Physics Enhancement Programme Phase 2
Selection Test 3 (Total 50 points)
11 February 2017

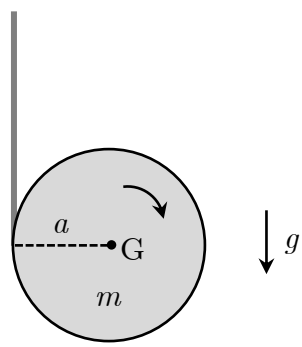
Physical Parameters

Planck's constant	$h = 4.136 \times 10^{-15}\text{eV} \cdot \text{s}$
Electron mass	$m_e = 9.109 \times 10^{-31}\text{kg}$
Proton mass	$m_p = 1.673 \times 10^{-27}\text{kg}$
Ground state energy of hydrogen	$E_0 = -13.6\ \text{eV}$
Bohr radius of hydrogen	$r_0 = 0.529\text{Å}$

1. Non-descending Yo-yo (6 points) 不降的摇摇（6 分）

Consider a yo-yo as a uniform cylinder of mass m, radius a, and moment of inertia mk^2 about the horizontal axis through its center of gravity G, where k is the radius of gyration. The free end of the string is held, and the yo-yo descends unwinding the string without slipping.

(a) Find the tension in the string in terms of a, g, k, and m, where g is the acceleration due to gravity. [3]



(b) In order to maintain the spinning yo-yo at the same vertical level, the free end of the string is pulled upward. Find the acceleration of the free end in terms of a, g, and k. [3]

2. Mirage (12 points) 海市蜃楼（12 分）

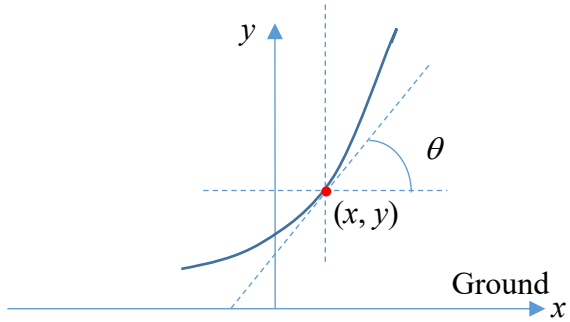
A light ray is travelling inside a medium with refractive index $n(y)$ varying with height y. At (x, y), the light ray makes an angle $\theta(y)$ with the horizontal direction, as shown in the figure below. Suppose the light ray passes through $(0, y_0)$, at which $\theta = \theta_0$.

In this problem, you may find it convenient to use the following defined functions:

$$\cosh x = \frac{1}{2}(e^x + e^{-x}) \text{ and } \sinh x = \frac{1}{2}(e^x - e^{-x}),$$

and their properties

$$\cosh^2 x - \sinh^2 x = 1, \frac{d}{dx}\cosh x = \sinh x, , \frac{d}{dx}\sinh x = \cosh x.$$



(a) Derive the differential equation satisfied by the trajectory of the light ray. [3]

(b) Suppose one can approximately let $n(y) = ay + b$ for $y \geq 0$, where $a, b > 0$. Find the equation of the trajectory. [Hint: The integral $\int \frac{du}{\sqrt{u^2-1}} = \cosh^{-1} u$ may be useful.] [5]

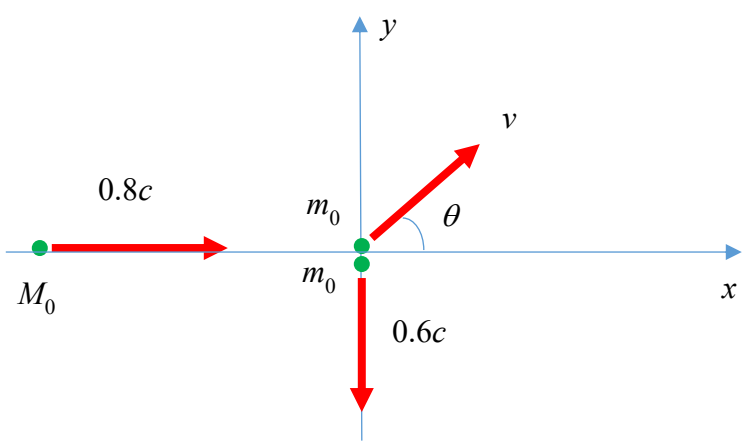
(c) Suppose the approximation in (c) is valid as a model of mirage. State the condition for the formation of mirage in terms of y_0, θ_0, a and b. [4]

