Partial differentiation

Table 12.2

Partial derivative	Rule	Example: $f(x,y,z) = 2x^3y + z^2$	
Partial derivative with respect to x: $\frac{\partial}{\partial x}$	Treat all variables as constants except for x	$\frac{\partial f}{\partial x} = 6x^2y$	(12.1a)
Partial derivative with respect to y: $\frac{\partial}{\partial y}$	Treat all variables as constants except for y	$\frac{\partial f}{\partial y} = 2x^3$	(12.1b)
Partial derivative with respect to z: $\frac{\partial}{\partial z}$	Treat all variables as constants except for z	$\frac{\partial f}{\partial z} = 2z$	(12.1c)

Wave equation

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

Example:

Show that the following functions are solutions of the wave equation.

$$y(x,t) = \sin(kx - \omega t)$$

$$y(x,t) = e^{i(kx-\omega t)}$$

$$y(x,t) = (x - vt)^n$$

$$y(x,t) = (x + vt)^n$$

$$y(x,t) = \frac{1}{A + x - vt}$$

In fact, we can show that any functions with the form,

$$y(x,t) = f(x+vt) + g(x-vt)$$

for any differentiable functions f(u) and g(u) are solutions of the wave equation.

$$y(x, t) = A\cos\left[\omega\left(\frac{x}{v} - t\right)\right] = A\cos\left[2\pi f\left(\frac{x}{v} - t\right)\right] \quad \begin{pmatrix}\text{sinusoidal wave} \\ \text{moving in} \\ +x\text{-direction}\end{pmatrix} \tag{15.3}$$

15.12 •• CALC Speed of Propagation vs. Particle Speed. (a) Show that Eq. (15.3) may be written as

$$y(x, t) = A\cos\left[\frac{2\pi}{\lambda}(x - vt)\right]$$

(b) Use $y(x, t)$ to find an expression for the transverse velocity v_y of a particle in the string on which the wave travels. (c) Find the maximum speed of a particle of the string. Under what circumstances is this equal to the propagation speed v? Less than v? Greater than v?

Example 3.2.

(1) Find the amplitude, frequency, wavelength, and speed of propagation of the wave described by the equation

$$y = 0.2 \cos [\pi(5t - 2x)].$$

Here, the units of length and time are taken to be meter and second, respectively.

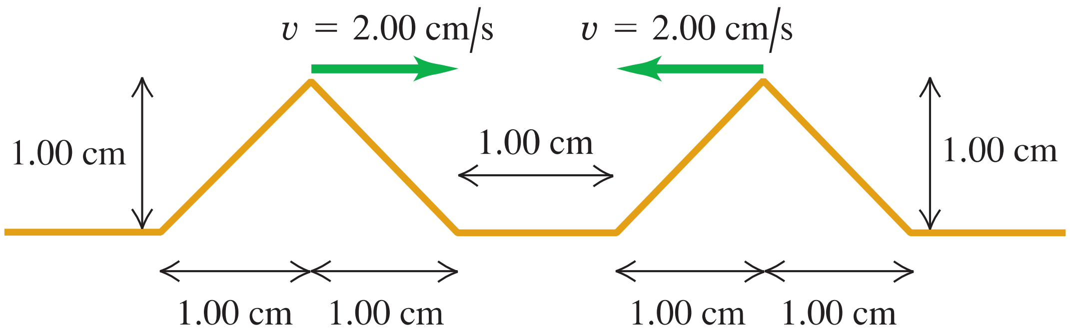
(2) When a sinusoidal wave of amplitude 0.1 m and frequency 2 Hz travels at a speed of 2 m/s in the $-x$ direction, derive the expression for the displacement y at position x at time t by using an integer, n. Here, we assume that the displacement at the origin ($x = 0$) at time $t = 0$ is zero, i.e., $y = 0$.

15.23 • A horizontal wire is stretched with a tension of 94.0 N, and the speed of transverse waves for the wire is 492 m/s. What must the amplitude of a traveling wave of frequency 69.0 Hz be in order for the average power carried by the wave to be 0.365 W?

