

Hong Kong Physics Olympiad 2010
2010 年香港物理奧林匹克競賽

Written Examination
筆試

Jointly Organized by

The Hong Kong Academy for Gifted Education
香港資優教育學院

The Education Bureau of HKSAR
香港特區政府教育局

The Hong Kong Physical Society
香港物理學會

The Hong Kong University of Science and Technology
香港科技大學

共同舉辦

May 23, 2010
2010 年 5 月 23 日

Rules and Regulations 競賽規則

1. All questions are in bilingual versions. You can answer in either Chinese or English.
所有題目均為中英對照。你可選擇以中文或英文作答。
2. The multiple-choice answer sheet will be collected 1.5 hours after the start of the contest. You can start answering the open-ended questions any time after you have completed the multiple-choice questions without waiting for further announcement.
選擇題的答題紙將于比賽開始後一小時三十分收回。若你在這之前已完成了選擇題，你亦可開始作答開放式題目，而無須等候任何宣佈。
3. Please follow the instructions on the multiple-choice answer sheet, and use a HB pencil to write your 8-digit Participant ID number in the field of "I.D. No.", and fill out the appropriate circles **fully**. After that, write your English name in the space provided and your Hong Kong ID number in the field of "Course number & Section No."
請依照選擇題答題紙的指示，用HB鉛筆在選擇題答題紙的I.D. No.欄上首先寫上你的8位數字參賽號碼，並把相應寫有數字的圓圈**完全塗黑**，然後在適當的空格上填上你的英文姓名，最後於"Course & Section No."欄內填上你的身分證號碼。
4. After you have made the choice in answering a multiple choice question, fill the corresponding circle on the multiple-choice answer sheet **fully** using a HB pencil.
選定選擇題的答案後，請將選擇題答題紙上相應的圓圈用HB鉛筆**完全塗黑**。
5. On the cover of the answer book, please write your Hong Kong ID number in the field of "Course Title", and write your English name in the field of "Student Name" and your 8-digit Participant ID number in the field of "Student Number". You can write your answers on both sides of the sheets in the answer book.
在答題簿封面上，請於 Course Title 欄中填上你的身分證號碼；請於 Student Name 欄中填上你的英文姓名；請於 Student Number 欄中填上你的8位數字參賽號碼。答題簿可雙面使用。
6. The information provided in the text and in the figure of a question should be put to use together.
解題時要將文字和簡圖提供的條件一起考慮。
7. Some open problems are quite long. Read the entire problem before attempting to solve them. If you cannot solve the whole problem, try to solve some parts of it. You can even use the answers in some unsolved parts as inputs to solve the others parts of a problem.
開放題較長，最好將整題閱讀完後才著手解題。若某些部分不會做，也可把它們的答案當作已知來做其它部分。

The following symbols and constants are used throughout the examination paper unless otherwise specified:

g – gravitational acceleration on Earth surface, $9.8 \text{ (m/s}^2\text{)}$
 G – gravitational constant, $6.67 \times 10^{-11} \text{ (N m}^2\text{/kg}^2\text{)}$
 e – charge of an electron, $-1.6 \times 10^{-19} \text{ (A s)}$
 ϵ_0 – electrostatic constant, $8.85 \times 10^{-12} \text{ (A s)/(V m)}$
 m_e – electron mass = $9.11 \times 10^{-31} \text{ kg}$
 c – speed of light in vacuum, $3.0 \times 10^8 \text{ m/s}$
 Radius of Earth = 6378 km
 Sun-Earth distance (= 1 Astronomical Unit (AU)) = $1.5 \times 10^{11} \text{ m}$
 Mass of Jupiter = $1.9 \times 10^{27} \text{ kg}$
 Mass of the sun = $1.99 \times 10^{30} \text{ kg}$
 Air Density = 1.2 kg/m^3
 Water Density = $1.0 \times 10^3 \text{ kg/m}^3$
 Standard atmosphere pressure $p_0 = 1.0 \times 10^5 \text{ N/m}^2$

