Waves and their properties

A wave is any disturbance that propagates from one region to another. A mechanical wave travels within some material called the medium. The wave speed v depends on the type of wave and the properties of the medium.

In a periodic wave, the motion of each point of the medium is periodic with frequency f and period T.The wavelength λ is the distance over which the wave pattern repeats, and the amplitude A is the maximum displacement of a particle in the medium. The product of λ and f equals the wave speed. A sinusoidal wave is a special periodic wave in which each point moves in simple harmonic motion.

$$v = \lambda f$$

Wave functions and wave dynamics

The wave function $y(x,t)$ describes the displacements of individual particles in the medium. Equations below give the wave equation for a sinusoidal wave traveling in the $+x$ direction. If the wave is moving in the $-x$ direction, the minus signs in the cosine functions are replaced by plus signs.

$$y(x,t) = A\cos\left[\omega\left(\frac{x}{v} - t\right)\right]$$

$$y(x,t) = A\cos 2\pi\left[\omega\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right]$$

$$y(x,t) = A\cos(kx - \omega t)$$

Where $k = 2\pi/\lambda$ and $\omega = 2\pi f = vk$.

The wave function obeys a partial differential equation called the wave equation as below.

$$\frac{\partial^2 y(x,t)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y(x,t)}{\partial t^2}$$

The speed of transverse waves on a string depends on the tension *F* and mass per unit length m.

Waves on a string:

$$v = \sqrt{\frac{F}{\mu}}$$

Wave power

Wave motion conveys energy from one region to another. For a sinusoidal mechanical wave, the average power P_{av} is proportional to the square of the wave amplitude and the square of the frequency.
For waves that spread out in three dimensions, the wave intensity I is inversely proportional to the square of the distance from the source.

Average power, sinusoidal wave:

$$P_{av} = \frac{1}{2}\sqrt{\mu F}\,\omega^2 A^2$$

Inverse-square law for intensity:

$$\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2}$$

Wave superposition

A wave reflects when it reaches a boundary of its medium. At any point where two or more waves overlap, the total displacement is the sum of the displacements of the individual waves (principle of superposition).

Principle of superposition:

$$y(x,t) = y_1(x,t) + y_2(x,t)$$

Standing waves on a string

When a sinusoidal wave is reflected from a fixed or free end of a stretched string, the incident and reflected waves combine to form a standing sinusoidal wave with nodes and antinodes.
Adjacent nodes are spaced a distance $\lambda/2$ apart, as are adjacent antinodes. When both ends of a string with length L are held fixed, standing waves can occur only when L is an integer multiple of $\lambda/2$. Each frequency with its associated vibration pattern is called a normal mode.

Standing wave on a string, fixed end at $x = 0$:

$$y(x,t) = (A_{SW} \sin kx) \sin \omega t$$

String fixed at both ends:

$$f_n = n\frac{v}{2L} = nf_1 \quad (n = 1,2,3, \dots .)$$

$$f_1 = \frac{1}{2L}\sqrt{\frac{F}{\mu}}$$

Sound waves

Sound consists of longitudinal waves in a medium. A sinusoidal sound wave is characterized by its frequency f and wavelength λ (or angular frequency ω and wave number k) and by its displacement amplitude A. The pressure amplitude $P_{\max}$ is directly proportional to the displacement amplitude, the wave number, and the bulk modulus B of the wave medium.
The speed of a sound wave in a fluid depends on the bulk modulus B and density ρ. If the fluid is an ideal gas, the speed can be expressed in terms of the temperature T, molar mass M, and ratio of heat capacities γ of the gas. The speed of longitudinal waves in a solid rod depends on the density and Young's modulus Y.

Sinusoidal sound wave:

$$P_{\max} = BkA$$

Longitudinal wave in a fluid:

$$v = \sqrt{\frac{B}{\rho}}$$

Sound wave in an ideal gas:

$$v = \sqrt{\frac{\gamma RT}{M}}$$

Longitudinal wave in a solid rod:

$$v = \sqrt{\frac{Y}{\rho}}$$

Intensity and sound intensity level

The intensity I of a sound wave is the time average rate at which energy is transported by the wave, per unit area. For a sinusoidal wave, the intensity can be expressed in terms of the displacement amplitude A or the pressure amplitude $P_{\max}$.

Intensity of a sinusoidal sound wave in a fluid:

$$I = \frac{1}{2}\sqrt{\rho B}\,\omega^2 A^2 = \frac{{P_{\max}}^2}{2\rho v} = \frac{{P_{\max}}^2}{2\sqrt{\rho B}}$$

The sound intensity level β of a sound wave is a logarithmic measure of its intensity. It is measured relative to I_0, an arbitrary intensity defined to be $10^{-12}\ W/m^2$. Sound intensity levels are expressed in decibels (dB).

Definition of sound intensity level:

$$\beta = (10\ dB)\log\frac{I}{I_0}$$

Standing sound waves

Standing sound waves can be set up in a pipe or tube. A closed end is a displacement node and a pressure antinode; an open end is a displacement antinode and a pressure node. For a pipe of length L open at both ends, the normal-mode frequencies are integer multiples of the sound speed divided by $2L$. For a stopped pipe (one that is open at only one end), the normal-mode frequencies are the odd multiples of the sound speed divided by $4L$.

Open pipe:

$$f_n = \frac{nv}{2L} \quad (n = 1,2,3,\dots)$$

Stopped pipe:

$$f_n = \frac{nv}{4L} \quad (n = 1,3,5,\dots)$$

A pipe or other system with normal-mode frequencies can be driven to oscillate at any frequency. A maximum response, or resonance, occurs if the driving frequency is close to one of the normal-mode frequencies of the system.

Interference

When two or more waves overlap in the same region of space, the resulting effects are called interference. The resulting amplitude can be either larger or smaller than the amplitude of each individual wave, depending on whether the waves are in phase (constructive interference) or out of phase (destructive interference).

Beats

Beats are heard when two tones with slightly different frequencies f_a and f_b are sounded together. The beat frequency f_{beat} is the difference between f_a and f_b.

Beat frequency:

$$f_{\text{beat}} = f_a - f_b$$

Doppler effect

The Doppler effect for sound is the frequency shift that occurs when there is motion of a source of sound, a listener, or both, relative to the medium. The source and listener frequencies f_s and f_L are related by the source and listener velocities v_s and v_L relative to the medium and to the speed of sound v.

Doppler effect, moving source and moving listener:

$$f_L = \frac{v + v_L}{v + v_s} f_s$$

Shock waves

A sound source moving with a speed v_s greater than the speed of sound v creates a shock wave. The wave front is a cone with angle α.

$$\sin \alpha = \frac{v}{v_s}$$

[END OF SUMMARY]