

Hong Kong Physics Olympiad 2011 (Senior Level)

2011 香港物理奧林匹克(高級組)

Suggested Solutions 答案及建議題解

Multiple Choice Questions

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d	b	b	e	c	a	b	a	e	c	a	d	a	e	a	a	a	d	e	b

MC1: (d)

$$\text{MC2: } F = -kx = (3m - \rho V)g, \quad k = -\frac{dF}{dx} = \rho g \frac{dV}{dx} = \rho g \pi R^2.$$

$$\omega = \sqrt{k/M} = \sqrt{\rho g \pi R^2 / (3m)}. \text{ (b)}$$

$$\text{It can be further simplified to } \rho \frac{2}{3} \pi R^3 = 3m \Rightarrow \omega = \sqrt{\frac{3g}{2R}}.$$

MC3: Using Archimedes' principle, volume of water displaced is $V_{\text{dis}} = (m_{\text{stone}} + \rho_{\text{ice}} V_{\text{ice}}) / \rho_{\text{water}}$.Suppose a volume of ΔV_{ice} melts, then the volume of water displaced decreases by $\rho_{\text{ice}} \Delta V_{\text{ice}} / \rho_{\text{water}}$. At the same time, since mass is conserved, the volume of water increases by $\rho_{\text{ice}} \Delta V_{\text{ice}} / \rho_{\text{water}}$.Hence there is no change in the water level. However, when all ice melts and the stone sinks to the bottom, the volume of water displaced is $m_{\text{stone}} / \rho_{\text{stone}} < m_{\text{stone}} / \rho_{\text{water}}$. Hence the water level falls. **Answer: (b)**MC4: Using Newton's law of universal gravitation and Newton's second law for circular orbits, $M \propto R^3 / T^2$. Comparing with Earth's orbit, where $M = 1$ solar mass, $R = 1$ AU and $T = 1$ year,

$$R = \sqrt[3]{1.5 \left(\frac{98}{365} \right)^2} = 0.48 \text{ AU}. \text{ Answer: (e)}$$

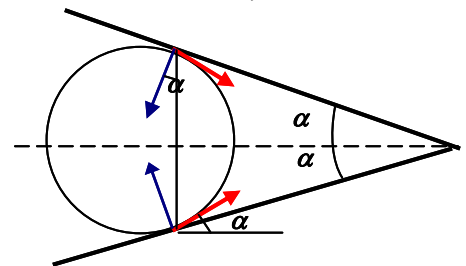
MC5: Denote the friction force and normal force acting on each blade as F_f and N , respectively. At the critical angle the static friction force should reach its maximal value which is $F = \mu N$.

The condition for the wire not to move is when all components of the forces adding up equal to zero. As shown in the force diagram acting on the wire, the vertical components cancel each other by symmetry, and the horizontal components cancel if

$$2F_f \cos \alpha = 2N \frac{\sin 2\alpha}{2}.$$

Thus, from the condition of the critical angle we obtain:

$$\mu = \tan \alpha. \text{ Answer: (c)}$$



MC6: (a)

MC7: Let v be the velocity of the particle of mass m . Kinetic energy of the compound pendulum:

$$K = \frac{1}{2} m v^2 + \frac{1}{2} (2m) \left(\frac{v}{2} \right)^2 = \frac{3}{4} m v^2$$

Potential energy of the compound pendulum:

$$U = -mgL \cos \theta - 2mg \frac{L}{2} \cos \theta = -2mgL \cos \theta$$

For small oscillations, $U \approx -2mgL\left(1 - \frac{\theta^2}{2}\right) = \text{constant} + mgL\theta^2$

For small oscillations, we can approximate the linear displacement by $x = L\theta$.

Hence $U = \frac{mg}{L}x^2$

Using the result of MC12, we can substitute $a = \frac{3}{2}m$, $b = \frac{2mg}{L}$ and $f = \frac{1}{2\pi}\sqrt{\frac{2mg/L}{3m/2}} = \frac{1}{2\pi}\sqrt{\frac{4g}{3L}}$.

Answer: (b)

MC8: Using Newton's second law, we find that the acceleration of the upward displacement is

$-g(\sin\alpha + \mu\cos\alpha)$. Hence $t_1 = \frac{v_1}{g(\sin\alpha + \mu\cos\alpha)}$ and $L = \frac{v_1^2}{2g(\sin\alpha + \mu\cos\alpha)}$.