除非特別注明，否則本卷將使用下列符號和常數：

g – 地球表面重力加速度, $9.8 \text{ (m/s}^2\text{)}$
 G – 萬有引力常數, $6.67 \times 10^{-11} \text{ (N m}^2\text{/kg}^2\text{)}$
 e – 電子電荷, $-1.6 \times 10^{-19} \text{ (A s)}$
 ϵ_0 – 靜電常數, $8.85 \times 10^{-12} \text{ (A s)/(V m)}$
 m_e – 電子質量, $9.11 \times 10^{-31} \text{ kg}$
 c – 真空光速, $3.0 \times 10^8 \text{ m/s}$
 地球半徑 = 6378 km
 太陽-地球距離 (= 1 天文單位) = $1.5 \times 10^{11} \text{ m}$
 木星質量 = $1.9 \times 10^{27} \text{ kg}$
 太陽質量 = $1.99 \times 10^{30} \text{ kg}$
 空氣密度 = 1.2 kg/m^3
 水密度 = $1.0 \times 10^3 \text{ kg/m}^3$
 標準大氣壓 $p_0 = 1.0 \times 10^5 \text{ N/m}^2$

The following conditions will be applied to all questions unless otherwise specified:

- 1) All objects are near Earth surface and the gravity is pointing downwards.
- 2) Neglect air resistance.
- 3) All speeds are much smaller than the speed of light.

除非特別注明，否則下列條件將適用於本卷所有問題：

- 1) 所有物體都處于地球表面，重力向下；
- 2) 忽略空氣阻力；
- 3) 所有速度均遠小於光速。

Multiple Choice Questions

(2 points each. Select one answer in each question.)

選擇題（每道題 2 分，每道題選擇一個答案。）

The MC questions with the '*' sign may require information on page-3.

帶 * 的選擇題可能需要用到第三頁上的資料。

MC1*

An opened parachute of mass 1.0 kg is coming straight down from the sky. Attached to the parachute is the upper end of a light spring scale, while a block of mass 10 kg is attached to its lower end of the scale. The scale reading is 80 N. The air resistance at the moment is approximately _____.

一個張開的質量為 1.0 kg 且豎直下降的降落傘下，掛有一個顯示讀數為 80N 的無重彈簧秤，彈簧秤的低端懸挂著一個質量為 10 kg 的重物。此時空氣阻力大約為 _____。

- (a) 55 N (b) 66 N (c) 77 N (d) 88 N (e) 99 N

MC2*

The wind speed is 100 Km/h. Estimate the force of the wind on an adult human being standing facing the wind. Choose the answer below that is closest to your result.

在風速為 100 公里/小時的情況下，一個迎風正面站著的成年人所受的風力大約是多少？選擇以下最接近你的結果的答案。

- (a) 0 N (b) 4630 N (c) 1.9 N (d) 39 N (e) 463 N

MC3*

The first extra-solar system discovered in 1995 consists of a solar type star and a planet with mass similar to Jupiter in the solar system. The period of the circular orbit of the planet is 4.2 days. Find the distance between the star and its planet **in terms of astronomical unit (A. U.)**.

于 1995 年首次發現的太陽系以外的行星系統由一個類似太陽的恆星和一個質量與木星相若的行星組成。行星的圓形軌道的周期為 4.2 天。求恆星-行星之間的距離（以天文單位表達）。

- (a) 0.2 (b) 9.5 (c) 0.05 (d) 15 (e) 3.0

MC4*

Following MC3, find the orbital speed of the star around the center of mass of the star-planet system.

接上題，求該恆星繞恆星-行星系統的質心的運動速率。

- (a) 2 m/s (b) 84 m/s (c) 62 m/s (d) 126 m/s (e) 312 m/s

MC5*

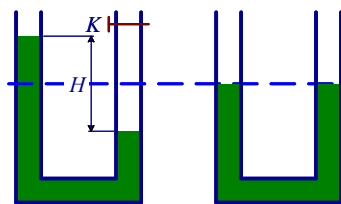
A small wooden sphere of density 0.8 g/cm^3 is attached to the end of a string of length 4.0 m in water. The other end of the string is attached to the bottom, forming a reverse pendulum. Ignore water friction. Find the period of the pendulum.

密度為 0.8 g/cm^3 的小木球系于長度為 4.0 m 的細繩的上端，細繩的下端系于水底，形成一個倒單擺。若不計阻力，求木球簡諧振動的周期。

- (a) 4 s (b) 2 s (c) 5 s (d) 1 s (e) 8 s

MC6

A U-tube of small and uniform cross section contains water of total length $4H$. The height difference between the water columns on the left and on the right is H when the valve K is closed. The valve is suddenly open, and water is flowing from left to right. Ignore friction. Find the speed of water when the heights of the left and the right water columns are the same.



