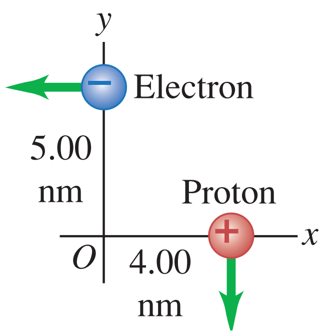
28.8 •• An electron and a proton are each moving at 845 km/s in perpendicular paths as shown in Fig. E28.8. At the instant when they are at the positions shown in the figure, find the magnitude and direction of (a) the total magnetic field they produce at the origin; (b) the magnetic field the electron produces at the location of the proton; (c) the total electric force and the total magnetic force that the electron exerts on the proton.

Figure **E28.8**



28.8. **IDENTIFY:** Both moving charges create magnetic fields, and the net field is the vector sum of the two. The magnetic force on a moving charge is $F_{\text{mag}} = qvB\sin\phi$ and the electrical force obeys Coulomb's law.

SET UP: The magnetic field due to a moving charge is $B = \dfrac{\mu_0}{4\pi}\dfrac{qv\sin\phi}{r^2}$.

EXECUTE: **(a)** Both fields are into the page, so their magnitudes add, giving

$$B = B_{\text{e}} + B_{\text{p}} = \frac{\mu_0}{4\pi}\left(\frac{ev}{r_{\text{e}}^2} + \frac{ev}{r_{\text{p}}^2}\right)\sin 90°$$

$$B = \frac{\mu_0}{4\pi}(1.60\times10^{-19}\,\text{C})(845{,}000\text{ m/s})\left[\frac{1}{(5.00\times10^{-9}\,\text{m})^2} + \frac{1}{(4.00\times10^{-9}\,\text{m})^2}\right]$$

$$B = 1.39\times10^{-3}\text{ T} = 1.39\text{ mT, into the page.}$$

(b) Using $B = \dfrac{\mu_0}{4\pi}\dfrac{qv\sin\phi}{r^2}$, where $r = \sqrt{41}$ nm and $\phi = 180° - \arctan(5/4) = 128.7°$, we get

$$B = \frac{4\pi\times10^{-7}\text{ T}\cdot\text{m/A}}{4\pi}\frac{(1.6\times10^{-19}\text{C})(845{,}000\text{ m/s})\sin 128.7°}{(\sqrt{41}\times10^{-9}\text{m})^2} = 2.58\times10^{-4}\text{ T, into the page.}$$

(c) $F_{\text{mag}} = qvB\sin 90° = (1.60\times10^{-19}\text{ C})(845{,}000\text{ m/s})(2.58\times10^{-4}\text{ T}) = 3.48\times10^{-17}\text{ N}$, in the $+x$-direction.

$F_{\text{elec}} = (1/4\pi\epsilon_0)e^2/r^2 = \dfrac{(9.00\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\text{C})^2}{(\sqrt{41}\times10^{-9}\text{m})^2} = 5.62\times10^{-12}\text{ N}$, at $51.3°$ below the $+x$-axis measured clockwise.

EVALUATE: The electric force is much stronger than the magnetic force.

28.28 • Four very long, current-carrying wires in the same plane intersect to form a square 40.0 cm on each side, as shown in Fig. E28.28. Find the magnitude and direction of the current I so that the magnetic field at the center of the square is zero.

Figure **E28.28**

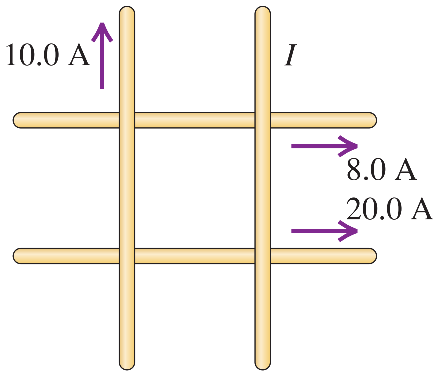
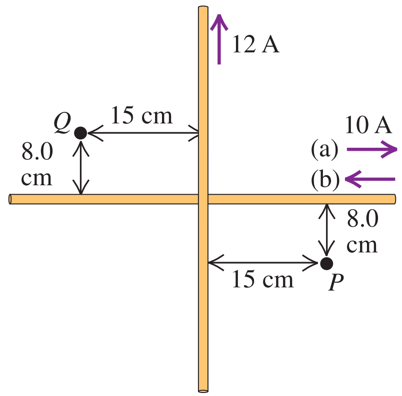


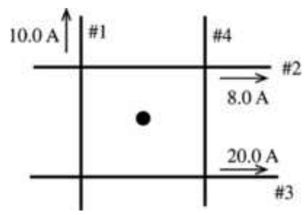
Figure **E28.29**



28.29 •• Two insulated wires perpendicular to each other in the same plane carry currents as shown in Fig. E28.29. Find the magnitude of the *net* magnetic field these wires produce at points P and Q if the 10.0 A-current is (a) to the right or (b) to the left.

28.28. **IDENTIFY:** Use Eq. (28.9) and the right-hand rule to determine the field due to each wire. Set the sum of the four fields equal to zero and use that equation to solve for the field and the current of the fourth wire.

SET UP: The three known currents are shown in Figure 28.28.



$\vec{B}_1 \otimes, \vec{B}_2 \otimes, \vec{B}_3 \odot$

$B = \dfrac{\mu_0 I}{2\pi r}$; $r = 0.200$ m for each wire

Figure 28.28

EXECUTE: Let $\odot$ be the positive z-direction. $I_1 = 10.0$ A, $I_2 = 8.0$ A, $I_3 = 20.0$ A. Then

$B_1 = 1.00\times10^{-5}$ T, $B_2 = 0.80\times10^{-5}$ T, and $B_3 = 2.00\times10^{-5}$ T.

