32.30 •• **Solar Sail 1.** During 2004, Japanese scientists successfully tested two solar sails. One had a somewhat complicated shape that we shall model as a disk 9.0 m in diameter and 7.5 μm thick. The intensity of solar energy at that location was about 1400 W/m^2. (a) What force did the sun's light exert on this sail, assuming that it struck perpendicular to the sail and that the sail was perfectly reflecting? (b) If the sail was made of magnesium, of density 1.74 g/cm^3, what acceleration would the sun's radiation give to the sail? (c) Does the acceleration seem large enough to be feasible for space flight? In what ways could the sail be modified to increase its acceleration?

32.30. **IDENTIFY:** We know the intensity of the solar light and the area over which it acts. We can use the light intensity to find the force the light exerts on the sail, and then use the sail's density to find its mass. Newton's second law will then give the acceleration of the sail.

SET UP: For a reflecting surface the pressure is $\frac{2I}{c}$. Pressure is force per unit area, and $F_{\text{net}} = ma$. The mass of the sail is its volume V times its density ρ. The area of the sail is πr^2, with $r = 4.5$ m. Its volume is $\pi r^2 t$, where $t = 7.5\times10^{-6}$ m is its thickness.

EXECUTE: **(a)** $F = \left(\frac{2I}{c}\right)A = \frac{2(1400\ \text{W/m}^2)}{3.00\times10^8\ \text{m/s}}\pi(4.5\ \text{m})^2 = 5.9\times10^{-4}\ \text{N}.$

(b) $m = \rho V = (1.74\times10^3\ \text{kg/m}^3)\pi(4.5\ \text{m})^2(7.5\times10^{-6}\ \text{m}) = 0.83\ \text{kg}.$

$$a = \frac{F}{m} = \frac{5.9\times10^{-4}\ \text{N}}{0.83\ \text{kg}} = 7.1\times10^{-4}\ \text{m/s}^2.$$

(c) With this acceleration it would take the sail $1.4\times10^6\ \text{s} = 16\ \text{days}$ to reach a speed of 1 km/s. This would be useful only in specialized applications. The acceleration could be increased by decreasing the mass of the sail, either by reducing its density or its thickness.

EVALUATE: The calculation assumed the only force on the sail is that due to the radiation pressure. The sun would also exert a gravitational force on the sail, which could be significant.

32.55 •• **CP** Interplanetary space contains many small particles referred to as *interplanetary dust.* Radiation pressure from the sun sets a lower limit on the size of such dust particles. To see the origin of this limit, consider a spherical dust particle of radius R and mass density ρ. (a) Write an expression for the gravitational force exerted on this particle by the sun (mass M) when the particle is a distance r from the sun. (b) Let L represent the luminosity of the sun, equal to the rate at which it emits energy in electromagnetic radiation. Find the force exerted on the (totally absorbing) particle due to solar radiation pressure, remembering that the intensity of the sun's radiation also depends on the distance r. The relevant area is the cross-sectional area of the particle, *not* the total surface area of the particle. As part of your answer, explain why this is so. (c) The mass density of a typical interplanetary dust particle is about 3000 kg/m^3. Find the particle radius R such that the gravitational and radiation forces acting on the particle are equal in magnitude. The luminosity of the sun is 3.9×10^{26} W. Does your answer depend on the distance of the particle from the sun? Why or why not? (d) Explain why dust particles with a radius less than that found in part (c) are unlikely to be found in the solar system. [*Hint:* Construct the ratio of the two force expressions found in parts (a) and (b).]

EVALUATE: A very large sail is needed, just to overcome the gravitational pull of the sun.

32.55. **IDENTIFY** and **SET UP:** The gravitational force is given by Eq. (13.2). Express the mass of the particle in terms of its density and volume. The radiation pressure is given by Eq. (32.32); relate the power output L of the sun to the intensity at a distance r. The radiation force is the pressure times the cross-sectional area of the particle.

EXECUTE: **(a)** The gravitational force is $F_g = G\frac{mM}{r^2}$. The mass of the dust particle is $m = \rho V = \rho \frac{4}{3}\pi R^3$.

Thus $F_g = \frac{4\rho G\pi M R^3}{3r^2}$.

