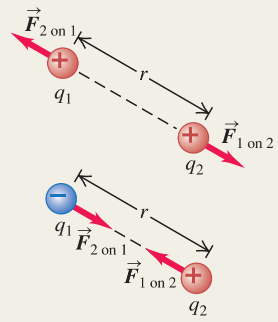
Coulomb's law: For charges q_1 and q_2 separated by a distance r, the magnitude of the electric force on either charge is proportional to the product $q_1 q_2$ and inversely proportional to r^2. The force on each charge is along the line joining the two charges—repulsive if q_1 and q_2 have the same sign, attractive if they have opposite signs. In SI units the unit of electric charge is the coulomb, abbreviated C. (See Examples 21.1 and 21.2.)

When two or more charges each exert a force on a charge, the total force on that charge is the vector sum of the forces exerted by the individual charges. (See Examples 21.3 and 21.4.)

$$F = \frac{1}{4\pi\epsilon_0}\frac{|q_1 q_2|}{r^2} \quad (21.2)$$

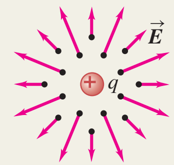
$$\frac{1}{4\pi\epsilon_0} = 8.988 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2$$



Electric field: Electric field $\vec{E}$, a vector quantity, is the force per unit charge exerted on a test charge at any point. The electric field produced by a point charge is directed radially away from or toward the charge. (See Examples 21.5–21.7.)

$$\vec{E} = \frac{\vec{F}_0}{q_0} \quad (21.3)$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r} \quad (21.7)$$

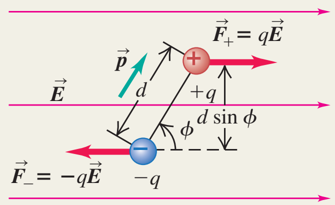


Electric dipoles: An electric dipole is a pair of electric charges of equal magnitude q but opposite sign, separated by a distance d. The electric dipole moment $\vec{p}$ has magnitude $p = qd$. The direction of $\vec{p}$ is from negative toward positive charge. An electric dipole in an electric field $\vec{E}$ experiences a torque $\vec{\tau}$ equal to the vector product of $\vec{p}$ and $\vec{E}$. The magnitude of the torque depends on the angle ϕ between $\vec{p}$ and $\vec{E}$. The potential energy U for an electric dipole in an electric field also depends on the relative orientation of $\vec{p}$ and $\vec{E}$. (See Examples 21.13 and 21.14.)

$$\tau = pE\sin\phi \quad (21.15)$$

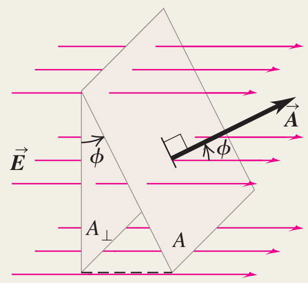
$$\vec{\tau} = \vec{p} \times \vec{E} \quad (21.16)$$

$$U = -\vec{p}\cdot\vec{E} \quad (21.18)$$



Electric flux: Electric flux is a measure of the "flow" of electric field through a surface. It is equal to the product of an area element and the perpendicular component of $\vec{E}$, integrated over a surface. (See Examples 22.1–22.3.)

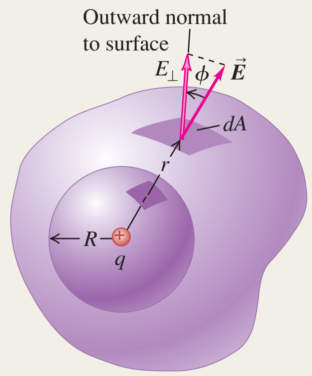
$$\Phi_E = \int E\cos\phi\, dA = \int E_{\perp}\, dA = \int \vec{E} \cdot d\vec{A} \qquad (22.5)$$



Gauss's law: Gauss's law states that the total electric flux through a closed surface, which can be written as the surface integral of the component of $\vec{E}$ normal to the surface, equals a constant times the total charge Q_{encl} enclosed by the surface. Gauss's law is logically equivalent to Coulomb's law, but its use greatly simplifies problems with a high degree of symmetry. (See Examples 22.4–22.10.)

When excess charge is placed on a conductor and is at rest, it resides entirely on the surface, and $\vec{E} = \mathbf{0}$ everywhere in the material of the conductor. (See Examples 22.11–22.13.)

$$\Phi_E = \oint E\cos\phi\, dA = \oint E_{\perp}\, dA = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0} \qquad (22.8), (22.9)$$



Electric field of various symmetric charge distributions: The following table lists electric fields caused by several symmetric charge distributions. In the table, q, Q, λ, and σ refer to the *magnitudes* of the quantities.

Charge Distribution	Point in Electric Field	Electric Field Magnitude
Single point charge q	Distance r from q	$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$
Charge q on surface of conducting sphere with radius R	Outside sphere, $r > R$	$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$
	Inside sphere, $r < R$	$E = 0$
Infinite wire, charge per unit length λ	Distance r from wire	$E = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{r}$
Infinite conducting cylinder with radius R, charge per unit length λ	Outside cylinder, $r > R$	$E = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{r}$
	Inside cylinder, $r < R$	$E = 0$
Solid insulating sphere with radius R, charge Q distributed uniformly throughout volume	Outside sphere, $r > R$	$E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$
	Inside sphere, $r < R$	$E = \frac{1}{4\pi\epsilon_0}\frac{Qr}{R^3}$
Infinite sheet of charge with uniform charge per unit area σ	Any point	$E = \frac{\sigma}{2\epsilon_0}$
Two oppositely charged conducting plates with surface charge densities $+\sigma$ and $-\sigma$	Any point between plates	$E = \frac{\sigma}{\epsilon_0}$
Charged conductor	Just outside the conductor	$E = \frac{\sigma}{\epsilon_0}$

Electric potential energy: The electric force caused by any collection of charges at rest is a conservative force. The work W done by the electric force on a charged particle moving in an electric field can be represented by the change in a potential-energy function U.

The electric potential energy for two point charges q and q_0 depends on their separation r. The electric potential energy for a charge q_0 in the presence of a collection of charges q_1, q_2, q_3 depends on the distance from q_0 to each of these other charges. (See Examples 23.1 and 23.2.)

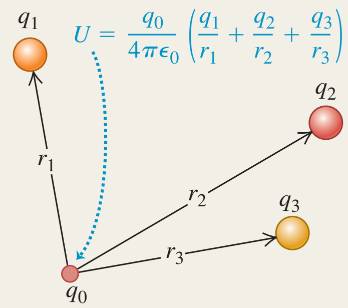
$$W_{a \to b} = U_a - U_b \qquad (23.2)$$

$$U = \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r} \qquad (23.9)$$

(two point charges)

$$U = \frac{q_0}{4\pi\epsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \cdots\right) = \frac{q_0}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i} \qquad (23.10)$$

(q_0 in presence of other point charges)



Electric potential: Potential, denoted by V, is potential energy per unit charge. The potential difference between two points equals the amount of work that would be required to move a unit positive test charge between those points. The potential V due to a quantity of charge can be calculated by summing (if the charge is a collection of point charges) or by integrating (if the charge is a distribution). (See Examples 23.3, 23.4, 23.5, 23.7, 23.11, and 23.12.)

