(34.24-T) The left end of a long glass rod 8.00 cm in diameter with an index of refraction of 1.60 , is ground and polished to a convex hemispherical surface with a radius of 4.00 cm . An object in the form of an arrow 1.50 mm tall, at right angles to the axis of the rod, is located on the axis 24.0 cm to the left of the vertex of the convex surface. Find the position and height of the image of the arrow formed by paraxial rays incident on the convex surface. Is the image erect or inverted?
(34.43-T) Two thin lenses with a focal length of magnitude 15.0 cm , the first diverging and the second converging, are located 11.3 cm apart. An object 1.60 mm tall is placed 25.0 cm to the left of the first (diverging) lens. (a) How far from this first lens is the final image formed? (b) Is the final image real or virtual? (c) What is the height of the final image? Is it erect or inverted?
34.24. IDENTIFY: $\frac{n_{a}}{s}+\frac{n_{b}}{s^{\prime}}=\frac{n_{b}-n_{a}}{R} . m=-\frac{n_{a} s}{n_{b} s}$.

SET UP: $\quad R=+4.00 \mathrm{~cm} . n_{a}=1.00 . n_{b}=1.60 . s=24.0 \mathrm{~cm}$.
EXECUTE: $\frac{1}{24.0 \mathrm{~cm}}+\frac{1.60}{s^{\prime}}=\frac{1.60-1.00}{4.00 \mathrm{~cm}} . s^{\prime}=+14.8 \mathrm{~cm} . m=-\frac{(1.00)(14.8 \mathrm{~cm})}{(1.60)(24.0 \mathrm{~cm})}=-0.385$.
$\left|y^{\prime}\right|=|m||y|=(0.385)(1.50 \mathrm{~mm})=0.578 \mathrm{~mm}$. The image is 14.8 cm to the right of the vertex and is 0.578 mm tall. $m<0$, so the image is inverted.

Evaluate: The image is real.
34.43. IDENTIFY: The first lens forms an image that is then the object for the second lens. We follow the same general procedure as in Problem 34.41.
SET UP: $\quad m_{\mathrm{tot}}=m_{1} m_{2} \cdot \frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f}$ gives $s^{\prime}=\frac{s f}{s-f}$.
EXECUTE: (a) Lens 1: $f_{1}=-15.0 \mathrm{~cm}, s_{1}=25.0 \mathrm{~cm} . s^{\prime}=\frac{(25.0 \mathrm{~cm})(-15.0 \mathrm{~cm})}{25.0 \mathrm{~cm}+15.0 \mathrm{~cm}}=-9.375 \mathrm{~cm}$.
$m_{1}=-\frac{s_{1}^{\prime}}{s_{1}}=-\frac{-9.375 \mathrm{~cm}}{25.0 \mathrm{~cm}}=+0.375$.

Lens 2: The image of lens 1 is 9.375 cm to the left of lens 1 so is 9.375 to the left of lens 2. $s_{2}=+20.675 \mathrm{~cm}$.
$f_{2}=+15.0 \mathrm{~cm} . s_{2}^{\prime}=\frac{(20.675 \mathrm{~cm})(15.0 \mathrm{~cm})}{20.675 \mathrm{~cm}-15.0 \mathrm{~cm}}=54.6 \mathrm{~cm} . m_{2}=-\frac{s_{2}^{\prime}}{s_{2}}=-\frac{54.6 \mathrm{~cm}}{20.675 \mathrm{~cm}}=-2.64$. The final image
is 54.6 cm to the right of lens 2 so is 65.9 cm to the right of the first lens.
(b) $s_{2}^{\prime}>0$ so the final image is real.
(c) $m_{\text {tot }}=m_{1} m_{2}=(+0.375)(-2.64)=-0.99$. The image is 1.78 mm tall and is inverted.

Evaluate: The light travels through the lenses in the direction from left to right. A real image for the second lens is to the right of that lens and a virtual image is to the left of the second lens.
(34.81-T) Figure shows an object and its image formed by a thin lens. (a) What is the focal length of the lens, and what type of lens is it? (b) What is the height of the image? Is it real or virtual?
(34.87-T) A convex mirror and a concave mirror are placed on the same optic axis, separated by a distance $\mathrm{L}=0.600 \mathrm{~m}$. The radius of curvature of each mirror has a magnitude of 0.360 m . A light source is located a distance x from the concave mirror, as shown in the figure. (a) What distance x will result in the rays from the source returning to the source returning to the source after reflecting first from the convex mirror and then from the concave mirror? (b) Repeat part (a) but now let the rays reflect first from the concave mirror and then from the convex one.