(b) For a totally absorbing surface $p_{rad} = \frac{I}{c}$. If L is the power output of the sun, the intensity of the solar radiation a distance r from the sun is $I = \frac{L}{4\pi r^2}$. Thus $p_{rad} = \frac{L}{4\pi c r^2}$. The force F_{rad} that corresponds to p_{rad} is in the direction of propagation of the radiation, so $F_{rad} = p_{rad} A_\perp$, where $A_\perp = \pi R^2$ is the component of area of the particle perpendicular to the radiation direction. Thus

$$F_{rad} = \left(\frac{L}{4\pi c r^2}\right)(\pi R^2) = \frac{LR^2}{4cr^2}.$$

(c) $F_g = F_{rad}$

$$\frac{4\rho G\pi M R^3}{3r^2} = \frac{LR^2}{4cr^2}$$

$$\left(\frac{4\rho G\pi M}{3}\right)R = \frac{L}{4c} \text{ and } R = \frac{3L}{16c\rho G\pi M}$$

$$R = \frac{3(3.9\times10^{26}\ \text{W})}{16(2.998\times10^8\ \text{m/s})(3000\ \text{kg/m}^3)(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\pi(1.99\times10^{30}\ \text{kg})}$$

$R = 1.9\times10^{-7}\ \text{m} = 0.19\ \mu\text{m}$.

EVALUATE: The gravitational force and the radiation force both have a r^{-2} dependence on the distance from the sun, so this distance divides out in the calculation of R.

(d) $\frac{F_{rad}}{F_g} = \left(\frac{LR^2}{4cr^2}\right)\left(\frac{3r^2}{4\rho G\pi m R^3}\right) = \frac{3L}{16c\rho G\pi MR}$. F_{rad} is proportional to R^2 and F_g is proportional to R^3, so this ratio is proportional to $1/R$. If $R < 0.20\ \mu\text{m}$ then $F_{rad} > F_g$ and the radiation force will drive the particles out of the solar system.

32.57 ••• **CP** Electromagnetic radiation is emitted by accelerating charges. The rate at which energy is emitted from an accelerating charge that has charge q and acceleration a is given by

$$\frac{dE}{dt} = \frac{q^2 a^2}{6\pi\epsilon_0 c^3}$$

where c is the speed of light. (a) Verify that this equation is dimensionally correct. (b) If a proton with a kinetic energy of 6.0 MeV is traveling in a particle accelerator in a circular orbit of radius 0.750 m, what fraction of its energy does it radiate per second? (c) Consider an electron orbiting with the same speed and radius. What fraction of its energy does it radiate per second?

32.58 ••• **CP** **The Classical Hydrogen Atom.** The electron in a hydrogen atom can be considered to be in a circular orbit with a radius of 0.0529 nm and a kinetic energy of 13.6 eV. If the electron behaved classically, how much energy would it radiate per second (see Challenge Problem 32.57)? What does this tell you about the use of classical physics in describing the atom?

32.57. IDENTIFY: The orbiting particle has acceleration $a = \dfrac{v^2}{R}$.

SET UP: $K = \frac{1}{2}mv^2$. An electron has mass $m_e = 9.11\times10^{-31}$ kg and a proton has mass $m_p = 1.67\times10^{-27}$ kg.

EXECUTE: **(a)** $\left[\dfrac{q^2a^2}{6\pi\epsilon_0 c^3}\right] = \dfrac{C^2(m/s^2)^2}{(C^2/N\cdot m^2)(m/s)^3} = \dfrac{N\cdot m}{s} = \dfrac{J}{s} = W = \left[\dfrac{dE}{dt}\right]$.

(b) For a proton moving in a circle, the acceleration is

$a = \dfrac{v^2}{R} = \dfrac{\frac{1}{2}mv^2}{\frac{1}{2}mR} = \dfrac{2(6.00\times10^6 \text{ eV})(1.6\times10^{-19} \text{ J/eV})}{(1.67\times10^{-27} \text{ kg})(0.75 \text{ m})} = 1.53\times10^{15} \text{ m/s}^2$. The rate at which it emits energy because of its acceleration is

$\dfrac{dE}{dt} = \dfrac{q^2a^2}{6\pi\epsilon_0 c^3} = \dfrac{(1.6\times10^{-19} \text{ C})^2(1.53\times10^{15} \text{ m/s}^2)^2}{6\pi\epsilon_0(3.0\times10^8 \text{ m/s})^3} = 1.33\times10^{-23} \text{ J/s} = 8.32\times10^{-5} \text{ eV/s}$.