The acceleration of the downward displacement is $g(\sin\alpha - \mu\cos\alpha)$ (in the downward

direction). Hence the time is given by $L = \frac{1}{2}g(\sin\alpha - \mu\cos\alpha)t_2^2$. Hence

$$t_2^2 = \frac{2L}{g(\sin\alpha - \mu\cos\alpha)} = \frac{v_1^2}{g^2(\sin\alpha - \mu\cos\alpha)(\sin\alpha + \mu\cos\alpha)} = t_1^2 \frac{\sin\alpha + \mu\cos\alpha}{\sin\alpha - \mu\cos\alpha}$$

Answer: (a)

MC9: **Sol.** $mv_0 = (m+M)v$, $v = \frac{m}{m+M}v_0 = 2\text{m/s}$,

$$\Delta E_k = \frac{1}{2}mv_0^2 - \frac{1}{2}(m+M)v^2 = \frac{1}{2} \times 1 \times 10^2 - \frac{1}{2} \times (1+4) \times 2^2 = 40\text{ J}$$

Answer: (e)

MC10: **Sol.** 已知 $m_1=1\text{kg}$, $m_2=3\text{kg}$, $F=6\text{N}$, $v_0=2\text{m/s}$, $d=1\text{m}$.

當兩物體速度相等時, 距離最小, 此時的速度為 v . 由動量守恆定律

$$m_2v_0 = (m_1 + m_2)v, v = \frac{m_2}{m_1 + m_2}v_0 = \frac{3}{1+3} \times 2 = 1.5\text{m/s}.$$

或距離最小時所用時間為 T , $v_1 = a_1T = v_0 + a_2T$, $6T = 2 - 2T$, $T = 0.25\text{s}$, $v = 1.5\text{m/s}$.

$$\Delta E_k = \frac{1}{2}m_2v_0^2 - \frac{1}{2}(m_1 + m_2)v^2 = \frac{1}{2} \times 3 \times 2^2 - \frac{1}{2} \times (1+3) \times 1.5^2 = 1.5\text{J} = F\Delta s, \Delta s = 0.25\text{m}.$$

兩物體間的最小距離 = $d - \Delta s = 0.75\text{m}$. **Answer: (c)**

$$\text{MC11: } \alpha = \tan^{-1}\left(\frac{2}{3}\tan\beta\right), \text{ (a)}$$

MC12: $a > d > c > b$, (d)

$$\text{MC13: } V(\rho_0 - \rho) = 100\text{kg} \Rightarrow \rho = 1.2 - 0.1 = 1.1\text{ kg/m}^3.$$

$$T = T_0\rho_0 / \rho = 293 \cdot 1.2 / 1.1 = 319.6\text{K} = 46.6^\circ\text{C} \quad \text{(a)}$$

MC14: Total reflection (e)

$$\text{MC15: } E = \sqrt{15}mg/q, \text{ (a)}$$

$$\text{MC16: } v = \frac{r^2 Bq}{2mR}, \text{ (a)}$$

MC17: 0, (a)

MC18: (d)

$$\text{MC19: } v = mgR / (B^2 L^2 N), \text{ (e)}$$

MC20: By energy conservation, $Q_1 = Q_2 = mgH$. (b)

Open Questions

Q 1 (10 points):

(a) For parallel connection, the total current I is the sum of the currents I_1 and I_2 through the two resistors, and their voltage U is the same. So $R_p = \frac{U}{I_1 + I_2} = U / \left(\frac{U}{R_1} + \frac{U}{R_2} \right) = \frac{R_1 R_2}{R_1 + R_2}$, (1.5 points)

For serial connection, the current I through the resistors is the same, and the total voltage U across the two resistors is sum of the voltages across each resistor. So

$$R_s = (U_1 + U_2) / I = R_1 + R_2; (1.5 \text{ points})$$

(b) For parallel connection, the total charge Q is the sum of the charge Q_1 and Q_2 on the two capacitors, and the voltage U is the same for both capacitors. So $C_p = (Q_1 + Q_2) / U = C_1 + C_2$, (1.5 points)

For serial connection, the charge Q on each capacitor is the same, because in the section of circuit between the two capacitors the total charge should be zero. The total voltage U is the sum of voltage across each capacitor. So $C_s = \frac{Q}{U_1 + U_2} = Q / \left(\frac{Q}{C_1} + \frac{Q}{C_2} \right) = \frac{C_1 C_2}{C_1 + C_2}$. (1.5 points)

