Hong Kong Physics Olympiad 2013
2013 年香港物理奧林匹克競賽

Jointly Organized by

The Hong Kong Academy for Gifted Education
香港資優教育學苑
The Education Bureau of the HKSAR Government
香港特別行政區政府教育局
The Physical Society of Hong Kong
香港物理學會
The Hong Kong University of Science and Technology
香港科技大學

共同舉辦

19 May, 2013
2013 年 5 月 19 日

Rules and Regulations 競賽規則

1. All questions are in bilingual versions. You can answer in either Chinese or English.

   所有題目均爲中英對照。你可選擇以中文或英文作答。

2. The multiple-choice answer sheet will be collected 1.5 hours after the start of the contest. You can start answering the open-ended questions any time after you have completed the multiple-choice questions without waiting for announcements.

   選擇題的答題紙將於比賽開始後一小時三十分收回。若你在這之前已完成了選擇題，你亦可開始作答開放式題目，而無須等候任何宣佈。

3. Please follow the instructions on the multiple-choice answer sheet, and use a HB pencil to write your 8-dight Participant ID Number in the field of “I. D. No.”, and fill out the appropriate circles **fully**. After that, write your English name in the space provided and your Hong Kong ID number in the field of “Course & Section No.”

   請依照選擇題答題紙的指示，用 HB 鉛筆在選擇題答題紙的 “I. D. No.” 欄上首先寫上你的 8 位數字參賽號碼，並把相應寫有數字的圓圈完全塗黑，然後在適當的空格填上你的英文姓名，最後於 “Course & Section No. ” 欄內填上你的身分證號碼。

4. After you have made the choice in answering a multiple choice question, fill the corresponding circle on the multiple-choice answer sheet **fully** using a HB pencil.

   選定選擇題的答案後，請將選擇題答題紙上相應的圓圈用 HB 鉛筆完全塗黑。

5. On the cover of the answer book, please write your Hong Kong ID number in the field of "Course Title", and write your English name in the field of "Student Name" and your 8-dight Participant I. D. Number in the field of "Student Number". You can write your answers on both sides of the sheets in the answer book.

   在答題簿封面上，請於 Course Title 欄中填上你的身分證號碼；請於 Student Name 欄中填上你的英文姓名；請於 Student Number 欄中填上你的 8 位數字參賽號碼。 答題簿可雙面使用。

6. The information provided in the text and in the figure of a question should be put to use together.

   解題時要將文字和簡圖提供的條件一起考慮。

7. Some open problems are quite long. Read the entire problem before attempting to solve them. If you cannot solve the whole problem, try to solve some parts of it. You can even use the answers in some unsolved parts as inputs to solve the others parts of a problem.

   開放題較長，最好將整題閱讀完後才著手解題。若某些部分不會做，也可把它們的答案當作已知來做其他部分。

The following symbols and constants are used throughout the examination paper unless otherwise specified:
除非特別注明，否則本卷將使用下列符號和常數：

Gravitational acceleration on Earth surface 地球表面重力加速度	g	9.8 m/s^2
Gravitational constant 萬有引力常數	G	6.67×10^{-11} N m^2/kg^2
Radius of Earth 地球半徑	R_E	6378 km
Sun-Earth distance 太陽-地球距離 (= 1 Astronomical Unit (AU)) (= 1 天文單位(AU))	r_E	1.5×10^{11} m
Mass of Sun 太陽質量	M_{Sun}	1.99×10^{30} kg
Mass of Earth 地球質量	M_E	5.98×10^{24} kg
Air Density 空氣密度	ρ_0	1.2 kg/m^3
Water Density 水密度	ρ_w	1.0×10^3 kg/m^3

Trigonometric identities:
三角學恆等式:

$$\sin(x+y) = \sin x \cos y + \cos x \sin y$$

$$\cos(x+y) = \cos x \cos y - \sin x \sin y$$

$$\sin 2x = 2\sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\sin x \cos y = \frac{1}{2}\left[\sin(x+y) + \sin(x-y)\right]$$

$$\cos x \cos y = \frac{1}{2}\left[\cos(x+y) + \cos(x-y)\right]$$

$$\sin x \sin y = \frac{1}{2}\left[\cos(x-y) - \cos(x+y)\right]$$

Multiple Choice Questions

(Select one answer in each question. For each question, 2 marks for correct answer, 0 mark for no answer, minus 0.25 mark for wrong answer, but the lowest mark of the multiple choice section is 0 mark.)