Therefore, the fraction of its energy that it radiates every second is

$\dfrac{(dE/dt)(1 \text{ s})}{E} = \dfrac{8.32\times10^{-5} \text{ eV}}{6.00\times10^6 \text{ eV}} = 1.39\times10^{-11}$.

(c) Carry out the same calculations as in part (b), but now for an electron at the same speed and radius. That means the electron's acceleration is the same as the proton, and thus so is the rate at which it emits energy, since they also have the same charge. However, the electron's initial energy differs from the proton's by the ratio of their masses: $E_e = E_p \dfrac{m_e}{m_p} = (6.00\times10^6 \text{ eV})\dfrac{(9.11\times10^{-31} \text{ kg})}{(1.67\times10^{-27} \text{ kg})} = 3273 \text{ eV}$. Therefore, the fraction of its energy that it radiates every second is $\dfrac{(dE/dt)(1 \text{ s})}{E} = \dfrac{8.32\times10^{-5} \text{ eV}}{3273 \text{ eV}} = 2.54\times10^{-8}$.

EVALUATE: The proton has speed $v = \sqrt{\dfrac{2E}{m_p}} = \sqrt{\dfrac{2(6.0\times10^6 \text{ eV})(1.60\times10^{-19} \text{ J/eV})}{1.67\times10^{-27} \text{ kg}}} = 3.39\times10^7 \text{ m/s}$. The electron has the same speed and kinetic energy 3.27 keV. The particles in the accelerator radiate at a much smaller rate than the electron in Problem 32.58 does, because in the accelerator the orbit radius is very much larger than in the atom, so the acceleration is much less.

32.58. **IDENTIFY:** The electron has acceleration $a = \dfrac{v^2}{R}$.

SET UP: $1\text{ eV} = 1.60\times10^{-19}\text{ C}$. An electron has $|q| = e = 1.60\times10^{-19}\text{ C}$.

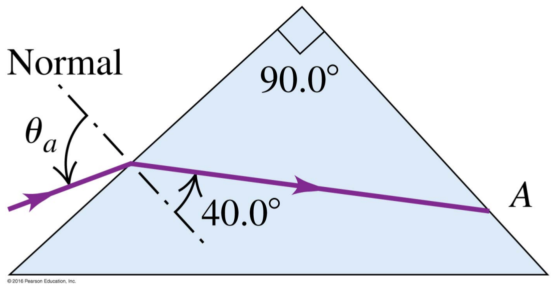
EXECUTE: For the electron in the classical hydrogen atom, its acceleration is

$a = \dfrac{v^2}{R} = \dfrac{\frac{1}{2}mv^2}{\frac{1}{2}mR} = \dfrac{2(13.6\text{ eV})(1.60\times10^{-19}\text{ J/eV})}{(9.11\times10^{-31}\text{ kg})(5.29\times10^{-11}\text{ m})} = 9.03\times10^{22}\text{ m/s}^2$. Then using the formula for the rate of energy emission given in Problem 32.57:

$\dfrac{dE}{dt} = \dfrac{q^2a^2}{6\pi\epsilon_0 c^3} = \dfrac{(1.60\times10^{-19}\text{ C})^2(9.03\times10^{22}\text{ m/s}^2)^2}{6\pi\epsilon_0(3.00\times10^8\text{ m/s})^3} = 4.64\times10^{-8}\text{ J/s} = 2.89\times10^{11}\text{ eV/s}$. This large value of $\dfrac{dE}{dt}$ would mean that the electron would almost immediately lose all its energy!

EVALUATE: The classical physics result in Problem 32.57 must not apply to electrons in atoms.

(33.42-T) A ray of light travelling in air is incident at angle θ_a on one face of a 90° prism made of glass. Part of the light refracts into the prism and strikes the opposite face at point A. If the ray at A is at the critical angle, what is the values of θ_a?



33.42. **IDENTIFY:** Because the prism is a right-angle prism, the normals at point A and at surface BC are perpendicular to each other (see Figure 33.42). Therefore the angle of incidence at A is 50.0°, and this is the critical angle at that surface. Apply Snell's law at A and at surface BC. For light incident at the critical angle, the angle of refraction is 90°.

Figure 33.42

SET UP: Apply Snell's law: $n_a \sin\theta_a = n_b \sin\theta_b$. Use n = 1.00 for air, and let n be the index of refraction of the glass.

