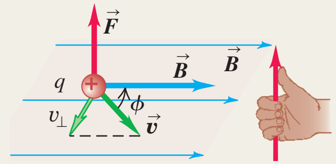
Magnetic forces: Magnetic interactions are fundamentally interactions between moving charged particles. These interactions are described by the vector magnetic field, denoted by $\vec{B}$. A particle with charge q moving with velocity $\vec{v}$ in a magnetic field $\vec{B}$ experiences a force $\vec{F}$ that is perpendicular to both $\vec{v}$ and $\vec{B}$. The SI unit of magnetic field is the tesla $(1\ \mathrm{T} = 1\ \mathrm{N/A \cdot m})$. (See Example 27.1.)

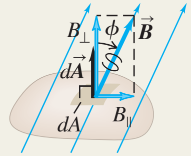
$$\vec{F} = q\vec{v} \times \vec{B} \qquad (27.2)$$



Magnetic field lines and flux: A magnetic field can be represented graphically by magnetic field lines. At each point a magnetic field line is tangent to the direction of $\vec{B}$ at that point. Where field lines are close together the field magnitude is large, and vice versa. Magnetic flux Φ_B through an area is defined in an analogous way to electric flux. The SI unit of magnetic flux is the weber $(1\ \mathrm{Wb} = 1\ \mathrm{T \cdot m^2})$. The net magnetic flux through any closed surface is zero (Gauss's law for magnetism). As a result, magnetic field lines always close on themselves. (See Example 27.2.)

$$\Phi_B = \int B_\perp\, dA = \int B\cos\phi\, dA = \int \vec{B} \cdot d\vec{A} \qquad (27.6)$$

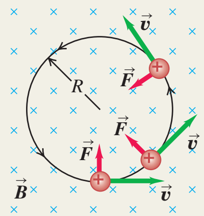
$$\oint \vec{B} \cdot d\vec{A} = 0 \quad \text{(closed surface)} \qquad (27.8)$$



Motion in a magnetic field: The magnetic force is always perpendicular to $\vec{v}$; a particle moving under the action of a magnetic field alone moves with constant speed. In a uniform field, a particle with initial velocity perpendicular to the field moves in a circle with radius R that depends on the magnetic field strength B and the particle mass m, speed v, and charge q. (See Examples 27.3 and 27.4.)

Crossed electric and magnetic fields can be used as a velocity selector. The electric and magnetic forces exactly cancel when $v = E/B$. (See Examples 27.5 and 27.6.)

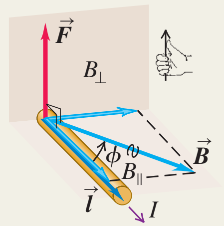
$$R = \frac{mv}{|q|B} \qquad (27.11)$$



Magnetic force on a conductor: A straight segment of a conductor carrying current I in a uniform magnetic field $\vec{B}$ experiences a force $\vec{F}$ that is perpendicular to both $\vec{B}$ and the vector $\vec{l}$, which points in the direction of the current and has magnitude equal to the length of the segment. A similar relationship gives the force $d\vec{F}$ on an infinitesimal current-carrying segment $d\vec{l}$. (See Examples 27.7 and 27.8.)

$$\vec{F} = I\vec{l} \times \vec{B} \tag{27.19}$$

$$d\vec{F} = I\,d\vec{l} \times \vec{B} \tag{27.20}$$

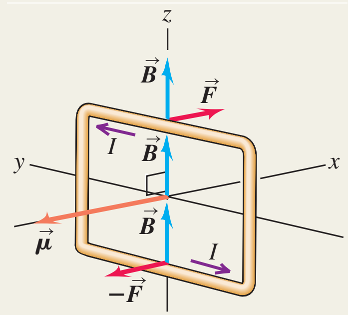


Magnetic torque: A current loop with area A and current I in a uniform magnetic field $\vec{B}$ experiences no net magnetic force, but does experience a magnetic torque of magnitude τ. The vector torque $\vec{\tau}$ can be expressed in terms of the magnetic moment $\vec{\mu} = I\vec{A}$ of the loop, as can the potential energy U of a magnetic moment in a magnetic field $\vec{B}$. The magnetic moment of a loop depends only on the current and the area; it is independent of the shape of the loop. (See Examples 27.9 and 27.10.)

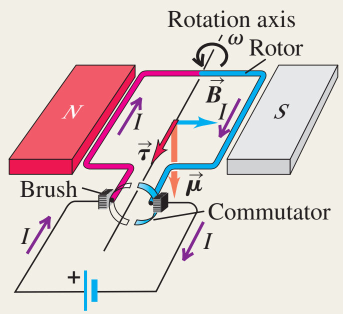
$$\tau = IBA\sin\phi \tag{27.23}$$

$$\vec{\tau} = \vec{\mu} \times \vec{B} \tag{27.26}$$

$$U = -\vec{\mu}\cdot\vec{B} = -\mu B\cos\phi \tag{27.27}$$

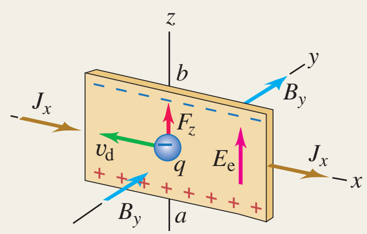


Electric motors: In a dc motor a magnetic field exerts a torque on a current in the rotor. Motion of the rotor through the magnetic field causes an induced emf called a back emf. For a series motor, in which the rotor coil is in parallel with coils that produce the magnetic field, the terminal voltage is the sum of the back emf and the drop Ir across the internal resistance. (See Example 27.11.)



