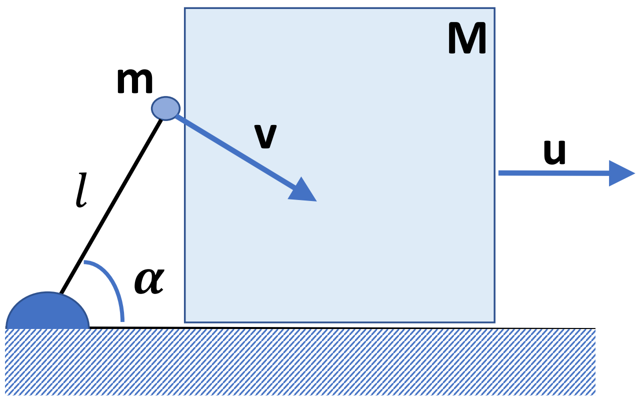
Physics Enhancement Programme Phase 1
Selection Test 1 (Total 100 points)
16 September 2017

1. (20 points) A massless rod with length l is connected to a smooth horizontal surface with a hinge; a point mass with mass m is attached at the end of the rod. Initially the rod is vertical (i.e. $\alpha = \pi/2$ in the figure) and the point mass rests against the block of mass M. The system is left to move freely and after some time the point mass loses contact with the surface of the block – at the moment when the rod forms an angle $\alpha = \pi/6$ with the horizontal. Let the speed of the point mass be v while the speed of the block is u at this moment. Notice that both the point mass and the block will have the same horizontal velocity and acceleration before losing contact. The interface between the point mass and the block is smooth.

a) Find the ratio u/v in terms of α.
b) What is the normal force between the point mass and the block at the moment of separation?
c) What is the tension in the rod at this moment?
d) Find the ratio of the masses M/m.
e) Find the velocity u of the block at this moment.



a) $u = v \sin \alpha$
b) The normal force is zero
c) since the block has zero acceleration (zero normal force), the sphere also has zero horizontal acceleration and hence the tension in the rod must be zero.

d) The energy conservation gives,

$$mgl(1 - \sin \alpha) = \frac{1}{2}mv^2 + \frac{1}{2}Mu^2$$

Finally, we look at the radical acceleration of the sphere,

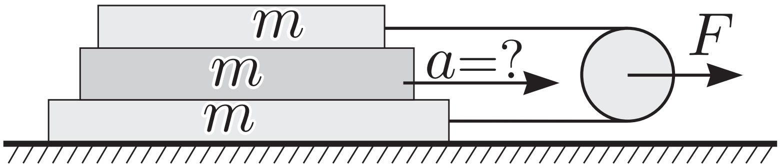
$$mg \sin \alpha = \frac{mv^2}{l} \quad \rightarrow \quad v^2 = \frac{gl}{2}$$

Hence, we get

$$\frac{M}{m} = 4$$

e) $u = \sqrt{\frac{gl}{8}}$

2. (20 points) A block of mass *m* lies on a frictionless horizontal table. On top of it lies another block of mass *m*, and on top of that – another block of mass *m*. A thread that connects the first and the third block has been extended around a weightless pulley. The threads are horizontal and the pulley is being pulled by a force *F*. The coefficients of static and kinetic friction between the blocks are both μ.



a) What is the acceleration of the second block if all three blocks move together? What is the maximum value of the force *F* to make it possible?
b) What is the acceleration of the second block if the top one slides and the bottom two stay together? What is the range of the force *F* to make it possible?
c) What is the acceleration of the second block if every blocks slide? What is the range of the force *F* to make it possible?

We can apply the 2nd law on each block separately.

$$T - f_1 = ma_1$$
$$f_1 + f_2 = ma_2$$
$$T - f_2 = ma_3$$

And since the pulley is massless, we have

$$2T = F$$

a) If all 3 blocks move together, we have $3ma = F$ or $a = F/3m$ and $f_1 = f_2 = F/3$. Hence $F \leq 6\mu mg$.

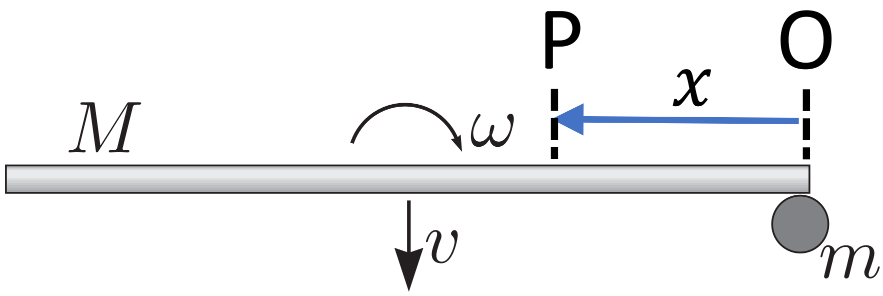
b) If the top one slides and the bottom two stay together, we have $\frac{F}{2} + \mu mg = 2ma_2$ or $a_2 = \frac{F}{4m} + \frac{1}{2}\mu g$.