EXECUTE: Apply Snell's law at point A.

n sin(50.0°) = (1.00) sin(90°) = 1.00.

n = 1.305.

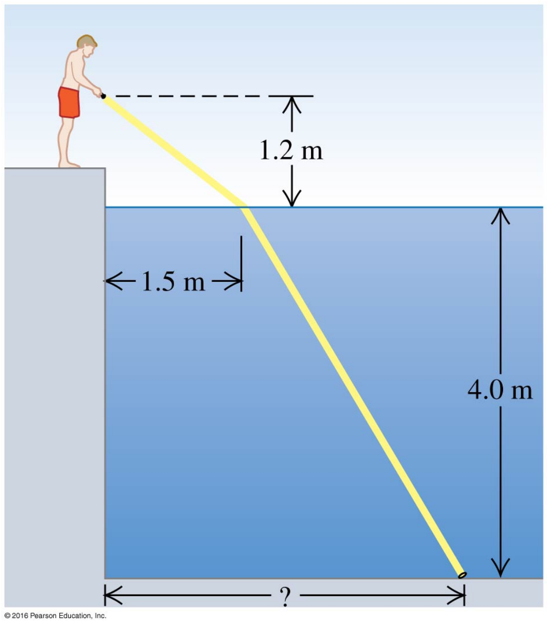
Now apply Snell's law at surface BC.

(1.00) $\sin\theta$ = (1.305) sin(40.0°).

θ = 57.0°.

EVALUATE: The critical angle at A would not be 50.0° if the prism were not a right-angle prism.

(33.44-T) After a long day of driving you take a late-night swim in a motel swimming pool. When you go to your room, you realise that you have lost your room key in the pool. You borrow a powerful flashlight and walk around the pool, shining the light into it. The light shines on the key, which is lying on the bottom of the pool, when the flashlight is held 1.2m above the water surface and is directed at the surface a horizontal distance of 1.5m from the edge. If the water here is 4.0 m deep, how far is the key from the edge of the pool?



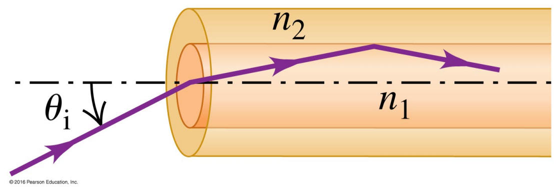
33.44. IDENTIFY: Apply Snell's law to the refraction of the light as it passes from water into air.

SET UP: $\theta_a = \arctan\left(\frac{1.5\text{ m}}{1.2\text{ m}}\right) = 51°$. $n_a = 1.00$. $n_b = 1.333$.

EXECUTE: $\theta_b = \arcsin\left(\frac{n_a}{n_b}\sin\theta_a\right) = \arcsin\left(\frac{1.00}{1.333}\sin 51°\right) = 36°$. Therefore, the distance along the bottom of the pool from directly below where the light enters to where it hits the bottom is $x = (4.0\text{ m})\tan\theta_b = (4.0\text{ m})\tan 36° = 2.9\text{ m}$. $x_{\text{total}} = 1.5\text{ m} + x = 1.5\text{ m} + 2.9\text{ m} = 4.4\text{ m}$.

EVALUATE: The light ray from the flashlight is bent toward the normal when it refracts into the water.

(33.46-T) Optical fibres are constructed with a cylindrical core surrounded by a sheath of cladding material. Common materials used are pure silica ($n_2 = 1.45$) for the cladding and silica doped with germanium ($n_1 = 1.465$) for the core. (a) What is the critical angle θ_{dirt} for light traveling in the core and reflecting at the interface with the cladding material? (b) The numerical aperture (NA) is defined as the angle of incidence θ_i at the flat end of the cable for which light is incident on the core-cladding interfaced at angle θ_{crit}. Show that $\sin\theta_i = \sqrt{n_1^2 - n_2^2}$.



33.46. IDENTIFY: Apply Snell's law. For light incident at the critical angle, the angle of refraction is 90°.

SET UP: Apply $n_a \sin\theta_a = n_b \sin\theta_b$ and use $n = 1.00$ for air.

EXECUTE: **(a)** Apply Snell's law at the interface between the cladding and the core. At that surface, the angle of incidence is the critical angle.

$n_1 \sin\theta_{crit} = n_2 \sin(90°) = n_2$.

$1.465 \sin\theta_{crit} = 1.450$.