The Hall effect: The Hall effect is a potential difference perpendicular to the direction of current in a conductor, when the conductor is placed in a magnetic field. The Hall potential is determined by the requirement that the associated electric field must just balance the magnetic force on a moving charge. Hall-effect measurements can be used to determine the sign of charge carriers and their concentration n. (See Example 27.12.)

$$nq = \frac{-J_x B_y}{E_z} \tag{27.30}$$



27.9 •• A group of particles is traveling in a magnetic field of unknown magnitude and direction. You observe that a proton moving at 1.50 km/s in the $+x$-direction experiences a force of 2.25×10^{-16} N in the $+y$-direction, and an electron moving at 4.75 km/s in the $-z$-direction experiences a force of 8.50×10^{-16} N in the $+y$-direction. (a) What are the magnitude and direction of the magnetic field? (b) What are the magnitude and direction of the magnetic force on an electron moving in the $-y$-direction at 3.20 km/s?

27.9. **IDENTIFY:** Apply $\vec{F} = q\vec{v} \times \vec{B}$ to the force on the proton and to the force on the electron. Solve for the components of $\vec{B}$ and use them to find its magnitude and direction.

SET UP: $\vec{F}$ is perpendicular to both $\vec{v}$ and $\vec{B}$. Since the force on the proton is in the $+y$-direction, $B_y = 0$ and $\vec{B} = B_x\hat{i} + B_z\hat{k}$. For the proton, $\vec{v}_p = (1.50 \text{ km/s})\hat{i} = v_p\hat{i}$ and $\vec{F}_p = (2.25\times10^{-16} \text{ N})\hat{j} = F_p\hat{j}$. For the electron, $\vec{v}_e = -(4.75 \text{ km/s})\hat{k} = -v_e\hat{k}$ and $\vec{F}_e = (8.50\times10^{-16} \text{ N})\hat{j} = F_e\hat{j}$. The magnetic force is $\vec{F} = q\vec{v} \times \vec{B}$.

EXECUTE: **(a)** For the proton, $\vec{F}_p = q\vec{v}_p \times \vec{B}$ gives $F_p\hat{j} = ev_p\hat{i} \times (B_x\hat{i} + B_z\hat{k}) = -ev_pB_z\hat{j}$. Solving for B_z gives $B_z = -\dfrac{F_p}{ev_p} = -\dfrac{2.25\times10^{-16} \text{ N}}{(1.60\times10^{-19} \text{ C})(1500 \text{ m/s})} = -0.9375 \text{ T}$. For the electron, $\vec{F}_e = -e\vec{v}_e \times \vec{B}$, which gives $F_e\hat{j} = (-e)(-v_e\hat{k}) \times (B_x\hat{i} + B_z\hat{k}) = ev_eB_x\hat{j}$. Solving for B_x gives $B_x = \dfrac{F_e}{ev_e} = \dfrac{8.50\times10^{-16} \text{ N}}{(1.60\times10^{-19} \text{ C})(4750 \text{ m/s})} = 1.118 \text{ T}$. Therefore $\vec{B} = 1.118 \text{ T}\hat{i} - 0.9375 \text{ T}\hat{k}$. The magnitude of the field is $B = \sqrt{B_x^2 + B_z^2} = \sqrt{(1.118 \text{ T})^2 + (-0.9375 \text{ T})^2} = 1.46 \text{ T}$. Calling θ the angle that the magnetic field makes with the $+x$-axis, we have $\tan\theta = \dfrac{B_z}{B_x} = \dfrac{-0.9375 \text{ T}}{1.118 \text{ T}} = -0.8386$, so $\theta = -40.0°$. Therefore the magnetic field is in the xz-plane directed at 40.0° from the $+x$-axis toward the $-z$-axis, having a magnitude of 1.46 T.

(b) $\vec{B} = B_x\hat{i} + B_z\hat{k}$ and $\vec{v} = (3.2 \text{ km/s})(-\hat{j})$.

$\vec{F} = q\vec{v} \times \vec{B} = (-e)(3.2 \text{ km/s})(-\hat{j}) \times (B_x\hat{i} + B_z\hat{k}) = e(3.2\times10^3 \text{ m/s})[B_x(-\hat{k}) + B_z\hat{i}]$.

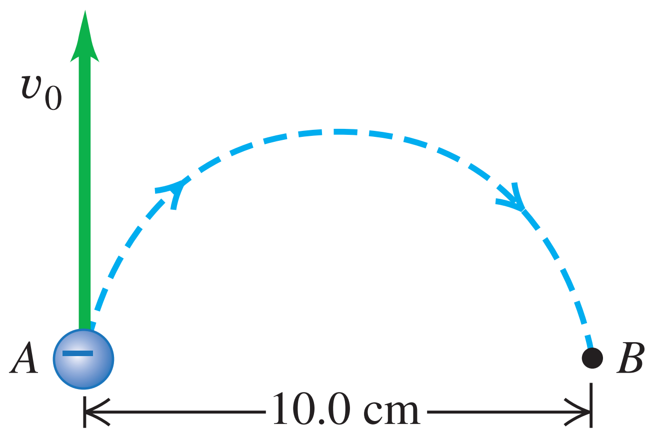
$\vec{F} = e(3.2\times10^3 \text{ m/s})(-1.118 \text{ T}\hat{k} - 0.9375 \text{ T}\hat{i}) = -4.80\times10^{-16} \text{ N}\hat{i} - 5.724\times10^{-16} \text{ N}\hat{k}$.

$F = \sqrt{F_x^2 + F_z^2} = 7.47\times10^{-16}$ N. Calling θ the angle that the force makes with the $-x$-axis, we have $\tan\theta = \dfrac{F_z}{F_x} = \dfrac{-5.724\times10^{-16} \text{ N}}{-4.800\times10^{-16} \text{ N}}$, which gives $\theta = 50.0°$. The force is in the xz-plane and is directed at 50.0° from the $-x$-axis toward either the $-z$-axis.

EVALUATE: The force on the electrons in parts (a) and (b) are comparable in magnitude because the electron speeds are comparable in both cases.

