Hong Kong Physics Olympiad 2012
2012 年香港物理奧林匹克競賽

(Senior Level 高級組)

Jointly Organized by

The Hong Kong Academy for Gifted Education
香港資優教育學院
The Education Bureau of HKSAR
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共同舉辦

March 17, 2012
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MC1-5 (b) (c) (d) (c) (e)
MC6-10 (a) (a) (e) (c) (b)

Q1 (10 points)

Heavy rain ~ 100 mm/hour = 100/3600 ≈ 0.03 kg/s = Φ (1 point)
Area of umbrella A ~ 1 m^2 (1 point)
Rain drop speed ~ 10 m/s (1 point)

$$ft = mv, \Rightarrow f_{rain} = \frac{mv}{t} = A \cdot \Phi \cdot v = 1 \cdot 0.03 \cdot 10 = 0.3N \quad \text{(2 points)}$$

Wing:
Wing speed is 10m/s, (1 point) $\Phi = 1.2 \cdot 10 = 12$ kg/s (1 point)

$f_{wind} = A \cdot \Phi \cdot v = 1 \cdot 12 \cdot 10 = 120N$. (3 points)

Any answers within a factor of 10 are fine.

Q2 (10 points)

$$L\cos\theta = (v_0 \cos\alpha)t \quad \text{(horizontally)},$$
$$L\sin\theta = (-v_0 \sin\alpha)t + \frac{1}{2}gt^2 \quad \text{(vertically)}. \qquad \text{(2 points)}$$

Other forms are fine as long as the expression for L below is correct.

Then we have,

$$L = f(\alpha) = \frac{2v_0^2}{g\cos^2\theta}(\sin\theta\cos^2\alpha + \cos\theta\sin\alpha\cos\alpha)$$
$$= \frac{2v_0^2}{g\cos^2\theta}[\sin\theta\frac{1}{2}(1+\cos 2\alpha) + \cos\theta\frac{1}{2}\sin 2\alpha]$$
$$= \frac{v_0^2}{g\cos^2\theta}[\sin\theta + (\sin\theta\cos 2\alpha + \cos\theta\sin 2\alpha)]$$

$$= \frac{v_0^2}{g\cos^2\theta}[\sin\theta + \sin(\theta + 2\alpha)]. \qquad \text{(6 points)}$$

$L_{\max} = 476m$ while $\alpha = 15^0$. (2 points)

Q3 (15 points). Point Dipole

$$\overrightarrow{v_a} = \overrightarrow{v_c} + \overrightarrow{\varpi} \times \overrightarrow{r_{ca}} = (v_x + \frac{1}{2}L\varpi\sin\theta)x + (v_y - \frac{1}{2}L\varpi\cos\theta)y,$$
$$\overrightarrow{v_b} = \overrightarrow{v_c} + \overrightarrow{\varpi} \times \overrightarrow{r_{cb}} = (v_x - \frac{1}{2}L\varpi\sin\theta)x + (v_y + \frac{1}{2}L\varpi\cos\theta)y.$$

Lorentz force $\vec{f} = q\vec{v} \times \vec{B}$.

$$\vec{f_x} = qB(v_y - \frac{1}{2}L\varpi \sin\theta)x + (-q)B(v_y + \frac{1}{2}L\varpi \cos\theta)x = -qBL\varpi \cos\theta x,$$

$$\vec{f_y} = qB(-v_x - \frac{1}{2}L\varpi \sin\theta)y + (-q)B(-v_x + \frac{1}{2}L\varpi \sin\theta)y = -qBL\varpi \sin\theta y$$

Net force $\vec{f} = \vec{f_x} + \vec{f_y} = -qBL\varpi(\cos\theta x + \sin\theta y)$. (net force is along the stick)

Net force only depends on rotation.

To caculate torques, only consider Lorentz forces perpendicular to the stick,

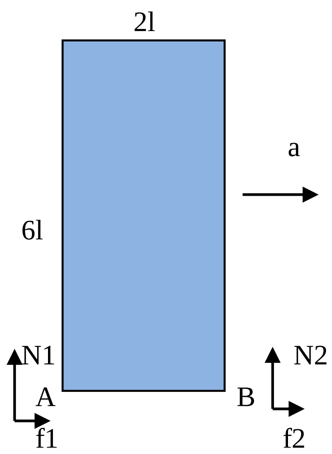
$\vec{M_a} = q(v_x \cos\theta \widehat{x + v_y} \sin\theta \widehat{y)) \times \vec{B}} \times \vec{r_{ca}},$

$\vec{M_b} = -q(v_x \cos\theta \widehat{x + v_y} \sin\theta \widehat{y)) \times \vec{B}} \times \vec{r_{cb}}.$

Net torque: $\vec{M} = \vec{M_a} + \vec{M_b} = qBL(v_x \cos\theta + v_y \sin\theta)\hat{z}.$

Net torque only depends on translation.