$\theta_{crit} = 81.8°$.

(b) Apply Snell's law at the flat end of the cable and then at the core-cladding interface. Call θ the angle of refraction at the flat end, and α the angle of incidence at the core-cladding interface. Because the flat end is perpendicular to the surface at the core-cladding interface, $\sin\alpha = \cos\theta$. (See Figure 33.46.)

Figure 33.46

At the flat end of the cable: $(1.00) \sin\theta_i = n_1 \sin\theta \quad \rightarrow \quad \sin\theta = \dfrac{\sin\theta_i}{n_1}$.

At the core-cladding interface: $n_1 \sin\alpha = n_2 \sin(90°) = n_2 \quad \rightarrow \quad n_1 \cos\theta = n_2 \quad \rightarrow \quad \cos\theta = n_2/n_1$.

Using the fact that $\sin^2\theta + \cos^2\theta = 1$, we get $\left(\dfrac{\sin\theta_i}{n_1}\right)^2 + \left(\dfrac{n_2}{n_1}\right)^2 = 1$. Solving for $\sin\theta_i$ gives

$\sin\theta_i = \sqrt{n_1^2 - n_2^2}$.

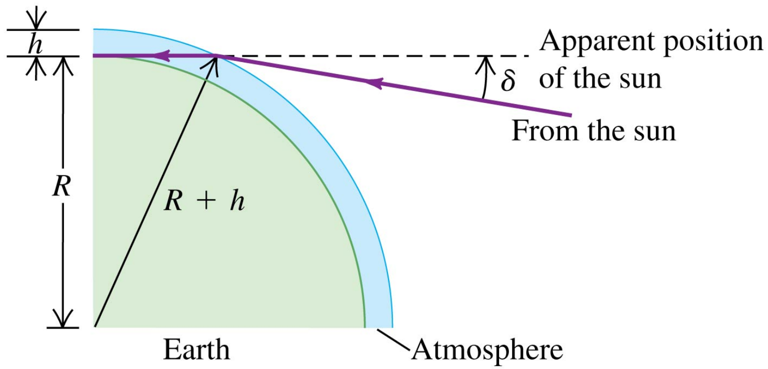
(c) Using the formula we just derived gives $\sin\theta_i = \sqrt{1.465^2 - 1.450^2} = 0.20911$, so $\theta_i = 12.1°$.

EVALUATE: If $n_2 > n_1$, the square root in (b) is not a real number, so there is no solution for θ_i. This is reasonable since total internal reflection will not occur unless $n_2 < n_1$.

(33.51-T) When the sun is either rising or setting and appears to be just on the horizon, it is in fact below the horizon. THe explanation for this seeming paradox is that light from the sun bends slightly when entering the earth's atmosphere. Since our perception is based on the idea that light travels in straight lines, we perceive the light to be coming from an apparent position that is an angle δ above the sun's true position. (a) Make the simplifying assumptions that the atmosphere has uniform density, and hence uniform index of refraction n, and extends to a height h above the earth's surface, at which point it abruptly stop. Show that the able δ is given by

$$\delta = \arcsin\left(\frac{nR}{R+h}\right) - \arcsin\left(\frac{R}{R+h}\right) \tag{1}$$

where $R = 6378km$ is the radius of the earth. (b) Calculate δ using $n = 1.0003$ and $h = 20km$. How does this compare to the angular radius of the sun, which is about one quarter of a degree? (In fact, a light ray from the sun bends gradually, not abruptly, since the density and refractive index of the atmosphere change gradually with altitude.)



33.51. IDENTIFY: Apply Snell's law to the refraction of the light as it enters the atmosphere.

SET UP: The path of a ray from the sun is sketched in Figure 33.51.

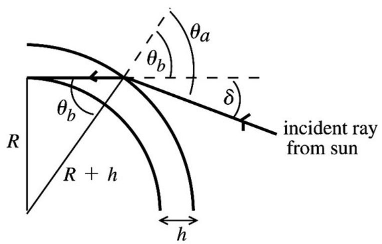


Figure 33.51

$\delta = \theta_a - \theta_b$.

