37.11 •• **Why Are We Bombarded by Muons?** Muons are unstable subatomic particles that decay to electrons with a mean lifetime of 2.2 μs. They are produced when cosmic rays bombard the upper atmosphere about 10 km above the earth's surface, and they travel very close to the speed of light. The problem we want to address is why we see any of them at the earth's surface. (a) What is the greatest distance a muon could travel during its 2.2-μs lifetime? (b) According to your answer in part (a), it would seem that muons could never make it to the ground. But the 2.2-μs lifetime is measured in the frame of the muon, and muons are moving very fast. At a speed of $0.999c$, what is the mean lifetime of a muon as measured by an observer at rest on the earth? How far would the muon travel in this time? Does this result explain why we find muons in cosmic rays? (c) From the point of view of the muon, it still lives for only 2.2 μs, so how does it make it to the ground? What is the thickness of the 10 km of atmosphere through which the muon must travel, as measured by the muon? Is it now clear how the muon is able to reach the ground?

37.11. **IDENTIFY** and **SET UP:** The $2.2\ \mu\text{s}$ lifetime is Δt_0 and the observer on earth measures Δt. The atmosphere is moving relative to the muon so in its frame the height of the atmosphere is l and l_0 is $10\ \text{km}$.

EXECUTE: **(a)** The greatest speed the muon can have is c, so the greatest distance it can travel in $2.2\times10^{-6}\ \text{s}$ is $d = vt = (3.00\times10^8\ \text{m/s})(2.2\times10^{-6}\ \text{s}) = 660\ \text{m} = 0.66\ \text{km}$.

(b) $\Delta t = \dfrac{\Delta t_0}{\sqrt{1-u^2/c^2}} = \dfrac{2.2\times10^{-6}\ \text{s}}{\sqrt{1-(0.999)^2}} = 4.9\times10^{-5}\ \text{s}$

$d = vt = (0.999)(3.00\times10^8\ \text{m/s})(4.9\times10^{-5}\ \text{s}) = 15\ \text{km}$

In the frame of the earth the muon can travel 15 km in the atmosphere during its lifetime.

(c) $l = l_0\sqrt{1-u^2/c^2} = (10\ \text{km})\sqrt{1-(0.999)^2} = 0.45\ \text{km}$

In the frame of the muon the height of the atmosphere is less than the distance it moves during its lifetime.

37.17 •• A pursuit spacecraft from the planet Tatooine is attempting to catch up with a Trade Federation cruiser. As measured by an observer on Tatooine, the cruiser is traveling away from the planet with a speed of $0.600c$. The pursuit ship is traveling at a speed of $0.800c$ relative to Tatooine, in the same direction as the cruiser. (a) For the pursuit ship to catch the cruiser, should the velocity of the cruiser relative to the pursuit ship be directed toward or away from the pursuit ship? (b) What is the speed of the cruiser relative to the pursuit ship?

37.17. **IDENTIFY:** The relativistic velocity addition formulas apply since the speeds are close to that of light.

SET UP: The relativistic velocity addition formula is $v_x' = \dfrac{v_x - u}{1 - \dfrac{uv_x}{c^2}}$.

EXECUTE: **(a)** For the pursuit ship to catch the cruiser, the distance between them must be decreasing, so the velocity of the cruiser relative to the pursuit ship must be directed toward the pursuit ship.

(b) Let the unprimed frame be Tatooine and let the primed frame be the pursuit ship. We want the velocity v' of the cruiser knowing the velocity of the primed frame u and the velocity of the cruiser v in the unprimed frame (Tatooine).

$$v_x' = \frac{v_x - u}{1 - \dfrac{uv_x}{c^2}} = \frac{0.600c - 0.800c}{1 - (0.600)(0.800)} = -0.385c$$

The result implies that the cruiser is moving toward the pursuit ship at $0.385c$.

EVALUATE: The nonrelativistic formula would have given $-0.200c$, which is considerably different from the correct result.

37.20 •• Two particles in a high-energy accelerator experiment are approaching each other head-on, each with a speed of $0.9520c$ as measured in the laboratory. What is the magnitude of the velocity of one particle relative to the other?

