

**Hong Kong Physics Olympiad 2013**  
**2013 年香港物理奧林匹克競賽**

**Jointly Organized by**

**The Hong Kong Academy for Gifted Education**  
**香港資優教育學苑**

**The Education Bureau of the HKSAR Government**  
**香港特別行政區政府教育局**

**The Physical Society of Hong Kong**  
**香港物理學會**

**The Hong Kong University of Science and Technology**  
**香港科技大學**

**共同舉辦**

**19 May, 2013**  
**2013 年 5 月 19 日**

**Rules and Regulations 競賽規則**

1. All questions are in bilingual versions. You can answer in either Chinese or English.  
所有題目均為中英對照。你可選擇以中文或英文作答。
2. The multiple-choice answer sheet will be collected 1.5 hours after the start of the contest. You can start answering the open-ended questions any time after you have completed the multiple-choice questions without waiting for announcements.  
選擇題的答題紙將於比賽開始後一小時三十分收回。若你在這之前已完成了選擇題，你亦可開始作答開放式題目，而無須等候任何宣佈。
3. Please follow the instructions on the multiple-choice answer sheet, and use a HB pencil to write your 8-digit Participant ID Number in the field of “I. D. No.”, and fill out the appropriate circles **fully**. After that, write your English name in the space provided and your Hong Kong ID number in the field of “Course & Section No.”

請依照選擇題答題紙的指示，用 HB 鉛筆在選擇題答題紙的 “I. D. No.” 欄上首先寫上你的 8 位數字參賽號碼，並把相應寫有數字的圓圈**完全塗黑**，然後在適當的空格填上你的英文姓名，最後於 “Course & Section No.” 欄內填上你的身分證號碼。

4. After you have made the choice in answering a multiple choice question, fill the corresponding circle on the multiple-choice answer sheet **fully** using a HB pencil.

選定選擇題的答案後，請將選擇題答題紙上相應的圓圈用 HB 鉛筆**完全塗黑**。

5. On the cover of the answer book, please write your Hong Kong ID number in the field of “Course Title”, and write your English name in the field of “Student Name” and your 8-digit Participant I. D. Number in the field of “Student Number”. You can write your answers on both sides of the sheets in the answer book.

在答題簿封面上，請於 Course Title 欄中填上你的身分證號碼；請於 Student Name 欄中填上你的英文姓名；請於 Student Number 欄中填上你的 8 位數字參賽號碼。答題簿可雙面使用。

6. The information provided in the text and in the figure of a question should be put to use together.

解題時要將文字和簡圖提供的條件一起考慮。

7. Some open problems are quite long. Read the entire problem before attempting to solve them. If you cannot solve the whole problem, try to solve some parts of it. You can even use the answers in some unsolved parts as inputs to solve the others parts of a problem.

開放題較長，最好將整題閱讀完後才著手解題。若某些部分不會做，也可把它們的答案當作已知來做其他部分。

**The following symbols and constants are used throughout the examination paper unless otherwise specified:**

除非特別注明，否則本卷將使用下列符號和常數：

Gravitational acceleration on Earth surface 地球表面重力加速度	$g$	$9.8 \text{ m/s}^2$
Gravitational constant 萬有引力常數	$G$	$6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2$
Radius of Earth 地球半徑	$R_E$	6378 km
Sun-Earth distance 太陽-地球距離 (= 1 Astronomical Unit (AU)) (= 1 天文單位(AU))	$r_E$	$1.5 \times 10^{11} \text{ m}$
Mass of Sun 太陽質量	$M_{\text{Sun}}$	$1.99 \times 10^{30} \text{ kg}$
Mass of Earth 地球質量	$M_E$	$5.98 \times 10^{24} \text{ kg}$
Air Density 空氣密度	$\rho_0$	$1.2 \text{ kg/m}^3$
Water Density 水密度	$\rho_w$	$1.0 \times 10^3 \text{ kg/m}^3$

**Trigonometric identities:**

三角學恆等式:

$$\sin(x + y) = \sin x \cos y + \cos x \sin y$$

$$\cos(x + y) = \cos x \cos y - \sin x \sin y$$

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\sin x \cos y = \frac{1}{2} [\sin(x + y) + \sin(x - y)]$$

$$\cos x \cos y = \frac{1}{2} [\cos(x + y) + \cos(x - y)]$$

$$\sin x \sin y = \frac{1}{2} [\cos(x - y) - \cos(x + y)]$$

### Multiple Choice Questions

(Select one answer in each question. For each question, 2 marks for correct answer, 0 mark for no answer, minus 0.25 mark for wrong answer, but the lowest mark of the multiple choice section is 0 mark.)

**選擇題** (每題選擇一個答案，每題答對 2 分，不答 0 分，答錯扣 0.25 分，但全部選擇題最低為 0 分。)

1. A massive rope of mass  $m$  and length  $L$ , as shown in the figure, rests on a horizontal table. If the coefficient of static friction between the table and the rope is  $\mu_s$ , what fraction of the rope can hang over the edge of the table without the rope sliding?

