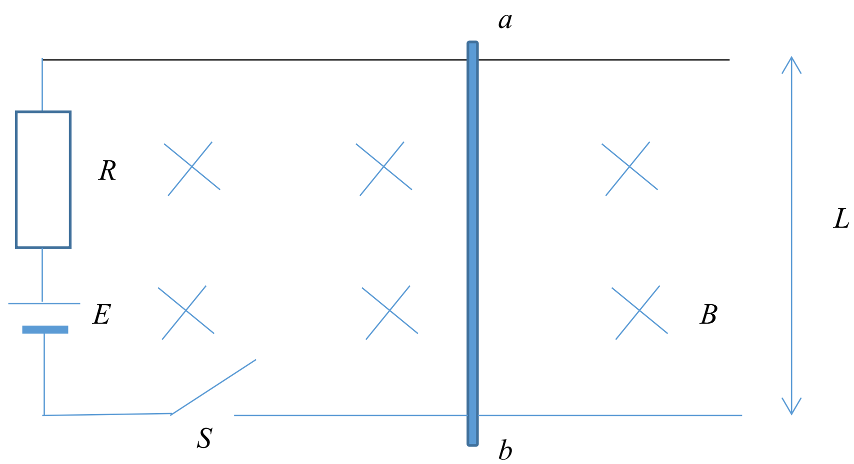
Physics Enhancement Programme Phase 2
Selection Test 4 (Total 50 points)
18 February 2017

1. **(9 points)** Consider the system as shown in the figure below,

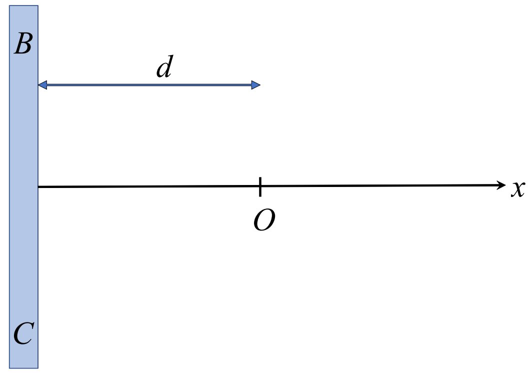


The electromotive force of the battery is E, with internal resistance r. The metal rod ab and the rails have negligible resistance. The metal rod can move on the rail. The kinetic friction is a constant force f. The circuit is inside a region of uniform magnetic field B pointing into the page.

(a) Find the terminal velocity v_T of the metal rod after the switch S is closed.
(b) The terminal velocity depends on B. Find the magnitude of the magnetic field so that v_T is maximum. What is the current in this case?
(c) If $r > R$, and when the velocity of the metal rod is v_p, the power output external to the battery is maximum $P_{\max}$, find the relation between $P_{\max}$ and v_p.

2. **(8 points)** An elastic string has a natural length of 1 m. It is assumed that when it is stretched, the restoring force obeys Hooke's law. One end of the string A is fixed at the ceiling, while the other end B is attached to a small ball with mass m = 0.2 kg. It is measured that after the ball is attached, the length of the string is stretched to 2 m at equilibrium. Now if the ball is released from rest at point A, after how long will it hit the ceiling? (g = 9.8 ms^{-2})

3. **(9 points)** In the figure, the origin O is a source which oscillates with amplitude A and angular frequency ω, that is, $y(0,t) = A\cos\omega t$. The oscillation generates a one-dimensional sinusoidal wave with wavelength λ along the x-axis of the stretched string with linear density ρ. BC is a dense reflecting plane where all incoming waves get reflected. $d = 5\lambda/4$ where λ is the wavelength.

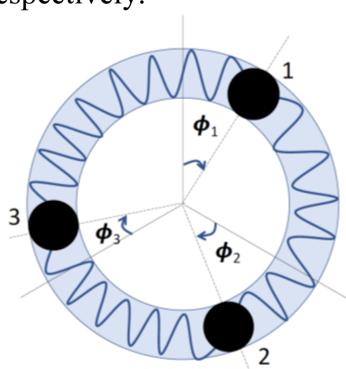


(a) Find the wave function for $0 > x > -d$ and $x > 0$.