34.81. IDENTIFY: $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f}$. The type of lens determines the sign of $f . m=\frac{y^{\prime}}{y}=-\frac{s^{\prime}}{s}$. The sign of $s^{\prime}$ depends on whether the image is real or virtual. $s=16.0 \mathrm{~cm}$.
SET UP: $s^{\prime}=-22.0 \mathrm{~cm} ; s^{\prime}$ is negative because the image is on the same side of the lens as the object.
EXECUTE: (a) $\frac{1}{f}=\frac{s+s^{\prime}}{s s^{\prime}}$ and $f=\frac{s s^{\prime}}{s+s^{\prime}}=\frac{(16.0 \mathrm{~cm})(-22.0 \mathrm{~cm})}{16.0 \mathrm{~cm}-22.0 \mathrm{~cm}}=+58.7 \mathrm{~cm} . f$ is positive so the lens is converging.
(b) $m=-\frac{s^{\prime}}{s}=-\frac{-22.0 \mathrm{~cm}}{16.0 \mathrm{~cm}}=1.38 . y^{\prime}=m y=(1.38)(3.25 \mathrm{~mm})=4.48 \mathrm{~mm} . \quad s^{\prime}<0$ and the image is virtual.

EVALUATE: A converging lens forms a virtual image when the object is closer to the lens than the focal point.
34.87. (a) IDENTIFY: Use $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f}$ to locate the image formed by each mirror. The image formed by the first mirror serves as the object for the second mirror.
SET UP: The positions of the object and the two mirrors are shown in Figure 34.87a.


$$
\begin{aligned}
& |R|=0.360 \mathrm{~m} . \\
& |f|=|R| / 2=0.180 \mathrm{~m} .
\end{aligned}
$$

## Figure 34.87a

EXECUTE: Image formed by convex mirror (mirror \#1):
convex means $f_{1}=-0.180 \mathrm{~m} ; s_{1}=L-x$.
$s_{1}^{\prime}=\frac{s_{1} f_{1}}{s_{1}-f_{1}}=\frac{(L-x)(-0.180 \mathrm{~m})}{L-x+0.180 \mathrm{~m}}=-(0.180 \mathrm{~m})\left(\frac{0.600 \mathrm{~m}-x}{0.780 \mathrm{~m}-x}\right)<0$.
The image is $(0.180 \mathrm{~m})\left(\frac{0.600 \mathrm{~m}-x}{0.780 \mathrm{~m}-x}\right)$ to the left of mirror \#1 so is
$0.600 \mathrm{~m}+(0.180 \mathrm{~m})\left(\frac{0.600 \mathrm{~m}-x}{0.780 \mathrm{~m}-x}\right)=\frac{0.576 \mathrm{~m}^{2}-(0.780 \mathrm{~m}) x}{0.780 \mathrm{~m}-x}$ to the left of mirror \#2.
Image formed by concave mirror (mirror \#2):
concave implies $f_{2}=+0.180 \mathrm{~m}$.
$s_{2}=\frac{0.576 \mathrm{~m}^{2}-(0.780 \mathrm{~m}) x}{0.780 \mathrm{~m}-x}$.
Rays return to the source implies $s_{2}^{\prime}=x$. Using these expressions in $s_{2}=\frac{s_{2}^{\prime} f_{2}}{s_{2}^{\prime}-f_{2}}$ gives
$\frac{0.576 \mathrm{~m}^{2}-(0.780 \mathrm{~m}) x}{0.780 \mathrm{~m}-x}=\frac{(0.180 \mathrm{~m}) x}{x-0.180 \mathrm{~m}}$.
$0.600 x^{2}-(0.576 \mathrm{~m}) x+0.10368 \mathrm{~m}^{2}=0$.
$x=\frac{1}{1.20}\left(0.576 \pm \sqrt{(0.576)^{2}-4(0.600)(0.10368)}\right) \mathrm{m}=\frac{1}{1.20}(0.576 \pm 0.288) \mathrm{m}$.
$x=0.72 \mathrm{~m}$ (impossible; can't have $x>L=0.600 \mathrm{~m}$ ) or $x=0.24 \mathrm{~m}$.
(b) SET UP: Which mirror is \#1 and which is \#2 is now reversed form part (a). This is shown in Figure 34.87b.


## Figure 34.87b

EXECUTE: Image formed by concave mirror (mirror \#1):
concave means $f_{1}=+0.180 \mathrm{~m} ; s_{1}=x$.
$s_{1}^{\prime}=\frac{s_{1} f_{1}}{s_{1}-f_{1}}=\frac{(0.180 \mathrm{~m}) x}{x-0.180 \mathrm{~m}}$.
The image is $\frac{(0.180 \mathrm{~m}) x}{x-0.180 \mathrm{~m}}$ to the left of mirror \#1, so
$s_{2}=0.600 \mathrm{~m}-\frac{(0.180 \mathrm{~m}) x}{x-0.180 \mathrm{~m}}=\frac{(0.420 \mathrm{~m}) x-0.180 \mathrm{~m}^{2}}{x-0.180 \mathrm{~m}}$.