37.20. **IDENTIFY** and **SET UP:** Let S be the laboratory frame and let S' be the frame of one of the particles, as shown in Figure 37.20. Let the positive x-direction for both frames be from particle 1 to particle 2. In the lab frame particle 1 is moving in the $+x$-direction and particle 2 is moving in the $-x$-direction. Then $u = 0.9520c$ and $v_x = -0.9520c$. v'_x is the velocity of particle 2 relative to particle 1.

EXECUTE: $v'_x = \dfrac{v_x - u}{1 - uv_x/c^2} = \dfrac{-0.9520c - 0.9520c}{1-(0.9520c)(-0.9520c)/c^2} = -0.9988c$. The speed of particle 2 relative to particle 1 is $0.9988c$. $v'_x < 0$ shows particle 2 is moving toward particle 1.

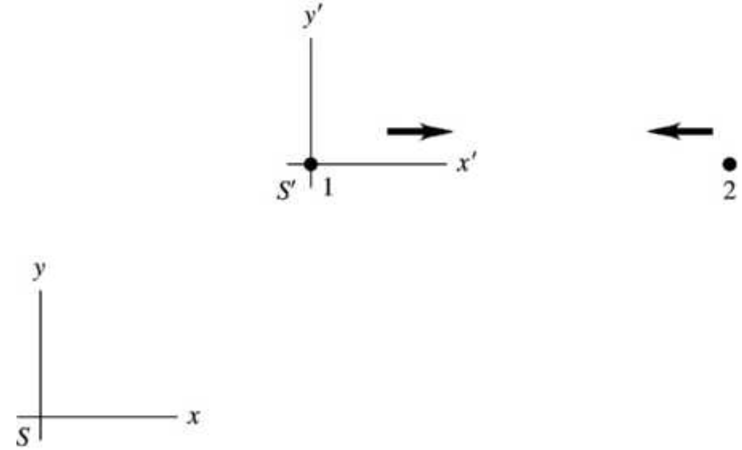


Figure 37.20

37.42 •• A 0.100-μg speck of dust is accelerated from rest to a speed of 0.900c by a constant 1.00×10^6 N force. (a) If the nonrelativistic mechanics is used, how far does the object travel to reach its final speed? (b) Using the correct relativistic treatment of Section 37.8, how far does the object travel to reach its final speed? (c) Which distance is greater? Why?

37.42. IDENTIFY: Since the final speed is close to the speed of light, there will be a considerable difference between the relativistic and nonrelativistic results.

SET UP: The nonrelativistic work-energy theorem is $F\Delta x = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$, and the relativistic formula for a constant force is $F\Delta x = (\gamma - 1)mc^2$.

EXECUTE: **(a)** Using the classical work-energy theorem and solving for Δx, we obtain

$$\Delta x = \frac{m(v^2 - v_0^2)}{2F} = \frac{(0.100\times10^{-9}\text{ kg})[(0.900)(3.00\times10^8\text{ m/s})]^2}{2(1.00\times10^6\text{ N})} = 3.65\text{ m}.$$

(b) Using the relativistic work-energy theorem for a constant force, we obtain

$$\Delta x = \frac{(\gamma - 1)mc^2}{F}.$$

For the given speed, $\gamma = \frac{1}{\sqrt{1 - 0.900^2}} = 2.29$, thus

$$\Delta x = \frac{(2.29 - 1)(0.100\times10^{-9}\text{ kg})(3.00\times10^8\text{ m/s})^2}{(1.00\times10^6\text{N})} = 11.6\text{ m}.$$

EVALUATE: **(c)** The distance obtained from the relativistic treatment is greater. As we have seen, more energy is required to accelerate an object to speeds close to c, so that force must act over a greater distance.

37.55 • **The Large Hadron Collider (LHC).** Physicists and engineers from around the world have come together to build the largest accelerator in the world, the Large Hadron Collider (LHC) at the CERN Laboratory in Geneva, Switzerland. The machine will accelerate protons to kinetic energies of 7 TeV in an underground ring 27 km in circumference. (For the latest news and more information on the LHC, visit www.cern.ch.) (a) What speed v will protons reach in the LHC? (Since v is very close to c, write $v = (1 - \Delta)c$ and give your answer in terms of Δ.) (b) Find the relativistic mass, m_{rel}, of the accelerated protons in terms of their rest mass.