(b) Find the time-averaged mechanical energy per unit length for $0 > x > -d$. (Hint: You can calculate the kinetic energy and potential energy of a small segment at x separately.)

(c) Find the time-averaged mechanical power transmitted by the oscillator.

4. **(8 points)** Three identical particles, each of mass m, are constrained to lie on a horizontal circle of radius R and are connected by identical springs lying on the circle, each of spring constant k and natural length $l = 2\pi R/3$. Initially, three particles are located at the vertices of an equilateral triangle. In the figure, ϕ_1, ϕ_2, ϕ_3 are the angular displacements of particles from their equilibrium position respectively.



(a) With small angular displacement, find the angular frequencies of oscillations in this system.

(b) Derive the general forms of $\{\phi_1(t), \phi_2(t), \phi_3(t)\}$ for these small oscillations.

(c) At $t = 0$, $\{\phi_1(0), \phi_2(0), \phi_3(0)\} = \{A, 2A, -3A\}$ and $\{\dot{\phi}_1(0), \dot{\phi}_2(0), \dot{\phi}_3(0)\} = \{0,0,0\}$. Derive $\{\phi_1(t), \phi_2(t), \phi_3(t)\}$ at any time $t > 0$.

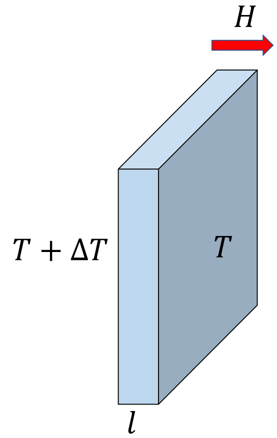
5. **Heat Transfer (8 points)**

An air-conditioner is a machine that extracts heat from indoor (cold-environment) and dumps into outdoor (hot environment) by doing work. Assume that the absolute temperatures of hot- and cold-environment are T_1 and T_2 respectively. If the heat extracted from the cold-environment is Q and the work done by the air-conditioner is W, we have

$$\frac{Q}{W} \leq \frac{T_2}{T_1 - T_2}$$

where the equality sign holds for the ideal air-conditioner.

An ideal air-conditioner is operating during the summer when the outdoor temperature is 35ºC and the indoor temperature is maintained at 20ºC. As the outdoor temperature is higher, heat is transferred from outdoor to indoor via conduction which satisfies the following condition: Consider a layer of material with thickness l, surface area S. The temperature difference between two sides of the material is ΔT. The heat transfer rate (heat energy transferred per second) through this conducting layer from high temperature to low temperature is



$$H = \kappa \frac{\Delta T}{l} S$$

where κ is the thermal conductivity which is a constant.

(a) If the heat can only be transferred into the indoor via the window glass with surface area $S = 5\text{m}^2$ and $l = 2\text{mm}$. And the thermal conductivity of glass is $\kappa = 0.75\text{W}/(\text{m}\cdot\text{K})$. Calculate the amount of work done by the air-conditioner in 12 hours.
(b) If the window glass is replaced by the double-layer glass where the thickness of each glass is $l = 2\text{mm}$. Between two glasses, there is a layer of air with thickness $l_0 = 0.5\text{mm}$ and the thermal conductivity of air is $\kappa_0 = 0.025\text{W}/(\text{m}\cdot\text{K})$. Calculate the amount of work-done by the air-conditioner in this case.
(c) In Hong Kong, the cost of electricity is \$0.8 per kilowatt hour (kW·h), calculate how much we can save by replacing the window with double-layer glass in every 12 hours.

6. (8 points) A plane of electromagnetic wave of frequency ω is normally incident on a thick glass of unit area where the permittivity and permeability are ε and μ respectively. The incident electric field has amplitude E and wavenumber k and the reflected field has amplitude rE. The transmitted electric field has amplitude E_m and wavenumber k_m.
   (a) Find E_m/B_m where B_m is the magnetic field amplitude in the glass. Assume that the glass is nonmagnetic so that $\mu = \mu_0$.
   (b) Find r by considering the boundary conditions at the plate and the result of (a).
   (c) Calculate the fraction of time-averaged power carried by the reflected and transmitted waves respectively.