

MCQ's

1(d), 2(e), 3(c), 4(d), 5(e), 6(b), 7(b), 8(a), 9(c), 10(c), 11(a), 12(a), 13(d), 14(e), 15(a), 16(b), 17(a), 18(b), 19(d), 20(a)

Open Q's

**Q1 (5 points)**

The work done by the electric field is proportional to the vertical displacement of the particle in the E-field. In case-B the magnetic field force has an upwards component so the vertical displacement of the particle is less than in case-A. Then, follow the energy conservation, we have  $v_a > v_b$ .

**Q2. (15 points)**

Solution:

$$PV = NRT, \quad (2 \text{ points})$$

$$NRT_1 = P_0LS \quad (2 \text{ points})$$

$$(P_0 + d\rho)\left(\frac{3}{4}L - d\right)S = P_0LS \quad (3 \text{ points})$$

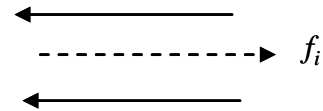
$$(P_0 - d\rho)(L - d)S = P_0LS/2 \quad (3 \text{ points})$$

Solving the two equations,  $\rho = 4\sqrt{3}P_0/L$ , and  $d = \frac{1}{4}(2 - \sqrt{3})L$  (5 points)

**Q3. (15 points)**

Solution:

As shown in figure below, the  $i^{\text{th}}$  page of book B subjects two frictional forces,  $f_i'$  (the page above  $i^{\text{th}}$  page) and  $f_i''$  (the page below  $i^{\text{th}}$  page)



The force required to pull out the  $i^{\text{th}}$  page from book A is given by  $f_i = f_i' + f_i''$  (3 points)

Since  $f_i' = 2(i-1)\mu mg$  and  $f_i'' = (2i-1)\mu mg$ , (3 points for each equation)

thus we have

$$f_i = (4i - 3)\mu mg. \dots (3 \text{ points})$$

From equation 1, when  $i = 1$ ,  $f_1 = 1\mu mg$ ; when  $i = 200$ ,  $f_{200} = 797 \mu mg$

Summing up all the forces,  $F = f_1 + f_2 + \dots + f_{200} = \frac{f_1 + f_{200}}{2} \times 200 = 79800\mu mg$

$$\therefore F = 79800 \times 0.3 \times 0.005 \times 9.8 = 1173N \quad (3 \text{ points})$$

**Q4. (15 points)**

Solution:

The spaceship moves under the influence of the Earth's gravity, given by

$$F = G \frac{MM_E}{R_1^2} \quad (2 \text{ points})$$

The net force acting on the spaceship is  $G \frac{MM_E}{R_1^2} - T = MR_1 \omega^2$  ....(1) (2 points)

where  $T$  is the tension of the communication cable. Similarly, for the astronaut,  
 $G \frac{mM_E}{R_2^2} + T = mR_2 \omega^2$  .....(2) (2 points)

Equation  $\omega$  from (1) and (2), we obtain

$$\frac{1}{MR_1} (G \frac{MM_E}{R_1^2} - T) = \frac{1}{mR_2} (G \frac{mM_E}{R_2^2} + T) \quad \dots(3) \quad (3 \text{ points})$$

From equation (3), we can easily find the tension  $T$ :

$$T = G \left( \frac{mM}{MR_1 + mR_2} \right) \left( \frac{R_2^3 - R_1^3}{R_1^2 R_2^2} \right) M_E \quad (2 \text{ points})$$

Using  $R_1 \approx R_2 \approx R$  ;  $R_2^3 - R_1^3 = (R_2 - R_1)(R_2^2 + R_1 R_2 + R_1^2) \approx 3R^2 L$

We can now rewrite  $T$  in the following form:

$$T = 3 \left( \frac{mM}{m+M} \right) \cdot L \cdot \frac{GM_E}{R^3} = 3 \left( \frac{L}{R} \right) \left( \frac{mM}{m+M} \right) g ; \text{ where } g = \frac{GM_E}{R^2} \quad (2 \text{ points})$$

Since  $M \gg m$ , we can write an even simpler formula as an estimate:

$$T = 3 \frac{mLg}{R} = 3 \frac{(110)(9.8)(100)}{6400 \times 10^3} \approx 0.05N \quad (2 \text{ points})$$

## 5. (10 points)

Solution:

$$a) \omega_\alpha = \frac{\sqrt{1 - v^2/c^2}}{1 + \frac{v \cos \theta}{c}} \omega \approx \left( 1 - \frac{v \cos \theta}{c} \right) \omega$$

For  $\theta = 0$ ,  $\omega_\alpha = (1 - v/c)\omega$ ,

For  $\theta = \pi$ ,  $\omega_\alpha = (1 + v/c)\omega$ . (2 points)

$$b) F = \Delta N p / t, \quad (2 \text{ points})$$

$$\frac{\Delta N}{t} = nA \left( \frac{1}{(\omega_\alpha^{(1)} - \omega_0)^2 + \gamma^2} - \frac{1}{(\omega_\alpha^{(2)} - \omega_0)^2 + \gamma^2} \right), (2 \text{ points})$$

$$F = nA \left( \frac{1}{(\omega_\alpha^{(1)} - \omega_0)^2 + \gamma^2} - \frac{1}{(\omega_\alpha^{(2)} - \omega_0)^2 + \gamma^2} \right) \frac{h\omega}{2\pi c}. (2 \text{ points})$$

$$c) F = nA \frac{h\omega}{\pi c^2} \frac{2\omega(\omega - \omega_0)}{(\gamma^2 + (\omega - \omega_0)^2)^2} v, \text{ hence, } \beta = nA \frac{h\omega}{\pi c^2} \frac{2\omega(\omega - \omega_0)}{(\gamma^2 + (\omega - \omega_0)^2)^2}. (2 \text{ points})$$