The potential difference between two points a and b, also called the potential of a with respect to b, is given by the line integral of $\vec{E}$. The potential at a given point can be found by first finding $\vec{E}$ and then carrying out this integral. (See Examples 23.6, 23.8, 23.9, and 23.10.)

$$V = \frac{U}{q_0} = \frac{1}{4\pi\epsilon_0}\frac{q}{r} \qquad (23.14)$$

(due to a point charge)

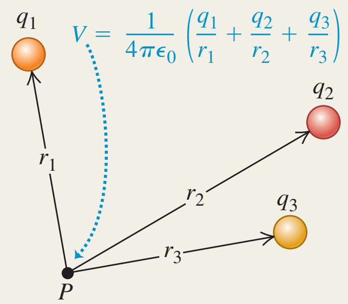
$$V = \frac{U}{q_0} = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i} \qquad (23.15)$$

(due to a collection of point charges)

$$V = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r} \qquad (23.16)$$

(due to a charge distribution)

$$V_a - V_b = \int_a^b \vec{E} \cdot d\vec{l} = \int_a^b E\cos\phi \, dl \qquad (23.17)$$



Finding electric field from electric potential: If the potential V is known as a function of the coordinates x, y, and z, the components of electric field $\vec{E}$ at any point are given by partial derivatives of V. (See Examples 23.13 and 23.14.)

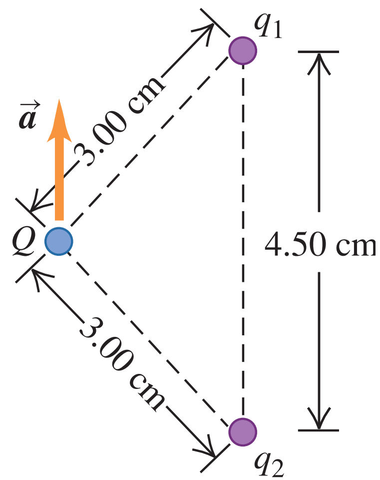
$$E_x = -\frac{\partial V}{\partial x} \quad E_y = -\frac{\partial V}{\partial y} \quad E_z = -\frac{\partial V}{\partial z} \tag{23.19}$$

$$\vec{E} = -\left(\hat{\imath}\frac{\partial V}{\partial x} + \hat{\jmath}\frac{\partial V}{\partial y} + \hat{k}\frac{\partial V}{\partial z}\right) \tag{23.20}$$

(vector form)

21.76 ••• Two point charges q_1 and q_2 are held in place 4.50 cm apart. Another point charge $Q = -1.75\ \mu C$ of mass 5.00 g is initially located 3.00 cm from each of these charges (Fig. P21.76) and released from rest. You observe that the initial acceleration of Q is 324 m/s^2 upward, parallel to the line connecting the two point charges. Find q_1 and q_2.

Figure **P21.76**



21.76. **IDENTIFY:** For the acceleration (and hence the force) on Q to be upward, as indicated, the forces due to q_1 and q_2 must have equal strengths, so q_1 and q_2 must have equal magnitudes. Furthermore, for the force to be upward, q_1 must be positive and q_2 must be negative.

SET UP: Since we know the acceleration of Q, Newton's second law gives us the magnitude of the force on it. We can then add the force components using $F = F_{Qq_1}\cos\theta + F_{Qq_2}\cos\theta = 2F_{Qq_1}\cos\theta$. The electrical force on Q is given by Coulomb's law, $F_{Qq_1} = \frac{1}{4\pi\epsilon_0}\frac{Qq_1}{r^2}$ (for q_1) and likewise for q_2.

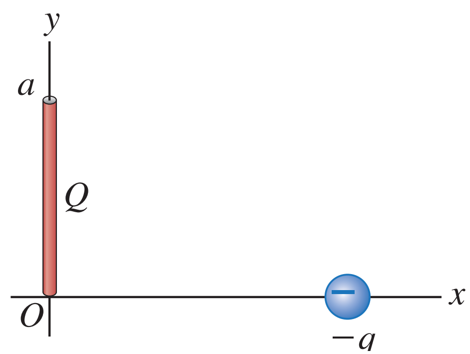
EXECUTE: First find the net force: $F = ma =$ $(0.00500\text{ kg})(324\text{ m/s}^2) = 1.62\text{ N}$. Now add the force components, calling θ the angle between the line connecting q_1 and q_2 and the line connecting q_1 and Q. $F = F_{Qq_1}\cos\theta + F_{Qq_2}\cos\theta = 2F_{Qq_1}\cos\theta$ and $F_{Qq_1} = \frac{F}{2\cos\theta} = \frac{1.62\text{ N}}{2\left(\frac{2.25\text{ cm}}{3.00\text{ cm}}\right)} = 1.08\text{ N}$. Now find the charges by solving for q_1 in Coulomb's law and use the fact that q_1 and q_2 have equal magnitudes but opposite signs. $F_{Qq_1} = \frac{1}{4\pi\epsilon_0}\frac{|Q|q_1}{r^2}$ and $q_1 = \frac{r^2 F_{Qq_1}}{\frac{1}{4\pi\epsilon_0}|Q|} = \frac{(0.0300\text{ m})^2(1.08\text{ N})}{(9.00\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(1.75\times10^{-6}\text{ C})} = 6.17\times10^{-8}\text{ C}$.

$q_2 = -q_1 = -6.17\times10^{-8}\text{ C}$.

EVALUATE: Simple reasoning allows us first to conclude that q_1 and q_2 must have equal magnitudes but opposite signs, which makes the equations much easier to set up than if we had tried to solve the problem in the general case. As Q accelerates and hence moves upward, the magnitude of the acceleration vector will change in a complicated way.

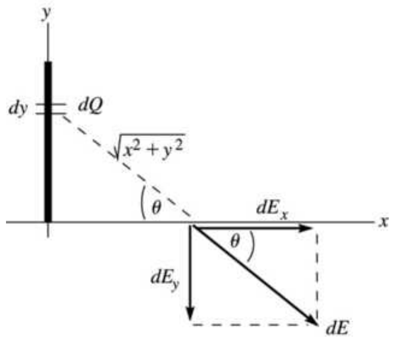
21.90 •• **CALC** Positive charge Q is distributed uniformly along the positive y-axis between $y = 0$ and $y = a$. A negative point charge $-q$ lies on the positive x-axis, a distance x from the origin (Fig. P21.90). (a) Calculate the x- and y-components of the electric field produced by the charge distribution Q at points on the positive x-axis. (b) Calculate the x- and y-components of the force that the charge distribution Q exerts on q. (c) Show that if $x \gg a$, $F_x \cong -Qq/4\pi\epsilon_0 x^2$ and $F_y \cong +Qqa/8\pi\epsilon_0 x^3$. Explain why this result is obtained.

Figure **P21.90**



1.90. **IDENTIFY:** Use Eq. (21.7) to calculate the electric field due to a small slice of the line of charge and integrate as in Example 21.10. Use Eq. (21.3) to calculate $\vec{F}$.

SET UP: The electric field due to an infinitesimal segment of the line of charge is sketched in Figure 21.90.



$$\sin\theta = \frac{y}{\sqrt{x^2+y^2}}$$

$$\cos\theta = \frac{x}{\sqrt{x^2+y^2}}$$

Figure 21.90

Slice the charge distribution up into small pieces of length dy. The charge dQ in each slice is $dQ = Q(dy/a)$. The electric field this produces at a distance x along the x-axis is dE. Calculate the components of $d\vec{E}$ and then integrate over the charge distribution to find the components of the total field.