37.55. **IDENTIFY:** Since the speed is very close to the speed of light, we must use the relativistic formula for kinetic energy.

SET UP: The relativistic formula for kinetic energy is $K = mc^2\left(\frac{1}{\sqrt{1-v^2/c^2}}-1\right)$ and the relativistic mass is $m_{\text{rel}} = \frac{m}{\sqrt{1-v^2/c^2}}$.

EXECUTE: **(a)** $K = 7\times10^{12}\ \text{eV} = 1.12\times10^{-6}\ \text{J}$. Using this value in the relativistic kinetic energy formula and substituting the mass of the proton for *m*, we get $K = mc^2\left(\frac{1}{\sqrt{1-v^2/c^2}}-1\right)$ which gives $\frac{1}{\sqrt{1-v^2/c^2}} = 7.45\times10^3$ and $1-\frac{v^2}{c^2} = \frac{1}{(7.45\times10^3)^2}$. Solving for *v* gives $1-\frac{v^2}{c^2} = \frac{(c+v)(c-v)}{c^2} = \frac{2(c-v)}{c}$, since $c+v \approx 2c$. Substituting $v = (1-\Delta)c$, we have $1-\frac{v^2}{c^2} = \frac{2(c-v)}{c} = \frac{2[c-(1-\Delta)c]}{c} = 2\Delta$. Solving for Δ gives $\Delta = \frac{1-v^2/c^2}{2} = \frac{\frac{1}{(7.45\times10^3)^2}}{2} = 9\times10^{-9}$, to one significant digit.

(b) Using the relativistic mass formula and the result that $\frac{1}{\sqrt{1-v^2/c^2}} = 7.45\times10^3$, we have $m_{\text{rel}} = \frac{m}{\sqrt{1-v^2/c^2}} = m\left(\frac{1}{\sqrt{1-v^2/c^2}}\right) = (7\times10^3)m$, to one significant digit.

EVALUATE: At such high speeds, the proton's mass is over 7000 times as great as its rest mass.

37.65 ••• Two events observed in a frame of reference S have positions and times given by (x_1, t_1) and (x_2, t_2), respectively. (a) Frame S' moves along the x-axis just fast enough that the two events occur at the same position in S'. Show that in S', the time interval $\Delta t'$ between the two events is given by

$$\Delta t' = \sqrt{(\Delta t)^2 - \left(\frac{\Delta x}{c}\right)^2}$$

where $\Delta x = x_2 - x_1$ and $\Delta t = t_2 - t_1$. Hence show that if $\Delta x > c\,\Delta t$, there is *no* frame S' in which the two events occur at the same point. The interval $\Delta t'$ is sometimes called the *proper time interval* for the events. Is this term appropriate? (b) Show that if $\Delta x > c\,\Delta t$, there is a different frame of reference S' in which the two events occur *simultaneously*. Find the distance between the two events in S'; express your answer in terms of Δx, Δt, and c. This distance is sometimes called a *proper length*. Is this term appropriate? (c) Two events are observed in a frame of reference S' to occur simultaneously at points separated by a distance of 2.50 m. In a second frame S moving relative to S' along the line joining the two points in S', the two events appear to be separated by 5.00 m. What is the time interval between the events as measured in S? [*Hint:* Apply the result obtained in part (b).]

37.65. **(a) IDENTIFY and SET UP:** Use the Lorentz coordinate transformation (Eq. 37.21) for (x_1, t_1) and (x_2, t_2):

$$x_1' = \frac{x_1 - ut_1}{\sqrt{1-u^2/c^2}},\quad x_2' = \frac{x_2 - ut_2}{\sqrt{1-u^2/c^2}}$$

$$t_1' = \frac{t_1 - ux_1/c^2}{\sqrt{1-u^2/c^2}},\quad t_2' = \frac{t_2 - ux_2/c^2}{\sqrt{1-u^2/c^2}}$$

Same point in S' implies $x_1' = x_2'$. What then is $\Delta t' = t_2' - t_1'$?