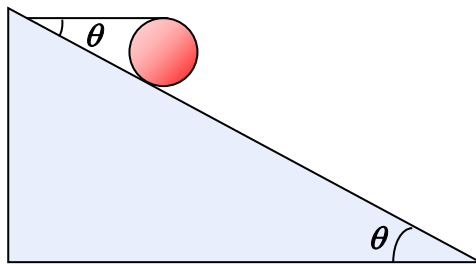
如圖所示，一根質量為  $m$ 、長度為  $L$  的繩纜，靜放在水平桌面上。若繩纜與桌面的靜止摩擦係數為  $\mu_s$ ，則懸垂於桌緣外的繩纜，可以達到全長的什麼分數，而繩纜仍然不至滑下？

- A.  $\frac{\mu_s}{1 + \mu_s}$       B.  $\frac{\mu_s}{1 + 2\mu_s}$       C.  $1 - \mu_s$       D.  $\sqrt{1 + \mu_s}$       E.  $\frac{2}{3}\mu_s$



2. As shown in the figure, a rigid sphere of mass  $m$  and radius  $R$  is held at rest by a horizontal string on an inclined plane with an inclination  $\theta$ . If the sphere does not move, what is the minimum coefficient of static friction  $\mu_s$  between the sphere and the incline?

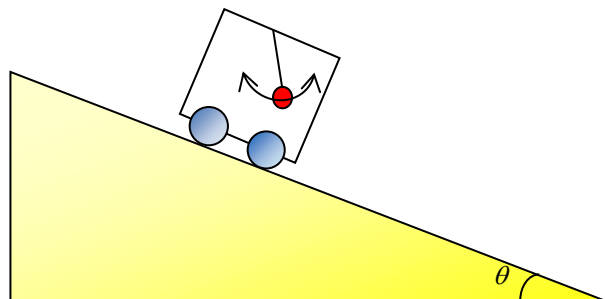
如圖所示，有質量為  $m$ 、半徑為  $R$  的球體，被一平行方向的繩索固定於傾角為  $\theta$  的斜面上。若球體不動，則球體與斜面之間的靜止摩擦係數，最小值應是什麼？



- A.  $\tan\theta$       B.  $\frac{1 - \cos\theta}{1 + \cos\theta}$       C.  $\frac{\cos\theta}{1 + \cos\theta}$       D.  $\frac{\sin\theta}{1 + \cos\theta}$       E.  $1 - \cos\theta$

3. A simple pendulum of length  $L$  is mounted in a massive cart that slides down a plane inclined at an angle  $\theta$  with the horizontal. Find the period  $T$  of small oscillations of this pendulum if the cart moves down the plane with acceleration  $a = g \sin\theta$ .

一部具相當質量的車上懸有一長度為  $L$  的簡單鐘擺，車子滑下傾角為  $\theta$  的斜面。當車子以加速度  $a = g \sin\theta$  在斜面向下運動時，求鐘擺以小幅度振動的周期  $T$ 。



A.  $2\pi \sqrt{\frac{L}{g \cos \theta}}$

B.  $2\pi \sqrt{\frac{L}{g \sqrt{2(1 + \sin \theta)}}}$

C.  $2\pi \sqrt{\frac{L \sin \theta}{g \cos \theta}}$

D.  $2\pi \sqrt{\frac{L}{g \sqrt{1 + 3 \sin^2 \theta}}}$

E.  $2\pi \sqrt{\frac{L}{g(1 - \sin \theta)}}$

4. A stone of mass  $M$  is tied to a string. It is whirled in a vertical circle of radius  $L$ . At the highest point of the circle, the speed of the stone is  $v$ . Find the tension of the string.  
 一塊質量為  $M$  的石子繫於一根繩子上。石子在空中以半徑為  $L$  的垂直圓圈旋轉。在圓圈的最高點，石子的速率是  $v$ 。求繩子的張力。

A.  $M \left( g + \frac{v^2}{L} \right)$

B.  $M \left( g + \frac{v^2}{2L} \right)$

C.  $M \left( g - \frac{v^2}{L} \right)$

D.  $M \frac{v^2}{L}$

E.  $M \left( \frac{v^2}{L} - g \right)$

5. A ring with mass  $m$  is hung vertically at the lower end of a uniform chain of total mass  $m$  and length  $L$ . Its upper end  $A$  is fixed, as shown in figure (a). The lower end  $B$  is raised until it is at the same position as  $A$ , and the ring slides to the midpoint of the string, as shown in figure (b). What is the minimum work required in this process?

有質量為  $m$  的小環，繫於一質量為  $m$ 、長度為  $L$  的均勻繩索的下端。繩索的上端固定，如圖(a)所示。繩索的下端被提到上端同樣高度，而小環滑到繩索的中點，如圖(b)所示。這過程所需的功，最小是多少？

A.  $mgL$

B.  $\frac{3}{4}mgL$

C.  $\frac{1}{2}mgL$

D.  $\frac{1}{4}mgL$

E.  $\frac{3}{2}mgL$

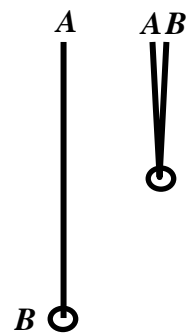


Fig. (a) Fig. (b)

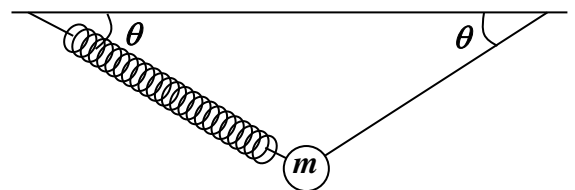
6. In a spacecraft orbiting around the Earth, an astronaut has a feeling of weightlessness. In Earth's reference frame, the explanation is
- the weight of the astronaut becomes zero.
  - the gravitational field inside the spacecraft becomes zero.
  - the net force acting on the astronaut becomes zero.
  - the astronaut is falling freely.
  - there is no change in the momentum of the astronaut.