Image formed by convex mirror (mirror \#2):
convex means $f_{2}=-0.180 \mathrm{~m}$.
rays return to the source means $s_{2}^{\prime}=L-x=0.600 \mathrm{~m}-x$.
$\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f}$ gives
$\frac{x-0.180 \mathrm{~m}}{(0.420 \mathrm{~m}) x-0.180 \mathrm{~m}^{2}}+\frac{1}{0.600 \mathrm{~m}-x}=-\frac{1}{0.180 \mathrm{~m}}$.
$\frac{x-0.180 \mathrm{~m}}{(0.420 \mathrm{~m}) x-0.180 \mathrm{~m}^{2}}=-\left(\frac{0.780 \mathrm{~m}-x}{0.180 \mathrm{~m}^{2}-(0.180 \mathrm{~m}) x}\right)$.
$0.600 x^{2}-(0.576 \mathrm{~m}) x+0.1036 \mathrm{~m}^{2}=0$.
This is the same quadratic equation as obtained in part (a), so again $x=0.24 \mathrm{~m}$.
EVALUATE: For $x=0.24 \mathrm{~m}$ the image is at the location of the source, both for rays that initially travel
from the source toward the left and for rays that travel from the source toward the right.
(34.89-T) In the figure, the candle is at the centre of curvature of the concave mirror, whose focal length is 10.0 cm . The converging lens has a focal length of 32.0 cm and is 85.0 cm to the right of the candle. The candle is viewed looking through the lens from the right.The lens forms two images of the candle. The first is formed by light passing directly through the lens. The second image is formed from the light that goes from the candle to the mirror, is reflected, and then passes through the lens. (a) For each of these two images, draw a principal-ray diagram that locaters the image. (b) For each image, answer the following questions: (i) Where is the image? (ii) Is the image real or virtual? (iii) Is the image erect or inverted with respect to the original object?

34.89. Identify: $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f}$ gives $s^{\prime}=\frac{s f}{s-f}$, for both the mirror and the lens.

SET UP: For the second image, the image formed by the mirror serves as the object for the lens. For the mirror, $f_{\mathrm{m}}=+10.0 \mathrm{~cm}$. For the lens, $f=32.0 \mathrm{~cm}$. The center of curvature of the mirror is
$R=2 f_{\mathrm{m}}=20.0 \mathrm{~cm}$ to the right of the mirror vertex.
ExECUTE: (a) The principal-ray diagrams from the two images are sketched in Figure 34.89. In Figure 34.89 b , only the image formed by the mirror is shown. This image is at the location of the candle so the principal-ray diagram that shows the image formation when the image of the mirror serves as the object for the lens is analogous to that in Figure 34.89a and is not drawn.
(b) Image formed by the light that passes directly through the lens: The candle is 85.0 cm to the left of the lens. $s^{\prime}=\frac{s f}{s-f}=\frac{(85.0 \mathrm{~cm})(32.0 \mathrm{~cm})}{85.0 \mathrm{~cm}-32.0 \mathrm{~cm}}=+51.3 \mathrm{~cm} . m=-\frac{s^{\prime}}{s}=-\frac{51.3 \mathrm{~cm}}{85.0 \mathrm{~cm}}=-0.604$. This image is 51.3 cm to the right of the lens. $s^{\prime}>0$ so the image is real. $m<0$ so the image is inverted. Image formed by the light that first reflects off the mirror: First consider the image formed by the mirror. The candle is 20.0 cm
to the right of the mirror, so $s=+20.0 \mathrm{~cm} . s^{\prime}=\frac{s f}{s-f}=\frac{(20.0 \mathrm{~cm})(10.0 \mathrm{~cm})}{20.0 \mathrm{~cm}-10.0 \mathrm{~cm}}=20.0 \mathrm{~cm}$.
$m_{1}=-\frac{s_{1}^{\prime}}{s_{1}}=-\frac{20.0 \mathrm{~cm}}{20.0 \mathrm{~cm}}=-1.00$. The image formed by the mirror is at the location of the candle, so
$s_{2}=+85.0 \mathrm{~cm}$ and $s_{2}^{\prime}=51.3 \mathrm{~cm} . m_{2}=-0.604 . m_{\text {tot }}=m_{1} m_{2}=(-1.00)(-0.604)=0.604$. The second image
is 51.3 cm to the right of the lens. $s_{2}^{\prime}>0$, so the final image is real. $m_{\text {tot }}>0$, so the final image is erect.

Evaluate: The two images are at the same place. They are the same size. One is erect and one is inverted.