From the diagram $\sin\theta_b = \dfrac{R}{R+h}$.

$$\theta_b = \arcsin\left(\frac{R}{R+h}\right).$$

EXECUTE: **(a)** Apply Snell's law to the refraction that occurs at the top of the atmosphere:

$n_a \sin\theta_a = n_b \sin\theta_b$

(a = vacuum of space, refractive index 1.0; b = atmosphere, refractive index n).

$$\sin\theta_a = n\sin\theta_b = n\left(\frac{R}{R+h}\right) \text{ so } \theta_a = \arcsin\left(\frac{nR}{R+h}\right).$$

$$\delta = \theta_a - \theta_b = \arcsin\left(\frac{nR}{R+h}\right) - \arcsin\left(\frac{R}{R+h}\right).$$

(b) $\dfrac{R}{R+h} = \dfrac{6.38\times10^6 \text{ m}}{6.38\times10^6 \text{ m} + 20\times10^3 \text{ m}} = 0.99688.$

$$\frac{nR}{R+h} = 1.0003(0.99688) = 0.99718.$$

$$\theta_b = \arcsin\left(\frac{R}{R+h}\right) = 85.47°.$$

$$\theta_a = \arcsin\left(\frac{nR}{R+h}\right) = 85.70°.$$

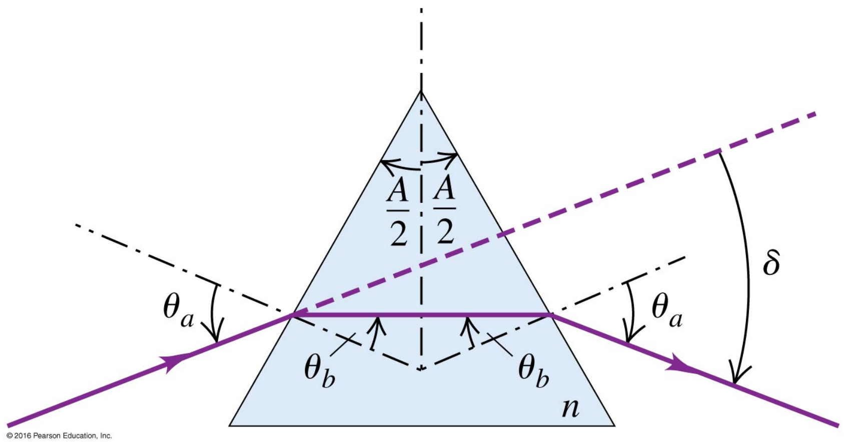
$$\delta = \theta_a - \theta_b = 85.70° - 85.47° = 0.23°.$$

EVALUATE: The calculated δ is about the same as the angular radius of the sun.

(33.53-T) The incident angle θ_a is chosen so that the light passes symmetrically through the prism, which has refractive index n and apex angle A. (a) Show that the angle of deviation δ (the angle between the initial and final directions of the ray) is given by

$$\sin\frac{A+\delta}{2} = n\sin\frac{A}{2} \tag{2}$$

(When the light passes through symmetrically, as shown, the angle of deviation is a minimum) (b) Use the result of part (a) to find the angle of deviation for a ray of light passing symmetrically through a prism having three equal angels ($A = 60°$) and $n = 1.52$ (c) A certain glass has a refractive index of 1.61 for red light (700nm) and 1.66 for violet light (400nm). If both colours pass through symmetrically, as described in part (a), and if $A = 60°$, find the difference between the angles of deviation for the two colour.



33.53. IDENTIFY: Apply Snell's law to the two refractions of the ray.

SET UP: Refer to the figure that accompanies the problem.

EXECUTE: **(a)** $n_a \sin\theta_a = n_b \sin\theta_b$ gives $\sin\theta_a = n_b \sin\frac{A}{2}$. But $\theta_a = \frac{A}{2} + \alpha$, so

$\sin\left(\frac{A}{2} + \alpha\right) = \sin\frac{A + 2\alpha}{2} = n\sin\frac{A}{2}$. At each face of the prism the deviation is α, so $2\alpha = \delta$ and

$\sin\frac{A + \delta}{2} = n\sin\frac{A}{2}$.

(b) From part (a), $\delta = 2\arcsin\left(n\sin\frac{A}{2}\right) - A$. $\delta = 2\arcsin\left((1.52)\sin\frac{60.0°}{2}\right) - 60.0° = 38.9°$.