Q4 (15 points). Refrigerator



(a) Rotation: $mgl = ma_{\max}(3l) \to a_{\max} = \frac{1}{3}g$

Translation: $\mu mg = ma_{\max} \to a_{\max} = \mu g \rangle \frac{1}{3}g$

Hence $a_{\max} = \frac{1}{3}g = 3.27m/s^2$

(b) $f_1 + f_2 = ma \quad N_1 + N_2 = mg$

Point A: $ma(3l) + N_2(2l) = mgl$

Then $N_1 = 0.65mg = 318.5N, \quad N_2 = 0.35mg = 171.5N$

$f_1 : f_2 = N_1 : N_2 \quad f_1 = 0.65ma = 31.85N \quad f_2 = 0.35ma = 17.15N$

Net forces: $\vec{F_A} = 318.5N\widehat{y + 31.85N}\widehat{x =} 320.8N(\tan\theta = 10)$

$\vec{F_B} = 171.5N\widehat{y + 17.15N}\widehat{x =} 172.4N(\tan\theta = 10)$

Q5 (15 points)

Block-Spring System

Force balance $\quad (a)\ kx = \mu Mg, \quad$ so $\quad x = \frac{\mu Mg}{k}$, (2 points)

Energy conservation $\frac{1}{2}Mv_0^2 = \frac{1}{2}kx^2 + \mu Mgx,$ so $v_0 = \mu g\sqrt{\frac{3M}{k}}$.(2 points)

$(b)\ kx = \mu Mg,$ so $x = \frac{\mu Mg}{k}$, (2 points)

Momentum Conservation $(M + M)v_f = Mv_0$ (1 point)

Energy Conservation $\frac{1}{2}Mv_0^2 = \frac{1}{2}(M + M)v_f^2 + \frac{1}{2}kx^2 + \mu Mgx, \Rightarrow v_0 = \mu g\sqrt{\frac{6M}{k}}$. (3 points)

Q6 (20 points)

(a)

At the final stage, the number of moles of gas in the tank is $n_f = \frac{PV}{RT} = \frac{0.5P_0 \cdot 10V_0}{RT} = 5n$. So N= 5 cylinders are needed. (5 points)

(b)

$P_0 N V_0 = P(NV_0 + V) \Rightarrow N = 0.5(N+10) \Rightarrow N = 10$. (5 points)

(c)

First one, $P_1 = \frac{P_0 V_0}{V+V_0} = \frac{1}{11}P_0$, $n_1 = \frac{Vn}{V+V_0} = \frac{10}{11}n$, (1 point)

Second, $P_2 = \frac{(n+n_1)RT}{V+V_0} = \frac{(1+10/11)n}{11V_0}\frac{P_0V_0}{n} = \frac{1}{11}(1+\frac{10}{11})P_0$, $n_2 = \frac{V(n+n_1)}{V+V_0} = \frac{10}{11}(1+\frac{10}{11})n$ (1 point)

Third, $P_3 = \frac{(n+n_2)RT}{V+V_0} = \frac{1}{11}(1+\frac{10}{11}+\frac{10\cdot 10}{11\cdot 11})P_0$, $n_3 = \frac{V(n+n_2)}{V+V_0} = \frac{10}{11}(1+\frac{10}{11}+\frac{10\cdot 10}{11\cdot 11})n$ (1 point)

$$P_2 - P_1 = \left(\frac{21}{11\cdot 11} - \frac{1}{11}\right)P_0 = \frac{10}{11\cdot 11}, \quad n_2 - n_1 = \frac{10\cdot 21}{11\cdot 11}n - \frac{10}{11}n = \frac{10}{11}\cdot\frac{10}{11}n$$

$$P_3 - P_2 = \frac{(11\cdot 11 + 10\cdot 21)}{11\cdot 11\cdot 11}P_0 - \frac{21}{11\cdot 11}P_0 = \frac{10\cdot 10}{11\cdot 11\cdot 11}P_0, \quad n_3 - n_2 = \frac{10\cdot 21}{11\cdot 11}n - \frac{10}{11}n = \frac{10\cdot 10\cdot 10}{11\cdot 11\cdot 11}n$$

$$n_N = \frac{(n+n_{N-1})V}{V+V_0} = \frac{10}{11}\left[1+\frac{10}{11}+\frac{10\cdot 10}{11\cdot 11}+\ldots+\left(\frac{10}{11}\right)^{N-1}\right]n = \frac{10}{11}\left[\frac{1-(10/11)^N}{1-10/11}\right]n = 10n[1-(10/11)^N],$$

(5 points)

$P_N = \frac{(n+n_{N-1})RT}{V+V_0} = [1-(10/11)^N]P_0$, $P_N = \frac{1}{2}P_0 \Rightarrow N = \frac{\ln 2}{\ln(11/10)} = 7.3$. (2 points)

So 8 cylinders are needed. (1 point)