EXECUTE: $dE = \dfrac{1}{4\pi\epsilon_0}\left(\dfrac{dQ}{x^2+y^2}\right) = \dfrac{Q}{4\pi\epsilon_0 a}\left(\dfrac{dy}{x^2+y^2}\right)$

$$dE_x = dE\cos\theta = \frac{Qx}{4\pi\epsilon_0 a}\left(\frac{dy}{(x^2+y^2)^{3/2}}\right)$$

$$dE_y = -dE\sin\theta = -\frac{Q}{4\pi\epsilon_0 a}\left(\frac{ydy}{(x^2+y^2)^{3/2}}\right)$$

$$E_x = \int dE_x = -\frac{Qx}{4\pi\epsilon_0 a}\int_0^a \frac{dy}{(x^2+y^2)^{3/2}} = \frac{Qx}{4\pi\epsilon_0 a}\left[\frac{1}{x^2}\frac{y}{\sqrt{x^2+y^2}}\right]_0^a = \frac{Q}{4\pi\epsilon_0 x}\frac{1}{\sqrt{x^2+a^2}}$$

$$E_y = \int dEy = -\frac{Q}{4\pi\epsilon_0 a}\int_0^a \frac{ydy}{(x^2+y^2)^{3/2}} = -\frac{Q}{4\pi\epsilon_0 a}\left[-\frac{1}{\sqrt{x^2+y^2}}\right]_0^a = -\frac{Q}{4\pi\epsilon_0 a}\left(\frac{1}{x}-\frac{1}{\sqrt{x^2+a^2}}\right)$$

(b) $\vec{F} = q_0\vec{E}$

$$F_x = -qE_x = \frac{-qQ}{4\pi\epsilon_0 x}\frac{1}{\sqrt{x^2+a^2}};\; F_y = -qE_y = \frac{qQ}{4\pi\epsilon_0 a}\left(\frac{1}{x}-\frac{1}{\sqrt{x^2+a^2}}\right)$$

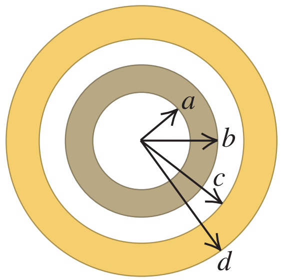
(c) For $x \gg a$, $\dfrac{1}{\sqrt{x^2+a^2}} = \dfrac{1}{x}\left(1+\dfrac{a^2}{x^2}\right)^{-1/2} = \dfrac{1}{x}\left(1-\dfrac{a^2}{2x^2}\right) = \dfrac{1}{x}-\dfrac{a^2}{2x^3}$

$$F_x \approx -\frac{qQ}{4\pi\epsilon_0 x^2},\; F_y \approx \frac{qQ}{4\pi\epsilon_0 a}\left(\frac{1}{x}-\frac{1}{x}+\frac{a^2}{2x^3}\right) = \frac{qQa}{8\pi\epsilon_0 x^3}$$

EVALUATE: For $x \gg a$, $F_y \ll F_x$ and $F \approx |F_x| = \dfrac{qQ}{4\pi\epsilon_0 x^2}$ and $\vec{F}$ is in the $-x$-direction. For $x \gg a$ the charge distribution Q acts like a point charge.

22.47 • **Concentric Spherical Shells.** A small conducting spherical shell with inner radius a and outer radius b is concentric with a larger conducting spherical shell with inner radius c and outer radius d (Fig. P22.47). The inner shell has total charge $+2q$, and the outer shell has charge $+4q$. (a) Calculate the electric field (magnitude and direction) in terms of q and the distance r from the common center of the two shells for (i) $r < a$; (ii) $a < r < b$; (iii) $b < r < c$; (iv) $c < r < d$; (v) $r > d$. Show your results in a graph of the radial component of $\vec{E}$ as a function of r. (b) What is the total charge on the (i) inner surface of the small shell; (ii) outer surface of the small shell; (iii) inner surface of the large shell; (iv) outer surface of the large shell?

Figure **P22.47**



22.47. **IDENTIFY:** Apply Gauss's law to a spherical Gaussian surface with radius r. Calculate the electric field at the surface of the Gaussian sphere.

(a) SET UP: (i) $r < a$: The Gaussian surface is sketched in Figure 22.47a.

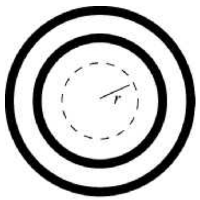


Figure 22.47a

EXECUTE: $\Phi_E = EA = E(4\pi r^2)$

$Q_{\text{encl}} = 0$; no charge is enclosed

$\Phi_E = \dfrac{Q_{\text{encl}}}{\epsilon_0}$ says

$E(4\pi r^2) = 0$ and $E = 0$.

(ii) $a < r < b$: Points in this region are in the conductor of the small shell, so $E = 0$.

(iii) **SET UP:** $b < r < c$: The Gaussian surface is sketched in Figure 22.47b.

Apply Gauss's law to a spherical Gaussian surface with radius $b < r < c$.

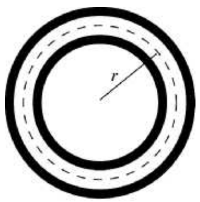


Figure 22.47b

EXECUTE: $\Phi_E = EA = E(4\pi r^2)$

The Gaussian surface encloses all of the small shell and none of the large shell, so $Q_{\text{encl}} = +2q$.

$\Phi_E = \dfrac{Q_{\text{encl}}}{\epsilon_0}$ gives $E(4\pi r^2) = \dfrac{2q}{\epsilon_0}$ so $E = \dfrac{2q}{4\pi\epsilon_0 r^2}$. Since the enclosed charge is positive the electric field is radially outward.

(iv) $c < r < d$: Points in this region are in the conductor of the large shell, so $E = 0$.

(v) **SET UP:** $r > d$: Apply Gauss's law to a spherical Gaussian surface with radius $r > d$, as shown in Figure 22.47c.

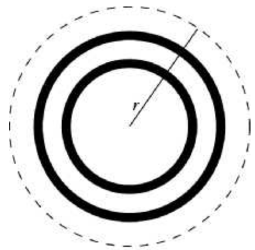


Figure 22.47c

EXECUTE: $\Phi_E = EA = E(4\pi r^2)$

The Gaussian surface encloses all of the small shell and all of the large shell, so $Q_{\text{encl}} = +2q + 4q = 6q$.

$\Phi_E = \dfrac{Q_{\text{encl}}}{\epsilon_0}$ gives $E(4\pi r^2) = \dfrac{6q}{\epsilon_0}$

$E = \dfrac{6q}{4\pi\epsilon_0 r^2}$. Since the enclosed charge is positive the electric field is radially outward.

The graph of E versus r is sketched in Figure 22.47d.

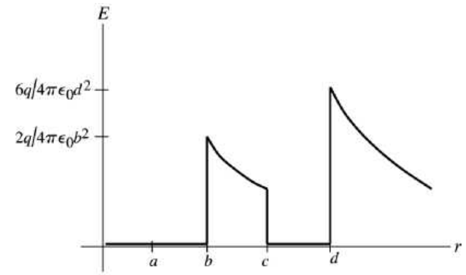


Figure 22.47d

(b) IDENTIFY and SET UP: Apply Gauss's law to a sphere that lies outside the surface of the shell for which we want to find the surface charge.

EXECUTE: (i) charge on inner surface of the small shell: Apply Gauss's law to a spherical Gaussian surface with radius $a < r < b$. This surface lies within the conductor of the small shell, where $E = 0$, so $\Phi_E = 0$. Thus by Gauss's law $Q_{\text{encl}} = 0$, so there is zero charge on the inner surface of the small shell.

(ii) charge on outer surface of the small shell: The total charge on the small shell is $+2q$. We found in part (i) that there is zero charge on the inner surface of the shell, so all $+2q$ must reside on the outer surface.

(iii) charge on inner surface of large shell: Apply Gauss's law to a spherical Gaussian surface with radius $c < r < d$. The surface lies within the conductor of the large shell, where $E = 0$, so $\Phi_E = 0$. Thus by Gauss's law $Q_{\text{encl}} = 0$. The surface encloses the $+2q$ on the small shell so there must be charge $-2q$ on the inner surface of the large shell to make the total enclosed charge zero.