橫截面均勻的 U 形管裝有總長為 $4H$ 的水，開始時閥門 K 閉合，左右支管內水面高度差為 H 。管內部橫截面積很小，摩擦阻力忽略不計。試求：突然打開閥門 K 後左右水面高度相同時，水的流動速率。

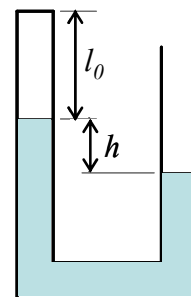
- (a) $\frac{1}{4}\sqrt{gH}$ (b) $\sqrt{\frac{gH}{8}}$ (c) $\frac{1}{2}\sqrt{gH}$ (d) $\sqrt{\frac{gH}{2}}$ (e) \sqrt{gH}

MC7

As shown, the left end of an upright U-shaped glass tube of uniform cross section is sealed and the right end is open. In the atmospheric pressure of $p_0 = 76 \text{ cmHg}$ and at $T_0 = 15^\circ\text{C}$, the length of the gas column sealed by the mercury is $l_0 = 8 \text{ cm}$ and the height difference between the left and the right mercury surfaces is $h = 4 \text{ cm}$.

When the temperature is raised to T_1 , the length of the gas column becomes $l_1 = 9 \text{ cm}$. Find T_1 .

如圖所示，粗細均勻豎直放置的 U 形玻璃管左上端閉封，右上端開口，在標準大氣壓 $p_0 = 76 \text{ cmHg}$ 下，當溫度 $T_0 = 15^\circ\text{C}$ 時，管內用水銀封閉著長 $l_0 = 8 \text{ cm}$ 的氣柱，這時兩管水銀面高度差 $h = 4 \text{ cm}$ 。當溫度升高至 T_1 時被封閉的氣柱長為 $l_1 = 9 \text{ cm}$ ，則 $T_1 = \underline{\hspace{2cm}}$ 。



- (a) 40°C (b) 60°C (c) 70°C (d) 50°C (e) 80°C

MC8

Following MC7, keep the temperature at T_1 , fill in mercury in the amount of length x from the open end such that the gas column length returns to $l_0 = 8 \text{ cm}$. Find x .

上題中若保持溫度 T_1 不變，再從開口端注入高度為 x 的水銀使左上端氣柱長變回 $l_0 = 8 \text{ cm}$ ，則 $x = \underline{\hspace{2cm}}$ 。

- (a) 11.25 cm (b) 13.25 cm (c) 14.35 cm (d) 8.25 cm (e) 7.25 cm

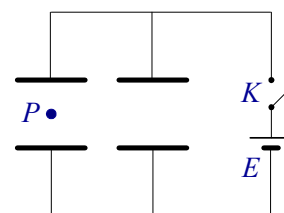
MC9

As shown, two parallel plate capacitors are placed horizontally and connected to a battery E . After charging the capacitors, the switch K is open. A charged particle is placed in the left capacitor and is at rest. If one reduces the distance between the plates of the right capacitor, the particle will then _____.

- (a) move horizontally (b) move upwards (c) move downwards
(d) move in circle (e) remain at rest

兩個水平放置的平行板電容器連接如圖，電池 E 對它們充電，然後將電鍵 K 斷開。在左邊電容器的兩板間放一帶電微粒 P ，它處於靜止平衡狀態。若將右邊電容器的兩極板間距離減小，微粒 P 將_____。

- (a) 做水平直線運動 (b) 做向上直綫運動
(c) 做向下直綫運動 (d) 做圓周運動 (e) 靜止不動



MC10

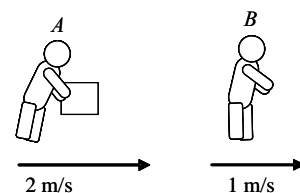
A $30 \mu\text{F}$ capacitor charged to 2.0 kV and a $60 \mu\text{F}$ capacitor charged to 1.0 kV are connected to each other, with the positive plate to the positive plate, and the negative plate to the negative plate. What is the final charge on the $30 \mu\text{F}$ capacitor?