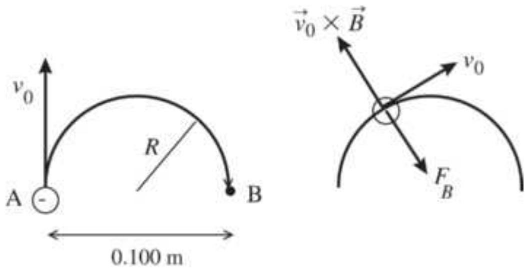
27.15 •• An electron at point A in Fig. E27.15 has a speed v_0 of 1.41×10^6 m/s. Find (a) the magnitude and direction of the magnetic field that will cause the electron to follow the semicircular path from A to B, and (b) the time required for the electron to move from A to B.

Figure **E27.15**



27.15. **(a) IDENTIFY:** Apply Eq. (27.2) to relate the magnetic force $\vec{F}$ to the directions of $\vec{v}$ and $\vec{B}$. The electron has negative charge so $\vec{F}$ is opposite to the direction of $\vec{v}\times\vec{B}$. For motion in an arc of a circle the acceleration is toward the center of the arc so $\vec{F}$ must be in this direction. $a = v^2/R$.

SET UP:



As the electron moves in the semicircle, its velocity is tangent to the circular path. The direction of $\vec{v}_0\times\vec{B}$ at a point along the path is shown in Figure 27.15.

Figure 27.15

EXECUTE: For circular motion the acceleration of the electron $\vec{a}_{\text{rad}}$ is directed in toward the center of the circle. Thus the force $\vec{F}_B$ exerted by the magnetic field, since it is the only force on the electron, must be radially inward. Since q is negative, $\vec{F}_B$ is opposite to the direction given by the right-hand rule for $\vec{v}_0\times\vec{B}$. Thus $\vec{B}$ is directed into the page. Apply Newton's second law to calculate the magnitude of $\vec{B}$:

$\sum\vec{F} = m\vec{a}$ gives $\sum F_{\text{rad}} = ma \quad F_B = m(v^2/R)$

$F_B = |q|vB\sin\phi = |q|vB$, so $|q|vB = m(v^2/R)$

$$B = \frac{mv}{|q|R} = \frac{(9.109\times10^{-31}\text{ kg})(1.41\times10^{6}\text{ m/s})}{(1.602\times10^{-19}\text{ C})(0.050\text{ m})} = 1.60\times10^{-4}\text{ T}$$

(b) IDENTIFY and SET UP: The speed of the electron as it moves along the path is constant. ($\vec{F}_B$ changes the direction of $\vec{v}$ but not its magnitude.) The time is given by the distance divided by v_0.

EXECUTE: The distance along the semicircular path is πR, so $t = \dfrac{\pi R}{v_0} = \dfrac{\pi(0.050\text{ m})}{1.41\times10^{6}\text{ m/s}} = 1.11\times10^{-7}$ s.

EVALUATE: The magnetic field required increases when v increases or R decreases and also depends on the mass to charge ratio of the particle.

27.27 •• A proton ($q = 1.60 \times 10^{-19}$ C, $m = 1.67 \times 10^{-27}$ kg) moves in a uniform magnetic field $\vec{B} = (0.500 \text{ T})\hat{\imath}$. At $t = 0$ the proton has velocity components $v_x = 1.50 \times 10^5$ m/s, $v_y = 0$, and $v_z = 2.00 \times 10^5$ m/s (see Example 27.4). (a) What are the magnitude and direction of the magnetic force acting on the proton? In addition to the magnetic field there is a uniform electric field in the $+x$-direction, $\vec{E} = (+2.00 \times 10^4 \text{ V/m})\hat{\imath}$. (b) Will the proton have a component of acceleration in the direction of the electric field? (c) Describe the path of the proton. Does the electric field affect the radius of the helix? Explain. (d) At $t = T/2$, where T is the period of the circular motion of the proton, what is the x-component of the displacement of the proton from its position at $t = 0$?

27.27. **(a) IDENTIFY and SET UP:** Eq. (27.4) gives the total force on the proton. At $t=0$,

$\vec{F}=q\vec{v}\times\vec{B}=q(v_x\hat{\boldsymbol{i}}+v_z\hat{\boldsymbol{k}})\times B_x\hat{\boldsymbol{i}}=qv_zB_x\hat{\boldsymbol{j}}.$

$\vec{F}=(1.60\times10^{-19}\text{ C})(2.00\times10^{5}\text{ m/s})(0.500\text{ T})\hat{\boldsymbol{j}}=(1.60\times10^{-14}\text{ N})\hat{\boldsymbol{j}}.$

(b) Yes. The electric field exerts a force in the direction of the electric field, since the charge of the proton is positive, and there is a component of acceleration in this direction.

(c) EXECUTE: In the plane perpendicular to $\vec{B}$ (the yz-plane) the motion is circular. But there is a velocity component in the direction of $\vec{B}$, so the motion is a helix. The electric field in the $+\hat{\boldsymbol{i}}$ direction exerts a force in the $+\hat{\boldsymbol{i}}$ direction. This force produces an acceleration in the $+\hat{\boldsymbol{i}}$ direction and this causes the pitch of the helix to vary. The force does not affect the circular motion in the yz-plane, so the electric field does not affect the radius of the helix.

(d) IDENTIFY and SET UP: Eq. (27.12) and $T=2\pi/\omega$ to calculate the period of the motion. Calculate a_x produced by the electric force and use a constant acceleration equation to calculate the displacement in the x-direction in time $T/2$.