(iv) charge on outer surface of large shell: The total charge on the large shell is $+4q$. We showed in part (iii) that the charge on the inner surface is $-2q$, so there must be $+6q$ on the outer surface.

EVALUATE: The electric field lines for $b < r < c$ originate from the surface charge on the outer surface of the inner shell and all terminate on the surface charge on the inner surface of the outer shell. These surface charges have equal magnitude and opposite sign. The electric field lines for $r > d$ originate from the surface charge on the outer surface of the outer sphere.

22.65 •• **CALC** A nonuniform, but spherically symmetric, distribution of charge has a charge density $\rho(r)$ given as follows:

$$\rho(r) = \rho_0(1 - r/R) \qquad \text{for } r \leq R$$
$$\rho(r) = 0 \qquad \text{for } r \geq R$$

where $\rho_0 = 3Q/\pi R^3$ is a positive constant. (a) Show that the total charge contained in the charge distribution is Q. (b) Show that the electric field in the region $r \geq R$ is identical to that produced by a point charge Q at $r = 0$. (c) Obtain an expression for the electric field in the region $r \leq R$. (d) Graph the electric-field magnitude E as a function of r. (e) Find the value of r at which the electric field is maximum, and find the value of that maximum field.

22.65. $\rho(r) = \rho_0(1 - r/R)$ for $r \le R$ where $\rho_0 = 3Q/\pi R^3$. $\rho(r) = 0$ for $r \ge R$

(a) IDENTIFY: The charge density varies with r inside the spherical volume. Divide the volume up into thin concentric shells, of radius r and thickness dr. Find the charge dq in each shell and integrate to find the total charge.
SET UP: The thin shell is sketched in Figure 22.65a.

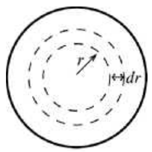


Figure 22.65a

EXECUTE: The volume of such a shell is $dV = 4\pi r^2 dr$.
The charge contained within the shell is $dq = \rho(r)dV = 4\pi r^2 \rho_0(1 - r/R)dr$.

The total charge Q in the charge distribution is obtained by integrating dq over all such shells into which the sphere can be subdivided:

$$Q = \int dq = \int_0^R 4\pi r^2 \rho_0 (1 - r/R)dr = 4\pi\rho_0 \int_0^R (r^2 - r^3/R)dr$$

$$Q = 4\pi\rho_0 \left[\frac{r^3}{3} - \frac{r^4}{4R}\right]_0^R = 4\pi\rho_0\left(\frac{R^3}{3} - \frac{R^4}{4R}\right) = 4\pi\rho_0(R^3/12) = 4\pi(3Q/\pi R^3)(R^3/12) = Q,$$ as was to be shown.

(b) IDENTIFY: Apply Gauss's law to a spherical surface of radius r, where $r > R$.
SET UP: The Gaussian surface is shown in Figure 22.65b.

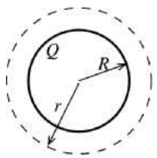


Figure 22.65b

EXECUTE: $\Phi_E = \dfrac{Q_{encl}}{\epsilon_0}$

$E(4\pi r^2) = \dfrac{Q}{\epsilon_0}$

$E = \dfrac{Q}{4\pi\epsilon_0 r^2}$; same as for point charge of charge Q.

(c) IDENTIFY: Apply Gauss's law to a spherical surface of radius r, where $r < R$:
SET UP: The Gaussian surface is shown in Figure 22.65c.

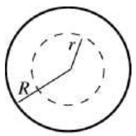


Figure 22.65c

EXECUTE: $\Phi_E = \dfrac{Q_{encl}}{\epsilon_0}$

$\Phi_E = E(4\pi r^2)$

To calculate the enclosed charge Q_{encl} use the same technique as in part (a), except integrate dq out to r rather than R. (We want the charge that is inside radius r.)

$$Q_{encl} = \int_0^r 4\pi r'^2 \rho_0\left(1 - \frac{r'}{R}\right)dr' = 4\pi\rho_0 \int_0^r \left(r'^2 - \frac{r'^3}{R}\right)dr'$$

$$Q_{encl} = 4\pi\rho_0\left[\frac{r'^3}{3} - \frac{r'^4}{4R}\right]_0^r = 4\pi\rho_0\left(\frac{r^3}{3} - \frac{r^4}{4R}\right) = 4\pi\rho_0 r^3\left(\frac{1}{3} - \frac{r}{4R}\right)$$

$$\rho_0 = \frac{3Q}{\pi R^3} \text{ so } Q_{encl} = 12Q\frac{r^3}{R^3}\left(\frac{1}{3} - \frac{r}{4R}\right) = Q\left(\frac{r^3}{R^3}\right)\left(4 - 3\frac{r}{R}\right).$$

Thus Gauss's law gives $E(4\pi r^2) = \dfrac{Q}{\epsilon_0}\left(\dfrac{r^3}{R^3}\right)\left(4 - 3\dfrac{r}{R}\right)$.

$$E = \frac{Qr}{4\pi\epsilon_0 R^3}\left(4 - \frac{3r}{R}\right), r \le R$$

(d) The graph of E versus r is sketched in Figure 22.65d.

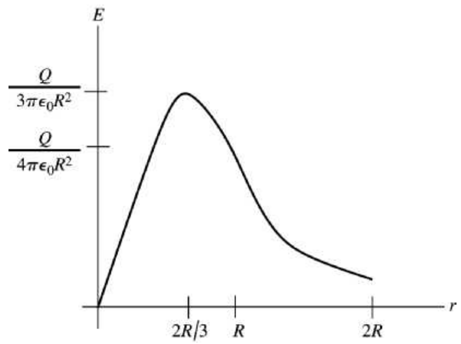


Figure 22.65d

(e) Where the electric field is a maximum, $\dfrac{dE}{dr} = 0$. Thus $\dfrac{d}{dr}\left(4r - \dfrac{3r^2}{R}\right) = 0$ so $4 - 6r/R = 0$ and $r = 2R/3$.

At this value of r, $E = \dfrac{Q}{4\pi\epsilon_0 R^3}\left(\dfrac{2R}{3}\right)\left(4 - \dfrac{3}{R}\dfrac{2R}{3}\right) = \dfrac{Q}{3\pi\epsilon_0 R^2}$.

EVALUATE: Our expressions for $E(r)$ for $r < R$ and for $r > R$ agree at $r = R$. The results of part (e) for the value of r where $E(r)$ is a maximum agrees with the graph in part (d).

23.89 ••• **CP** In experiments in which atomic nuclei collide, head-on collisions like that described in Problem 23.82 do happen, but “near misses” are more common. Suppose the alpha particle in Problem 23.82 was not “aimed” at the center of the lead nucleus, but had an initial nonzero angular momentum (with respect to the stationary lead nucleus) of magnitude $L = p_0 b$, where p_0 is the magnitude of the initial momentum of the alpha particle and $b = 1.00 \times 10^{-12}$ m. What is the distance of closest approach? Repeat for $b = 1.00 \times 10^{-13}$ m and $b = 1.00 \times 10^{-14}$ m.

23.89. IDENTIFY: Angular momentum and energy must be conserved.

SET UP: At the distance of closest approach the speed is not zero. $E = K + U$. $q_1 = 2e$, $q_2 = 82e$.