15.24 •• A light wire is tightly stretched with tension F. Transverse traveling waves of amplitude A and wavelength λ_1 carry average power $P_{\text{av},1} = 0.400$ W. If the wavelength of the waves is doubled, so $\lambda_2 = 2\lambda_1$, while the tension F and amplitude A are not altered, what then is the average power $P_{\text{av},2}$ carried by the waves?

15.32 • **Interference of Triangular Pulses.** Two triangular wave pulses are traveling toward each other on a stretched string as shown in Fig. E15.32. Each pulse is identical to the other and travels at 2.00 cm/s. The leading edges of the pulses are 1.00 cm apart at $t = 0$. Sketch the shape of the string at $t = 0.250$ s, $t = 0.500$ s, $t = 0.750$ s, $t = 1.000$ s, and $t = 1.250$ s.

Figure **E15.32**



15.38 • **CALC** **Wave Equation and Standing Waves.** (a) Prove by direct substitution that $y(x, t) = (A_{SW} \sin kx) \sin \omega t$ is a solution of the wave equation, Eq. (15.12), for $v = \omega/k$. (b) Explain why the relationship $v = \omega/k$ for *traveling* waves also applies to *standing* waves.

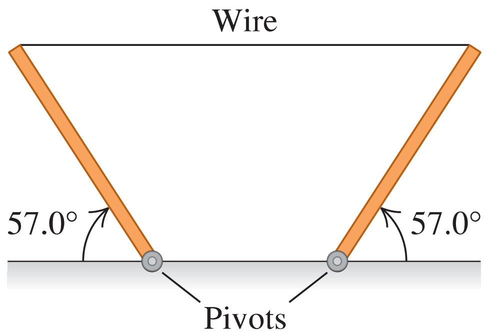
15.39 • **CALC** Let $y_1(x, t) = A \cos(k_1 x - \omega_1 t)$ and $y_2(x, t) = A \cos(k_2 x - \omega_2 t)$ be two solutions to the wave equation, Eq. (15.12), for the same v. Show that $y(x, t) = y_1(x, t) + y_2(x, t)$ is also a solution to the wave equation.

15.59 ••• **CP** The lower end of a uniform bar of mass 45.0 kg is attached to a wall by a frictionless hinge. The bar is held by a horizontal wire attached at its upper end so that the bar makes an angle of 30.0° with the wall. The wire has length 0.330 m and mass 0.0920 kg. What is the frequency of the fundamental standing wave for transverse waves on the wire?

15.60 ••• **CP** You are exploring a newly discovered planet. The radius of the planet is 7.20×10^7 m. You suspend a lead weight from the lower end of a light string that is 4.00 m long and has mass 0.0280 kg. You measure that it takes 0.0600 s for a transverse pulse to travel from the lower end to the upper end of the string. On earth, for the same string and lead weight, it takes 0.0390 s for a transverse pulse to travel the length of the string. The weight of the string is small enough that its effect on the tension in the string can be neglected. Assuming that the mass of the planet is distributed with spherical symmetry, what is its mass?

15.62 ••• **CP** A 5.00-m, 0.732-kg wire is used to support two uniform 235-N posts of equal length (Fig. P15.62). Assume that the wire is essentially horizontal and that the speed of sound is 344 m/s. A strong wind is blowing, causing the wire to vibrate in its 5th overtone. What are the frequency and wavelength of the sound this wire produces?