EXECUTE: Calculate the period T: $\omega=|q|B/m$

$$T=\frac{2\pi}{\omega}=\frac{2\pi m}{|q|B}=\frac{2\pi(1.67\times10^{-27}\text{ kg})}{(1.60\times10^{-19}\text{ C})(0.500\text{ T})}=1.312\times10^{-7}\text{ s. Then } t=T/2=6.56\times10^{-8}\text{ s.}$$

$v_{0x}=1.50\times10^{5}\text{ m/s}$

$$a_x=\frac{F_x}{m}=\frac{(1.60\times10^{-19}\text{ C})(2.00\times10^{4}\text{ V/m})}{1.67\times10^{-27}\text{ kg}}=+1.916\times10^{12}\text{ m/s}^2$$

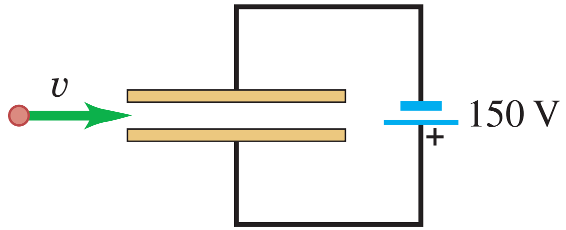
$x-x_0=v_{0x}t+\tfrac{1}{2}a_xt^2$

$x-x_0=(1.50\times10^{5}\text{ m/s})(6.56\times10^{-8}\text{ s})+\tfrac{1}{2}(1.916\times10^{12}\text{ m/s}^2)(6.56\times10^{-8}\text{ s})^2=1.40\text{ cm}$

EVALUATE: The electric and magnetic fields are in the same direction but produce forces that are in perpendicular directions to each other.

27.31 •• A 150-V battery is connected across two parallel metal plates of area 28.5 cm^2 and separation 8.20 mm. A beam of alpha particles (charge $+2e$, mass 6.64×10^{-27} kg) is accelerated from rest through a potential difference of 1.75 kV and enters the region between the plates perpendicular to the electric field, as shown in Fig. E27.31. What magnitude and direction of magnetic field are needed so that the alpha particles emerge undeflected from between the plates?

Figure **E27.31**



positively and negatively charged particles.

27.31. **IDENTIFY:** For the alpha particles to emerge from the plates undeflected, the magnetic force on them must exactly cancel the electric force. The battery produces an electric field between the plates, which acts on the alpha particles.

SET UP: First use energy conservation to find the speed of the alpha particles as they enter the region between the plates: $qV = 1/2\ mv^2$. The electric field between the plates due to the battery is $E = V_b d$. For the alpha particles not to be deflected, the magnetic force must cancel the electric force, so $qvB = qE$, giving $B = E/v$.

EXECUTE: Solve for the speed of the alpha particles just as they enter the region between the plates. Their charge is $2e$.

$$v_\alpha = \sqrt{\frac{2(2e)V}{m}} = \sqrt{\frac{4(1.60\times10^{-19}\ \text{C})(1750\text{V})}{6.64\times10^{-27}\ \text{kg}}} = 4.11\times10^5\ \text{m/s}$$

The electric field between the plates, produced by the battery, is

$$E = V_b/d = (150\ \text{V})/(0.00820\ \text{m}) = 18{,}300\ \text{V/m}$$

The magnetic force must cancel the electric force:

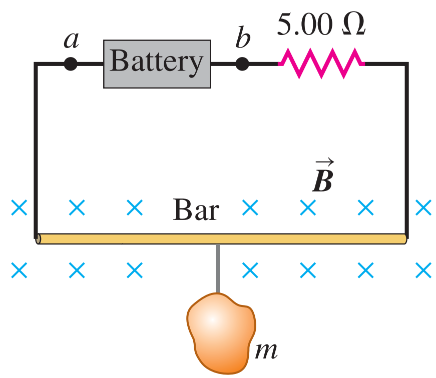
$$B = E/v_\alpha = (18{,}300\ \text{V/m})/(4.11\times10^5\ \text{m/s}) = 0.0445\ \text{T}$$

The magnetic field is perpendicular to the electric field. If the charges are moving to the right and the electric field points upward, the magnetic field is out of the page.

EVALUATE: The sign of the charge of the alpha particle does not enter the problem, so negative charges of the same magnitude would also not be deflected.

27.42 • **Magnetic Balance.** The circuit shown in Fig. E27.42 is used to make a magnetic balance to weigh objects. The mass m to be measured is hung from the center of the bar that is in a uniform magnetic field of 1.50 T, directed into the plane of the figure. The battery voltage can be adjusted to vary the current in the circuit. The horizontal bar is 60.0 cm long and is made of extremely light-weight material. It is connected to the battery by thin vertical wires that can support no appreciable tension; all the weight of the suspended mass m is supported by the magnetic force on the bar. A resistor with $R = 5.00\ \Omega$ is in series with the bar; the resistance of the rest of the circuit is much less than this. (a) Which point, a or b, should be the positive terminal of the battery? (b) If the maximum terminal voltage of the battery is 175 V, what is the greatest mass m that this instrument can measure?

Figure **E27.42**



27.42. **IDENTIFY:** The magnetic force $\vec{F}_B$ must be upward and equal to mg. The direction of $\vec{F}_B$ is determined by the direction of I in the circuit.

SET UP: $F_B = IlB\sin\phi$, with $\phi = 90°$. $I = \dfrac{V}{R}$, where V is the battery voltage.

EXECUTE: **(a)** The forces are shown in Figure 27.42. The current I in the bar must be to the right to produce $\vec{F}_B$ upward. To produce current in this direction, point a must be the positive terminal of the battery.

(b) $F_B = mg$. $IlB = mg$. $m = \dfrac{IlB}{g} = \dfrac{VlB}{Rg} = \dfrac{(175\text{ V})(0.600\text{ m})(1.50\text{ T})}{(5.00\ \Omega)(9.80\text{ m/s}^2)} = 3.21\text{ kg}.$

EVALUATE: If the battery had opposite polarity, with point a as the negative terminal, then the current would be clockwise and the magnetic force would be downward.