太空船環繞地球運動，其中的太空人感到沒有重量。在地球的參照系內，這現象的解釋是

- 太空人的重量變為零。
- 太空船內的引力場變為零。
- 作用於太空人身上的力，總和為零。
- 太空人在自由下墜。
- 太空人的動量沒有改變。

7. In the figure, a mass  $m$  is hung by a light spring and a light string at the ceiling. Both the spring and the string make an angle  $\theta$  with the horizontal at equilibrium. If the string is suddenly cut, what is the instantaneous acceleration of the mass  $m$ ?

如圖所示，有質量為  $m$  的物體，被輕量的彈簧和輕量的繩索繫於天花板上。在平衡狀態下，彈簧和繩索與水平方向都成角度  $\theta$ 。若繩索突然斷了，物體的瞬時加速度是多少？



A.  $\frac{2g}{\sin \theta}$

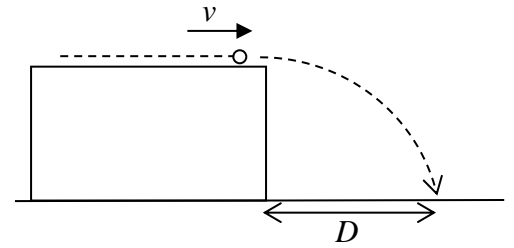
B.  $\frac{g}{\sin \theta}$

C.  $\frac{g}{2 \sin \theta}$

D.  $\frac{g}{\cos \theta}$

E.  $\frac{g}{2 \cos \theta}$

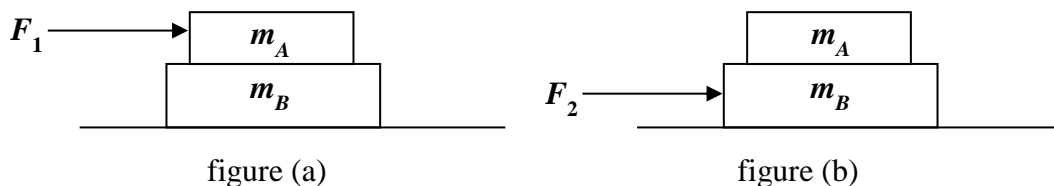
8. A particle is projected horizontally from the edge of a smooth table with initial speed  $v$ . The particle hits the ground at a horizontal distance  $D$  from the table. Different values of  $v$  are used and the corresponding values of  $D$  are recorded. Which of the following pair of quantities will give a straight-line curve?



有粒子從平滑桌子的邊緣以初速  $v$  拋射向地面，粒子在水平距離  $D$  處著地。不同的  $v$  值得到不同的  $D$  值。下面哪一對數量具有線性關係？

- A.  $v$  and  $D$                       B.  $v^2$  and  $D$                       C.  $v$  and  $D^2$   
 D.  $v$  and  $\frac{1}{D}$                       E.  $v$  and  $\frac{1}{\sqrt{D}}$
9. Which of the following facts is/are direct evidence(s) supporting Newton's first law of motion?  
 (1) A feather and a coin spend equal time to reach the ground when dropped from the same height on the Moon surface.  
 (2) A satellite orbits around the Earth with uniform speed without supply of fuel.  
 (3) A man is thrown forward on a bus which stops suddenly.  
 下面哪些事實可以作為牛頓第一定律的直接證據？  
 (1) 一根羽毛和一枚硬幣從相同高度掉到月球表面，所需的時間相同。  
 (2) 衛星以均速環繞地球運動，不用消耗燃料。  
 (3) 當巴士突然停下來，乘客被拋向前。  
 A. (3) only                      B. (1) and (2) only                      C. (1) and (3) only                      D. (2) and (3) only                      E. (1), (2) and (3)
10. Two masses,  $m_A$  and  $m_B$  ( $m_B > m_A$ ) are put on a smooth horizontal table as shown. The maximum static friction between the two masses is  $f$ . A gradually increasing horizontal force acts on  $m_A$  and the two masses accelerate together. The masses start to slip over each other when the force attains  $F_1$ . (figure (a)) If initially, the force acts on  $m_B$  instead, the masses start to slip over each other when the force attains  $F_2$ . (figure (b)) Compare  $F_1$  and  $F_2$ .

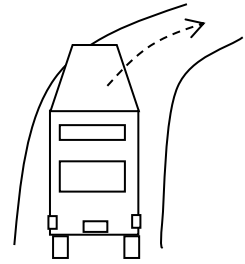
如圖所示，兩物體， $m_A$  和  $m_B$  ( $m_B > m_A$ )，被置於平滑的水平桌子上。兩物體之間的最大摩擦力為  $f$ 。有一沿水平方向作用於  $m_A$  的外力逐漸增加，使兩物體一起加速。當外力達到  $F_1$  時，兩物體之間產生滑動（圖(a)）。若從起首考慮，外力作用於  $m_B$  而非  $m_A$ ，則當外力達到  $F_2$  時，兩物體之間產生滑動（圖(b)）。試比較  $F_1$  和  $F_2$ 。



- A.  $F_1 > 2F_2$                       B.  $F_1 > F_2$                       C.  $F_1 = F_2$                       D.  $F_1 < F_2$                       E.  $F_1 < F_2/2$
11. Planet  $P$  is moving in a circular orbit around a star  $X$ , while in another stellar system, planet  $Q$  is moving in a circular orbit around a star  $Y$ . The orbital radius of  $P$  is twice that of  $Q$  and the orbital period of  $P$  is also twice that of  $Q$ . Find the ratio of mass of  $X$  to that of  $Y$ .  
 行星  $P$  以圓形軌道環繞恆星  $X$ ，而在另一恆星系，行星  $Q$  以圓形軌道環繞恆星  $Y$ 。 $P$  的軌道半徑為  $Q$  的兩倍， $P$  的軌道周期也為  $Q$  的兩倍。求  $X$  與  $Y$  的質量比。  
 A. 8:1                      B. 4:1                      C. 2:1                      D. 1:1                      E. 1:2

12. A bus is turning around a corner on a horizontal road. The diagram shows the rear view of the bus, which is turning to its right. Which of the following diagram best shows the force diagram of the bus?