(c) As the chain is infinitely long, it does not matter if one section is taken away.

$$\text{Therefore } \frac{1}{C^T} = \frac{1}{2C + C^T} + \frac{1}{C}, (3 \text{ points})$$

$$\text{Solving the equation one gets } C^T = (\sqrt{3} - 1)C. (1 \text{ point})$$

Q 2 (14 points):

(a)

Note that for the whole system the net force in the horizontal direction is zero. The impulse will give a total momentum to the system and its center of mass will move at constant speed $\frac{m_1 v_0}{m_1 + m_2}$, while the block and the ball will move back and forth relative to the center of mass.

Two ways to determine the vibration frequency are given below.

Method 1: (4 points)

Take the following coordinate system.

x_1 is the coordinate of the ball relative to the block. So $x_1 = L\theta$.

x_2 is the coordinate of the block relative to the floor.

$$\begin{cases} -m_1 g \theta = m_1 (\ddot{x}_1 + \ddot{x}_2), & (\text{Newton's equation for the ball}) \\ m_1 (\ddot{x}_1 + \ddot{x}_2) + m_2 \ddot{x}_2 = 0, & (\text{zero acceleration for center of mass.}) \end{cases}'$$

$$\text{Solving the two equations, } \ddot{x}_2 = -\frac{m_1}{m_1 + m_2} \ddot{x}_1 = -\frac{m_1}{m_1 + m_2} L \ddot{\theta}$$

$$-g\theta = \left(1 - \frac{m_1}{m_1 + m_2} \right) \ddot{x}_1 = \frac{m_2}{m_1 + m_2} L \ddot{\theta} \Rightarrow \omega = \sqrt{\frac{g(m_1 + m_2)}{m_2 L}}.$$

Method 2: (4 points)

Both coordinates are relative to the floor.

$$\begin{cases} x_1 - x_2 = L\theta \\ m_1 x_1 + m_2 x_2 = 0 \end{cases} \Rightarrow \begin{cases} x_1 = \frac{m_2 L \theta}{m_1 + m_2} \\ x_2 = -\frac{m_1 L \theta}{m_1 + m_2} \end{cases},$$

$$E_k = \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 \dot{x}_2^2 = \frac{m_1 m_2 L^2 \dot{\theta}^2}{2(m_1 + m_2)},$$

$$U = \frac{1}{2} m_1 g L \theta^2.$$

$$\text{Thus } \omega = \sqrt{\frac{g(m_1 + m_2)}{m_2 l}}.$$

We now proceed to get the equations of motion of the ball and the block.

The general solution is $\theta(t) = A \sin(\omega t) + B \cos(\omega t)$.

Consider the initial conditions $\theta(0) = 0$, $\dot{\theta}(0) = v_0 / L$, thus

$$\theta(t) = \frac{v_0}{\omega L} \sin(\omega t) = v_0 \sqrt{\frac{m_2}{Lg(m_1 + m_2)}} \sin(\omega t), \text{ and}$$

$$x_1 = \frac{m_2 v_0}{m_1 + m_2} \sqrt{\frac{Lm_2}{g(m_1 + m_2)}} \sin(\omega t),$$

or, in the reference system of the floor,

$$\begin{cases} x_1 = \frac{m_2 v_0}{m_1 + m_2} \sqrt{\frac{Lm_2}{g(m_1 + m_2)}} \sin(\omega t) + \frac{m_1 v_0}{m_1 + m_2} t \\ x_2 = -\frac{m_1 v_0}{m_1 + m_2} \sqrt{\frac{Lm_2}{g(m_1 + m_2)}} \sin(\omega t) + \frac{m_1 v_0}{m_1 + m_2} t \end{cases} . \text{ (3 points)}$$

(b) Throughout the process, momentum and energy are always conserved. (7 points)

Take all coordinates to be relative to the floor. As the rod mass is ignored, the force of rod on the ball is always along the rod so it does not produce a torque to the rod. One can therefore replace the rod by a massless thread, or assuming the ball is moving on a smooth circular rail centered at the hinge.