一個 $30 \mu\text{F}$ 的電容充電至 2.0 kV 的電壓，一個 $60 \mu\text{F}$ 的電容充電至 1.0 kV 的電壓。現將兩電容的正極與正極相連，負極與負極相連，問在 $30 \mu\text{F}$ 的電容上有多少電荷？

- (a) 50 mC (b) 200 mC (c) 40 mC (d) 60 mC (e) 0 mC

MC11

Two astronauts, A and B , both with mass of 60 kg , are moving along a straight line in the same direction in a “weightless” spaceship. Relative to the spaceship the speed of A is 2 m/s and that of B is 1 m/s . A is carrying a bag of mass 5 kg with him. To avoid collision with B , A throws the bag with a speed v relative to the spaceship towards B and B catches it. Find the minimum value of v .



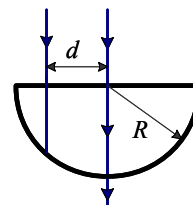
兩個宇航員 A 、 B ，每個的質量為 60 kg ，在一‘無重’的飛船

裏沿同一直線向同一方向飄移。 A 相對於飛船的速率為 2 m/s ， B 相對於飛船的速率為 1 m/s 。為避免相撞， A 將質量為 5 kg 的背包以相對於飛船 v 的速度扔向 B 。 B 接住背包。求 v 的最小值。

- (a) 7.8 m/s (b) 26.0 m/s (c) 14.0 m/s (d) 9.2 m/s (e) 0

MC12

As shown, a narrow beam of light is incident onto a semi-circular glass cylinder of radius R . Light can exit the cylinder when the beam is at the center. When the beam is moved parallelly to a distance d from the central line, no light can exit the cylinder from its lower surface. Find the refractive index of the glass.



如圖所示，一束細光線從中間射入半徑為 R 的半圓柱形玻璃磚，可以

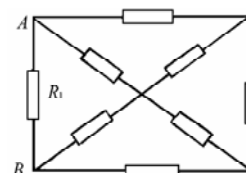
觀察到有光綫從玻璃磚射出。現把入射光綫平行地移動，當入射光距離中心綫為 d 時，從玻璃磚下表面射出的光綫剛好消失。求玻璃的折射率。

- (a) $\frac{R}{d}$ (b) $\frac{d}{R}$ (c) $\frac{R}{\sqrt{R^2 - d^2}}$ (d) $\frac{\sqrt{R^2 - d^2}}{R}$ (e) $\frac{R^2}{R^2 - d^2}$

MC13

As shown, the circuit is made of 8 different resistors. It is found that when $R_1 = 4 \Omega$, the resistance between A and B is 2Ω . Now replace R_1 by a 6Ω resistor, what is the resistance between A and B ?

如圖所示電路由 8 個不同的電阻組成，已知 $R_1 = 4 \Omega$ 時，測得 A 、 B 間的總電阻為 2Ω 。現將 R_1 換成 6Ω 的電阻，則 A 、 B 間的總電阻是多少 Ω ？



- (a) 1 (b) 2 (c) 6 (d) 2.4 (e) 1.5

MC14*

To reach the fire at the height of 30 meters , what should be the initial speed of water jet on the ground?

為將水射到 30 米 高處的火場，在地面的水的初速度須為多少？

- (a) 30 m/s (b) 12 m/s (c) 300 m/s (d) 5 m/s (e) 24 m/s

MC15*

Following MC14, the area of the cross section of the water hose is 10 cm^2 . What is the recoil force?

接上題，消防水槍口的截面積為 10 cm^2 ，求水的反衝力。

- (a) 588 N (b) 60 N (c) 705 N (d) 24 N (e) 5 N

MC16

Three point charges, each with charge Q , are placed on the apexes of an equilateral triangle of length R . The potential energy of the system is then $\frac{K}{4\pi\epsilon_0} \frac{Q^2}{R}$, where K is _____.

三個點電荷，每個帶電 Q ，分別放在一個邊長為 R 的等邊三角形的三個頂點上。系統的勢能可表達為 $\frac{K}{4\pi\epsilon_0} \frac{Q^2}{R}$ 。求 K 。

- (a) 2 (b) 3 (c) 6 (d) 1 (e) 12

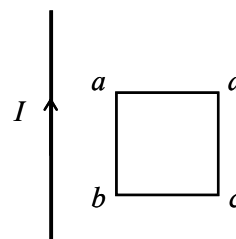
MC17

As shown, the electric current in a straight wire is increasing with time.

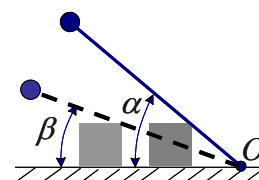
The direction of the electric current in the square wire loop is _____.