EXECUTE: $x_1' = x_2'$ implies $x_1 - ut_1 = x_2 - ut_2$

$$u(t_2 - t_1) = x_2 - x_1 \text{ and } u = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$

From the time transformation equations,

$$\Delta t' = t_2' - t_1' = \frac{1}{\sqrt{1-u^2/c^2}}(\Delta t - u\Delta x/c^2)$$

Using the result that $u = \dfrac{\Delta x}{\Delta t}$ gives

$$\Delta t' = \frac{1}{\sqrt{1-(\Delta x)^2/((\Delta t)^2c^2)}}(\Delta t - (\Delta x)^2/((\Delta t)c^2))$$

$$\Delta t' = \frac{\Delta t}{\sqrt{(\Delta t)^2 - (\Delta x)^2/c^2}}(\Delta t - (\Delta x)^2/((\Delta t)c^2))$$

$$\Delta t' = \frac{(\Delta t)^2 - (\Delta x)^2/c^2}{\sqrt{(\Delta t)^2 - (\Delta x)^2/c^2}} = \sqrt{(\Delta t)^2 - (\Delta x/c)^2}, \text{ as was to be shown.}$$

This equation doesn't have a physical solution (because of a negative square root) if $(\Delta x/c)^2 > (\Delta t)^2$ or $\Delta x \geq c\Delta t$.

(b) IDENTIFY and SET UP: Now require that $t_2' = t_1'$ (the two events are simultaneous in S') and use the Lorentz coordinate transformation equations.

EXECUTE: $t_2' = t_1'$ implies $t_1 - ux_1/c^2 = t_2 - ux_2/c^2$

$$t_2 - t_1 = \left(\frac{x_2 - x_1}{c^2}\right)u \text{ so } \Delta t = \left(\frac{\Delta x}{c^2}\right)u \text{ and } u = \frac{c^2\Delta t}{\Delta x}$$

From the Lorentz transformation equations,

$$\Delta x' = x_2' - x_1' = \left(\frac{1}{\sqrt{1-u^2/c^2}}\right)(\Delta x - u\Delta t).$$

Using the result that $u = c^2\Delta t/\Delta x$ gives

$$\Delta x' = \frac{1}{\sqrt{1-c^2(\Delta t)^2/(\Delta x)^2}}(\Delta x - c^2(\Delta t)^2/\Delta x)$$

$$\Delta x' = \frac{\Delta x}{\sqrt{(\Delta x)^2 - c^2(\Delta t)^2}}(\Delta x - c^2(\Delta t)^2/\Delta x)$$

$$\Delta x' = \frac{(\Delta x)^2 - c^2(\Delta t)^2}{\sqrt{(\Delta x)^2 - c^2(\Delta t)^2}} = \sqrt{(\Delta x)^2 - c^2(\Delta t)^2}$$

(c) IDENTIFY and SET UP: The result from part (b) is $\Delta x' = \sqrt{(\Delta x)^2 - c^2(\Delta t)^2}$.

Solve for Δt: $(\Delta x')^2 = (\Delta x)^2 - c^2(\Delta t)^2$

EXECUTE: $\Delta t = \dfrac{\sqrt{(\Delta x)^2 - (\Delta x')^2}}{c} = \dfrac{\sqrt{(5.00\text{ m})^2 - (2.50\text{ m})^2}}{2.998\times10^8\text{ m/s}} = 1.44\times10^{-8}\text{ s}$

EVALUATE: This provides another illustration of the concept of simultaneity (Section 37.2): events observed to be simultaneous in one frame are not simultaneous in another frame that is moving relative to the first.