有巴士在平路上拐彎，圖中所示為巴士右轉時，從後看的示意圖。下面哪張作用力圖最為正確？

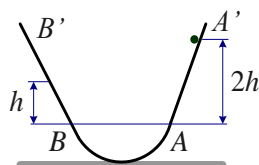


- A. B. C.
- D. E.

13. A cylindrical trough is placed on a horizontal plane. The two edges of the trough are connected with smooth inclined planes  $AA'$  and  $BB'$  at locations  $A$  and  $B$  respectively. The trough edges and the inclined planes are tangential at the connections, and  $A, B$  are located at the same horizontal level, as shown in the figure. A small mass slides freely down the slope  $A'A$  from a height of  $2h$  above the horizontal level  $AB$ . It enters the trough at position  $A$ , reaches  $B$  and then slides upward along slope  $BB'$  until it reaches a height of  $h$  above the horizontal level  $AB$ . It then slides downwards along slope  $B'B$  and enters the trough again. Neglecting air resistance, the small mass

一個圓弧形槽的底部放在水平地面上，槽的兩側與光滑斜坡  $AA'$ 、 $BB'$  相切，相切處  $A$ 、 $B$  位於同一水平面內，槽與斜坡在豎直平面內的截面如圖所示。一小物塊從斜坡  $A'A$  上距水平面  $AB$  的高度為  $2h$  處沿斜坡自由滑下，並自  $A$  處入槽內，到達  $B$  後沿斜坡  $BB'$  向上滑行，到達距水平面  $AB$  高度為  $h$  的位置；接著小物塊沿斜坡  $B'B$  滑下並從  $B$  處進入槽內。若不考慮空氣阻力，則小物塊

- |  |                        |
|--|------------------------|
| A. reaches a height lower than $A$               | A. 到達低於 $A$ 的位置        |
| B. just reaches the location $A$                 | B. 恰好到達 $A$ 處          |
| C. passes $A$ and reaches a height less than $h$ | C. 過 $A$ 處後上升高度小於 $h$  |
| D. passes $A$ and reaches the height $h$         | D. 過 $A$ 處後上升高度等於 $h$  |
| E. passes $A$ and reaches the height $2h$        | E. 過 $A$ 處後上升高度等於 $2h$ |



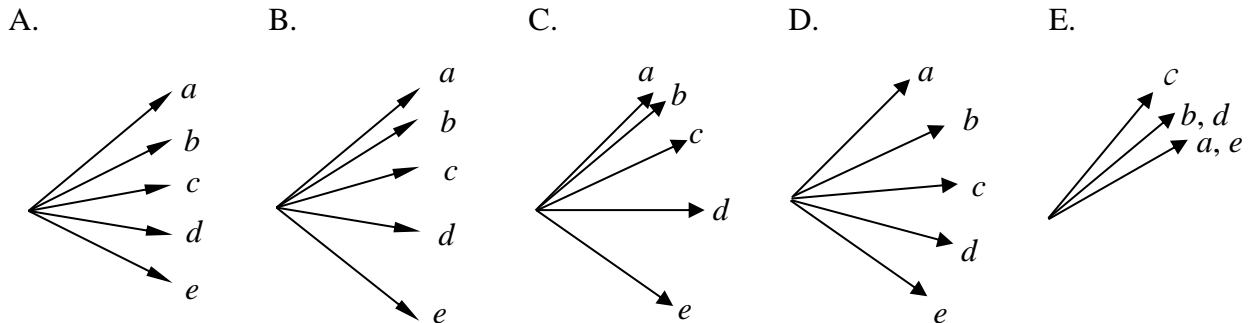
14. Consider a satellite of mass  $m$  orbiting around the Earth in a circular orbit of radius  $R_S$ . Given the radius of the Earth is  $R_E$ , and the gravitational acceleration on Earth's surface is  $g$ , and atmospheric resistance can be neglected, then the energy required to launch the satellite from Earth's surface is

一顆質量為  $m$  的人造衛星以圓形軌道環繞地球，軌道半徑為  $R_S$ 。設地球半徑為  $R_E$ ，地面重力加速度為  $g$ ，大氣層對衛星的阻力忽略不計，則從地面發射該人造衛星所需能量為

- A.  $mgR_E \left(1 - \frac{R_E}{2R_S}\right)$       B.  $mgR_E \left(1 + \frac{R_E}{2R_S}\right)$       C.  $mgR_E \left(\frac{1}{2} - \frac{R_E}{2R_S}\right)$   
 D.  $mgR_E \left(\frac{1}{2} + \frac{R_E}{2R_S}\right)$       E.  $mgR_E \left(\frac{R_S}{R_E} - 1\right)$

15. The velocity vectors of an object performing projectile motion are drawn from time instants  $a$  to  $e$  at fixed time intervals. Which of the following gives a possible drawing?