3. Decay of a Relativistic Particle (12 points) 相对论粒子的衰变（12 分）

As shown in the figure below, a particle with rest mass M_0 initially moving with speed $0.8c$ in the positive x direction decays into two particles with equal rest mass m_0. One of the two particles moves with speed $0.6c$ in the $-y$ direction. Find

(a) the speed v and angle θ of the other particle,

(b) the ratio m_0/M_0.



4. A Tauon-Hydrogen Atom (8 points) Tau 粒子-氢原子（8 分）

The particle tauon (τ) has the same charge as that of an electron but is 210 times more massive. A tauon with initial speed of 8.0×10^5 m/s is captured by a proton to form a τ-hydrogen atom.

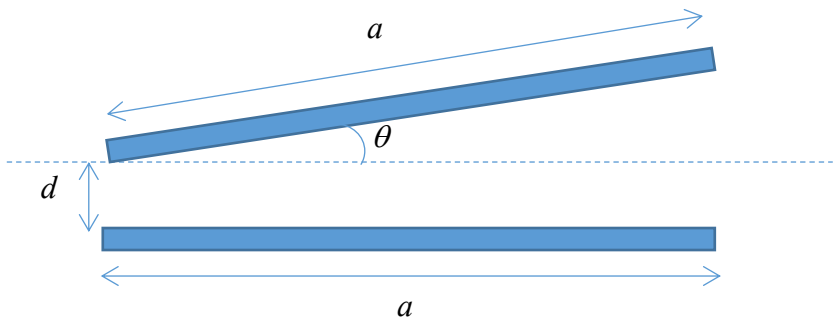
(a) When the atom decays to ground state, radiation will be emitted. What is the wavelength (in angstrom, Å) of the radiation?

(b) To excite a τ-hydrogen atom in ground state to excited states, what is the maximum possible wavelength (in Å)?

(c) What is the distance (in Å) between the tauon and the proton when the atom is in ground state?

5. Inclined Capacitor (6 points) 斜面电容器（6 分）

As shown in the figure, a capacitor is formed by two square plates both with side length a, with minimum separation $d \ll a$. When the inclination angle θ is very small, derive the capacitance up to first order in θ.



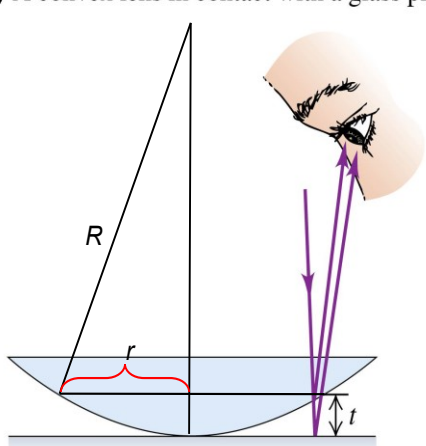
6. Newton's Rings(6 points) 牛顿环（6 分）

Figure (a) shows a convex lens with radius of curvature R lying on a flat glass plate (both glasses have the refractive index $n_g = 1.5$) and illuminated from above by light with wavelength λ. Figure (b) (a photograph taken from above the lens) shows that circular interference fringes (called Newton's rings) appear, associated with the variable thickness d of the air film between the lens and the plate.

(a) Find the radii r of the interference maxima assuming $r \ll R$. [3]

(b) Now, the entire setup is immersed in water (with refractive index $n_w = 1.33$). What are the new values of the radii r of the interference maxima assuming $r \ll R$. [3]

(a) A convex lens in contact with a glass plane



(b) Newton's rings: circular interference fringes