EXECUTE: $mv_1 b = mv_2 r_2$. $E_1 = E_2$ gives $E_1 = \frac{1}{2}mv_2^{\ 2} + \frac{kq_1 q_2}{r_2}$. $E_1 = 11\ \text{MeV} = 1.76\times10^{-12}$ J. r_2 is the distance of closest approach. Substituting in for $v_2 = v_1\left(\frac{b}{r_2}\right)$ we find $E_1 = E_1\frac{b^2}{r_2^{\ 2}} + \frac{kq_1 q_2}{r_2}$.

$(E_1)r_2^2 - (kq_1 q_2)r_2 - E_1 b^2 = 0$. For $b = 10^{-12}$ m, $r_2 = 1.01\times10^{-12}$ m. For $b = 10^{-13}$ m, $r_2 = 1.11\times10^{-13}$ m. And for $b = 10^{-14}$ m, $r_2 = 2.54\times10^{-14}$ m.

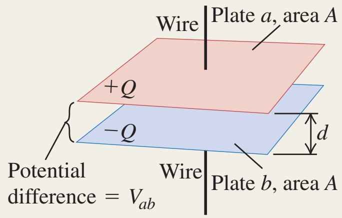
EVALUATE: As b decreases the collision is closer to being head-on and the distance of closest approach decreases. Problem 23.82 shows that the distance of closest approach is 2.15×10^{-14} m when $b = 0$.

Capacitors and capacitance: A capacitor is any pair of conductors separated by an insulating material. When the capacitor is charged, there are charges of equal magnitude Q and opposite sign on the two conductors, and the potential V_{ab} of the positively charged conductor with respect to the negatively charged conductor is proportional to Q. The capacitance C is defined as the ratio of Q to V_{ab}. The SI unit of capacitance is the farad (F): $1\ \mathrm{F} = 1\ \mathrm{C/V}$.

A parallel-plate capacitor consists of two parallel conducting plates, each with area A, separated by a distance d. If they are separated by vacuum, the capacitance depends only on A and d. For other geometries, the capacitance can be found by using the definition $C = Q/V_{ab}$. (See Examples 24.1–24.4.)

$$C = \frac{Q}{V_{ab}} \tag{24.1}$$

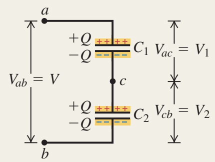
$$C = \frac{Q}{V_{ab}} = \epsilon_0 \frac{A}{d} \tag{24.2}$$

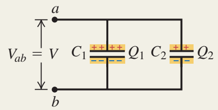


Capacitors in series and parallel: When capacitors with capacitances $C_1, C_2, C_3, \ldots$ are connected in series, the reciprocal of the equivalent capacitance C_{eq} equals the sum of the reciprocals of the individual capacitances. When capacitors are connected in parallel, the equivalent capacitance C_{eq} equals the sum of the individual capacitances. (See Examples 24.5 and 24.6.)

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots \tag{24.5}$$
(capacitors in series)

$$C_{eq} = C_1 + C_2 + C_3 + \cdots \tag{24.7}$$
(capacitors in parallel)

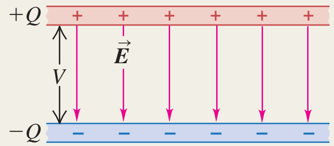




Energy in a capacitor: The energy U required to charge a capacitor C to a potential difference V and a charge Q is equal to the energy stored in the capacitor. This energy can be thought of as residing in the electric field between the conductors; the energy density u (energy per unit volume) is proportional to the square of the electric-field magnitude. (See Examples 24.7–24.9.)

$$U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV \qquad (24.9)$$

$$u = \frac{1}{2}\epsilon_0 E^2 \qquad (24.11)$$



Dielectrics: When the space between the conductors is filled with a dielectric material, the capacitance increases by a factor K, called the dielectric constant of the material. The quantity $\epsilon = K\epsilon_0$ is called the permittivity of the dielectric. For a fixed amount of charge on the capacitor plates, induced charges on the surface of the dielectric decrease the electric field and potential difference between the plates by the same factor K. The surface charge results from polarization, a microscopic rearrangement of charge in the dielectric. (See Example 24.10.)

Under sufficiently strong fields, dielectrics become conductors, a situation called dielectric breakdown. The maximum field that a material can withstand without breakdown is called its dielectric strength.

In a dielectric, the expression for the energy density is the same as in vacuum but with ϵ_0 replaced by $\epsilon = K\epsilon$. (See Example 24.11.)

Gauss's law in a dielectric has almost the same form as in vacuum, with two key differences: $\vec{E}$ is replaced by $K\vec{E}$ and Q_{encl} is replaced by $Q_{encl\text{-}free}$, which includes only the free charge (not bound charge) enclosed by the Gaussian surface. (See Example 24.12.)

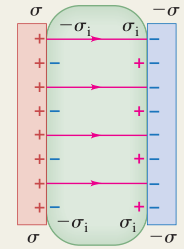
$$C = KC_0 = K\epsilon_0\frac{A}{d} = \epsilon\frac{A}{d}$$

(parallel-plate capacitor filled with dielectric) (24.19)

$$u = \frac{1}{2}K\epsilon_0 E^2 = \frac{1}{2}\epsilon E^2 \qquad (24.20)$$

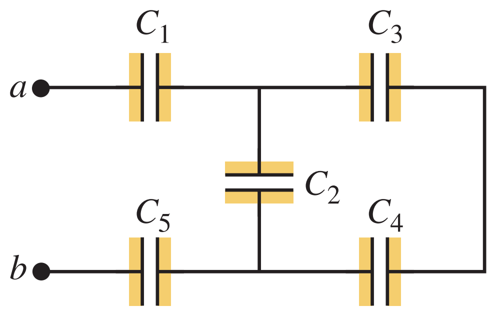
$$\oint K\vec{E}\cdot d\vec{A} = \frac{Q_{encl\text{-}free}}{\epsilon_0} \qquad (24.23)$$

Dielectric between plates



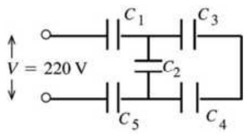
24.57 • In Fig. P24.57, $C_1 = C_5 = 8.4\ \mu\text{F}$ and $C_2 = C_3 = C_4 = 4.2\ \mu\text{F}$. The applied potential is $V_{ab} = 220$ V. (a) What is the equivalent capacitance of the network between points a and b? (b) Calculate the charge on each capacitor and the potential difference across each capacitor.

Figure **P24.57**



24.57. **(a) IDENTIFY:** Replace series and parallel combinations of capacitors by their equival

SET UP: The network is sketched in Figure 24.57a.

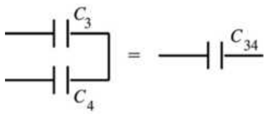


$C_1 = C_5 = 8.4\ \mu\text{F}$
$C_2 = C_3 = C_4 = 4.2\ \mu\text{F}$

Figure 24.57a

EXECUTE: Simplify the circuit by replacing the capacitor combinations by their equiv

are in series and can be replaced by C_{34} (Figure 24.57b):

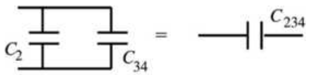


$$\frac{1}{C_{34}} = \frac{1}{C_3} + \frac{1}{C_4}$$
$$\frac{1}{C_{34}} = \frac{C_3 + C_4}{C_3 C_4}$$

Figure 24.57b

$$C_{34} = \frac{C_3 C_4}{C_3 + C_4} = \frac{(4.2\ \mu\text{F})(4.2\ \mu\text{F})}{4.2\ \mu\text{F} + 4.2\ \mu\text{F}} = 2.1\ \mu\text{F}$$

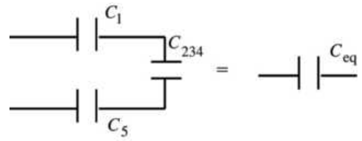
C_2 and C_{34} are in parallel and can be replaced by their equivalent (Figure 24.57c):



$C_{234} = C_2 + C_{34}$
$C_{234} = 4.2\ \mu\text{F} + 2.1\ \mu\text{F}$
$C_{234} = 6.3\ \mu\text{F}$

Figure 24.57c

C_1, C_5 and C_{234} are in series and can be replaced by C_{eq} (Figure 24.57d):



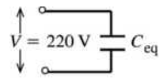
$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_5} + \frac{1}{C_{234}}$$
$$\frac{1}{C_{\text{eq}}} = \frac{2}{8.4\ \mu\text{F}} + \frac{1}{6.3\ \mu\text{F}}$$
$$C_{\text{eq}} = 2.5\ \mu\text{F}$$

Figure 24.57d

EVALUATE: For capacitors in series the equivalent capacitor is smaller than any of those in series. For capacitors in parallel the equivalent capacitance is larger than any of those in parallel.