$$\begin{cases} m_1 v_1 + m_2 v_2 = m_1 v_0 \\ \frac{1}{2} m_1 (v_1^2 + v_2^2) + \frac{1}{2} m_2 v_2^2 + mgh = \frac{1}{2} m_1 v_0^2 \end{cases}$$

$$(i) v_y = 0, \quad h = 0 \Rightarrow \begin{cases} v_1 = v_0 \\ v_2 = 0 \end{cases}, \text{ and } F = m_1 v_0^2 / L, \quad T = F + m_1 g = m_1 \left(\frac{v_0^2}{L} + g \right), \text{ and The force of the}$$

floor on the block is $N = m_2 g + T = (m_1 + m_2) g + m_1 v_0^2 / L$. (2 points)

$$(ii) h = L, \quad v_1 = v_2 = v_x \Rightarrow \begin{cases} v_x = \frac{m_1 v_0}{m_1 + m_2} \\ v_y = \sqrt{\frac{m_2 v_0^2}{m_1 + m_2} - 2gL} \end{cases},$$

$$T = F = \frac{m_1 v_y^2}{L} = \frac{m_1 m_2 v_0^2}{(m_1 + m_2)L} - 2m_1 g$$

$N = (m_1 + m_2) g$ (2 points)

$$(iii) v_y = 0, \quad h = 2L \Rightarrow \begin{cases} v_1 = \frac{m_1^2 v_0 - \sqrt{m_1^2 m_2 (m_2 v_0^2 - 4(m_1 + m_2) gL)}}{m_1 (m_1 + m_2)} \\ v_2 = \frac{m_1 m_2 v_0 - \sqrt{m_1^2 m_2 (m_2 v_0^2 - 4(m_1 + m_2) gL)}}{m_2 (m_1 + m_2)} \end{cases},$$

$$T = F - m_1 g = m_1 \left(\frac{(v_1 - v_2)^2}{L} - g \right) = m_1 \left(\frac{v_0^2}{L} - \left(\frac{4m_1}{m_2} + 5 \right) g \right)$$

$$N = m_2 g - T = (m_1 + m_2) \left(\frac{4m_1 + m_2}{m_2} \right) g - m_1 v_0^2 / L. \text{ (2 points)}$$

Q 3: (10 points)

Method-1

$$\vec{\tau} = \vec{M} \times \vec{B}, \text{ and } \vec{F} = e\vec{v} \times \vec{B}.$$

Maintaining the relative direction between spin and velocity requires simply that

$$\tau / I = F / P$$

$$\frac{geB}{2m} = \frac{evB}{mv} = \frac{eB}{m}, \text{ so } g = 2.$$

Method-2

The electron will move in a circle in the magnetic field. We need to find the period of such circular motion first. Let the radius of the circle be R . Then $evB = mv^2 / R \Rightarrow v = eBR / m$. The

$$\text{period is } T_e = 2\pi R / v = \frac{2\pi m R}{eBR} = \frac{2\pi m}{eB}.$$

For the electron spin \vec{S} , the torque is always perpendicular to the spin, so the spin will rotate. Suppose the spin rotate by a small angle $\Delta\theta$ over a short time period Δt , the change of angular momentum is $\Delta S = S\Delta\theta$, similar to circular motion where the change of velocity is $\Delta v = v\Delta\theta$. The angular speed of the spin rotation is then $\omega = \Delta\theta / \Delta t$.

$$\text{Using the equation for torque and angular momentum, } \frac{geSB}{2m} = \tau = \frac{\Delta S}{\Delta t} = \frac{S\Delta\theta}{\Delta t} = \omega S.$$

$$\text{The rotation period of the spin is } T_s = \frac{2\pi}{\omega} = \frac{4\pi m}{geB}.$$

$$\text{Finally, } T_s = T_e \Rightarrow g = 2.$$

Q4 (12 points)(a) At the same pressure, $T\rho = \text{Constant}$, (1 point)

The density at 40°C is $\rho_0 = 1.2 \times \frac{293}{313} = 1.12 \text{ kg/m}^3$ (1 point)