把一物體在拋射過程中不同時間的速度矢量畫出來，其中時刻  $a$  到  $e$  的間隔相同。下面哪張圖是可能的？



16. It is known that the gravitational acceleration is  $g = GM_E / R_S^2$  and is directed vertically towards Earth's centre. Due to Earth's rotation, the acceleration  $\mathbf{f}$  of a freely falling object in Hong Kong has a magnitude  $f$  different from  $g$ , and  $\mathbf{f}$  is no longer pointing vertically downwards. In fact,

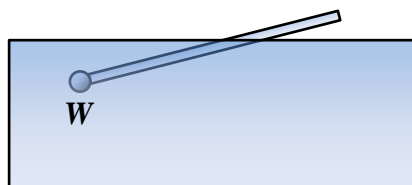
已知重力加速度為  $g = GM_E / R_S^2$  並指向地球中心。因著地球轉動，在香港的自由落體加速度則為  $\mathbf{f}$ 。它的數值  $f$  與  $g$  不同，而  $\mathbf{f}$  的方向也不是完全垂直。其實，

- |   |                               |
|---|-------------------------------|
| A. $f < g$ and $\mathbf{f}$ has a Northward component | A. $f < g$ 而 $\mathbf{f}$ 偏向北 |
| B. $f < g$ and $\mathbf{f}$ has a Southward component | B. $f < g$ 而 $\mathbf{f}$ 偏向南 |
| C. $f > g$ and $\mathbf{f}$ has a Northward component | C. $f > g$ 而 $\mathbf{f}$ 偏向北 |
| D. $f > g$ and $\mathbf{f}$ has a Southward component | D. $f > g$ 而 $\mathbf{f}$ 偏向南 |
| E. $f > g$ and $\mathbf{f}$ has a Eastward component  | E. $f > g$ 而 $\mathbf{f}$ 偏向東 |

17. A uniform rod floats in water. A ball with weight  $W$  is attached to one end of the rod, and the volume of the ball is negligible. This structure causes the rod to float at an inclined position, with the other end remaining above the water surface, as shown in the figure. If the part of the rod above the water surface is  $1/n$  of the total length, calculate the weight of the rod.

一根均勻桿浮在水中，一端附著重量為  $W$  而體積可忽略不計的小球。這結構令桿子傾斜地浮著，致另一端保持在水面上。若水面上的桿長為全長的  $1/n$ ，求桿的重量。

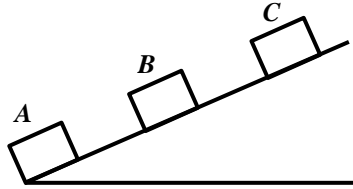
- A.  $W(n+1)$       B.  $Wn$       C.  $W(n-1)$       D.  $W/(n+1)$       E.  $W/(n-1)$



18. A block has an initial kinetic energy of 128 J. It slides up from point A at the bottom of the inclined plane with uniform deceleration. When it passes point B, its kinetic energy is reduced by 80 J, and its mechanical energy is reduced by 35 J. Calculate the work done against friction when the block moves from A to the highest point C on the inclined plane.

一滑塊以 128 J 的初動能，從斜面底端的 A 點沿斜面向上作勻減速直線運動，它經過 B 點時，動能減少了 80 J，機械能減少了 35 J，求滑塊從 A 到最高點 C 對摩擦力所作的功。

- A. 42 J      B. 48 J      C. 56 J      D. 72 J      E. 128 J



19. A block of mass  $m$  is placed on a smooth horizontal surface and is attached to a spring of force constant  $k$ . When the block is pulled sideways and released, it undergoes simple harmonic motion. At the equilibrium position, its velocity is 3 m/s. Calculate its velocity when it moves to the position at two-third of the amplitude from the equilibrium position.

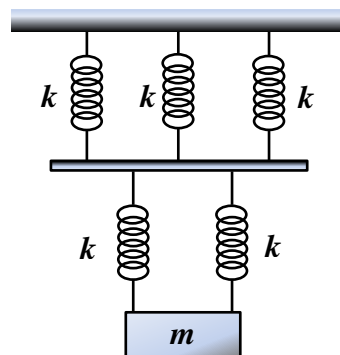
一質量為  $m$  的方塊置於平滑水平面，並繫著彈性係數為  $k$  的彈簧。當方塊被拉向一旁再放鬆時，方塊即進行簡諧運動。方塊在平衡位置時的速度為 3 m/s，求在振幅三分之二處的速度。

- A.  $\sqrt{2}$  m/s      B.  $\sqrt{3}$  m/s      C. 2 m/s      D.  $\sqrt{5}$  m/s      E.  $\sqrt{6}$  m/s

20. As shown in the figure, a block of mass  $m$  is hung from the ceiling by the system of springs consisting of two layers. The upper layer consists of 3 springs in parallel, and the lower layer consists of 2 springs in parallel. The force constants of all springs are  $k$ . Calculate the frequency of the vertical oscillations of the block.