(b) IDENTIFY and **SET UP:** In each equivalent network apply the rules for Q and V for capacitors in series and parallel; start with the simplest network and work back to the original circuit.

EXECUTE: The equivalent circuit is drawn in Figure 24.57e.



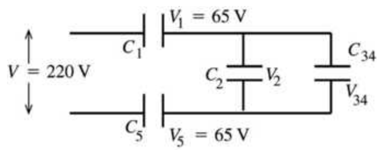
$Q_{\text{eq}} = C_{\text{eq}} V$
$Q_{\text{eq}} = (2.5\ \mu\text{F})(220\ \text{V}) = 550\ \mu\text{C}$

Figure 24.57e

$Q_1 = Q_5 = Q_{234} = 550\ \mu\text{C}$ (capacitors in series have same charge)

$$V_1 = \frac{Q_1}{C_1} = \frac{550\ \mu\text{C}}{8.4\ \mu\text{F}} = 65\ \text{V}$$
$$V_5 = \frac{Q_5}{C_5} = \frac{550\ \mu\text{C}}{8.4\ \mu\text{F}} = 65\ \text{V}$$
$$V_{234} = \frac{Q_{234}}{C_{234}} = \frac{550\ \mu\text{C}}{6.3\ \mu\text{F}} = 87\ \text{V}$$

Now draw the network as in Figure 24.57f.

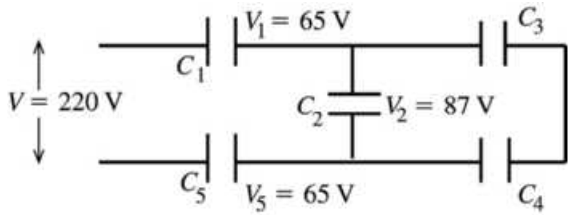


$V_2 = V_{34} = V_{234} = 87\ \text{V}$
capacitors in parallel have the same potential

Figure 24.57f

$Q_2 = C_2 V_2 = (4.2\ \mu\text{F})(87\ \text{V}) = 370\ \mu\text{C}$
$Q_{34} = C_{34} V_{34} = (2.1\ \mu\text{F})(87\ \text{V}) = 180\ \mu\text{C}$

Finally, consider the original circuit (Figure 24.57g).



$Q_3 = Q_4 = Q_{34} = 180\ \mu\text{C}$

capacitors in series have the same charge

Figure 24.57g

$$V_3 = \frac{Q_3}{C_3} = \frac{180\ \mu\text{C}}{4.2\ \mu\text{F}} = 43\ \text{V}$$

$$V_4 = \frac{Q_4}{C_4} = \frac{180\ \mu\text{C}}{4.2\ \mu\text{F}} = 43\ \text{V}$$

Summary: $Q_1 = 550\ \mu\text{C},\ V_1 = 65\ \text{V}$

$Q_2 = 370\ \mu\text{C},\ V_2 = 87\ \text{V}$

$Q_3 = 180\ \mu\text{C},\ V_3 = 43\ \text{V}$

$Q_4 = 180\ \mu\text{C},\ V_4 = 43\ \text{V}$

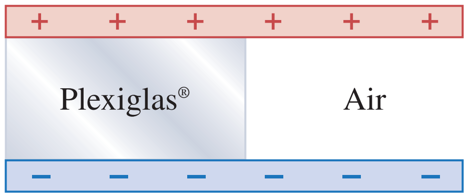
$Q_5 = 550\ \mu\text{C},\ V_5 = 65\ \text{V}$

EVALUATE: $V_3 + V_4 = V_2$ and $V_1 + V_2 + V_5 = 220\ \text{V}$ (apart from some small rounding error)

$Q_1 = Q_2 + Q_3$ and $Q_5 = Q_2 + Q_4$

24.72 •• A parallel-plate capacitor is made from two plates 12.0 cm on each side and 4.50 mm apart. Half of the space between these plates contains only air, but the other half is filled with Plexiglas® of dielectric constant 3.40 (Fig. P24.72). An 18.0-V battery is connected across the plates. (a) What is the capacitance of this combination? (*Hint:* Can you think of this capacitor as equivalent to two capacitors in parallel?) (b) How much energy is stored in the capacitor? (c) If we remove the Plexiglas® but change nothing else, how much energy will be stored in the capacitor?

Figure **P24.72**



24.72. **IDENTIFY:** The capacitor is equivalent to two capacitors in parallel, as shown in Figure 24.72.

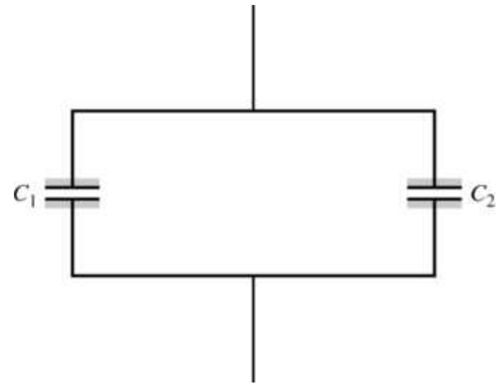


Figure 24.72

SET UP: Each of these two capacitors have plates that are 12.0 cm by 6.0 cm. For a parallel-plate capacitor with dielectric filling the volume between the plates, $C = K\epsilon_0 \frac{A}{d}$. For two capacitors in parallel, $C = C_1 + C_2$. The energy stored in a capacitor is $U = \frac{1}{2}CV^2$.

EXECUTE: **(a)** $C = C_1 + C_2$.

$$C_2 = \epsilon_0 \frac{A}{d} = \frac{(8.854\times10^{-12}\text{ F/m})(0.120\text{ m})(0.060\text{ m})}{4.50\times10^{-3}\text{ m}} = 1.42\times10^{-11}\text{ F}.$$

$C_1 = KC_2 = (3.40)(1.42\times10^{-11}\text{ F}) = 4.83\times10^{-11}\text{ F}$. $C = C_1 + C_2 = 6.25\times10^{-11}\text{ F} = 62.5\text{ pF}$.

(b) $U = \frac{1}{2}CV^2 = \frac{1}{2}(6.25\times10^{-11}\text{ F})(18.0\text{ V})^2 = 1.01\times10^{-8}\text{ J}$.

(c) Now $C_1 = C_2$ and $C = 2(1.42\times10^{-11}\text{ F}) = 2.84\times10^{-11}\text{ F}$.

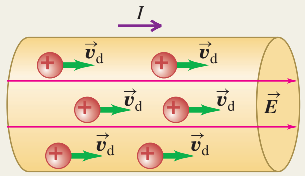
$U = \frac{1}{2}CV^2 = \frac{1}{2}(2.84\times10^{-11}\text{ F})(18.0\text{ V})^2 = 4.60\times10^{-9}\text{ J}$.