   In this case, the static friction $2\mu mg \geq f_2 = \frac{F}{4} - \frac{1}{2}\mu mg$ or $6\mu mg \leq F \leq 10\mu mg$

c) If every block slides, $a_2 = 3\mu g$ and $F > 10\mu mg$

3. (20 points) A uniform rod with mass M and length l is sliding on ice surface while rotating. The velocity of the rod's center of mass is v, its angular velocity is ω. At the moment when the rod is perpendicular to the velocity of its center of mass, the rod hits a stationary puck with mass m located at the origin O. The rod is straight and its mass density is uniform.

a) Find the resultant velocity of the rod at point P which is at distance x from the origin.
b) Find the total angular momentum of the system with respect to the origin before the collision. (Hint: The moment of inertia of a rod of length l at the center of mass is $I = \frac{1}{12}ml^2$.)
c) Find the ratio of ω/v in terms of l if the rod stays at rest while the puck slides away after the collision.
d) Find the ratio of the masses M/m in the situation. The collisions are perfectly elastic.



a) Combining the rotational and the translational motion, the resultant velocity is

$$v' = v + \left[\frac{l}{2} - x\right]\omega$$

b) With respect to the point of impact, the angular momentum is conversed and is equal to zero if the rod stays in place.

The angular momentum before the collision,

$$L = \lambda \int_0^l \left(v + \left[\frac{l}{2} - x\right]\omega\right) x dx = \frac{Mvl}{2} - \frac{Ml^2}{12}\omega$$

c) After the collision, the angular momentum is zero. Hence we have $\omega = 6v/l$

d) From the conservation of energy and momentum,

$$Mv = mV$$

$$\frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mV^2$$

eliminating v and V, we get. M/m=4

4. (10 points) If a man blowing a whistle of frequency 500 Hz moves away from a stationary observer towards a fixed wall, in a direction perpendicular to the wall at 2.00m/s, determine the beat frequency heard by the stationary observer if the speed of sound in air is *c*=330m/s.

The Doppler effect is

$$f_L = \left(\frac{v}{v + v_S}\right) f_S$$

where f_S=500Hz.

As the source is moving away from the observer, v_S=2m/s

$$f_{L1} = \left(\frac{330}{332}\right) \times 500$$

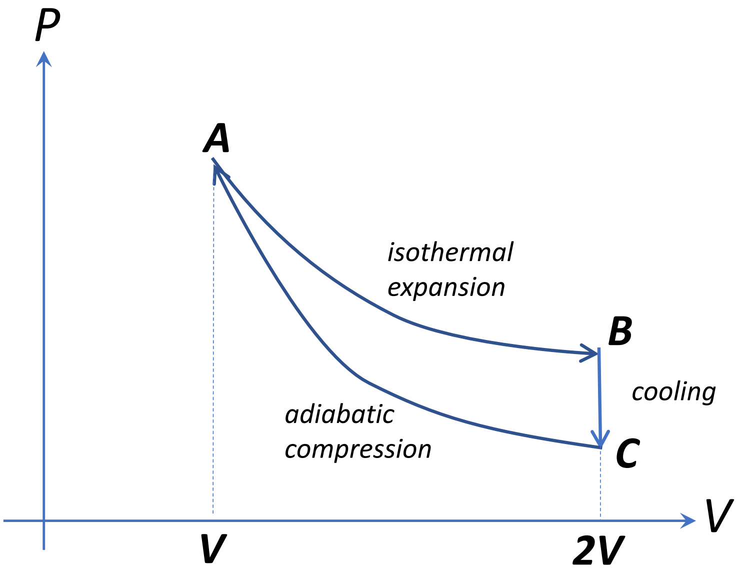
The reflected wave is as the source is moving towards the observer, v_S=-2m/s

$$f_{L2} = \left(\frac{330}{328}\right) \times 500$$

The beat frequency is 6Hz

5. (30 points) A cylinder with a piston contains 0.25 mol of oxygen at 2.4×10^5Pa and 335K initially at point A in the figure below. The oxygen may be treated as a diatomic ideal gas with $\gamma = 1.4$. The gas first expands isothermally (constant temperature) to point B with 2 times the original volume. It is then cooled at constant volume to point C, and finally it is compressed adiabatically to its original volume and pressure. (The universal gas constant $R = 8.314$J/mol·K)

   a) Compute pressure P_B after the isothermal expansion at B.
   b) Calculate the change in the internal energy, the work done and the heat absorbed by the gas during the isothermal expansion A→B.
   c) Compute the pressure P_C and temperature T_C at C so that the gas can be compressed adiabatically to its original volume and pressure at A.
   d) Compute the change in the internal energy, the work done and the heat absorbed by the gas during the cooling process B→C.
   e) Compute the change in the internal energy, the work done and the heat absorbed by the gas during the adiabatic compression C→A.
   f) What is the efficiency of this heat engine?



Solution:
a) During the isothermal expansion, PV=nRT=const, P_2=P_1/2=1.2×10^5Pa.

b) Since the internal energy of an ideal gas depends solely on temperature, the change of internal energy is zero during isothermal expansion.
The work done by the gas

$$W_{1\to2} = \int_V^{2V} pdV = nRT\int_V^{2V}\frac{dV}{V} = nRT_1\ln 2 = 482.6J$$

From the first law, heat absorbed by the gas Q=W=482.6J

c) During the adiabatic compression, we have PV^{γ}=const and $TV^{\gamma-1}$=const.
So $P_3 = 0.909\times10^5$Pa and $T_3 = 253.9K$.

d) During the cooling, work done is zero.

$C_V = \frac{5}{2}R = 20.785$J/mol·K

Heat absorbed $Q = nC_V(T_3 - T_1)$=-421.4J

And the change in internal energy = -421.4J

e) During the adiabatic compression, heat absorption is zero.

After a cycle, the change in the internal energy must be zero.

Hence, $\Delta U = 421.4J$

And the work done by the gas

$$W_{C\rightarrow A} = Q - \Delta U = 0 - 421.4J = -421.4J$$

e) Efficiency = (Net work done)/(Heat absorbed) = =(482.6 - 421.4)/482.6 = 12.7%