豎直電綫裏的電流 I 隨時間增加。電綫邊上的四方綫圈裏的電流的方向是_____。

- (a) $abcd$ (b) $adcb$ (c) $adbc$ (d) $acbd$ (e) $bcad$

**MC18**

A cubic block of mass m and side length b is placed on a smooth floor. A smooth and rigid rod of length L and with negligible mass is leaning against the block. A sphere of mass M is attached to the upper end of the rod. The lower end of the rod is fixed at point O , but the rod can rotate freely around the point within the vertical plane. Initially the angle between the rod and the floor is α while the system is at rest. After releasing, find the speed of the block when the angle between the rod and the floor is β .



在光滑水平面上放置一個質量為 m 、邊長為 b 的正方體滑塊。滑塊上擱有一長為 L 的無重光滑直杆，杆的一端用光滑鉸鏈連結于地面上 O 點，使桿可繞 O 點在豎直平面內自由轉動；另一端鑲著一個質量為 M 的金屬球。開始時杆和滑塊均靜止，杆和水平面夾角為 α 。試求：放開杆後當杆轉動到杆和水平面夾角變為 β 時滑塊的速度。

- (a) $b \sqrt{\frac{MgL(\sin \alpha - \sin \beta)}{mb^2 + ML^2 \sin^4 \beta}}$ (b) $b \sqrt{\frac{2MgL(\sin \alpha - \sin \beta)}{mb^2 + ML^2 \sin^4 \beta}}$
- (c) $b \sqrt{\frac{2MgL(\sin \alpha - \sin \beta)}{mb^2 + ML^2 \cos^4 \beta}}$ (d) $b \sqrt{\frac{MgL(\sin \alpha - \sin \beta)}{mb^2 + ML^2 \cos^4 \beta}}$ (e) 0

MC19

In a traffic accident, a taxi runs into a bus initially at rest. The two vehicles are locked together after impact and slide for a distance of 4 m. It is known that the frictional force between the locked vehicles and the ground is 25000 N. Find the speed of the taxi just before impact. Mass of the taxi = 1500 kg. Mass of the bus = 6000 kg.

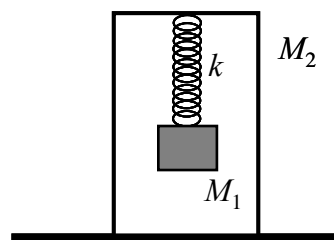
在一起交通事故中，一輛質量為 1500 kg 的士撞上一輛停著的巴士。巴士的質量為 6000 公斤。相撞後兩車貼在一起向前沖了 4 m。已知兩車與地面的磨擦力為 25000N。求的士在撞車前的速度。

- (a) 23.1 km/h (b) 46.2 km/h (c) 69.5 km/h (d) 93 km/h (e) 0

MC20

A block of mass M_1 is hanged by a light spring of force constant k to the top bar of a reverse U-frame of mass M_2 on the floor. The block is pulled down from its equilibrium position by a distance x and then released. Find the minimum value of x such that the reverse U-frame will leave the floor momentarily.

一質量為 M_1 的物塊吊在一無重彈簧的下端。彈簧的彈力係數為 k ，上端系在質量為 M_2 的倒 U-架的橫梁上。倒 U-架放在地面上。現將物塊從它的平衡位置向下拉一距離 x ，然後釋放，求能使倒 U-架瞬時離開地面的 x 值。



- (a) $x = (M_1 + M_2)g / k$ (b) $x = (2M_1 + M_2)g / k$
 (c) $x = (M_1 + 2M_2)g / k$ (d) $x = M_1g / k$ (e) $x = M_2g / k$

《End of MC's 選擇題完》

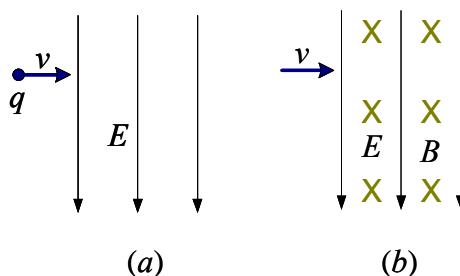
Open Problems 開放題

Total 5 problems 共 5 題

The Open Problem(s) with the ‘*’ sign may require information on page-3.
帶 * 的開放題可能需要用到第三頁上的資料。

Q1 (5 points)

As shown in Fig. (a), a charged particle enters a region of uniform electric field E with initial horizontal velocity v . When it exits the region, its speed is v_a . In the second case, the same particle with the same initial velocity enters the region where a magnetic field B ($v < E/B$) is added perpendicularly to the original electric field E , as shown in Fig. (b). When it exits the region, its speed is v_b . Compare v_a with v_b , which one is larger? Give brief explanation in no more than 5 lines.