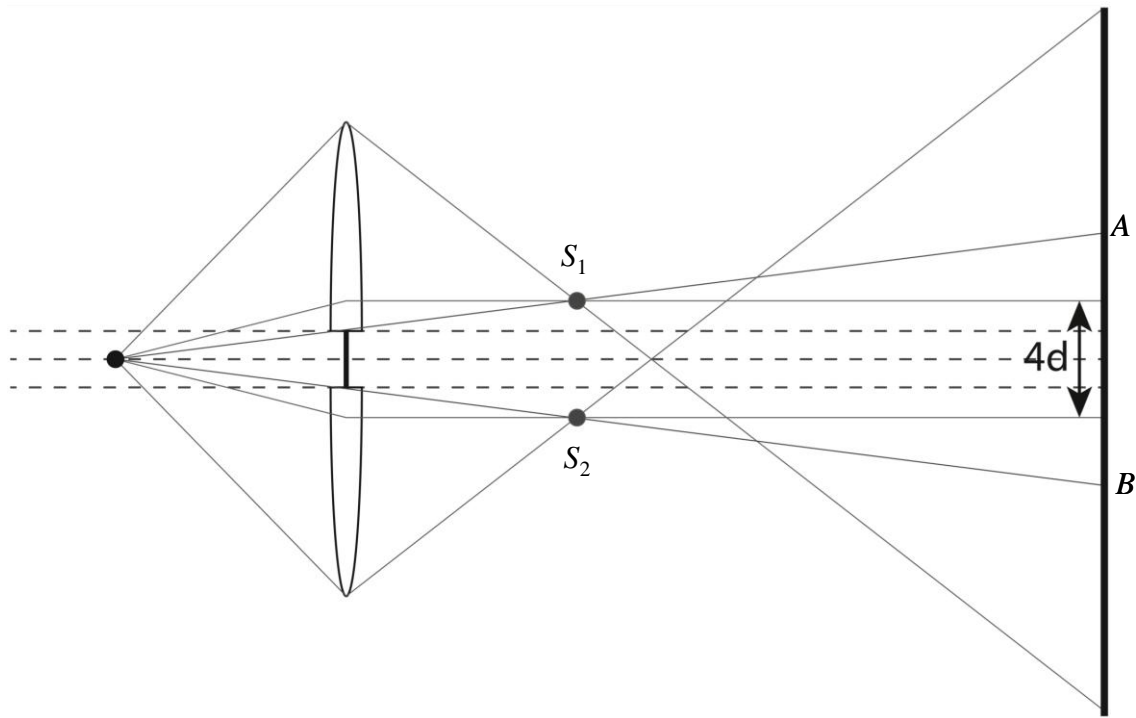
(b) The fraction of water vapor at 40°C at sea level $\eta_1 = \frac{55.35}{760} \times 90\%$ (2 points)

For adiabatic process, $PV^{7/5} = \text{constant}$, and for ideal gas $PV = nkT$, so $P = CT^{7/2}$

The fraction of water at 5°C at high altitude $\eta_2 = \frac{6.5}{760 \left(\frac{278}{313} \right)^{7/2}}$ (2 points)

$$\text{Rain} = (\eta_1 - \eta_2) \times \rho \times V = \left(\frac{55.35}{760} \times 90\% - \frac{6.5}{760 \left(\frac{278}{313} \right)^{7/2}} \right) \times 1.12 = (0.0665 - 0.013) \times 1.12 = 60 \text{ g (3 points)}$$

(c) Height: $278 = 313 \left(1 - \frac{2 \times 1.12 \times 9.8 \times h}{7 \times 1.03 \times 10^5} \right) \Rightarrow h = 3673 \text{ m}$ (3 points)

Q 5: (14 points)**(a) (7 points)**

Using the lens formula, we know that there are two images S_1 and S_2 at distance $2f$ from the split lens and their distance is $4d$. (1 point) The two images are equivalent to the two small holes illuminated by a point source in a typical Young's interference experiment. Their interference effect produces the fringes on the screen.

$\tilde{L} = L - f$. Let ξ be the distance of the observation position on the screen to the optical axis.

$$\delta_1 = \frac{1}{\lambda} \sqrt{(\xi - 2d)^2 + \tilde{L}^2} = \frac{\tilde{L}}{\lambda} \sqrt{\left(\frac{\xi - 2d}{\tilde{L}}\right)^2 + 1} \approx \frac{\tilde{L}}{\lambda} \left(1 + \left(\frac{\xi - 2d}{\tilde{L}}\right)^2\right),$$

$$\delta_2 = \frac{\tilde{L}}{\lambda} \left(1 + \left(\frac{\xi + 2d}{\tilde{L}}\right)^2\right). \text{ And then optical path difference is } \Delta = |\delta_1 - \delta_2| = \frac{4d}{\tilde{L}} \xi. \text{ (2 points)}$$