$B_{1z} = -1.00\times10^{-5}$ T, $B_{2z} = -0.80\times10^{-5}$ T, $B_{3z} = +2.00\times10^{-5}$ T

$B_{1z} + B_{2z} + B_{3z} + B_{4z} = 0$

$B_{4z} = -(B_{1z} + B_{2z} + B_{3z}) = -2.0\times10^{-6}$ T

To give $\vec{B}_4$ in the $\otimes$ direction the current in wire 4 must be toward the bottom of the page.

$$B_4 = \frac{\mu_0 I}{2\pi r} \text{ so } I_4 = \frac{rB_4}{(\mu_0/2\pi)} = \frac{(0.200\text{ m})(2.0\times10^{-6}\text{ T})}{(2\times10^{-7}\text{ T}\cdot\text{m/A})} = 2.0\text{ A}$$

EVALUATE: The fields of wires #2 and #3 are in opposite directions and their net field is the same as due to a current $20.0\text{ A} - 8.0\text{ A} = 12.0\text{ A}$ in one wire. The field of wire #4 must be in the same direction as that of wire #1, and $10.0\text{ A} + I_4 = 12.0\text{ A}$.

28.29. **IDENTIFY:** The net magnetic field at any point is the vector sum of the magnetic fields of the two wires.

SET UP: For each wire $B = \frac{\mu_0 I}{2\pi r}$ and the direction of $\vec{\boldsymbol{B}}$ is determined by the right-hand rule described in the text. Let the wire with 12.0 A be wire 1 and the wire with 10.0 A be wire 2.

EXECUTE: **(a)** *Point Q*: $B_1 = \frac{\mu_0 I_1}{2\pi r_1} = \frac{(4\pi \times 10^{-7}\,\text{T} \cdot \text{m/A})(12.0\ \text{A})}{2\pi(0.15\ \text{m})} = 1.6 \times 10^{-5}\ \text{T}.$

The direction of $\vec{\boldsymbol{B}}_1$ is out of the page. $B_2 = \frac{\mu_0 I_2}{2\pi r_2} = \frac{(4\pi \times 10^{-7}\,\text{T} \cdot \text{m/A})(10.0\ \text{A})}{2\pi(0.80\ \text{m})} = 2.5 \times 10^{-5}\ \text{T}.$

The direction of $\vec{\boldsymbol{B}}_2$ is out of the page. Since $\vec{\boldsymbol{B}}_1$ and $\vec{\boldsymbol{B}}_2$ are in the same direction, $B = B_1 + B_2 = 4.1 \times 10^{-5}\ \text{T}$ and $\vec{\boldsymbol{B}}$ is directed out of the page.

Point P: $B_1 = 1.6 \times 10^{-5}\ \text{T},$ directed into the page. $B_2 = 2.5 \times 10^{-5}\ \text{T},$ directed into the page.

$B = B_1 + B_2 = 4.1 \times 10^{-5}\ \text{T}$ and $\vec{\boldsymbol{B}}$ is directed into the page.

(b) $\vec{\boldsymbol{B}}_1$ is the same as in part (a), out of the page at *Q* and into the page at *P*. The direction of $\vec{\boldsymbol{B}}_2$ is reversed from what it was in (a) so is into the page at *Q* and out of the page at *P*.

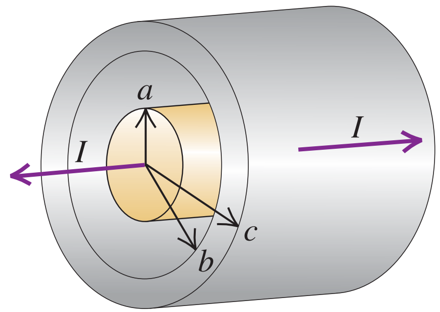
Point Q: $\vec{\boldsymbol{B}}_1$ and $\vec{\boldsymbol{B}}_2$ are in opposite directions so $B = B_2 - B_1 = 2.5 \times 10^{-5}\ \text{T} - 1.6 \times 10^{-5}\ \text{T} = 9.0 \times 10^{-6}\ \text{T}$ and $\vec{\boldsymbol{B}}$ is directed into the page.

Point P: $\vec{\boldsymbol{B}}_1$ and $\vec{\boldsymbol{B}}_2$ are in opposite directions so $B = B_2 - B_1 = 9.0 \times 10^{-6}\ \text{T}$ and $\vec{\boldsymbol{B}}$ is directed out of the page.

EVALUATE: Points *P* and *Q* are the same distances from the two wires. The only difference is that the fields point in either the same direction or in opposite directions.

28.45 • **Coaxial Cable.** A solid conductor with radius a is supported by insulating disks on the axis of a conducting tube with inner radius b and outer radius c (Fig. E28.45). The central conductor and tube carry equal currents I in opposite directions. The currents are distributed uniformly over the cross sections of each conductor. Derive an expression for the magnitude of the magnetic field (a) at points outside the central, solid conductor but inside the tube $(a < r < b)$ and (b) at points outside the tube $(r > c)$.

Figure **E28.45**



28.45. **IDENTIFY:** Apply Ampere's law.

SET UP: To calculate the magnetic field at a distance r from the center of the cable, apply Ampere's law to a circular path of radius r. By symmetry, $\oint \vec{B} \cdot d\vec{l} = B(2\pi r)$ for such a path.

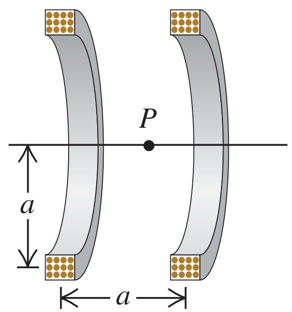
EXECUTE: **(a)** For $a < r < b, I_{\text{encl}} = I \Rightarrow \oint \vec{B} \cdot d\vec{l} = \mu_0 I \Rightarrow B 2\pi r = \mu_0 I \Rightarrow B = \dfrac{\mu_0 I}{2\pi r}$.

(b) For $r > c$, the enclosed current is zero, so the magnetic field is also zero.

EVALUATE: A useful property of coaxial cables for many applications is that the current carried by the cable doesn't produce a magnetic field outside the cable.

28.75 • **CALC** **Helmholtz Coils.** Figure 28.75 is a sectional view of two circular coils with radius a, each wound with N turns of wire carrying a current I, circulating in the same direction in both coils. The coils are separated by a distance a equal to their radii. In this configuration the coils are called Helmholtz coils; they produce a very uniform magnetic field in the region between them. (a) Derive the expression for the magnitude B of the magnetic field at a point on the axis a distance x to the right of point P, which is midway between the coils. (b) Graph B versus x for $x = 0$ to $x = a/2$. Compare this graph to one for the magnetic field due to the right-hand coil alone. (c) From part (a), obtain an expression for the magnitude of the magnetic field at point P. (d) Calculate the magnitude of the magnetic field at P if $N = 300$ turns, $I = 6.00$ A, and $a = 8.00$ cm. (e) Calculate dB/dx and d^2B/dx^2 at $P(x = 0)$. Discuss how your results show that the field is very uniform in the vicinity of P.