(c) If two colors have different indices of refraction for the glass, then the deflection angles for them will differ:

$$\delta_{\text{red}} = 2\arcsin\left((1.61)\sin\frac{60.0°}{2}\right) - 60.0° = 47.2°.$$

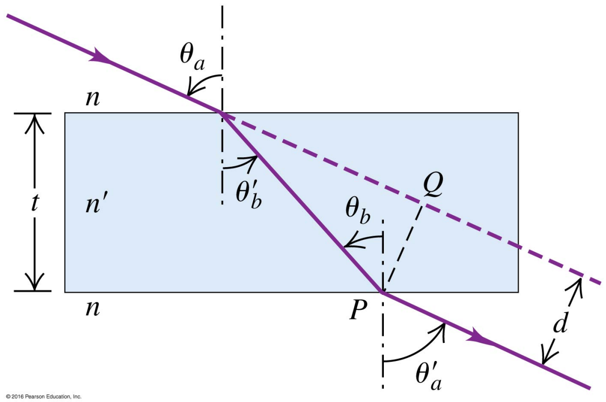
$$\delta_{\text{violet}} = 2\arcsin\left((1.66)\sin\frac{60.0°}{2}\right) - 60.0° = 52.2° \Rightarrow \Delta\delta = 52.2° - 47.2° = 5.0°.$$

EVALUATE: The violet light has a greater refractive index and therefore the angle of deviation is greater for the violet light.

(33.54-T) Light is incident in air at an angle θ_a on the upper surface of a transparent plat, the surfaces of the plate begin plane and parallel to each other. (a) Prove that $\theta_a = \theta'_a$. (b) Show that this is true for any number of different parallel plates. (c) Prove that the lateral displacement d of the emergent beam is given by the relationship

$$d = t\frac{\sin(\theta_a - \theta'_b)}{\cos\theta'_b} \tag{3}$$

where t is the thickness of the plate. (d) A ray of light is incident at an angle of 66° on one surface of a glass plate 2.4cm thick with an index of refraction of 1.80. The medium on either side of the plate is air. Find the lateral displacement between the incident and emergent rays.



33.54. IDENTIFY: Apply Snell's law to each refraction.

SET UP: Refer to the angles and distances defined in the figure that accompanies the problem.

EXECUTE: **(a)** For light in air incident on a parallel-faced plate, Snell's Law yields:

$n\sin\theta_a = n'\sin\theta_b' = n'\sin\theta_b = n\sin\theta_a' \Rightarrow \sin\theta_a = \sin\theta_a' \Rightarrow \theta_a = \theta_a'$.

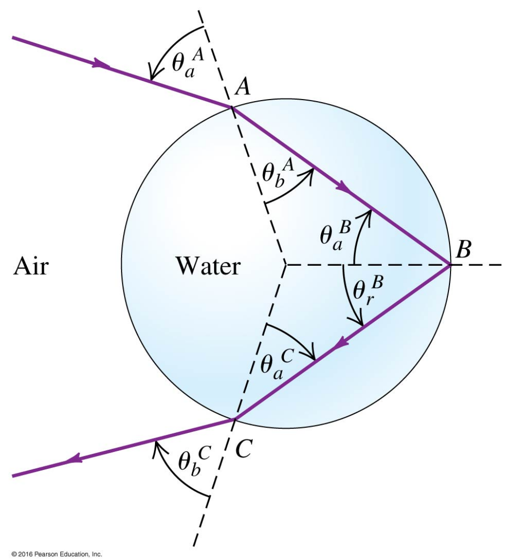
(b) Adding more plates just adds extra steps in the middle of the above equation that always cancel out. The requirement of parallel faces ensures that the angle $\theta_n' = \theta_n$ and the chain of equations can continue.

(c) The lateral displacement of the beam can be calculated using geometry:

$$d = L\sin(\theta_a - \theta_b') \text{ and } L = \frac{t}{\cos\theta_b'} \Rightarrow d = \frac{t\sin(\theta_a - \theta_b')}{\cos\theta_b'}.$$

(d) $\theta_b' = \arcsin\left(\frac{n\sin\theta_a}{n'}\right) = \arcsin\left(\frac{\sin 66.0°}{1.80}\right) = 30.5°$ and $d = \frac{(2.40\text{ cm})\sin(66.0° - 30.5°)}{\cos 30.5°} = 1.62\text{ cm}.$