Figure **P15.62**

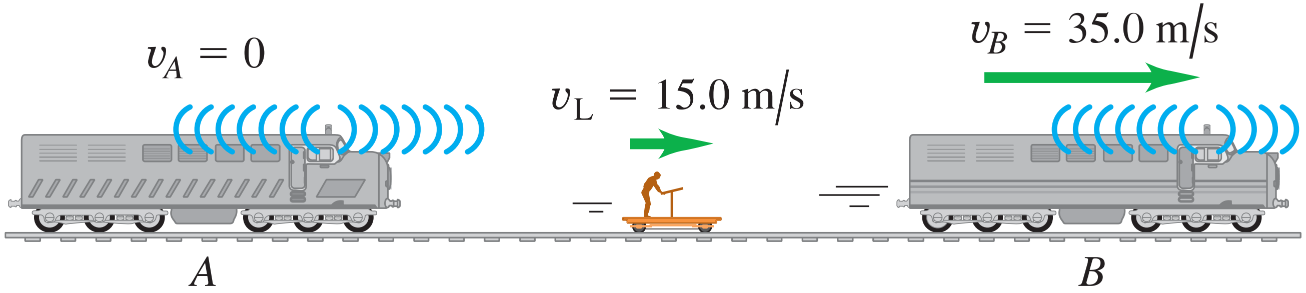


15.74 ••• **CALC** A guitar string is vibrating in its fundamental mode, with nodes at each end. The length of the segment of the string that is free to vibrate is 0.386 m. The maximum transverse acceleration of a point at the middle of the segment is $8.40 \times 10^3 \text{ m/s}^2$ and the maximum transverse velocity is 3.80 m/s. (a) What is the amplitude of this standing wave? (b) What is the wave speed for the transverse traveling waves on this string?

16.39 •• **Tuning a Violin.** A violinist is tuning her instrument to concert A (440 Hz). She plays the note while listening to an electronically generated tone of exactly that frequency and hears a beat of frequency 3 Hz, which increases to 4 Hz when she tightens her violin string slightly. (a) What was the frequency of the note played by her violin when she heard the 3-Hz beat? (b) To get her violin perfectly tuned to concert A, should she tighten or loosen her string from what it was when she heard the 3-Hz beat?

16.45 • Two train whistles, A and B, each have a frequency of 392 Hz. A is stationary and B is moving toward the right (away from A) at a speed of 35.0 m/s. A listener is between the two whistles and is moving toward the right with a speed of 15.0 m/s (Fig. E16.45). No wind is blowing. (a) What is the frequency from A as heard by the listener? (b) What is the frequency from B as heard by the listener? (c) What is the beat frequency detected by the listener?

Figure **E16.45**



16.56 • The shock-wave cone created by the space shuttle at one instant during its reentry into the atmosphere makes an angle of 58.0° with its direction of motion. The speed of sound at this altitude is $331\ \text{m/s}$. (a) What is the Mach number of the shuttle at this instant, and (b) how fast (in m/s and in mi/h) is it traveling relative to the atmosphere? (c) What would be its Mach number and the angle of its shock-wave cone if it flew at the same speed but at low altitude where the speed of sound is $344\ \text{m/s}$?

16.64 ••• **CP** **A New Musical Instrument.** You have designed a new musical instrument of very simple construction. Your design consists of a metal tube with length L and diameter $L/10$. You have stretched a string of mass per unit length μ across the open end of the tube. The other end of the tube is closed. To produce the musical effect you're looking for, you want the frequency of the third-harmonic standing wave on the string to be the same as the fundamental frequency for sound waves in the air column in the tube. The speed of sound waves in this air column is v_s. (a) What must be the tension of the string to produce the desired effect? (b) What happens to the sound produced by the instrument if the tension is changed to twice the value calculated in part (a)? (c) For the tension calculated in part (a), what other harmonics of the string, if any, are in resonance with standing waves in the air column?

16.82 •• On a clear day you see a jet plane flying overhead. From the apparent size of the plane, you determine that it is flying at a constant altitude h. You hear the sonic boom at time T after the plane passes directly overhead. Show that if the speed of sound v is the same at all altitudes, the speed of the plane is

$$v_S = \frac{hv}{\sqrt{h^2 - v^2T^2}}$$

(*Hint:* Trigonometric identities will be useful.)

Example 3.5. Let us discuss a sinusoidal wave moving in the $+x$ direction.

$$y_1(x, t) = A \sin \frac{2\pi}{T}\left(t - \frac{x}{v}\right).$$

A wall located at $x = L$ reflects this wave. Answer the following questions for each of the following two cases: (a) The wall is a fixed end. (b) The wall is a free end.

(1) Find the expression for the reflected wave.
(2) Find the expression for the resultant wave produced by the incident wave and the reflected wave.

Note that a fixed end is an end where the amplitude of the resultant wave of the incident and reflected waves vanishes at all times, whereas a free end is an end where the displacement of the reflected wave is equal to that of the incident wave.