The width of the fringe $\Delta\xi$ is then

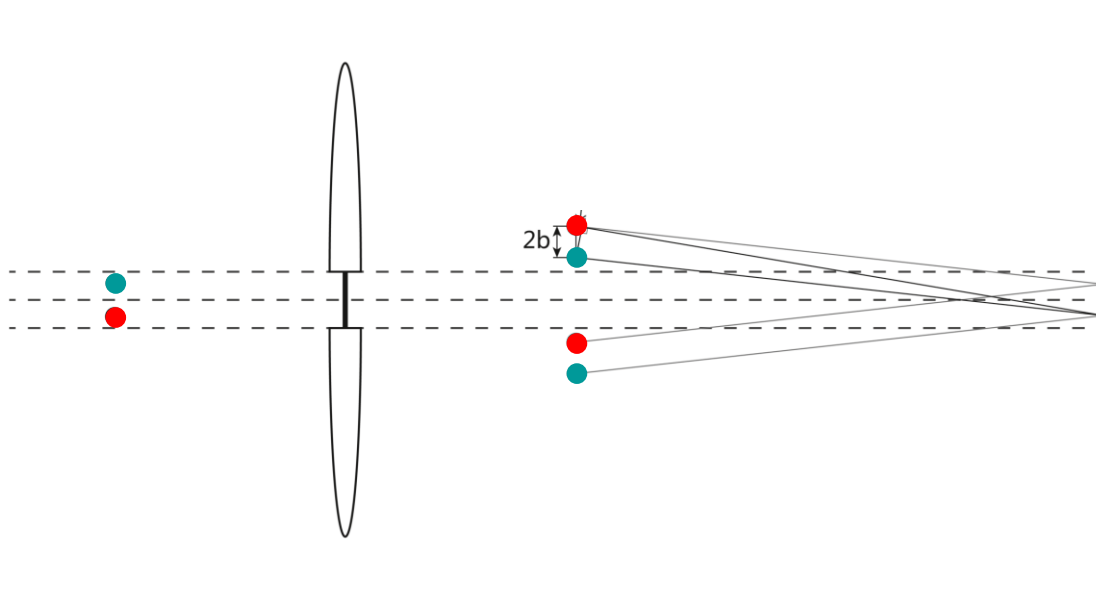
$$\frac{4d}{\tilde{L}} \Delta\xi = \lambda \Rightarrow \Delta\xi = \frac{\lambda \tilde{L}}{4d} = \frac{500 \times 10^9 \times (1 - 0.2)}{4 \times 10^{-3}} = 1.0 \times 10^{-4} \text{ m} = 0.1 \text{ mm}.$$

Now we need to know the width of the area where interference occurs, or where the light from S_1 overlap with the light from S_2 . From the figure we see that the area is between point-A and point-B. By simple geometry, we get the distance between point-A and point-B to be

$$D = (3f + L) \frac{d}{f} = 1600 \times 1 / 200 = 8 \text{ mm} \text{ (1 point)}$$

$$\text{So Number} = \frac{D}{\Delta\xi} = \frac{8}{0.1} = 80. \text{ (2 points)}$$

(b)



Each source (red or green) will form a pair of images, just like in (a). Each pair forms its own set of interference fringes on the screen. When the red set bright fringes overlap with the dark fringes of the green pair, then the light intensity on the screen is uniform and no fringes can be observed. This means that at any point on the screen, the optical path difference from the red pair should differ from that from the green pair by at least half the wavelength.

The distance between each pair of interfering images is still $4d$. The center position of the red pair from the optical axis is $((2d + b) - (2d - b))/2 = b$. At a position above the optical axis by ξ , the optical path difference is $\Delta_R = \frac{4d}{\tilde{L}}(\xi - b)$.

Likewise, the center position of the green pair from the optical axis is $-b$. At the same position above the optical axis by ξ the optical path difference is $\Delta_G = \frac{4d}{\tilde{L}}(\xi + b)$ (2 points)

For fringes to disappear, we must have $\frac{8db}{\tilde{L}} = \Delta_G - \Delta_R = \frac{\lambda}{2}$.

Finally, $b = \frac{\lambda \tilde{L}}{16d} = \frac{5 \times 10^{-4} \times 800}{16} = 0.025 \text{ mm}$. (3 points)