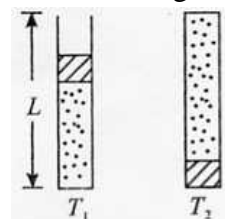


題 1 (5 分)

如(a)圖所示，一個帶電粒子以水平速度 v 進入勻強電場區域。當粒子離開電場區時，粒子的速率為 v_a 。若加磁場 B 在同一區域，($v < E/B$)，電場和磁場正交重疊，如(b)圖所示，粒子仍以初速度 v 進入場區。當粒子離開場區時，粒子的速率為 v_b 。比較 v_a 和 v_b ，哪個大？（需簡單給出理由。答案最好不要超過 5 行。）

Q2 (15 points)

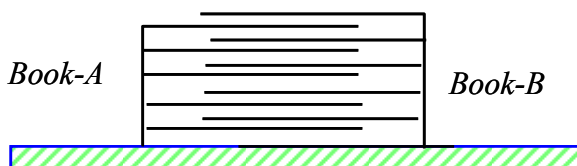
Air is filled in a vertical cylinder of length L . The lower end of the cylinder is sealed and the upper end is open. The air pressure outside is p_0 . A piston, which can move freely up and down the cylinder without gas leakage, is placed from the open end and sits down on the gas column. At equilibrium the length from the upper end of the cylinder to the outer surface of the piston is $L/4$. The temperature of the air inside is kept at T_1 in the process. Now, lower the temperature of the air inside to $T_2 = T_1/2$, and keep the temperature constant while turning the cylinder upside down. At equilibrium the outer surface of the piston is right at the open end edge of the cylinder. Find the thickness and the mass density of the piston.



題 2 (15 分) 一個豎直放置的、長度為 L 的圓筒下端封閉，上端與壓強為 p_0 的大氣相通。初始時筒內氣體溫度為 T_1 。現將一個可沿筒壁自由滑動的、厚度為 d 的活塞從上端放進圓筒，活塞下滑過程中氣體溫度保持不變且沒有氣體漏出，平衡後活塞外表面比圓筒上端低 $L/4$ 。將圓筒下部氣體溫度降至 $T_2 = T_1/2$ ，在保持溫度不變的條件下將筒倒置，平衡後活塞下端與圓筒下端剛好平齊。求活塞厚度 d 和活塞質量密度 ρ 。

Q3 (15 points)

As shown in the figure, the pages of two identical books A and B are overlapping on one another. The mass of each book is 1000 g and the number of pages of each book is 200. The friction coefficient between the pages is $\mu = 0.3$. Book- A is fixed on the table. A horizontal force F is applied to book- B . Determine the minimum value of F to pull book- B out.

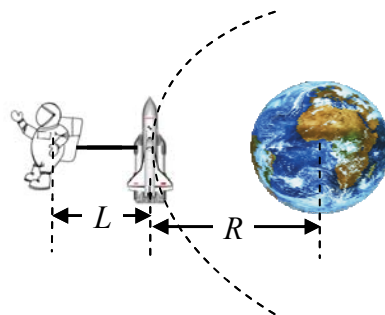


題 3 (15 分)

如圖，兩本相同的書，每本 200 頁，重 1000 g，每頁互相交叉疊在一起，頁面間的摩擦係數為 $\mu = 0.3$ 。書-A 固定在桌面上。求能將書-B 拉出的最小水平力 F 。

Q4* (15 points)

As shown in the figure, an astronaut of total mass $m = 110$ kg was doing a spacewalk when his jetpack failed. His only connection to the spaceship was the communication cable wire of length $L = 100$ m. Assume the radius of the orbit R is the Earth's radius. Also assume that the astronaut and the spaceship underwent circular motion with the common angular velocity of ω and remained on a straight line projecting from the Earth's center. The mass of the spaceship is much larger than that of the astronaut. Find the tension in the cable.