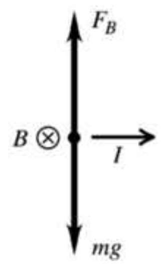
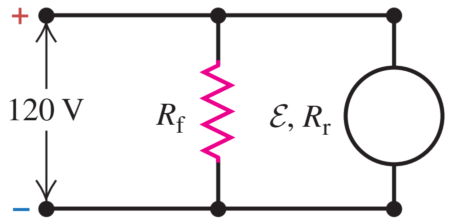


Figure 27.42

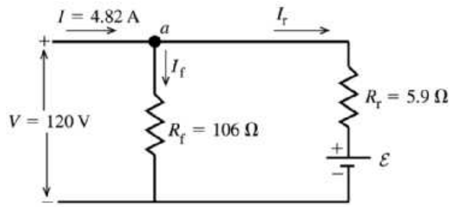
27.51 •• In a shunt-wound dc motor with the field coils and rotor connected in parallel (Fig. E27.51), the resistance R_f of the field coils is 106 Ω, and the resistance R_r of the rotor is 5.9 Ω. When a potential difference of 120 V is applied to the brushes and the motor is running at full speed delivering mechanical power, the current supplied to it is 4.82 A. (a) What is the current in the field coils? (b) What is the current in the rotor? (c) What is the induced emf developed by the motor? (d) How much mechanical power is developed by this motor?

Figure **E27.51**



27.51. **IDENTIFY:** The circuit consists of two parallel branches with the potential difference of 120 V applied across each. One branch is the rotor, represented by a resistance R_r and an induced emf that opposes the applied potential. Apply the loop rule to each parallel branch and use the junction rule to relate the currents through the field coil and through the rotor to the 4.82 A supplied to the motor.

SET UP: The circuit is sketched in Figure 27.51.



$\mathcal{E}$ is the induced emf developed by the motor. It is directed so as to oppose the current through the rotor.

Figure 27.51

EXECUTE: **(a)** The field coils and the rotor are in parallel with the applied potential difference V, so $V = I_f R_f$. $I_f = \dfrac{V}{R_f} = \dfrac{120\text{ V}}{106\ \Omega} = 1.13\text{ A}.$

(b) Applying the junction rule to point a in the circuit diagram gives $I - I_f - I_r = 0.$

$I_r = I - I_f = 4.82\text{ A} - 1.13\text{ A} = 3.69\text{ A}.$

(c) The potential drop across the rotor, $I_r R_r + \mathcal{E}$, must equal the applied potential difference V: $V = I_r R_r + \mathcal{E}$

$\mathcal{E} = V - I_r R_r = 120\text{ V} - (3.69\text{ A})(5.9\ \Omega) = 98.2\text{ V}$

(d) The mechanical power output is the electrical power input minus the rate of dissipation of electrical energy in the resistance of the motor:

electrical power input to the motor

$P_{\text{in}} = IV = (4.82\text{ A})(120\text{ V}) = 578\text{ W}$

electrical power loss in the two resistances

$P_{\text{loss}} = I_f^2 R_f + I_r^2 R_r = (1.13\text{ A})^2(106\ \Omega) + (3.69\text{ A})^2(5.9\ \Omega) = 216\text{ W}$

mechanical power output

$P_{\text{out}} = P_{\text{in}} - P_{\text{loss}} = 578\text{ W} - 216\text{ W} = 362\text{ W}$

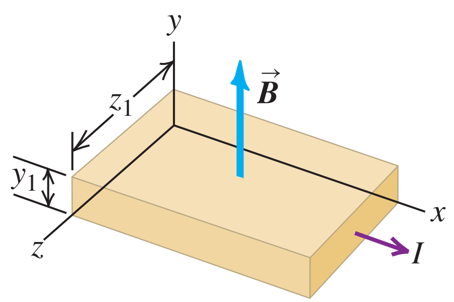
The mechanical power output is the power associated with the induced emf $\mathcal{E}$.

$P_{\text{out}} = P_{\mathcal{E}} = \mathcal{E} I_r = (98.2\text{ V})(3.69\text{ A}) = 362\text{ W},$ which agrees with the above calculation.

EVALUATE: The induced emf reduces the amount of current that flows through the rotor. This motor differs from the one described in Example 27.11. In that example the rotor and field coils are connected in series and in this problem they are in parallel.

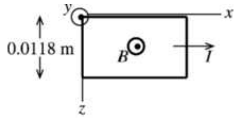
27.53 • Figure E27.53 shows a portion of a silver ribbon with $z_1 = 11.8$ mm and $y_1 = 0.23$ mm, carrying a current of 120 A in the $+x$-direction. The ribbon lies in a uniform magnetic field, in the y-direction, with magnitude 0.95 T. Apply the simplified model of the Hall effect presented in Section 27.9. If there are 5.85×10^{28} free electrons per cubic meter, find (a) the magnitude of the drift velocity of the electrons in the x-direction; (b) the magnitude and direction of the electric field in the z-direction due to the Hall effect; (c) the Hall emf.

Figure **E27.53**



27.53. **IDENTIFY:** The drift velocity is related to the current density by Eq. (25.4). The electric field is determined by the requirement that the electric and magnetic forces on the current-carrying charges are equal in magnitude and opposite in direction.

(a) SET UP: The section of the silver ribbon is sketched in Figure 27.53a.



$J_x = n|q|v_d$

so $v_d = \dfrac{J_x}{n|q|}$

Figure 27.53a

EXECUTE: $J_x = \dfrac{I}{A} = \dfrac{I}{y_1 z_1} = \dfrac{120\text{ A}}{(0.23\times10^{-3}\text{ m})(0.0118\text{ m})} = 4.42\times10^{7}\text{ A/m}^2$

$$v_d = \frac{J_x}{n|q|} = \frac{4.42\times10^{7}\text{ A/m}^2}{(5.85\times10^{28}/\text{m}^3)(1.602\times10^{-19}\text{ C})} = 4.7\times10^{-3}\text{ m/s} = 4.7\text{ mm/s}$$

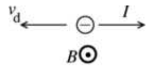
(b) magnitude of $\vec{E}$

$|q|E_z = |q|v_d B_y$

$E_z = v_d B_y = (4.7\times10^{-3}\text{ m/s})(0.95\text{ T}) = 4.5\times10^{-3}\text{ V/m}$

direction of $\vec{E}$

The drift velocity of the electrons is in the opposite direction to the current, as shown in Figure 27.53b.