37.73 ••• **CALC** **Lorentz Transformation for Acceleration.** Using a method analogous to the one in the text to find the Lorentz transformation formula for velocity, we can find the Lorentz transformation for *acceleration.* Let frame S' have a constant x-component of velocity u relative to frame S. An object moves relative to frame S along the x-axis with instantaneous velocity v_x and instantaneous acceleration a_x. (a) Show that its instantaneous acceleration in frame S' is

$$a'_x = a_x\left(1 - \frac{u^2}{c^2}\right)^{3/2}\left(1 - \frac{uv_x}{c^2}\right)^{-3}$$

[*Hint:* Express the acceleration in S' as $a'_x = dv'_x/dt'$. Then use Eq. (37.21) to express dt' in terms of dt and dx, and use Eq. (37.22) to express dv'_x in terms of u and dv_x. The velocity of the object in S is $v_x = dx/dt$.] (b) Show that the acceleration in frame S can be expressed as

$$a_x = a'_x\left(1 - \frac{u^2}{c^2}\right)^{3/2}\left(1 + \frac{uv'_x}{c^2}\right)^{-3}$$

where $v'_x = dx'/dt'$ is the velocity of the object in frame S'.

measured experimentally.

37.73. **IDENTIFY** and **SET UP:** Follow the procedure specified in the hint.

EXECUTE: **(a)** $a' = \dfrac{dv}{dt'}$. $dt' = \gamma(dt - u\,dx/c^2)$. $dv' = \dfrac{dv}{(1 - uv/c^2)} + \dfrac{v-u}{(1-uv/c^2)^2}\dfrac{u}{c^2}dv$

$$\frac{dv'}{dv} = \frac{1}{1-uv/c^2} + \frac{v-u}{(1-uv/c^2)^2}\left(\frac{u}{c^2}\right).\quad dv' = dv\left(\frac{1}{1-uv/c^2} + \frac{(v-u)u/c^2}{(1-uv/c^2)^2}\right) = dv\left(\frac{1-u^2/c^2}{(1-uv/c^2)^2}\right)$$

$$a' = \frac{dv\dfrac{(1-u^2/c^2)}{(1-uv/c^2)^2}}{\gamma dt - u\gamma dx/c^2} = \frac{dv}{dt}\frac{(1-u^2/c^2)}{(1-uv/c^2)^2}\frac{1}{\gamma(1-uv/c^2)} = a(1-u^2/c^2)^{3/2}\,(1-uv/c^2)^{-3}.$$

(b) Changing frames from $S' \to S$ just involves changing

$$a \to a',\ v \to -v' \Rightarrow a = a'(1-u^2/c^2)^{3/2}\left(1 + \frac{uv'}{c^2}\right)^{-3}.$$

EVALUATE: a'_x depends not only on a_x and u, but also on v_x, the component of the velocity of the object in frame S.

37.74 ••• CALC A Realistic Version of the Twin Paradox. A rocket ship leaves the earth on January 1, 2100. Stella, one of a pair of twins born in the year 2075, pilots the rocket (reference frame S'); the other twin, Terra, stays on the earth (reference frame S). The rocket ship has an acceleration of constant magnitude g in its own reference frame (this makes the pilot feel at home, since it simulates the earth's gravity). The path of the rocket ship is a straight line in the $+x$-direction in frame S. (a) Using the results of Challenge Problem 37.73, show that in Terra's earth frame S, the rocket's acceleration is

$$\frac{du}{dt} = g\left(1 - \frac{u^2}{c^2}\right)^{3/2}$$

where u is the rocket's instantaneous velocity in frame S. (b) Write the result of part (a) in the form $dt = f(u)\,du$, where $f(u)$ is a function of u, and integrate both sides. (*Hint:* Use the integral given in Problem 37.63.) Show that in Terra's frame, the time when Stella attains a velocity v_{1x} is

$$t_1 = \frac{v_{1x}}{g\sqrt{1 - v_{1x}^{\,2}/c^2}}$$

(c) Use the time dilation formula to relate dt and dt' (infinitesimal time intervals measured in frames S and S', respectively). Combine this result with the result of part (a) and integrate as in part (b) to show the following: When Stella attains a velocity v_{1x} relative to Terra, the time t'_1 that has elapsed in frame S' is

$$t'_1 = \frac{c}{g}\,\text{arctanh}\left(\frac{v_{1x}}{c}\right)$$

Here arctanh is the inverse hyperbolic tangent. (*Hint:* Use the integral given in Challenge Problem 5.124.) (d) Combine the results of parts (b) and (c) to find t_1 in terms of t'_1, g, and c alone. (e) Stella accelerates in a straight-line path for five years (by her clock), slows down at the same rate for five years, turns around, accelerates for five years, slows down for five years, and lands back on the earth. According to Stella's clock, the date is January 1, 2120. What is the date according to Terra's clock?