EVALUATE: The plexiglass increases the capacitance and that increases the energy stored for the same voltage across the capacitor.

Current and current density: Current is the amount of charge flowing through a specified area, per unit time. The SI unit of current is the ampere $(1\text{ A} = 1\text{ C/s})$. The current I through an area A depends on the concentration n and charge q of the charge carriers, as well as on the magnitude of their drift velocity $\vec{v}_d$. The current density is current per unit cross-sectional area. Current is usually described in terms of a flow of positive charge, even when the charges are actually negative or of both signs. (See Example 25.1.)

$$I = \frac{dQ}{dt} = n|q|v_d A \qquad (25.2)$$

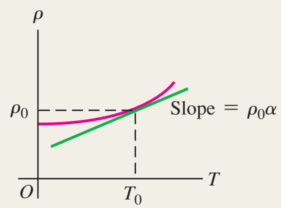
$$\vec{J} = nq\vec{v}_d \qquad (25.4)$$



Resistivity: The resistivity ρ of a material is the ratio of the magnitudes of electric field and current density. Good conductors have small resistivity; good insulators have large resistivity. Ohm's law, obeyed approximately by many materials, states that ρ is a constant independent of the value of E. Resistivity usually increases with temperature; for small temperature changes this variation is represented approximately by Eq. (25.6), where α is the temperature coefficient of resistivity.

$$\rho = \frac{E}{J} \qquad (25.5)$$

$$\rho(T) = \rho_0[1 + \alpha(T - T_0)] \qquad (25.6)$$

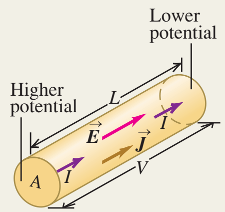


Metal: ρ increases with increasing T.

Resistors: The potential difference V across a sample of material that obeys Ohm's law is proportional to the current I through the sample. The ratio $V/I = R$ is the resistance of the sample. The SI unit of resistance is the ohm $(1\ \Omega = 1\text{ V/A})$. The resistance of a cylindrical conductor is related to its resistivity ρ, length L, and cross-sectional area A. (See Examples 25.2 and 25.3.)

$$V = IR \qquad (25.11)$$

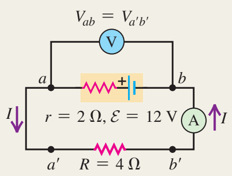
$$R = \frac{\rho L}{A} \qquad (25.10)$$



Circuits and emf: A complete circuit has a continuous current-carrying path. A complete circuit carrying a steady current must contain a source of electromotive force (emf) $\mathcal{E}$. The SI unit of electromotive force is the volt (1 V). Every real source of emf has some internal resistance r, so its terminal potential difference V_{ab} depends on current. (See Examples 25.4–25.7.)

$$V_{ab} = \mathcal{E} - Ir \quad (25.15)$$

(source with internal resistance)



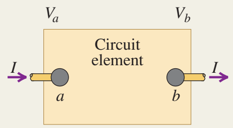
Energy and power in circuits: A circuit element with a potential difference $V_a - V_b = V_{ab}$ and a current I puts energy into a circuit if the current direction is from lower to higher potential in the device, and it takes energy out of the circuit if the current is opposite. The power P equals the product of the potential difference and the current. A resistor always takes electrical energy out of a circuit. (See Examples 25.8–25.10.)

$$P = V_{ab}I \quad (25.17)$$

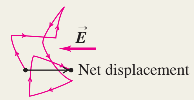
(general circuit element)

$$P = V_{ab}I = I^2R = \frac{V_{ab}^2}{R} \quad (25.18)$$

(power into a resistor)



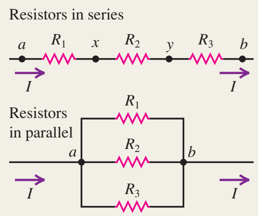
Conduction in metals: The microscopic basis of conduction in metals is the motion of electrons that move freely through the metallic crystal, bumping into ion cores in the crystal. In a crude classical model of this motion, the resistivity of the material can be related to the electron mass, charge, speed of random motion, density, and mean free time between collisions. (See Example 25.11.)



Resistors in series and parallel: When several resistors $R_1, R_2, R_3, \ldots$ are connected in series, the equivalent resistance R_{eq} is the sum of the individual resistances. The same *current* flows through all the resistors in a series connection. When several resistors are connected in parallel, the reciprocal of the equivalent resistance R_{eq} is the sum of the reciprocals of the individual resistances. All resistors in a parallel connection have the same *potential difference* between their terminals. (See Examples 26.1 and 26.2.)

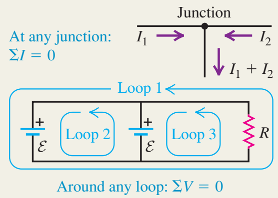
$$R_{eq} = R_1 + R_2 + R_3 + \cdots \qquad (26.1)$$
(resistors in series)

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots \qquad (26.2)$$
(resistors in parallel)



Kirchhoff's rules: Kirchhoff's junction rule is based on conservation of charge. It states that the algebraic sum of the currents into any junction must be zero. Kirchhoff's loop rule is based on conservation of energy and the conservative nature of electrostatic fields. It states that the algebraic sum of potential differences around any loop must be zero. Careful use of consistent sign rules is essential in applying Kirchhoff's rules. (See Examples 26.3–26.7.)

$$\sum I = 0 \quad \text{(junction rule)} \qquad (26.5)$$
$$\sum V = 0 \quad \text{(loop rule)} \qquad (26.6)$$



R-C circuits: When a capacitor is charged by a battery in series with a resistor, the current and capacitor charge are not constant. The charge approaches its final value asymptotically and the current approaches zero asymptotically. The charge and current in the circuit are given by Eqs. (26.12) and (26.13). After a time $\tau = RC$, the charge has approached within $1/e$ of its final value. This time is called the time constant or relaxation time of the circuit. When the capacitor discharges, the charge and current are given as functions of time by Eqs. (26.16) and (26.17). The time constant is the same for charging and discharging. (See Examples 26.12 and 26.13.)

Capacitor charging:

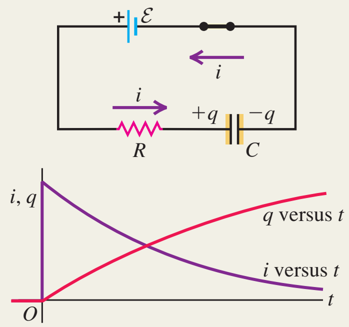
$$q = C\mathcal{E}\left(1 - e^{-t/RC}\right) = Q_f\left(1 - e^{-t/RC}\right) \qquad (26.12)$$

$$i = \frac{dq}{dt} = \frac{\mathcal{E}}{R}e^{-t/RC} = I_0 e^{-t/RC} \qquad (26.13)$$

Capacitor discharging:

$$q = Q_0 e^{-t/RC} \qquad (26.16)$$

$$i = \frac{dq}{dt} = -\frac{Q_0}{RC}e^{-t/RC} = I_0 e^{-t/RC} \qquad (26.17)$$



25.53 • A "540-W" electric heater is designed to operate from 120-V lines. (a) What is its resistance? (b) What current does it draw? (c) If the line voltage drops to 110 V, what power does the heater take? (Assume that the resistance is constant. Actually, it will change because of the change in temperature.) (d) The heater coils are metallic, so that the resistance of the heater decreases with decreasing temperature. If the change of resistance with temperature is taken into account, will the electrical power consumed by the heater be larger or smaller than what you calculated in part (c)? Explain.