Figure **P28.75**



28.75. **IDENTIFY:** Find the vector sum of the fields due to each loop.

SET UP: For a single loop $B = \dfrac{\mu_0 I a^2}{2(x^2 + a^2)^{3/2}}$. Here we have two loops, each of N turns, and measuring the field along the x-axis from between them means that the "x" in the formula is different for each case:

EXECUTE:

$$\text{Left coil: } x \to x + \frac{a}{2} \Rightarrow B_\text{l} = \frac{\mu_0 N I a^2}{2((x + a/2)^2 + a^2)^{3/2}}.$$

$$\text{Right coil: } x \to x - \frac{a}{2} \Rightarrow B_\text{r} = \frac{\mu_0 N I a^2}{2((x - a/2)^2 + a^2)^{3/2}}.$$

So, the total field at a point a distance x from the point between them is

$$B = \frac{\mu_0 N I a^2}{2}\left(\frac{1}{((x + a/2)^2 + a^2)^{3/2}} + \frac{1}{((x - a/2)^2 + a^2)^{3/2}}\right).$$

(b) B versus x is graphed in Figure 28.75. Figure 28.75a is the total field and Figure 28.75b is the field from the right-hand coil.

(c) At point P, $x = 0$ and $B = \dfrac{\mu_0 N I a^2}{2}\left(\dfrac{1}{((a/2)^2 + a^2)^{3/2}} + \dfrac{1}{((-a/2)^2 + a^2)^{3/2}}\right) = \dfrac{\mu_0 N I a^2}{(5a^2/4)^{3/2}} = \left(\dfrac{4}{5}\right)^{3/2}\dfrac{\mu_0 N I}{a}$

(d) $B = \left(\dfrac{4}{5}\right)^{3/2}\dfrac{\mu_0 N I}{a} = \left(\dfrac{4}{5}\right)^{3/2}\dfrac{\mu_0 (300)(6.00\text{ A})}{(0.080\text{ m})} = 0.0202\text{ T}.$

(e) $\dfrac{dB}{dx} = \dfrac{\mu_0 N I a^2}{2}\left(\dfrac{-3(x + a/2)}{((x + a/2)^2 + a^2)^{5/2}} + \dfrac{-3(x - a/2)}{((x - a/2)^2 + a^2)^{5/2}}\right)$. At $x = 0$,

$$\left.\frac{dB}{dx}\right|_{x=0} = \frac{\mu_0 N I a^2}{2}\left(\frac{-3(a/2)}{((a/2)^2 + a^2)^{5/2}} + \frac{-3(-a/2)}{((-a/2)^2 + a^2)^{5/2}}\right) = 0.$$

$$\frac{d^2B}{dx^2} = \frac{\mu_0 N I a^2}{2}\left(\frac{-3}{((x + a/2)^2 + a^2)^{5/2}} + \frac{6(x + a/2)^2(5/2)}{((x + a/2)^2 + a^2)^{7/2}} + \frac{-3}{((x - a/2)^2 + a^2)^{5/2}} + \frac{6(x - a/2)^2(5/2)}{((x - a/2)^2 + a^2)^{7/2}}\right)$$

At $x = 0$,

$$\left.\frac{d^2B}{dx^2}\right|_{x=0} = \frac{\mu_0 N I a^2}{2}\left(\frac{-3}{((a/2)^2 + a^2)^{5/2}} + \frac{6(a/2)^2(5/2)}{((a/2)^2 + a^2)^{7/2}} + \frac{-3}{((a/2)^2 + a^2)^{5/2}} + \frac{6(-a/2)^2(5/2)}{((a/2)^2 + a^2)^{7/2}}\right) = 0.$$

EVALUATE: Since both first and second derivatives are zero, the field can only be changing very slowly.

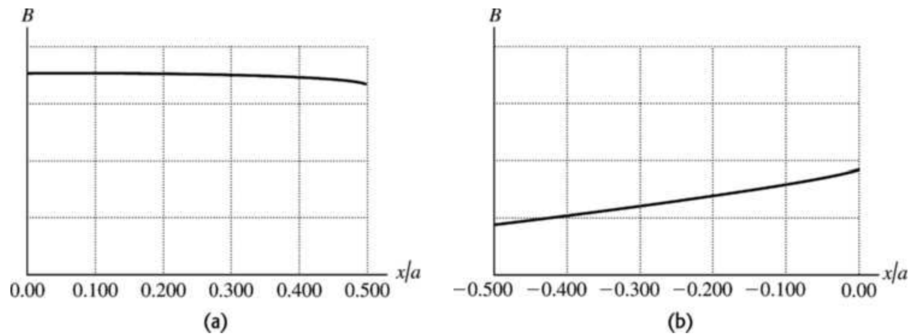
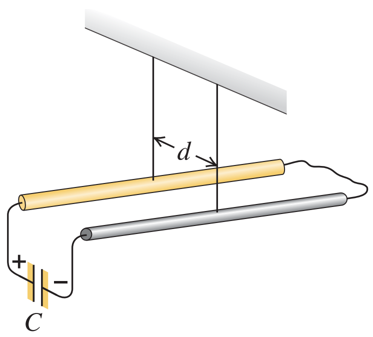


Figure 28.75

28.87 ••• **CP** Two long, straight conducting wires with linear mass density λ are suspended from cords so that they are each horizontal, parallel to each other, and a distance d apart. The back ends of the wires are connected to each other by a slack, low-resistance connecting wire. A charged capacitor (capacitance C) is now added to the system; the positive plate of the capacitor (initial charge $+Q_0$) is connected to the front end of one of the wires, and the negative plate of the capacitor (initial charge $-Q_0$) is connected to the front end of the other wire (Fig. P28.87). Both of these connections are also made by slack, low-resistance wires. When the connection is made, the wires are pushed aside by the repulsive force between the wires, and each wire has an initial horizontal velocity of magnitude v_0. Assume that the time constant for the capacitor to discharge is negligible compared to the time it takes for any appreciable displacement in the position of the wires to occur. (a) Show that the initial speed v_0 of either wire is given by

Figure **P28.87**



$$v_0 = \frac{\mu_0 Q_0^2}{4\pi \lambda RCd}$$

where R is the total resistance of the circuit. (b) To what height h will each wire rise as a result of the circuit connection?