37.74. **IDENTIFY and SET UP:** Follow the procedures specified in the problem.

EXECUTE: **(a)** The speed v' is measured relative to the rocket, and so for the rocket and its occupant, $v' = 0.$ The acceleration as seen in the rocket is given to be $a' = g,$ and so the acceleration as measured on the earth is $a = \dfrac{du}{dt} = g\left(1 - \dfrac{u^2}{c^2}\right)^{3/2}.$

(b) With $v_1 = 0$ when $t = 0,$

$$dt = \frac{1}{g}\frac{du}{(1-u^2/c^2)^{3/2}}. \quad \int_0^{t_1} dt = \frac{1}{g}\int_0^{v_1}\frac{du}{(1-u^2/c^2)^{3/2}}. \quad t_1 = \frac{v_1}{g\sqrt{1-v_1^2/c^2}}.$$

(c) $dt' = \gamma dt = dt/\sqrt{1-u^2/c^2},$ so the relation in part (b) between dt and $du,$ expressed in terms of dt' and $du,$ is $dt' = \gamma dt = \dfrac{1}{\sqrt{1-u^2/c^2}}\dfrac{du}{g(1-u^2/c^2)^{3/2}} = \dfrac{1}{g}\dfrac{du}{(1-u^2/c^2)^2}.$

Integrating as above (perhaps using the substitution $z = u/c$) gives $t_1' = \dfrac{c}{g}\operatorname{arctan\,h}\left(\dfrac{v_1}{c}\right).$ For those who wish to avoid inverse hyperbolic functions, the above integral may be done by the method of partial fractions; $gdt' = \dfrac{du}{(1+u/c)(1-u/c)} = \dfrac{1}{2}\left[\dfrac{du}{1+u/c} + \dfrac{du}{1-uc}\right],$ which integrates to $t_1' = \dfrac{c}{2g}\ln\left(\dfrac{c+v_1}{c-v_1}\right).$

(d) Solving the expression from part (c) for v_1 in terms of t_1, $(v_1/c) = \tanh(gt_1'/c)$, so that $\sqrt{1-(v_1/c)^2} = 1/\cosh(gt_1'/c)$, using the appropriate indentities for hyperbolic functions. Using this in the expression found in part (b), $t_1 = \frac{c}{g}\frac{\tanh(gt_1'/c)}{1/\cosh(gt_1'/c)} = \frac{c}{g}\sinh(gt_1'/c)$, which may be rearranged slightly as $\frac{gt_1}{c} = \sinh\left(\frac{gt'}{c}\right)$. If hyperbolic functions are not used, v_1 in terms of t_1' is found to be $\frac{v_1}{c} = \frac{e^{gt_1'/c} - e^{-gt_1'/c}}{e^{gt_1'/c} + e^{-gt_1'/c}}$ which is the same as $\tanh(gt_1'/c)$. Inserting this expression into the result of part (b) gives, after much algebra, $t_1 = \frac{c}{2g}(e^{gt_1'/c} - e^{-gt_1'/c})$, which is equivalent to the expression found using hyperbolic functions.

(e) After the first acceleration period (of 5 years by Stella's clock), the elapsed time on earth is

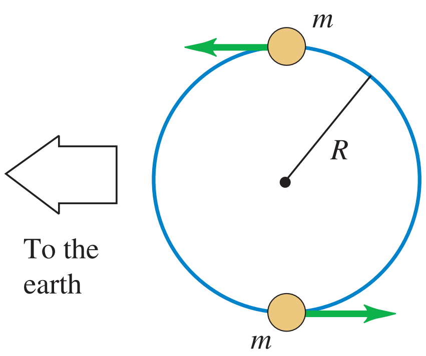
$$t_1' = \frac{c}{g}\sinh(gt_1'/c) = 2.65\times 10^9 \text{ s} = 84.0 \text{ yr}.$$

The elapsed time will be the same for each of the four parts of the voyage, so when Stella has returned, Terra has aged 336 yr and the year is 2436. (Keeping more precision than is given in the problem gives February 7 of that year.)