[Hint: For $R \gg L$, $(R+L)^3 - R^3 \approx 3R^2L$]

題 4* (15 分)

如圖，一質量為 $m = 110$ kg 的宇航員在空間漫步時，推進系統突然失靈，只剩下一條長為 $L = 100$ m 的通訊電纜與飛船連接。設飛船軌道半徑約等于地球的半徑，宇航員和飛船以同樣的角速度 ω 作圓周運動，宇航員、飛船、地球中心始終保持在同一直線上，飛船質量比宇航員大很多，求通訊電纜的張力。

[提示：當 $R \gg L$ 時， $(R+L)^3 - R^3 \approx 3R^2L$]

Q5 (10 points)

A photon of angular frequency ω carries energy $h\omega/2\pi$ and momentum $h\omega/2\pi c$, where h is the Planck Constant, and c is the speed of light in vacuum. When a photon meets an atom, it may be absorbed by the atom. The probability P for an atom to absorb a photon depends on

the photon frequency *in the rest frame* of the atom ω_a , namely $P(\omega_a) = \frac{A}{(\omega_a - \omega_0)^2 + \gamma^2}$,

where A , γ , and ω_0 are constants. Shortly after absorbing the photon, the atom re-emits a photon of the same frequency in an arbitrary direction. Within a very short time the atom absorbs many photons from a laser beam in one direction and re-emits photons in all directions. Therefore, on average over time, the atom experiences a force which can slow down its speed. This is the principle of cooling atoms with laser.

- An atom is moving in the positive x direction with speed v ($\ll c$), and a photon with frequency ω is moving in the negative x direction. What is the frequency of the photon ω_a in the rest frame of the atom? (2 points)
- Suppose the atom is moving in the positive x direction with velocity v , and two identical laser beams shine along the positive x direction and the negative x direction, respectively. There are n photons from each beam colliding with the atom per unit time. Find the expression for the average force F acting on the atom. If the expression contains ω_a , you should replace it with the answer in (a). (6 points)
- For small v , we have $|\omega - \omega_a| \ll |\omega - \omega_0|$, the force in (b) can then be expressed as $F = -\beta v$. Find the expression for β . (2 points)

(Hint: For small $\delta\omega$, $\frac{A}{(\omega + \delta\omega)^2 + \gamma^2} \approx \frac{A}{\omega^2 + \gamma^2} - \frac{2\omega A \delta\omega}{(\omega^2 + \gamma^2)^2}$.)

題 5 的中文版在下頁，敬請留意。

題 5 (10 分)

一角頻率為 ω 的光子具有能量 $h\omega/2\pi$ 、動量 $h\omega/2\pi c$ ，其中 h 為普郎克常數， c 是真空中光速。當光子遇上原子時，光子可能被原子吸收。光子被原子吸收的機率（或然率, Probability）與在原子是靜止的參照系裏的光子的角頻率 ω_a 有關，其表達式為

$$P(\omega_a) = \frac{A}{(\omega_a - \omega_0)^2 + \gamma^2},$$

其中 A 、 γ 、 ω_0 為常數。原子在吸收了一個光子之後，很快就

朝一任意方向發出一個頻率相同的光子。在一短時間內，該原子可以吸收很多從同一方向來的雷射光子，同時向各個方向不斷發射光子。因此，經時間平均，原子受到一個力，使它的運動減慢，這就是用雷射冷卻原子的工作原理。

- (a) 一原子以速度 v ($\ll c$) 沿正 x 方向運動，一頻率為 ω 的光子沿反方向運動，求在原子是靜止的參照系裏光子的頻率 ω_a 。(2 分)
- (b) 設一原子仍以速度 v 沿正 x 方向運動，兩束相同的雷射分別沿正 x 方向和反 x 方向照射，每束雷射每單位時間有 n 個光子與原子相遇。求該原子受到的平均力 F 。若你的表達式裏含有 ω_a ，你必須用 (a) 的答案將它代替。(6 分)
- (c) 若 v 很小，則 $|\omega - \omega_a| \ll |\omega - \omega_0|$ ，(b) 中的力可表達成 $F = -\beta v$ 。求 β 。(2 分)

(提示：當 $\delta\omega$ 很小時， $\frac{A}{(\omega + \delta\omega)^2 + \gamma^2} \approx \frac{A}{\omega^2 + \gamma^2} - \frac{2\omega A \delta\omega}{(\omega^2 + \gamma^2)^2}$ 。)

《END 完》