28.87. **IDENTIFY:** The current-carrying wires repel each other magnetically, causing them to accelerate horizontally. Since gravity is vertical, it plays no initial role.

SET UP: The magnetic force per unit length is $\dfrac{F}{L}=\dfrac{\mu_0}{2\pi}\dfrac{I^2}{d}$, and the acceleration obeys the equation $F/L = m/L\ a$. The rms current over a short discharge time is $I_0/\sqrt{2}$.

EXECUTE: **(a)** First get the force per unit length:

$$\frac{F}{L}=\frac{\mu_0}{2\pi}\frac{I^2}{d}=\frac{\mu_0}{2\pi d}\left(\frac{I_0}{\sqrt{2}}\right)^2=\frac{\mu_0}{4\pi d}\left(\frac{V}{R}\right)^2=\frac{\mu_0}{4\pi d}\left(\frac{Q_0}{RC}\right)^2$$

Now apply Newton's second law using the result above: $\dfrac{F}{L}=\dfrac{m}{L}a=\lambda a=\dfrac{\mu_0}{4\pi d}\left(\dfrac{Q_0}{RC}\right)^2$. Solving for a gives

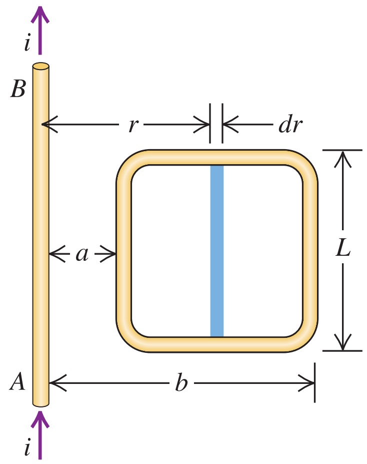
$a=\dfrac{\mu_0 Q_0^2}{4\pi\lambda R^2C^2d}$. From the kinematics equation $v_x = v_{0x}+a_x t$, we have $v_0 = at = aRC = \dfrac{\mu_0 Q_0^2}{4\pi\lambda RCd}$.

(b) Conservation of energy gives $\frac{1}{2}mv_0^2 = mgh$ and $h=\dfrac{v_0^2}{2g}=\dfrac{\left(\dfrac{\mu_0 Q_0^2}{4\pi\lambda RCd}\right)^2}{2g}=\dfrac{1}{2g}\left(\dfrac{\mu_0 Q_0^2}{4\pi\lambda RCd}\right)^2$.

EVALUATE: Once the wires have swung apart, we would have to consider gravity in applying Newton's second law.

29.7 • **CALC** The current in the long, straight wire AB shown in Fig. E29.7 is upward and is increasing steadily at a rate di/dt. (a) At an instant when the current is i, what are the magnitude and direction of the field $\vec{B}$ at a distance r to the right of the wire? (b) What is the flux $d\Phi_B$ through the narrow, shaded strip? (c) What is the total flux through the loop? (d) What is the induced emf in the loop? (e) Evaluate the numerical value of the induced emf if $a = 12.0$ cm, $b = 36.0$ cm, $L = 24.0$ cm, and $di/dt = 9.60$ A/s.

Figure **E29.7**



29.7. **IDENTIFY:** Calculate the flux through the loop and apply Faraday's law.

SET UP: To find the total flux integrate $d\Phi_B$ over the width of the loop. The magnetic field of a long straight wire, at distance r from the wire, is $B = \dfrac{\mu_0 I}{2\pi r}$. The direction of $\vec{\boldsymbol{B}}$ is given by the right-hand rule.

EXECUTE: **(a)** $B = \dfrac{\mu_0 i}{2\pi r}$, into the page.

(b) $d\Phi_B = BdA = \dfrac{\mu_0 i}{2\pi r} Ldr.$

(c) $\Phi_B = \int_a^b d\Phi_B = \dfrac{\mu_0 iL}{2\pi}\int_a^b \dfrac{dr}{r} = \dfrac{\mu_0 iL}{2\pi}\ln(b/a).$

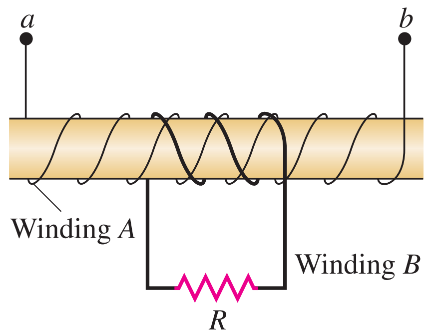
(d) $|\varepsilon| = \dfrac{d\Phi_B}{dt} = \dfrac{\mu_0 L}{2\pi}\ln(b/a)\dfrac{di}{dt}.$

(e) $|\varepsilon| = \dfrac{\mu_0 (0.240 \text{ m})}{2\pi}\ln(0.360/0.120)(9.60 \text{ A/s}) = 5.06\times 10^{-7} \text{ V}.$

EVALUATE: The induced emf is proportional to the rate at which the current in the long straight wire is changing

29.18 • A cardboard tube is wrapped with two windings of insulated wire wound in opposite directions, as shown in Fig. E29.18. Terminals *a* and *b* of winding *A* may be connected to a battery through a reversing switch. State whether the induced current in the resistor *R* is from left to right or from right to left in the following circumstances: (a) the current in winding *A* is from *a* to *b* and is increasing; (b) the current in winding *A* is from *b* to *a* and is decreasing; (c) the current in winding *A* is from *b* to *a* and is increasing.

Figure **E29.18**



29.18. **IDENTIFY:** Apply Lenz's law.

SET UP: The field of the induced current is directed to oppose the change in flux in the primary circuit.

EXECUTE: **(a)** The magnetic field in A is to the left and is increasing. The flux is increasing so the field due to the induced current in B is to the right. To produce magnetic field to the right, the induced current flows through R from right to left.