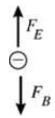


$\vec{v}\times\vec{B}\uparrow$

$\vec{F}_B = q\vec{v}\times\vec{B} = -e\vec{v}\times\vec{B}\downarrow$

Figure 27.53b

The directions of the electric and magnetic forces on an electron in the ribbon are shown in Figure 27.53c.



$\vec{F}_E$ must oppose $\vec{F}_B$ so $\vec{F}_E$ is in the $-z$-direction.

Figure 27.53c

$\vec{F}_E = q\vec{E} = -e\vec{E}$ so $\vec{E}$ is opposite to the direction of $\vec{F}_E$ and thus $\vec{E}$ is in the $+z$-direction.

(c) The Hall emf is the potential difference between the two edges of the strip (at $z = 0$ and $z = z_1$) that results from the electric field calculated in part (b). $\mathcal{E}_{\text{Hall}} = Ez_1 = (4.5\times10^{-3}\text{ V/m})(0.0118\text{ m}) = 53\,\mu\text{V}$.

EVALUATE: Even though the current is quite large the Hall emf is very small. Our calculated Hall emf is more than an order of magnitude larger than in Example 27.12. In this problem the magnetic field and current density are larger than in the example, and this leads to a larger Hall emf.

27.58 •• **Magnetic Moment of the Hydrogen Atom.** In the Bohr model of the hydrogen atom (see Section 38.5), in the lowest energy state the electron orbits the proton at a speed of 2.2×10^6 m/s in a circular orbit of radius 5.3×10^{-11} m. (a) What is the orbital period of the electron? (b) If the orbiting electron is considered to be a current loop, what is the current I? (c) What is the magnetic moment of the atom due to the motion of the electron?

27.58. **IDENTIFY:** The period is $T = 2\pi r/v,$ the current is Q/t and the magnetic moment is $\mu = IA.$

SET UP: The electron has charge $-e.$ The area enclosed by the orbit is $\pi r^2.$

EXECUTE: **(a)** $T = 2\pi r/v = 1.5\times10^{-16}$ s

(b) Charge $-e$ passes a point on the orbit once during each period, so $I = Q/t = e/t = 1.1$ mA.

(c) $\mu = IA = I\pi r^2 = 9.3\times10^{-24}\ \text{A}\cdot\text{m}^2$

EVALUATE: Since the electron has negative charge, the direction of the current is opposite to the direction of motion of the electron.

27.68 •• Mass Spectrograph. A mass spectrograph is used to measure the masses of ions, or to separate ions of different masses (see Section 27.5). In one design for such an instrument, ions with mass m and charge q are accelerated through a potential difference V. They then enter a uniform magnetic field that is perpendicular to their velocity, and they are deflected in a semicircular path of radius R. A detector measures where the ions complete the semicircle and from this it is easy to calculate R. (a) Derive the equation for calculating the mass of the ion from measurements of B, V, R, and q. (b) What potential difference V is needed so that singly ionized ^{12}C atoms will have $R = 50.0$ cm in a 0.150-T magnetic field? (c) Suppose the beam consists of a mixture of ^{12}C and ^{14}C ions. If v and B have the same values as in part (b), calculate the separation of these two isotopes at the detector. Do you think that this beam separation is sufficient for the two ions to be distinguished? (Make the assumption described in Problem 27.67 for the masses of the ions.)

27.68. **IDENTIFY:** Apply conservation of energy to the acceleration of the ions and Newton's second law to their motion in the magnetic field.

SET UP: The singly ionized ions have $q = +e$. A ^{12}C ion has mass 12 u and a ^{14}C ion has mass 14 u, where $1\ \text{u} = 1.66\times10^{-27}$ kg.

EXECUTE: **(a)** During acceleration of the ions, $qV = \frac{1}{2}mv^2$ and $v = \sqrt{\frac{2qV}{m}}$. In the magnetic field,

$$R = \frac{mv}{qB} = \frac{m\sqrt{2qV/m}}{qB} \text{ and } m = \frac{qB^2R^2}{2V}.$$

(b) $V = \dfrac{qB^2R^2}{2m} = \dfrac{(1.60\times10^{-19}\ \text{C})(0.150\ \text{T})^2(0.500\ \text{m})^2}{2(12)(1.66\times10^{-27}\ \text{kg})} = 2.26\times10^4\ \text{V}$

(c) The ions are separated by the differences in the diameters of their paths. $D = 2R = 2\sqrt{\dfrac{2Vm}{qB^2}}$, so

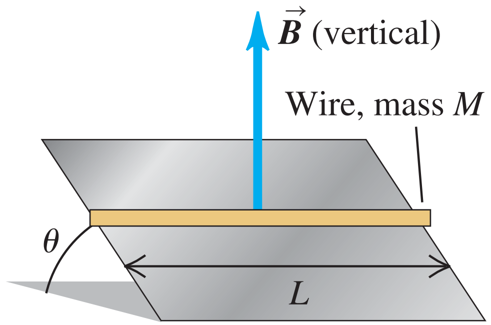
$$\Delta D = D_{14} - D_{12} = 2\sqrt{\frac{2Vm}{qB^2}}\bigg|_{14} - 2\sqrt{\frac{2Vm}{qB^2}}\bigg|_{12} = 2\sqrt{\frac{2V(1\ \text{u})}{qB^2}}(\sqrt{14} - \sqrt{12}).$$

$\Delta D = 2\sqrt{\dfrac{2(2.26\times10^4\ \text{V})(1.66\times10^{-27}\ \text{kg})}{(1.6\times10^{-19}\ \text{C})(0.150\ \text{T})^2}}(\sqrt{14} - \sqrt{12}) = 8.01\times10^{-2}\ \text{m}$. This is about 8 cm and is easily distinguishable.