25.53. **IDENTIFY:** $P = I^2 R = \frac{V^2}{R} = VI$. $V = IR$.

SET UP: The heater consumes 540 W when $V = 120$ V. Energy $= Pt$.

EXECUTE: **(a)** $P = \frac{V^2}{R}$ so $R = \frac{V^2}{P} = \frac{(120 \text{ V})^2}{540 \text{ W}} = 26.7\ \Omega$

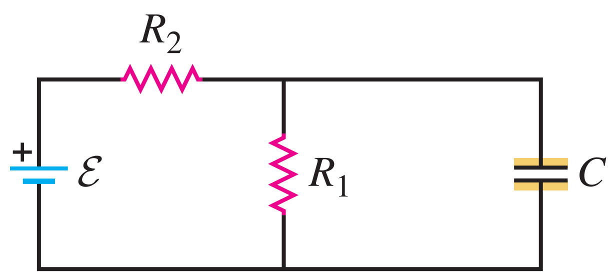
(b) $P = VI$ so $I = \frac{P}{V} = \frac{540 \text{ W}}{120 \text{ V}} = 4.50$ A

(c) Assuming that *R* remains $26.7\ \Omega$, $P = \frac{V^2}{R} = \frac{(110 \text{ V})^2}{26.7\ \Omega} = 453$ W. *P* is smaller by a factor of $(110/120)^2$.

EVALUATE: **(d)** With the lower line voltage the current will decrease and the operating temperature will decrease. *R* will be less than $26.7\ \Omega$ and the power consumed will be greater than the value calculated in part (c).

25.83 •• **CP** Consider the circuit shown in Fig. P25.83. The emf source has negligible internal resistance. The resistors have resistances $R_1 = 6.00\ \Omega$ and $R_2 = 4.00\ \Omega$. The capacitor has capacitance $C = 9.00\ \mu F$. When the capacitor is fully charged, the magnitude of the charge on its plates is $Q = 36.0\ \mu C$. Calculate the emf $\mathcal{E}$.

Figure **P25.83**



25.83. **IDENTIFY:** No current flows through the capacitor when it is fully charged.

SET UP: With the capacitor fully charged, $I = \dfrac{\mathcal{E}}{R_1 + R_2}$. $V_R = IR$ and $V_C = Q/C$.

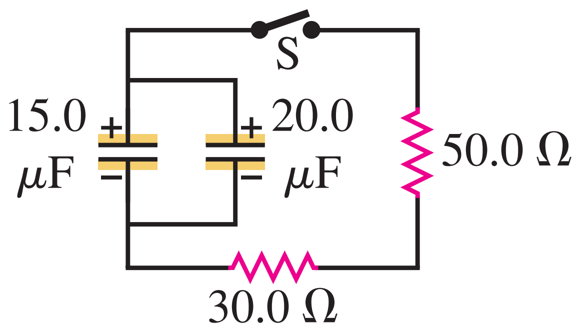
EXECUTE: $V_C = \dfrac{Q}{C} = \dfrac{36.0\ \mu\text{C}}{9.00\ \mu\text{F}} = 4.00\ \text{V}$. $V_{R_1} = V_C = 4.00\ \text{V}$ and $I = \dfrac{V_{R_1}}{R_1} = \dfrac{4.00\ \text{V}}{6.00\ \Omega} = 0.667\ \text{A}$.

$V_{R_2} = IR_2 = (0.667\ \text{A})(4.00\ \Omega) = 2.668\ \text{V}$. $\mathcal{E} = V_{R_1} + V_{R_2} = 4.00\ \text{V} + 2.668\ \text{V} = 6.67\ \text{V}$.

EVALUATE: When a capacitor is fully charged, it acts like an open circuit and prevents any current from flowing though it.

26.43 •• **CP** In the circuit shown in Fig. E26.43 both capacitors are initially charged to 45.0 V. (a) How long after closing the switch S will the potential across each capacitor be reduced to 10.0 V, and (b) what will be the current at that time?

Figure **E26.43**



26.43. IDENTIFY: The capacitors, which are in parallel, will discharge exponentially through the resistors.

SET UP: Since V is proportional to Q, V must obey the same exponential equation as Q, $V = V_0 e^{-t/RC}$. The current is $I = (V_0/R)\, e^{-t/RC}$.

EXECUTE: **(a)** Solve for time when the potential across each capacitor is 10.0 V:

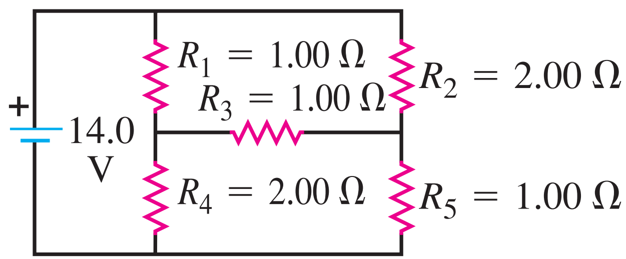
$$t = -RC \ln(V/V_0) = -(80.0\ \Omega)(35.0\ \mu\text{F}) \ln(10/45) = 4210\ \mu\text{s} = 4.21\ \text{ms}$$

(b) $I = (V_0/R)\, e^{-t/RC}$. Using the above values, with $V_0 = 45.0$ V, gives $I = 0.125$ A.

EVALUATE: Since the current and the potential both obey the same exponential equation, they are both reduced by the same factor (0.222) in 4.21 ms.

26.66 • (a) Find the current through the battery and each resistor in the circuit shown in Fig. P26.66. (b) What is the equivalent resistance of the resistor network?

Figure **P26.66**



26.66. **IDENTIFY:** Apply the loop and junction rules.

SET UP: Use the currents as defined on the circuit diagram in Figure 26.66 and obtain three equations to solve for the currents.

EXECUTE: Left loop: $14 - I_1 - 2(I_1 - I_2) = 0$ and $3I_1 - 2I_2 = 14$.

Top loop: $-2(I - I_1) + I_2 + I_1 = 0$ and $-2I + 3I_1 + I_2 = 0$.

Bottom loop: $-(I - I_1 + I_2) + 2(I_1 - I_2) - I_2 = 0$ and $-I + 3I_1 - 4I_2 = 0$.

Solving these equations for the currents we find: $I = I_{\text{battery}} = 10.0$ A; $I_1 = I_{R_1} = 6.0$ A; $I_2 = I_{R_3} = 2.0$ A.

So the other currents are: $I_{R_2} = I - I_1 = 4.0$ A; $I_{R_4} = I_1 - I_2 = 4.0$ A; $I_{R_5} = I - I_1 + I_2 = 6.0$ A.

(b) $R_{\text{eq}} = \dfrac{V}{I} = \dfrac{14.0\text{ V}}{10.0\text{ A}} = 1.40\ \Omega.$

EVALUATE: It isn't possible to simplify the resistor network using the rules for resistors in series and parallel. But the equivalent resistance is still defined by $V = IR_{\text{eq}}$.

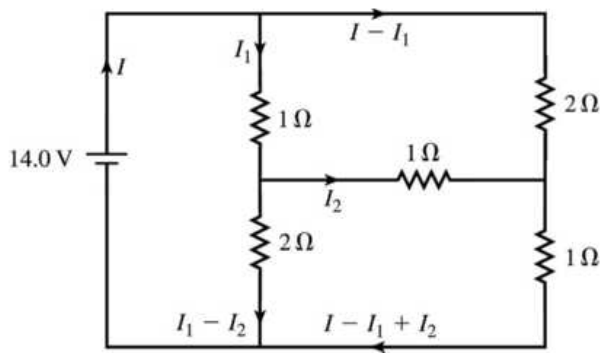


Figure 26.66