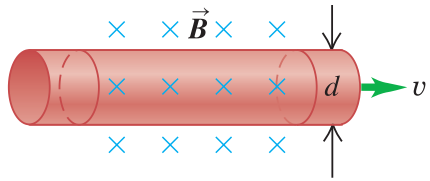
(b) The magnetic field in A is to the right and is decreasing. The flux is decreasing so the field due to the induced current in B is to the right. To produce magnetic field to the right the induced current flows through R from right to left.

(c) The magnetic field in A is to the right and is increasing. The flux is increasing so the field due to the induced current in B is to the left. To produce magnetic field to the left the induced current flows through R from left to right.

EVALUATE: The direction of the induced current depends on the direction of the external magnetic field and whether the flux due to this field is increasing or decreasing.

29.32 •• BIO Measuring Blood Flow. Blood contains positive and negative ions and thus is a conductor. A blood vessel, therefore, can be viewed as an electrical wire. We can even picture the flowing blood as a series of parallel conducting slabs whose thickness is the diameter d of the vessel moving with speed v. (See Fig. E29.32.) (a) If the blood vessel is placed in a magnetic field B perpendicular to the vessel, as in the figure, show that the motional potential difference induced across it is $\mathcal{E} = vBd$. (b) If you expect that the blood will be flowing at 15 cm/s for a vessel 5.0 mm in diameter, what strength of magnetic field will you need to produce a potential difference of 1.0 mV? (c) Show that the volume rate of flow (R) of the blood is equal to $R = \pi\mathcal{E}d/4B$. (*Note:* Although the method developed here is useful in measuring the rate of blood flow in a vessel, it is limited to use in surgery because measurement of the potential $\mathcal{E}$ must be made directly across the vessel.)

Figure **E29.32**



29.32. **IDENTIFY:** A motional emf is induced across the blood vessel.

SET UP and SOLVE: **(a)** Each slab of flowing blood has maximum width d and is moving perpendicular to the field with speed v. $\varepsilon = vBL$ becomes $\varepsilon = vBd$.

(b) $B = \dfrac{\varepsilon}{vd} = \dfrac{1.0\times10^{-3}\ \text{V}}{(0.15\ \text{m/s})(5.0\times10^{-3}\ \text{m})} = 1.3\ \text{T}.$

(c) The blood vessel has cross-sectional area $A = \pi d^2/4$. The volume of blood that flows past a cross section of the vessel in time t is $\pi(d^2/4)vt$. The volume flow rate is volume/time $= R = \pi d^2 v/4$. $v = \dfrac{\varepsilon}{Bd}$

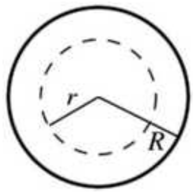
so $R = \dfrac{\pi d^2}{4}\left(\dfrac{\varepsilon}{Bd}\right) = \dfrac{\pi\varepsilon d}{4B}.$

EVALUATE: A very strong magnetic field (1.3 T) is required to produce a small potential difference of only 1 mV.

29.36 •• A long, thin solenoid has 900 turns per meter and radius 2.50 cm. The current in the solenoid is increasing at a uniform rate of 60.0 A/s. What is the magnitude of the induced electric field at a point near the center of the solenoid and (a) 0.500 cm from the axis of the solenoid; (b) 1.00 cm from the axis of the solenoid?

29.36. **IDENTIFY:** Use Eq. (29.10) to calculate the induced electric field E at a distance r from the center of the solenoid. Away from the ends of the solenoid, $B = \mu_0 nI$ inside and $B = 0$ outside.

(a) SET UP: The end view of the solenoid is sketched in Figure 29.36.



Let R be the radius of the solenoid.

Figure 29.36

Apply $\oint \vec{E} \cdot d\vec{l} = -\dfrac{d\Phi_B}{dt}$ to an integration path that is a circle of radius r, where $r < R$. We need to calculate just the magnitude of E so we can take absolute values.

EXECUTE: $\left|\oint \vec{E} \cdot d\vec{l}\right| = E(2\pi r)$

$$\Phi_B = B\pi r^2, \left|-\frac{d\Phi_B}{dt}\right| = \pi r^2 \left|\frac{dB}{dt}\right|$$

$\left|\oint \vec{E} \cdot d\vec{l}\right| = \left|-\dfrac{d\Phi_B}{dt}\right|$ implies $E(2\pi r) = \pi r^2 \left|\dfrac{dB}{dt}\right|$

$$E = \tfrac{1}{2} r \left|\frac{dB}{dt}\right|$$

$$B = \mu_0 nI, \text{ so } \frac{dB}{dt} = \mu_0 n \frac{dI}{dt}$$

Thus $E = \tfrac{1}{2} r \mu_0 n \dfrac{dI}{dt} = \tfrac{1}{2}(0.00500 \text{ m})(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(900 \text{ m}^{-1})(60.0 \text{ A/s}) = 1.70 \times 10^{-4} \text{ V/m}.$

(b) $r = 0.0100 \text{ cm}$ is still inside the solenoid so the expression in part (a) applies.

$$E = \tfrac{1}{2} r \mu_0 n \frac{dI}{dt} = \tfrac{1}{2}(0.0100 \text{ m})(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(900 \text{ m}^{-1})(60.0 \text{ A/s}) = 3.39 \times 10^{-4} \text{ V/m}$$

EVALUATE: Inside the solenoid E is proportional to r, so E doubles when r doubles.

29.45 • **CALC** **Displacement Current in a Wire.** A long, straight, copper wire with a circular cross-sectional area of 2.1 mm^2 carries a current of 16 A. The resistivity of the material is $2.0 \times 10^{-8}\ \Omega \cdot \mathrm{m}$. (a) What is the uniform electric field in the material? (b) If the current is changing at the rate of 4000 A/s, at what rate is the electric field in the material changing? (c) What is the displacement current density in the material in part (b)? (*Hint:* Since K for copper is very close to 1, use $\epsilon = \epsilon_0$.) (d) If the current is changing as in part (b), what is the magnitude of the magnetic field 6.0 cm from the center of the wire? Note that both the conduction current and the displacement current should be included in the calculation of B. Is the contribution from the displacement current significant?

29.45. **IDENTIFY:** Ohm's law relates the current in the wire to the electric field in the wire. $j_D = \epsilon \frac{dE}{dt}$. Use Eq. (29.15) to calculate the magnetic fields.