EVALUATE: Stella has aged only 20 yrs, much less than Terra.

37.75 ••• CP Determining the Masses of Stars. Many of the stars in the sky are actually *binary stars,* in which two stars orbit about their common center of mass. If the orbital speeds of the stars are high enough, the motion of the stars can be detected by the Doppler shifts of the light they emit. Stars for which this is the case are called *spectroscopic binary stars*. Figure P37.75 shows the simplest case of a spectroscopic binary star: two identical stars, each with mass m, orbiting their center of mass in a circle of radius R. The plane of the stars' orbits is edge-on to the line of sight of an observer on the earth. (a) The light produced by heated hydrogen gas in a laboratory on the earth has a frequency of 4.568110×10^{14} Hz. In the light received from the stars by a telescope on the earth, hydrogen light is observed to vary in frequency between 4.567710×10^{14} Hz and 4.568910×10^{14} Hz. Determine whether the binary star system as a whole is moving toward or away from the earth, the speed of this motion, and the orbital speeds of the stars. (*Hint:* The speeds involved are much less than c, so you may use the approximate result $\Delta f/f = u/c$ given in Section 37.6.)

Figure **P37.75**



(b) The light from each star in the binary system varies from its maximum frequency to its minimum frequency and back again in 11.0 days. Determine the orbital radius R and the mass m of each star. Give your answer for m in kilograms and as a multiple of the mass of the sun, 1.99×10^{30} kg. Compare the value of R to the distance from the earth to the sun, 1.50×10^{11} m. (This technique is actually used in astronomy to determine the masses of stars. In practice, the problem is more complicated because the two stars in a binary system are usually not identical, the orbits are usually not circular, and the plane of the orbits is usually tilted with respect to the line of sight from the earth.)

37.75. **IDENTIFY:** Apply the Doppler effect equation.

SET UP: At the two positions shown in the figure given in the problem, the velocities of the star relative to the earth are $u+v$ and $u-v$, where u is the velocity of the center of mass and v is the orbital velocity.

EXECUTE: **(a)** $f_0 = 4.568110\times10^{14}$ Hz; $f_+ = 4.568910\times10^{14}$ Hz; $f_- = 4.567710\times10^{14}$ Hz

$$\left.\begin{array}{l} f_+ = \sqrt{\dfrac{c+(u+v)}{c-(u+v)}}\,f_0 \\[2ex] f_- = \sqrt{\dfrac{c+(u-v)}{c-(u-v)}}\,f_0 \end{array}\right\} \Rightarrow \begin{array}{l} f_+^2(c-(u+v)) = f_0^2\,(c+(u+v)) \\ f_-^2(c-(u-v)) = f_0^2(c+(u-v)) \end{array}$$

$(u+v) = \dfrac{(f_+/f_0)^2-1}{(f_+/f_0)^2+1}c$ and $(u-v) = \dfrac{(f_-^2/f_0^2)-1}{(f_-^2/f_0^2)+1}c$. $u+v = 5.25\times10^4$ m/s and $u-v = -2.63\times10^4$ m/s.

This gives $u = +1.31\times10^4$ m/s (moving toward at 13.1 km/s) and $v = 3.94\times10^4$ m/s.

(b) $v = 3.94\times10^4$ m/s; $T = 11.0$ days. $2\pi R = vt \Rightarrow$

$R = \dfrac{(3.94\times10^4\text{ m/s})(11.0\text{ days})(24\text{ hrs/day})(3600\text{ sec/hr})}{2\pi} = 5.96\times10^9$ m. This is about 0.040 times the earth-sun distance.