(33.60-T) A rainbow is produced by the reflection of sunlight by spherical drops of water in the air. In the figure, we show a ray that refracts into a drop at point A, is reflected from the back surface of the drop at point B, and refracts back into the air at point C. The angles of incidence and refraction, θ_a and θ_b, are shown at point A and C, and the angles of incidence and reflection, θ_a and θ_c are shown at point B. (a) Show that $\theta_a^B = \theta_b^A$, $\theta_a^C = \theta_b^A$, and $\theta_b^C = \theta_a^A$. (b) Show that the angle in radians between the ray before it enters the drop at A and after it exits at C (i.e. the total angular deflection f the ray) is $\Delta = 2\theta_a^A - 4\theta_b^A + \pi$. (Hint: Find the angular deflections that occurs at A, B and C, and add them to get Δ) (c) Use Snell's law to write Δ in terms of θ_a^A and n, the refractive index of the water in the drop. (d) A rainbow will form when the angular deflection Δ is stationary in the incident angle θ_a^A - that is, when $\frac{d\Delta}{d\theta_a^A} = 0$. If this condition is satisfied, all the rays with incident angles close to θ_a^Awill be sent back in the same direction, producing a bright zone in the sky. Let θ_1 be the value of θ_a^A for which this occurs. Show that $\cos^2\theta_1 = \frac{1}{3}(n^2 - 1)$. (Hint: You may find the derivation formula $\frac{d \arcsin u(x)}{dx} = (1 - u^2)^{-1/2}\frac{du}{dx}$ helpful.) (e) The index of refraction in water is 1.342 for violet light and 1.330 for red light. Use the results of parts (c) and (d) to find θ_1 and Δ for violet and red light. When you view the rainbow, which colour, red or violet, is higher above the horizon?



33.60. IDENTIFY: Apply Snell's law to each refraction.

SET UP: Refer to the figure that accompanies the problem.

EXECUTE: **(a)** By the symmetry of the triangles, $\theta_b^A = \theta_a^B$, and $\theta_a^C = \theta_r^B = \theta_a^B = \theta_b^A$. Therefore,

$\sin\theta_b^C = n\sin\theta_a^C = n\sin\theta_b^A = \sin\theta_a^A = \theta_b^C = \theta_a^A$.

(b) The total angular deflection of the ray is $\Delta = \theta_a^A - \theta_b^A + \pi - 2\theta_a^B + \theta_b^C - \theta_a^C = 2\theta_a^A - 4\theta_b^A + \pi$.

(c) From Snell's law, $\sin\theta_a^A = n\sin\theta_b^A \Rightarrow \theta_b^A = \arcsin\left(\frac{1}{n}\sin\theta_a^A\right)$.

$$\Delta = 2\theta_a^A - 4\theta_b^A + \pi = 2\theta_a^A - 4\arcsin\left(\frac{1}{n}\sin\theta_a^A\right) + \pi.$$

(d)

$$\frac{d\Delta}{d\theta_a^A} = 0 = 2 - 4\frac{d}{d\theta_a^A}\left(\arcsin\left(\frac{1}{n}\sin\theta_a^A\right)\right) \Rightarrow 0 = 2 - \frac{4}{\sqrt{1-\frac{\sin^2\theta_1}{n^2}}}\cdot\left(\frac{\cos\theta_1}{n}\right)\cdot 4\left(1-\frac{\sin^2\theta_1}{n^2}\right) = \left(\frac{16\cos^2\theta_1}{n^2}\right).$$

$4\cos^2\theta_1 = n^2 - 1 + \cos^2\theta_1$. $3\cos^2\theta_1 = n^2 - 1$. $\cos^2\theta_1 = \frac{1}{3}(n^2-1)$.

(e) For violet: $\theta_1 = \arccos\left(\sqrt{\tfrac{1}{3}(n^2-1)}\right) = \arccos\left(\sqrt{\tfrac{1}{3}(1.342^2-1)}\right) = 58.89°$.

$\Delta_{\text{violet}} = 139.2° \Rightarrow \theta_{\text{violet}} = 40.8°$.

For red: $\theta_1 = \arccos\left(\sqrt{\tfrac{1}{3}(n^2-1)}\right) = \arccos\left(\sqrt{\tfrac{1}{3}(1.330^2-1)}\right) = 59.58°$.

$\Delta_{\text{red}} = 137.5° \Rightarrow \theta_{\text{red}} = 42.5°$.

EVALUATE: The angles we have calculated agree with the values given in Figure 33.19d in the textbook. θ_1 is larger for red than for violet, so red in the rainbow is higher above the horizon.