EVALUATE: The speed of the ^{12}C ion is $v = \sqrt{\dfrac{2(1.60\times10^{-19}\ \text{C})(2.26\times10^4\ \text{V})}{12(1.66\times10^{-27}\ \text{kg})}} = 6.0\times10^5$ m/s. This is very fast, but well below the speed of light, so relativistic mechanics is not needed.

27.69 •• A straight piece of conducting wire with mass M and length L is placed on a frictionless incline tilted at an angle θ from the horizontal (Fig. P27.69). There is a uniform, vertical magnetic field $\vec{B}$ at all points (produced by an arrangement of magnets not shown in the figure). To keep the wire from sliding down the incline, a voltage source is attached to the ends of the wire. When just the right amount of current flows through the wire, the wire remains at rest. Determine the magnitude and direction of the current in the wire that will cause the wire to remain at rest. Copy the figure and draw the direction of the current on your copy. In addition, show in a free-body diagram all the forces that act on the wire.

Figure **P27.69**



27.69. **IDENTIFY:** The force exerted by the magnetic field is given by Eq. (27.19). The net force on the wire must be zero.

SET UP: For the wire to remain at rest the force exerted on it by the magnetic field must have a component directed up the incline. To produce a force in this direction, the current in the wire must be directed from right to left in Figure P27.69 in the textbook. Or, viewing the wire from its left-hand end the directions are shown in Figure 27.69a.

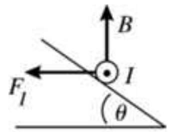


Figure 27.69a

The free-body diagram for the wire is given in Figure 27.69b.

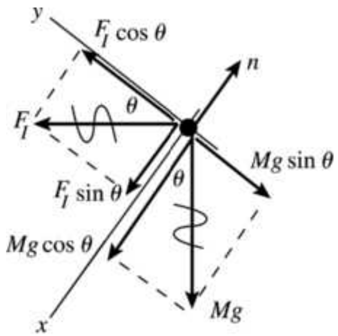


Figure 27.69b

EXECUTE: $\sum F_y = 0$

$F_I \cos\theta - Mg\sin\theta = 0$

$F_I = ILB\sin\phi$

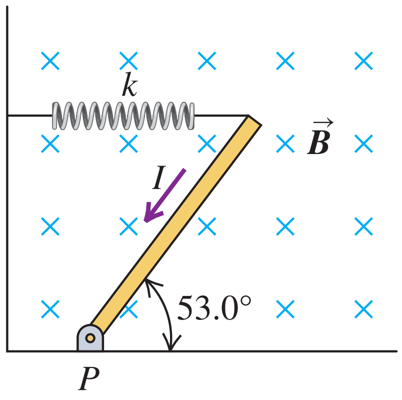
$\phi = 90°$ since $\vec{B}$ is perpendicular to the current direction.

Thus $(ILB)\ \cos\theta - Mg\sin\theta = 0$ and $I = \dfrac{Mg\tan\theta}{LB}$.

EVALUATE: The magnetic and gravitational forces are in perpendicular directions so their components parallel to the incline involve different trig functions. As the tilt angle θ increases there is a larger component of *Mg* down the incline and the component of F_I up the incline is smaller; *I* must increase with θ to compensate. As $\theta \to 0$, $I \to 0$ and as $\theta \to 90°$, $I \to \infty$.

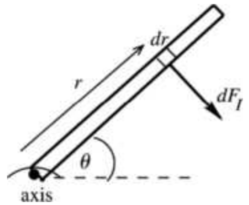
27.79 •• **CP CALC** A thin, uniform rod with negligible mass and length 0.200 m is attached to the floor by a frictionless hinge at point P (Fig. P27.79). A horizontal spring with force constant $k = 4.80$ N/m connects the other end of the rod to a vertical wall. The rod is in a uniform magnetic field $B = 0.340$ T directed into the plane of the figure. There is current $I = 6.50$ A in the rod, in the direction shown. (a) Calculate the torque due to the magnetic force on the rod, for an axis at P. Is it correct to take the total magnetic force to act at the center of gravity of the rod when calculating the torque? Explain. (b) When the rod is in equilibrium and makes an angle of 53.0° with the floor, is the spring stretched or compressed? (c) How much energy is stored in the spring when the rod is in equilibrium?

Figure **P27.79**



27.79. **IDENTIFY:** Use Eq. (27.20) to calculate the force and then the torque on each small section of the rod and integrate to find the total magnetic torque. At equilibrium the torques from the spring force and from the magnetic force cancel. The spring force depends on the amount x the spring is stretched and then $U = \frac{1}{2}kx^2$ gives the energy stored in the spring.

(a) SET UP:



Divide the rod into infinitesimal sections of length dr, as shown in Figure 27.79.

Figure 27.79

EXECUTE: The magnetic force on this section is $dF_I = IBdr$ and is perpendicular to the rod. The torque $d\tau$ due to the force on this section is $d\tau = rdF_I = IBr\,dr$. The total torque is $\int d\tau = IB\int_0^l r\,dr = \frac{1}{2}Il^2B = 0.0442\ \text{N}\cdot\text{m}$, clockwise.

(b) SET UP: F_I produces a clockwise torque so the spring force must produce a counterclockwise torque. The spring force must be to the left; the spring is stretched.