SET UP: Ohm's law says $E = \rho J$. Apply Ohm's law to a circular path of radius r.

EXECUTE: **(a)** $E = \rho J = \frac{\rho I}{A} = \frac{(2.0\times10^{-8}\ \Omega\cdot\text{m})(16\ \text{A})}{2.1\times10^{-6}\ \text{m}^2} = 0.15\ \text{V/m}.$

(b) $\frac{dE}{dt} = \frac{d}{dt}\left(\frac{\rho I}{A}\right) = \frac{\rho}{A}\frac{dI}{dt} = \frac{2.0\times10^{-8}\ \Omega\cdot\text{m}}{2.1\times10^{-6}\ \text{m}^2}(4000\ \text{A/s}) = 38\ \text{V/m}\cdot\text{s}.$

(c) $j_D = \epsilon_0 \frac{dE}{dt} = \epsilon_0(38\ \text{V/m}\cdot\text{s}) = 3.4\times10^{-10}\ \text{A/m}^2.$

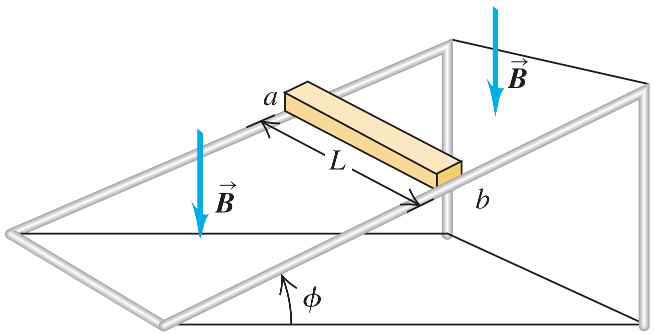
(d) $i_D = j_D A = (3.4\times10^{-10}\ \text{A/m}^2)(2.1\times10^{-6}\ \text{m}^2) = 7.14\times10^{-16}\ \text{A}.$ Eq. (29.15) applied to a circular path of radius r gives $B_D = \frac{\mu_0 I_D}{2\pi r} = \frac{\mu_0(7.14\times10^{-16}\ \text{A})}{2\pi(0.060\ \text{m})} = 2.38\times10^{-21}\ \text{T},$ and this is a negligible contribution.

$$B_C = \frac{\mu_0 I_C}{2\pi r} = \frac{\mu_0}{2\pi}\frac{(16\ \text{A})}{(0.060\ \text{m})} = 5.33\times10^{-5}\ \text{T}.$$

EVALUATE: In this situation the displacement current is much less than the conduction current.

29.77 ••• A metal bar with length L, mass m, and resistance R is placed on frictionless metal rails that are inclined at an angle ϕ above the horizontal. The rails have negligible resistance. A uniform magnetic field of magnitude B is directed downward as shown in Fig. P29.77. The bar is released from rest and slides down the rails. (a) Is the direction of the current induced in the bar from a to b or from b to a? (b) What is the terminal speed of the bar? (c) What is the induced current in the bar when the terminal speed has been reached? (d) After the terminal speed has been reached, at what rate is electrical energy being converted to thermal energy in the resistance of the bar? (e) After the terminal speed has been reached, at what rate is work being done on the bar by gravity? Compare your answer to that in part (d).

Figure **P29.77**



29.77. **IDENTIFY:** The motion of the bar produces an induced current and that results in a magnetic force on the bar.

SET UP: $\vec{F}_B$ is perpendicular to $\vec{B}$, so is horizontal. The vertical component of the normal force equals $mg\cos\phi,$ so the horizontal component of the normal force equals $mg\tan\phi.$

EXECUTE: **(a)** As the bar starts to slide, the flux is decreasing, so the current flows to increase the flux, which means it flows from a to b. $F_B = iLB = \dfrac{LB}{R}\mathcal{E} = \dfrac{LB}{R}\dfrac{d\Phi_B}{dt} = \dfrac{LB}{R}B\dfrac{dA}{dt} = \dfrac{LB^2}{R}(vL\cos\phi) = \dfrac{vL^2B^2}{R}\cos\phi.$

(b) At the terminal speed the horizontal forces balance, so $mg\tan\phi = \dfrac{v_\text{t}L^2B^2}{R}\cos\phi$ and $v_\text{t} = \dfrac{Rmg\tan\phi}{L^2B^2\cos\phi}.$

(c) $i = \dfrac{\mathcal{E}}{R} = \dfrac{1}{R}\dfrac{d\Phi_B}{dt} = \dfrac{1}{R}B\dfrac{dA}{dt} = \dfrac{B}{R}(v_\text{t}L\cos\phi) = \dfrac{v_\text{t}LB\cos\phi}{R} = \dfrac{mg\tan\phi}{LB}.$

(d) $P = i^2R = \dfrac{Rm^2g^2\tan^2\phi}{L^2B^2}.$

(e) $P_\text{g} = Fv_\text{t}\cos(90° - \phi) = mg\left(\dfrac{Rmg\tan\phi}{L^2B^2\cos\phi}\right)\sin\phi$ and $P_\text{g} = \dfrac{Rm^2g^2\tan^2\phi}{L^2B^2}.$

EVALUATE: The power in part (e) equals that in part (d), as is required by conservation of energy.

30.15 •• **Inductance of a Solenoid.** (a) A long, straight solenoid has N turns, uniform cross-sectional area A, and length l. Show that the inductance of this solenoid is given by the equation $L = \mu_0 AN^2/l$. Assume that the magnetic field is uniform inside the solenoid and zero outside. (Your answer is approximate because B is actually smaller at the ends than at the center. For this reason, your answer is actually an upper limit on the inductance.) (b) A metallic laboratory spring is typically 5.00 cm long and 0.150 cm in diameter and has 50 coils. If you connect such a spring in an electric circuit, how much self-inductance must you include for it if you model it as an ideal solenoid?

30.15. **IDENTIFY:** Use the definition of inductance and the geometry of a solenoid to derive its self-inductace.

SET UP: The magnetic field inside a solenoid is $B = \mu_0 \frac{N}{l} i$, and the definition of self-inductance is $L = \frac{N\Phi_B}{i}$.