如圖所示，天花板下一組兩層的彈簧懸吊著質量為  $m$  的方塊。上層有 3 個並排的彈簧，下層則有 2 個並排的彈簧。所有彈簧的彈力常數都是  $k$ 。求方塊上下振動時的頻率。

- A.  $\frac{1}{2\pi} \sqrt{\frac{k}{5m}}$       B.  $\frac{1}{2\pi} \sqrt{\frac{4k}{5m}}$       C.  $\frac{1}{2\pi} \sqrt{\frac{5k}{6m}}$       D.  $\frac{1}{2\pi} \sqrt{\frac{6k}{5m}}$       E.  $\frac{1}{2\pi} \sqrt{\frac{5k}{2m}}$



《END OF MC's 選擇題完》



## Open Problems 開放題

### Total 5 problems 共 5 題

The Open Problem(s) with the “\*” sign may require information on page 2.  
帶 \* 的開放題可能需要用到第二頁上的資料。

#### 1\*. James Bond Ski Chased by a Killer 殺手追殺占士邦 (10 marks)

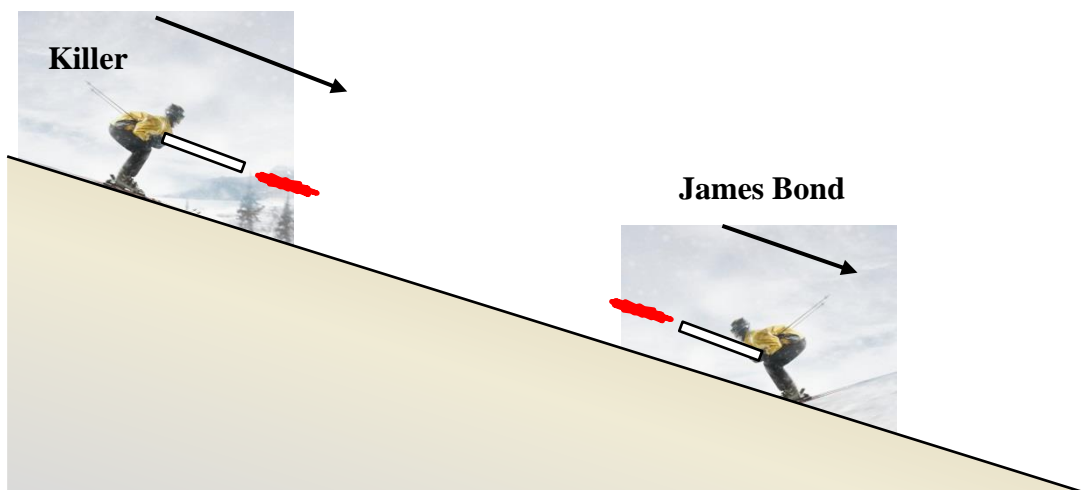
In James Bond movie “The Spy Who Loved Me” (鐵金剛勇破海底城), James skied down a snowy slope in an attempt to escape from the killer. Unfortunately, the killer had a higher skiing speed than James’s; they are 45 m/s and 40 m/s respectively. Now consider an alternative version of the movie. James noted that he and the killer carried the same kind of rifle and estimated that their masses  $M$  (including body mass, skis, weapon, backpack, etc) were about the same. Recalling high school physics knowledge, James realised that each time he fired his weapon back at the killer, his momentum would change; whereas when the killer fired, the killer’s momentum would also change. Every time James fired a bullet, the killer would fire back accordingly.

- (a) How many bullets James had to fire in order to assure that the killer couldn’t catch up with him? Assume that all bullets missed their targets (otherwise this exercise would terminate). Given  $M = 100$  kg, mass and the muzzle velocity of a bullet are  $m = 0.02$  kg and  $v = 500$  m/s respectively.
- (b) Traveling with the final velocity obtained in part (a), James Bond escaped by sliding down a cliff at an inclination of  $20^\circ$ . After 20 seconds, he opened his parachute and landed safely. The average air drag during his fall is 600 N in both the vertical and horizontal directions, as long as the velocity components are nonzero. Calculate the landing position of James Bond and the height of the cliff. (Neglect the distance he traveled with a parachute.)

在占士邦電影“鐵金剛勇破海底城”裡，占士邦滑下雪坡，企圖逃離殺手的追殺。不幸殺手的滑速比占士邦高，分別為 45 m/s 和 40 m/s。現考慮電影的另一版本。

占士邦留意到殺手和他採用同樣的手槍，又推測兩人的質量  $M$  (包括身體、滑雪裝備、武器、背包等) 相若。他想起中學學到的物理學，理解到每當他向對方開火，他的動量就會改變；同樣當對方開火，對方的動量也會改變。每次占士邦發射一顆子彈，對方也同樣回射一顆。

- (a) 占士邦需要發射多少顆子彈，才能保證殺手不會追上他？設  $M = 100$  kg，子彈的質量為  $m = 0.02$  kg，手槍射擊速度為  $v = 500$  m/s。
- (b) 占士邦以(a)部的最終速度，滑下傾角為  $20^\circ$  的懸崖，成功逃脫。20 秒後，他打開降傘，安全著陸。設他在下墜過程中，垂直方向和水平方向的空氣阻力平均值均為 600 N(只要該速度的分量為非零)，計算占士邦著陸的位置和懸崖的高度。(可忽略降傘滑翔的距離。)



## 2. The Bicycle 自行車 (10 marks)

A student rides a bicycle on a slope of inclination  $\theta$ . Due to air drag, he found that the bicycle can barely move down the slope without his pedaling. He would like to estimate the power he needs to drive the bicycle up the same slope at a uniform velocity.

To achieve this, he measured that during the up-slope drive, one of his feet pedaled  $N$  cycles in a time interval  $T$  (assuming that the pedaling is continuous and at a uniform rate). He also obtained the following data: the total mass of the bicycle and the rider  $m$ , length of pedal crank  $L$ , radius of gear 1  $R_1$ , radius of gear 2  $R_2$ , radius of rear wheel  $R_3$ , as shown in the figure.

It is given that the air drags during the up-slope and down-slope drives have the same magnitude, and there are no slippings between the wheels and the slope during both the up-slope and down-slope drives. The energy loss due to the relative motion of the bicycle components is negligible.