(a)

(b)
(34.61-T) A telescope is constructed from two converging lenses with focal lengths of 95.0 cm and 20.0 cm , the 95.0 cm lens being used as the objective. Both the object being viewed and the final image are at infinity. (a) Find the angular magnification for the telescope. (b) Find the height of the image formed by the objective of a building 60.0 m tall, 3.5 km away. (c) What is the angular size of the final image as viewed by an eye very close to the eyepiece?
34.61. (a) IDENTIFY and SET UP: Use $M=-\frac{f_{1}}{f_{2}}$, with $f_{1}=95.0 \mathrm{~cm}$ (objective) and $f_{2}=20.0 \mathrm{~cm}$
(eyepiece).
EXECUTE: $\quad M=-\frac{f_{1}}{f_{2}}=-\frac{95.0 \mathrm{~cm}}{20.0 \mathrm{~cm}}=-4.75$.
(b) Identify: Use $m=\frac{y^{\prime}}{y}=-\frac{s^{\prime}}{s}$ to calculate $y^{\prime}$.

SET UP: $s=3.50 \times 10^{3} \mathrm{~m}$.
$s^{\prime}=f_{1}=95.0 \mathrm{~cm}$ (since $s$ is very large, $s^{\prime} \approx f$ ).
EXECUTE: $m=-\frac{s^{\prime}}{s}=-\frac{0.950 \mathrm{~m}}{3.50 \times 10^{3} \mathrm{~m}}=-2.714 \times 10^{-4}$.
$\left|y^{\prime}\right|=|m \| y|=\left(2.714 \times 10^{-4}\right)(60.0 \mathrm{~m})=0.0163 \mathrm{~m}=1.63 \mathrm{~cm}$.
(c) Identify and SET UP: Use $M=\frac{\theta^{\prime}}{\theta}$ and the angular magnification $M$ obtained in part (a) to calculate $\theta^{\prime}$. The angular size $\theta$ of the image formed by the objective (object for the eyepiece) is its height divided by its distance from the objective.
EXECUTE: The angular size of the object for the eyepiece is $\theta=\frac{0.0163 \mathrm{~m}}{0.950 \mathrm{~m}}=0.0171 \mathrm{rad}$.
(Note that this is also the angular size of the object for the objective: $\theta=\frac{60.0 \mathrm{~m}}{3.50 \times 10^{3} \mathrm{~m}}=0.0171 \mathrm{rad}$. For a thin lens the object and image have the same angular size and the image of the objective is the object for the eyepiece.) $M=\frac{\theta^{\prime}}{\theta}$, so the angular size of the image is $\theta^{\prime}=M \theta=-(4.75)(0.0171 \mathrm{rad})=-0.081 \mathrm{rad}$. (The minus sign shows that the final image is inverted.)
Evaluate: The lateral magnification of the objective is small; the image it forms is much smaller than the object. But the total angular magnification is larger than 1.00 ; the angular size of the final image viewed by the eye is 4.75 times larger than the angular size of the original object, as viewed by the unaided eye.
(34.100-T) The Galilean Telescope In the figure, we show a digram of a Galilean telescope, or opera glass, with both the object and its final image at infinity. The image I serves as a virtual object for the eyepiece. The final image is virtual and erect. (a) Prove that the angular magnification is $M=-f_{1} / f_{2}$. (b) A Galilean telescope is to be constructed with the same objective lens as in Exercise 34.61. What focal length should the eyepiece have if this telescope is to have the same magnitude of angular magnification as the one in Exercise 34.61? (c) Compare the length of the telescopes.

34.100. Identify: For $u$ and $u^{\prime}$ as defined in Figure P 34.100 in the textbook, $M=\frac{u^{\prime}}{u}$.

SET UP: $f_{2}$ is negative. From Figure P 34.100 in the textbook, the length of the telescope is $f_{1}+f_{2}$, since $f_{2}$ is negative.

EXECUTE: (a) From the figure, $u=\frac{y}{f_{1}}$ and $u^{\prime}=\frac{y}{\left|f_{2}\right|}=-\frac{y}{f_{2}}$. The angular magnification is $M=\frac{u^{\prime}}{u}=-\frac{f_{1}}{f_{2}}$.
(b) $M=-\frac{f_{1}}{f_{2}} \Rightarrow f_{2}=-\frac{f_{1}}{M}=-\frac{95.0 \mathrm{~cm}}{6.33}=-15.0 \mathrm{~cm}$.
(c) The length of the telescope is $95.0 \mathrm{~cm}-15.0 \mathrm{~cm}=80.0 \mathrm{~cm}$, compared to the length of 110 cm for the telescope in Exercise 34.61.
Evaluate: An advantage of this construction is that the telescope is somewhat shorter.

