangs from a thermally insulating thread (Sphere B). Equal amounts of heat are transferred to the 2 spheres. Identify the form(s) of energy that is/are transferred to or from the spheres (kinetic energy, potential energy, and/or internal energy?). Which sphere has the higher temperature, or both the spheres have the same temperature?
Hint: A very small amount of temperature change is considered as a temperature change.

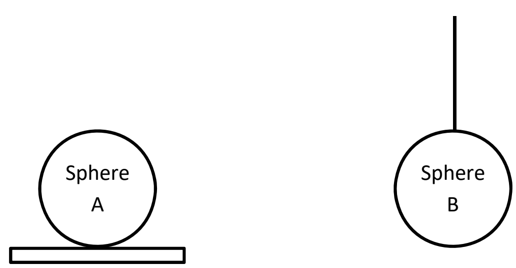



Fig. Q1

2. [ 28 Marks ] Corrections to the oscillation period of pendulum

For small oscillations, the period ($T$) of a pendulum (mass of bob $m$, and massless rope) is approximately $T \approx 2\pi\sqrt{L/g}$, where $L$ is the length of pendulum, and g is the acceleration due to gravity. However, for a more accurate result, the oscillation period is a function of swing amplitude $\theta_o$ (the maximum angle that the pendulum swings away from vertical).

(a) Find the exact expression for the period $T$. Hint: you may consider the pendulum motion swings from $\theta_o$ to $\theta$ (Fig. Q2). Express your answer in integral form, and express your answer in terms of $L$, g, $\theta$, and $\theta_o$.

Hint: Take zero potential energy at the lowest point on the pendulum path.

(b) By using Taylor series expansion to expand your integrand [your answer in Part (a)] in power of $\theta_o$, find the period of the pendulum up to the second order in $\theta_o^2$. Comment whether the period approximated by $T \approx 2\pi\sqrt{L/g}$ is underestimated (低估) or overestimated (高估).

Hint:

i. $\cos(x) = 1 - 2\sin^2(x/2)$.

ii. $\int\left[1 + k\sin^2(x)\right]dx = \frac{kx}{2} - \frac{k}{4}\sin(2x) + x + \text{constant}$, where $k$ is a constant.

iii. Change of variable $\sin(y) = \sin(\theta/2)/\sin(\theta_o/2)$ may simplify your calculation.

iv. Series expansion at $x = 0$: $\frac{1}{\sqrt{1-a^2x^2}} = 1 + \frac{a^2}{2}x^2 + \frac{3a^4}{8}x^4 + \frac{5a^6}{16}x^6 + \frac{35a^8}{128}x^8 + \cdots\cdots$

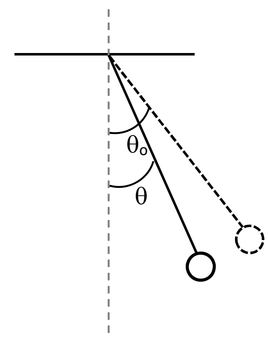



Fig. Q2

3. [ 9 Marks ] Charged thin rod(s)

(a) As illustrated in Fig. Q3a, calculate the electric field at a point (labelled as '•', distance $d$ from one end) along the long axis of a rod (length $L$, uniform positive charge per unit length $\sigma$, total charge $Q$). In addition, find the electric fields under the limiting cases of very short rod and very long rod. Express your answer in terms of $k_e$ (Coulomb Constant), $\sigma$, $L$, $d$, and/or $Q$.

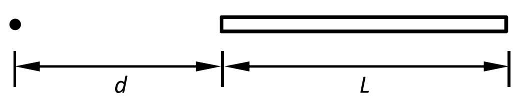



Fig. Q3a

(b) Two thin identical rods (length $L$) carry equal charges +Q uniformly distributed along the lengths (Fig. Q3b). The rods lie along the same axis with the separation d from their centers. Find the force exerted by the left rod on the right rod. Express your answer in terms of $k_e$ (Coulomb Constant), $Q$, $L$, and $d$.

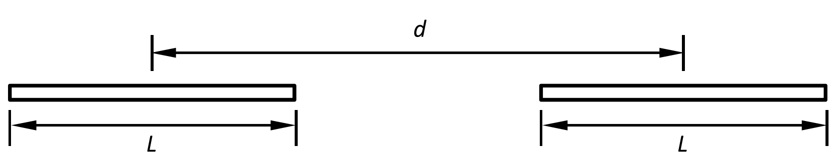
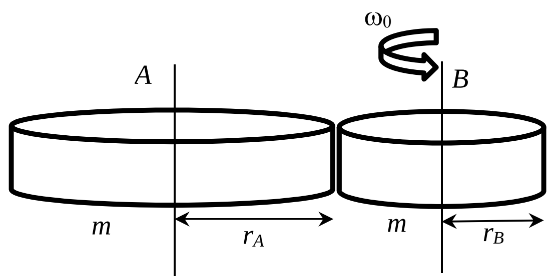



Fig. Q3b

4. [ 19 Marks ] Two steel uniform circular wheels *A* and *B* of equal mass *m* and of radii $r_A$ and $r_B$ respectively, considered nearly perfectly rigid without any elastic hysteresis, are brought into contact. Initially *B* has an angular velocity $\omega_o$ while the other at rest. They then rotate without slipping along their own smooth fixed axes passing through their centers.




To determine how they will be eventually, we employ a simple toy model that the wheels are virtually spur gears with microscopic teeth. They are brought into contact in the manner that *A* sits on its axis all the time, and *B*, rotating with its initial angular velocity, is dropped quasi-statically along its axis so perfectly that *B* meshes with *A* in negligible time without any teeth pressing each other at $t = 0$.

(a) Write down the equations of motion of *A* and *B* in first order approximation in terms of time-dependent functions of angular velocities for $t \geq 0$ up to a constant. State the physical meaning of the constant.
(b) Find their steady-state angular velocities.
(c) Give brief analytic and/or quantitative comments on the results in (b) compared to the daily observations.

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