(a) Derive an expression for the force needed to drive the bicycle up-slope at uniform velocity.

(b) Derive an expression for the power needed to drive the bicycle up-slope at uniform velocity.

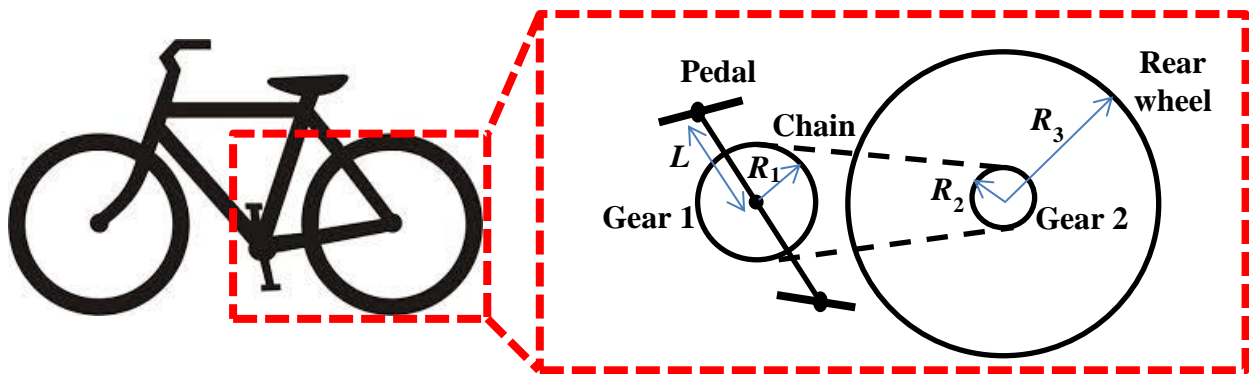
某同學選了一個傾角為  $\theta$  的斜坡，因有風阻，他騎在自行車上剛好能在不踩踏板的情況下讓自行車沿斜坡勻速向下行駛。現在他想估測沿此斜坡向上勻速行駛時的功率。

為此，他數出在上坡過程中某一隻腳蹬踏板的圈數  $N$  (設不間斷的勻速蹬踏)，並測得所用的時間  $T$ ，再測得下列相關資料：自行車和人的總質量  $m$ 、踏板桿的長度  $L$ 、輪盤半徑  $R_1$ 、飛輪半徑  $R_2$ 、車後輪半徑  $R_3$ ，如圖所示。

已知上、下坡過程中的風阻大小相等，不論是在上坡還是下坡過程中，車輪與坡面接觸處都無滑動。不計自行車內部各部件之間因相對運動而消耗的能量。

(a) 試導出駕駛自行車勻速上坡所需作用力的表達式。

(b) 試導出估測功率的表達式。



## 3. The Ships 輪船 (10 marks)

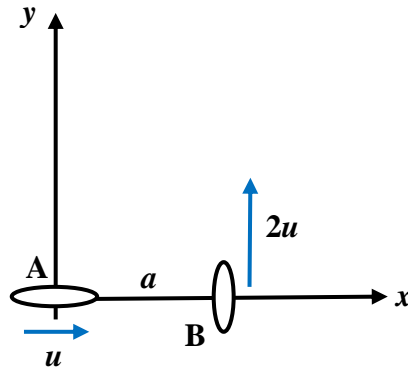
Consider two ships on the sea as shown in the figure. Ship A moves with velocity  $u$  directed to East. Ship B moves with velocity  $2u$  directed to North. At time  $t = 0$ , ship B crosses the path of ship A at a distance  $a$  in front of ship A.

(a) Find the shortest distance between the ships, and the time they reaches this position.

(b) Suppose at  $t = 0$ , the speed of ship A remains at  $u$ , but its direction can be adjusted. What should be the direction of ship A such that the shortest distance between the two ships is minimum, and how much is this minimum shorter than the result obtained in part (a)?

如圖所示，海面上有兩艘輪船，船 A 以速度  $u$  向正東方向航行，船 B 以速度  $2u$  向正北方向航行。在時刻  $t = 0$  時，船 B 恰好同時經過船 A 的航線並位於船 A 的前方，船 B 到船 A 的距離為  $a$ 。

- (a) 求兩船最接近的距離，和到達這距離的時刻。  
 (b) 假如在時刻  $t = 0$ ，船 A 的速率仍是  $u$ ，但方向可以自由調節，那麼船 A 應走什麼方向，兩船最接近的距離才是最短，這最短距離比(a)部得出的距離短多少？



#### 4\*. STEP (15 marks)

According to Newton's second law of motion,  $F = m_I a$ , where  $m_I$  is the *inertial mass*. According to Newton's law of universal gravitation, the gravitational force between Earth and an object is  $F = GM_E m_G / R^2$ , where  $m_G$  is the *gravitational mass* of the object (here  $R$  is the distance between Earth's center and the object). Presently, it is widely accepted that  $m_G = m_I$ , but some physicists would like to test the validity of this assumption. If there is a difference between  $m_I$  and  $m_G$ , even as small as one part in  $10^{18}$ , our present understanding about gravity has to be revised. Hence they proposed a satellite experiment called STEP to measure the mass ratio  $r = m_G/m_I$ . (STEP represents Satellite Test of the Equivalence Principle.)