EXECUTE: **(a)** $B = \mu_0 \frac{N}{l} i$, $L = \frac{N\Phi_B}{i}$, and $\Phi_B = \frac{\mu_0 N A i}{l}$. Combining these expressions gives

$$L = \frac{N\Phi_B}{i} = \frac{\mu_0 N^2 A}{l}.$$

(b) $L = \frac{\mu_0 N^2 A}{l}$. $A = \pi r^2 = \pi(0.0750\times10^{-2}\ \text{m})^2 = 1.767\times10^{-6}\ \text{m}^2$.

$$L = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(50)^2(1.767\times10^{-6}\ \text{m}^2)}{5.00\times10^{-2}\ \text{m}} = 1.11\times10^{-7}\ \text{H} = 0.111\ \mu\text{H}.$$

EVALUATE: This is a physically reasonable value for self-inductance.

30.37 •• An L-C circuit containing an 80.0-mH inductor and a 1.25-nF capacitor oscillates with a maximum current of 0.750 A. Calculate: (a) the maximum charge on the capacitor and (b) the oscillation frequency of the circuit. (c) Assuming the capacitor had its maximum charge at time $t = 0$, calculate the energy stored in the inductor after 2.50 ms of oscillation.

30.37. **IDENTIFY:** Apply energy conservation and Eqs. (30.22) and (30.23).

SET UP: If I is the maximum current, $\frac{1}{2}LI^2 = \frac{Q^2}{2C}$. For the inductor, $U_L = \frac{1}{2}Li^2$.

EXECUTE: **(a)** $\frac{1}{2}LI^2 = \frac{Q^2}{2C}$ gives $Q = I\sqrt{LC} = (0.750\text{ A})\sqrt{(0.0800\text{ H})(1.25\times 10^{-9}\text{ F})} = 7.50\times 10^{-6}\text{ C}$.

(b) $\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(0.0800\text{ H})(1.25\times 10^{-9}\text{ F})}} = 1.00\times 10^{5}\text{ rad/s}$. $f = \frac{\omega}{2\pi} = 1.59\times 10^{4}\text{ Hz}$.

(c) $q = Q$ at $t = 0$ means $\phi = 0$. $i = -\omega Q\sin(\omega t)$, so

$i = -(1.00\times 10^{5}\text{ rad/s})(7.50\times 10^{-6}\text{ C})\sin([1.00\times 10^{5}\text{ rad/s}][2.50\times 10^{-3}\text{ s}]) = 0.7279\text{ A}$.

$U_L = \frac{1}{2}Li^2 = \frac{1}{2}(0.0800\text{ H})(0.7279\text{ A})^2 = 0.0212\text{ J}$.

EVALUATE: The total energy of the system is $\frac{1}{2}LI^2 = 0.0225\text{ J}$. At $t = 2.50\text{ ms}$, the current is close to its maximum value and most of the system's energy is stored in the inductor.

30.47 •• **Solar Magnetic Energy.** Magnetic fields within a sunspot can be as strong as 0.4 T. (By comparison, the earth's magnetic field is about 1/10,000 as strong.) Sunspots can be as large as 25,000 km in radius. The material in a sunspot has a density of about 3×10^{-4} kg/m^3. Assume μ for the sunspot material is μ_0. If 100% of the magnetic-field energy stored in a sunspot could be used to eject the sunspot's material away from the sun's surface, at what speed would that material be ejected? Compare to the sun's escape speed, which is about 6×10^5 m/s. (*Hint:* Calculate the kinetic energy the magnetic field could supply to 1 m^3 of sunspot material.)

30.47. **IDENTIFY:** Set $U_B = K$, where $K = \frac{1}{2}mv^2$.

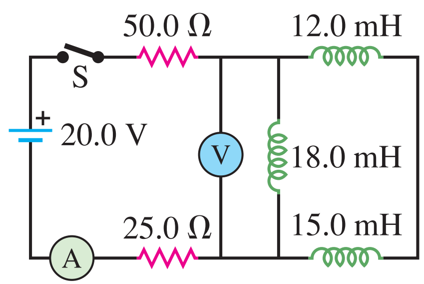
SET UP: The energy density in the magnetic field is $u_B = B^2/2\mu_0$. Consider volume $V = 1\ \text{m}^3$ of sunspot material.

EXECUTE: The energy density in the sunspot is $u_B = B^2/2\mu_0 = 6.366\times10^4\ \text{J/m}^3$. The total energy stored in volume V of the sunspot is $U_B = u_B V$. The mass of the material in volume V of the sunspot is $m = \rho V$. $K = U_B$ so $\frac{1}{2}mv^2 = U_B$. $\frac{1}{2}\rho V v^2 = u_B V$. The volume divides out, and $v = \sqrt{2u_B/\rho} = 2\times10^4\ \text{m/s}$.

EVALUATE: The speed we calculated is about 30 times smaller than the escape speed.

30.61 ••• **CP** In the circuit shown in Fig. P30.61, the switch has been open for a long time and is suddenly closed. Neither the battery nor the inductors have any appreciable resistance. (a) What do the ammeter and voltmeter read just after S is closed? (b) What do the ammeter and the voltmeter read after S has been closed a very long time? (c) What do the ammeter and the voltmeter read 0.115 ms after S is closed?

Figure **P30.61**



30.61. **IDENTIFY:** The current through an inductor doesn't change abruptly. After a long time the current isn't changing and the voltage across each inductor is zero.

SET UP: First combine the inductors.

EXECUTE: **(a)** Just after the switch is closed there is no current in the inductors. There is no current in the resistors so there is no voltage drop across either resistor. A reads zero and V reads 20.0 V.

(b) After a long time the currents are no longer changing, there is no voltage across the inductors, and the inductors can be replaced by short-circuits. The circuit becomes equivalent to the circuit shown in Figure 30.61a. $I = (20.0\text{ V})/(75.0\ \Omega) = 0.267\text{ A}$. The voltage between points a and b is zero, so the voltmeter reads zero.