EXECUTE: Find x, the amount the spring is stretched:

$\sum\tau = 0$, axis at hinge, counterclockwise torques positive

$$(kx)l\sin 53^\circ - \frac{1}{2}Il^2B = 0$$

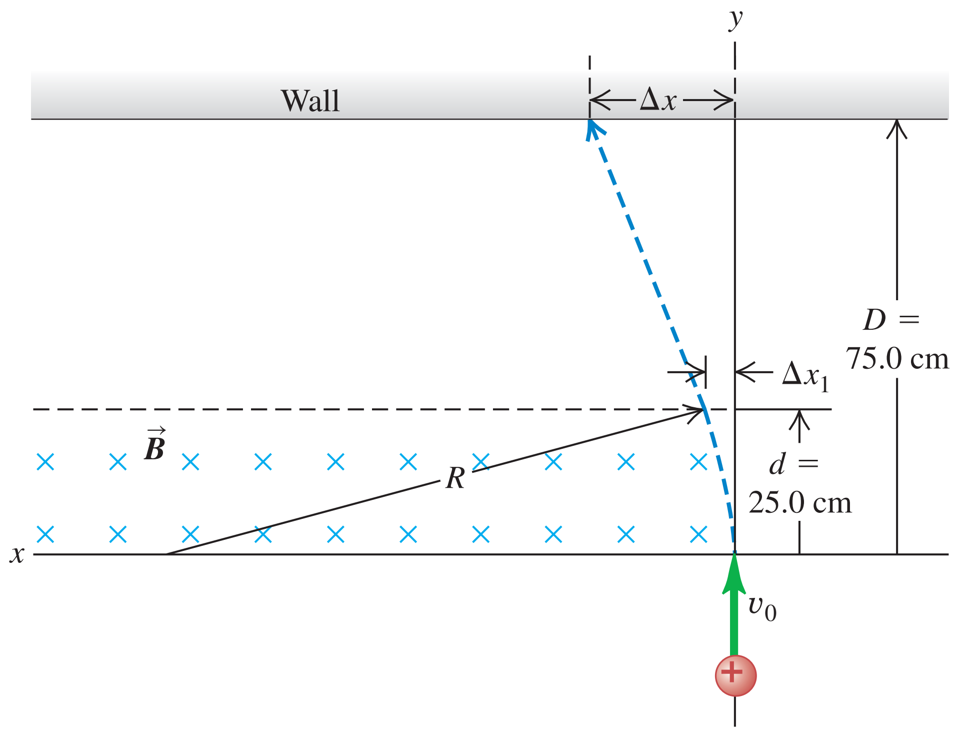
$$x = \frac{IlB}{2k\sin 53.0^\circ} = \frac{(6.50\ \text{A})(0.200\ \text{m})(0.340\ \text{T})}{2(4.80\ \text{N/m})\sin 53.0^\circ} = 0.05765\ \text{m}$$

(c) $U = \frac{1}{2}kx^2 = 7.98\times 10^{-3}\ \text{J}$

EVALUATE: The magnetic torque calculated in part (a) is the same torque calculated from a force diagram in which the total magnetic force $F_I = IlB$ acts at the center of the rod. We didn't include a gravity torque since the problem said the rod had negligible mass.

27.89 ••• A particle with charge 2.15 μC and mass 3.20×10^{-11} kg is initially traveling in the $+y$-direction with a speed $v_0 = 1.45 \times 10^5$ m/s. It then enters a region containing a uniform magnetic field that is directed into, and perpendicular to, the page in Fig. P27.89. The magnitude of the field is 0.420 T. The region extends a distance of 25.0 cm along the initial direction of travel; 75.0 cm from the point of entry into the magnetic field region is a wall. The length of the field-free region is thus 50.0 cm. When the charged particle enters the magnetic field, it follows a curved path whose radius of curvature is R. It then leaves the magnetic field after a time t_1, having been deflected a distance Δx_1. The particle then travels in the field-free region and strikes the wall after undergoing a total deflection Δx. (a) Determine the radius R of the curved part of the path. (b) Determine t_1, the time the particle spends in the magnetic field. (c) Determine Δx_1, the horizontal deflection at the point of exit from the field. (d) Determine Δx, the total horizontal deflection.

Figure **P27.89**



27.89. **IDENTIFY** and **SET UP:** In the magnetic field, $R = \frac{mv}{qB}$. Once the particle exits the field it travels in a straight line. Throughout the motion the speed of the particle is constant.

EXECUTE: **(a)** $R = \frac{mv}{qB} = \frac{(3.20\times10^{-11}\text{ kg})(1.45\times10^{5}\text{ m/s})}{(2.15\times10^{-6}\text{ C})(0.420\text{ T})} = 5.14\text{ m}.$

(b) See Figure 27.89. The distance along the curve, d, is given by $d = R\theta$. $\sin\theta = \frac{0.25\text{ m}}{5.14\text{ m}}$, so $\theta = 2.79° = 0.0486\text{ rad}$. $d = R\theta = (5.14\text{ m})(0.0486\text{ rad}) = 0.25\text{ m}$. And

$t = \frac{d}{v} = \frac{0.25\text{ m}}{1.45\times10^{5}\text{ m/s}} = 1.72\times10^{-6}\text{s}.$

(c) $\Delta x_1 = d\tan(\theta/2) = (0.25\text{ m})\tan(2.79°/2) = 6.08\times10^{-3}\text{ m}.$

(d) $\Delta x = \Delta x_1 + \Delta x_2$, where Δx_2 is the horizontal displacement of the particle from where it exits the field region to where it hits the wall. $\Delta x_2 = (0.50\text{ m})\tan 2.79° = 0.0244\text{ m}$. Therefore,

$\Delta x = 6.08\times10^{-3}\text{ m} + 0.0244\text{ m} = 0.0305\text{ m}.$

EVALUATE: d is much less than R, so the horizontal deflection of the particle is much smaller than the distance it travels in the y-direction.

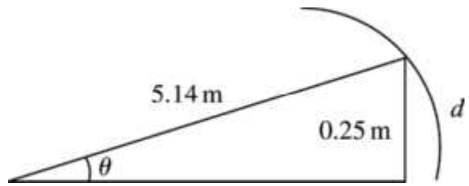


Figure 27.89