Also the gravitational force between them (a distance of $2R$) must equal the centripetal force from the center of mass:

$\dfrac{(Gm^2)}{(2R)^2} = \dfrac{mv^2}{R} \Rightarrow m = \dfrac{4Rv^2}{G} = \dfrac{4(5.96\times10^9\text{ m})(3.94\times10^4\text{ m/s})^2}{6.672\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2} = 5.55\times10^{29}\text{ kg} = 0.279\, m_{\text{sun}}.$

EVALUATE: u and v are both much less than c, so we could have used the approximate expression $\Delta f = \pm f_0 v_{\text{rev}}/c$, where v_{rev} is the speed of the source relative to the observer.

37.77 ••• CP Kaon Production. In high-energy physics, new particles can be created by collisions of fast-moving projectile particles with stationary particles. Some of the kinetic energy of the incident particle is used to create the mass of the new particle. A proton–proton collision can result in the creation of a negative kaon (K^-) and a positive kaon (K^+)

$$p + p \rightarrow p + p + \mathrm{K}^- + \mathrm{K}^+$$

(a) Calculate the minimum kinetic energy of the incident proton that will allow this reaction to occur if the second (target) proton is initially at rest. The rest energy of each kaon is 493.7 MeV, and the rest energy of each proton is 938.3 MeV. (*Hint:* It is useful here to work in the frame in which the total momentum is zero. But note that the Lorentz transformation must be used to relate the velocities in the laboratory frame to those in the zero-total-momentum frame.) (b) How does this calculated minimum kinetic energy compare with the total rest mass energy of the created kaons? (c) Suppose that instead the two protons are both in motion with velocities of equal magnitude and opposite direction. Find the minimum combined kinetic energy of the two protons that will allow the reaction to occur. How does this calculated minimum kinetic energy compare with the total rest mass energy of the created kaons? (This example shows that when colliding beams of particles are used instead of a stationary target, the energy requirements for producing new particles are reduced substantially.)

37.77. IDENTIFY: Apply conservation of total energy, in the frame in which the total momentum is zero (the center of momentum frame).

SET UP: In the center of momentum frame, the two protons approach each other with equal velocities (since the protons have the same mass). After the collision, the two protons are at rest—but now there are kaons as well. In this situation the kinetic energy of the protons must equal the total rest energy of the two kaons.

EXECUTE: **(a)** $2(\gamma_{\text{cm}}-1)m_{\text{p}}c^2 = 2m_{\text{k}}c^2 \Rightarrow \gamma_{\text{cm}} = 1+\dfrac{m_{\text{k}}}{m_{\text{p}}} = 1.526.$ The velocity of a proton in the center of momentum frame is then $v_{\text{cm}} = c\sqrt{\dfrac{\gamma_{\text{cm}}^2-1}{\gamma_{\text{cm}}^2}} = 0.7554c.$

To get the velocity of this proton in the lab frame, we must use the Lorentz velocity transformations. This is the same as "hopping" into the proton that will be our target and asking what the velocity of the projectile proton is. Taking the lab frame to be the unprimed frame moving to the left, $u = v_{\text{cm}}$ and $v' = v_{\text{cm}}$ (the velocity of the projectile proton in the center of momentum frame).

$$v_{\text{lab}} = \frac{v'+u}{1+\dfrac{uv'}{c^2}} = \frac{2v_{\text{cm}}}{1+\dfrac{v_{\text{cm}}^2}{c^2}} = 0.9619c \Rightarrow \gamma_{\text{lab}} = \frac{1}{\sqrt{1-\dfrac{v_{\text{lab}}^2}{c^2}}} = 3.658 \Rightarrow K_{\text{lab}} = (\gamma_{\text{lab}}-1)m_{\text{p}}c^2 = 2494\text{ MeV}.$$

(b) $\dfrac{K_{\text{lab}}}{2m_{\text{k}}} = \dfrac{2494\text{ MeV}}{2(493.7\text{ MeV})} = 2.526.$

(c) The center of momentum case considered in part (a) is the same as this situation. Thus, the kinetic energy required *is* just twice the rest mass energy of the kaons. $K_{\text{cm}} = 2(493.7\text{ MeV}) = 987.4\text{ MeV}.$

EVALUATE: The colliding beam situation of part (c) offers a substantial advantage over the fixed target experiment in part (b). It takes less energy to create two kaons in the proton center of momentum frame.