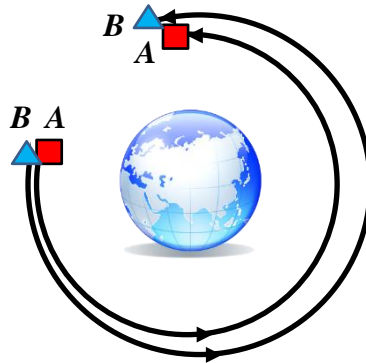
In the proposed experiment, several test bodies are enclosed in a vacuum box in a satellite that orbits around the Earth. The box protects the test bodies from outside disturbances and all forces from the satellite acting on the test bodies have been carefully eliminated, so that each test body can be considered as a mini-satellite orbiting around the Earth.

- (a) The proposed satellite has a circular orbit with a period of 24 hours. Calculate its orbital radius  $R$ . Express your answer in multiples of  $R_E$ , the radius of Earth.  
 (b) Consider test bodies A and B with mass ratios  $r_A$  and  $r_B$  respectively, as shown in the figure. Suppose the two bodies have the same position at a point on the orbit. When body A completes one orbit, what is the displacement of body B relative to body A?  
 (c) Simplify your result in part (b) using the approximation  $(1 + x)^n \approx 1 + nx$  when  $|x| \ll 1$ . Suppose the position sensors in the satellite can detect position changes as little as  $10^{-15}$  m. What is the duration of the satellite flight before differences in the mass ratio of the order  $10^{-18}$  can be detected? Express your answer in hours.

根據牛頓運動第二定律， $F = m_I a$ ，其中  $m_I$  是慣性質量。根據牛頓萬有引力定律，地球作用於物體的引力是  $F = GM_E m_G / R^2$ ，其中  $m_G$  是引力質量（這裡  $R$  是地球中心與物體的距離）。現時普遍接納  $m_G = m_I$ ，但有物理學家希望驗證這假設的真確性。如果  $m_I$  與  $m_G$  有差別，即使是  $10^{18}$  分之一那麼微小，我們對萬有引力的理解也要改寫。所以他們提出名為 STEP 的衛星實驗以量度質量比  $r = m_G/m_I$ 。（STEP 代表 Satellite Test of the Equivalence Principle。）

在提出的實驗中，環繞地球的衛星中有一真空箱，數個實驗物封在其中。箱子保護實驗物免受外力干擾，衛星作用在實驗物上的所有作用力都被仔細隔絕，使每一實驗物都可考慮成環繞地球的袖珍衛星。

- (a) 提出的衛星軌道為圓形，周期為 24 小時。試計算其軌道半徑  $R$ 。答案應以地球半徑  $R_E$  的倍數為單位。
- (b) 考慮實驗物  $A$  和  $B$ ，質量比分別為  $r_A$  和  $r_B$ ，如圖所示。設兩物在軌道某點位置相同。當物體  $A$  完成一周，物體  $B$  相對於物體  $A$  的位移是多少？
- (c) 試用  $|x| \ll 1$  時的近似  $(1+x)^n \approx 1+nx$  簡化(b)部的結果。設衛星上的位置感應器可檢測到小至  $10^{-15}$  m 的位移。衛星航行多久，才能檢測到  $10^{-18}$  量階的質量比？答案應以小時為單位。



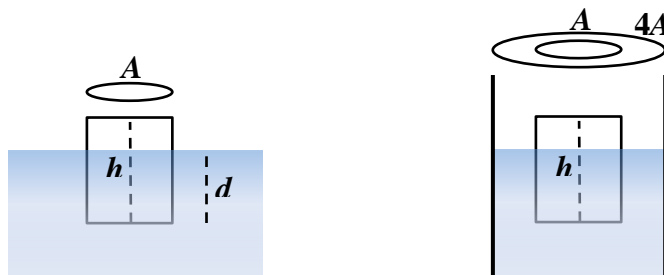
### 5. The Floating Ice 浮冰 (15 marks)

As shown in the left figure, a cylindrical piece of ice floats in water. Its cross sectional area is  $A$  and its height is  $h$ . The density of ice and water are  $\rho_I$  and  $\rho_W$  respectively.

- (a) Find  $d$ , the depth of ice immersed in water.
- (b) Suppose the ice is pushed slightly in the vertical direction. Find the frequency of oscillations. You may assume that the motion of water is negligible.
- (c) Suppose the ice floats in water in a container of cross-section area  $4A$ , as shown in the right figure. The ice is displaced from equilibrium by  $z$  in the vertical direction. (i) Calculate the change in the total potential energy of the system, up to order  $z^2$ . (ii) Calculate the kinetic energy of the system during the push when the ice moves at velocity  $v$ . You may assume that the water below the bottom level of the ice does not move, and the water above the bottom level of the ice moves with the same velocity. (iii) Hence find the frequency of oscillations.

如左圖所示，一圓柱形的冰塊浮在水里。它的橫切面積是  $A$ ，高度是  $h$ 。冰和水的密度分別為  $\rho_I$  和  $\rho_W$ 。

- (a) 求冰塊浸在水里的深度  $d$ 。
- (b) 設冰塊在垂直方向被輕推，求它的振動頻率。你可假設水的流動可以忽略。
- (c) 如右圖所示，設冰塊浮在容器的水裡，容器的橫切面積為  $4A$ 。冰塊在垂直方向被推，相對於平衡態的位移為  $z$ 。(i) 試計算系統總位能的改變，算至  $z^2$  階。(ii) 試計算當在冰塊被推過程中，冰塊速率為  $v$  時的系統總動能。你可假設低於冰塊底部水平的水不動，而高於冰塊底部水平的水以同一速度流動。(iii) 由此找出系統的振動頻率。



《END 完》