(c) Combine the inductor network into its equivalent, as shown in Figure 30.61b. $R = 75.0\ \Omega$ is the equivalent resistance. Eq. (30.14) says $i = (\mathcal{E}/R)(1 - e^{-t/\tau})$ with $\tau = L/R = (10.8\text{ mH})/(75.0\ \Omega) = 0.144\text{ ms}$. $\mathcal{E} = 20.0\text{ V}$, $R = 75.0\ \Omega$, $t = 0.115\text{ ms}$ so $i = 0.147\text{ A}$. $V_R = iR = (0.147\text{ A})(75.0\ \Omega) = 11.0\text{ V}$. $20.0\text{ V} - V_R - V_L = 0$ and $V_L = 20.0\text{ V} - V_R = 9.0\text{ V}$. The ammeter reads 0.147 A and the voltmeter reads 9.0 V.

EVALUATE: The current through the battery increases from zero to a final value of 0.267 A. The voltage across the inductor network drops from 20.0 V to zero.

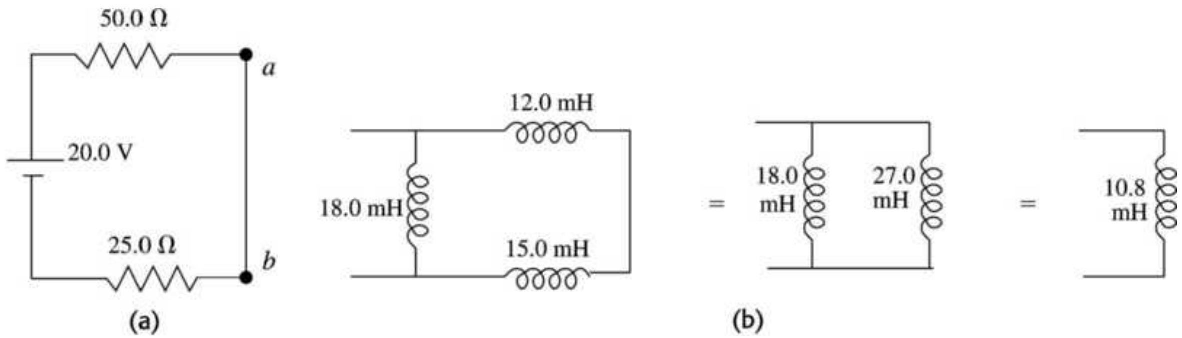


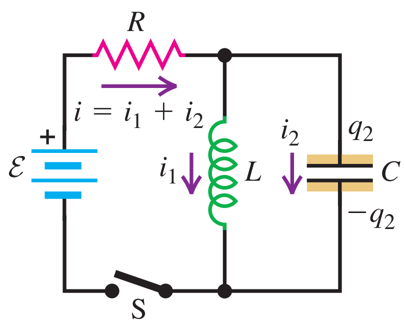
Figure 30.61

30.76 ••• **CP CALC** Consider the circuit shown in Fig. P30.76. The circuit elements are as follows: $\mathcal{E} = 32.0$ V, $L = 0.640$ H, $C = 2.00\ \mu$F, and $R = 400\ \Omega$. At time $t = 0$, switch S is closed. The current through the inductor is i_1, the current through the capacitor branch is i_2, and the charge on the capacitor is q_2. (a) Using Kirchhoff's rules, verify the circuit equations

$$R(i_1 + i_2) + L\left(\frac{di_1}{dt}\right) = \mathcal{E}$$

$$R(i_1 + i_2) + \frac{q_2}{C} = \mathcal{E}$$

Figure **P30.76**



(b) What are the initial values of i_1, i_2, and q_2? (c) Show by direct substitution that the following solutions for i_1 and q_2 satisfy the circuit equations from part (a). Also, show that they satisfy the initial conditions

$$i_1 = \left(\frac{\mathcal{E}}{R}\right)[1 - e^{-\beta t}\{(2\omega RC)^{-1}\sin(\omega t) + \cos(\omega t)\}]$$

$$q_2 = \left(\frac{\mathcal{E}}{\omega R}\right)e^{-\beta t}\sin(\omega t)$$

where $\beta = (2RC)^{-1}$ and $\omega = [(LC)^{-1} - (2RC)^{-2}]^{1/2}$. (d) Determine the time t_1 at which i_2 first becomes zero.

30.76. **IDENTIFY:** Follow the steps specified in the problem.

SET UP: The current in an inductor does not change abruptly.

EXECUTE: **(a)** Using Kirchhoff's loop rule on the left and right branches:

Left: $\mathcal{E}-(i_1+i_2)R-L\dfrac{di_1}{dt}=0 \Rightarrow R(i_1+i_2)+L\dfrac{di_1}{dt}=\mathcal{E}.$

Right: $\mathcal{E}-(i_1+i_2)R-\dfrac{q_2}{C}=0 \Rightarrow R(i_1+i_2)+\dfrac{q_2}{C}=\mathcal{E}.$

(b) Initially, with the switch just closed, $i_1=0, i_2=\dfrac{\mathcal{E}}{R}$ and $q_2=0.$

(c) The substitution of the solutions into the circuit equations to show that they satisfy the equations is a somewhat tedious exercise but straightforward exercise. We will show that the initial conditions are satisfied: At $t=0, q_2=\dfrac{\mathcal{E}}{\omega R}e^{-\beta t}\sin(\omega t)=\dfrac{\mathcal{E}}{\omega R}\sin(0)=0.$

$i_1(t)=\dfrac{\mathcal{E}}{R}(1-e^{-\beta t}[(2\omega RC)^{-1}\sin(\omega t)+\cos(\omega t)] \Rightarrow i_1(0)=\dfrac{\mathcal{E}}{R}(1-[\cos(0)])=0.$

(d) When does i_2 first equal zero? $\omega=\sqrt{\dfrac{1}{LC}-\dfrac{1}{(2RC)^2}}=625\text{ rad/s}.$

$i_2(t)=0=\dfrac{\mathcal{E}}{R}e^{-\beta t}[-(2\omega RC)^{-1}\sin(\omega t)+\cos(\omega t)] \Rightarrow -(2\omega RC)^{-1}\tan(\omega t)+1=0$ and

$\tan(\omega t)=+2\omega RC=+2(625\text{ rad/s})(400\ \Omega)(2.00\times 10^{-6}\text{ F})=+1.00.$

$\omega t=\arctan(+1.00)=+0.785 \Rightarrow t=\dfrac{0.785}{625\text{ rad/s}}=1.256\times 10^{-3}\text{s}.$

EVALUATE: As $t\to\infty$, $i_1\to\mathcal{E}/R$, $q_2\to 0$ and $i_2\to 0.$