選擇題（每題選擇一個答案，每題答對 2 分，不答 0 分，答錯扣 0.25 分，但全部選擇題最低為 0 分。）

1. A massive rope of mass m and length L, as shown in the figure, rests on a horizontal table. If the coefficient of static friction between the table and the rope is μ_s, what fraction of the rope can hang over the edge of the table without the rope sliding?
如圖所示，一根質量為 m、長度為 L 的繩纜，靜放在水平桌面上。若繩纜與桌面的靜止摩擦係數為μ_s，則懸垂於桌緣外的繩纜，可以達到全長的什麼分數，而繩纜仍然不至滑下？

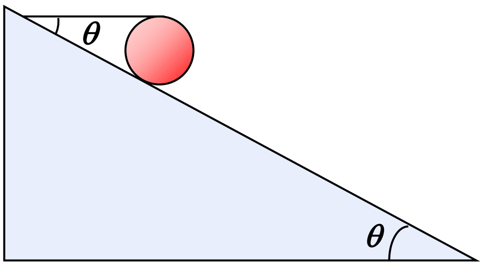
A. $\dfrac{\mu_s}{1+\mu_s}$ B. $\dfrac{\mu_s}{1+2\mu_s}$ C. $1 - \mu_s$ D. $\sqrt{1+\mu_s}$ E. $\dfrac{2}{3}\mu_s$

Solution:
Assume that a length rL is on the table, so the length $(1 - r)L$ is the part of the rope which hangs over the edge of the table.
The tension in the rope at the edge of the table is then $(1 - r)mg$, and the friction force on the part of the rope on the table is $f = \mu_s\,(rmg)$. This must be the same as the tension in the rope at the edge of the table, so $(1 - r)mg = \mu_s(rmg)$ and $r = 1/(1+ \mu_s)$.
The fraction that hangs over the edge is

$$\frac{(1-r)L}{L} = \frac{\mu_s}{1+\mu_s}$$

Note: the result is independent of L and mg of the rope.
Answer: A.

2. As shown in the figure, a rigid sphere of mass m and radius R is held at rest by a horizontal string on an inclined plane with an inclination θ. If the sphere does not move, what is the minimum coefficient of static friction μ_s, between the sphere and the incline?
如圖所示，有質量為 m、半徑為 R 的球體，被一平行方向的繩索固定於傾角為θ的斜面上。若球體不動，則球體與斜面之間的靜止摩擦係數，最小值應是什麼？



A. $\tan\theta$ B. $\dfrac{1-\cos\theta}{1+\cos\theta}$ C. $\dfrac{\cos\theta}{1+\cos\theta}$ D. $\dfrac{\sin\theta}{1+\cos\theta}$ E. $1 - \cos\theta$

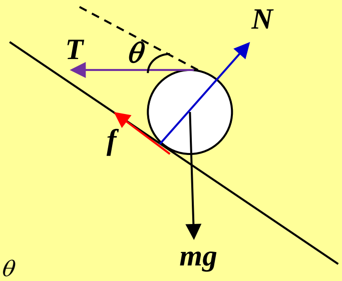
Solution:
Consider the torques about an axis through the center of the sphere:
$TR - fR = 0 \;\Rightarrow\; T = f$

Apply $\Sigma F_x = 0$ to the sphere:

$f + T\cos\theta - mg\sin\theta = 0$

$f(1+\cos\theta) = mg\sin\theta$

$f = \dfrac{mg\sin\theta}{1+\cos\theta}$



Similarly, apply $\Sigma F_y = 0$ to the sphere:

$N - mg\cos\theta - T\sin\theta = 0$

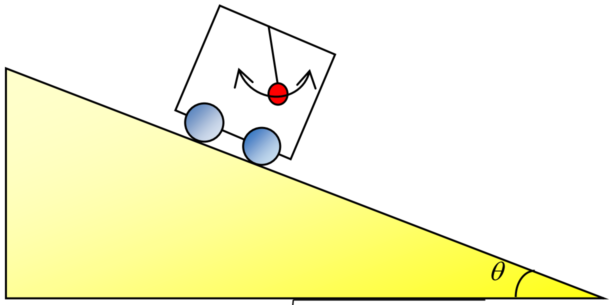
$N = mg\cos\theta + f\sin\theta = mg\cos\theta + \left(\dfrac{mg\sin^2\theta}{1+\cos\theta}\right) = mg\cdot\dfrac{\cos\theta+\cos^2\theta+\sin^2\theta}{1+\cos\theta}$

$= mg\cdot\dfrac{\cos\theta+1}{1+\cos\theta} = mg$

$\therefore\ f \le \mu_s N \quad \Rightarrow \mu_s \ge \dfrac{f}{N} = \dfrac{mg\sin\theta}{1+\cos\theta} \div mg = \dfrac{\sin\theta}{1+\cos\theta}$

Answer: D.

3. A simple pendulum of length L is mounted in a massive cart that slides down a plane inclined at an angle θ with the horizontal. Find the period T of small oscillations of this pendulum if the cart moves down the plane with acceleration $a = g\sin\theta$.
一部具相當質量的車上懸有一長度為 L 的簡單鐘擺，車子滑下傾角為 θ 的斜面。當車子以加速度 $a = g\sin\theta$ 在斜面向下運動時，求鐘擺以小幅度振動的周期 T。

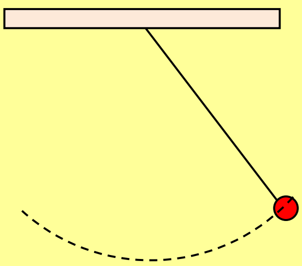


A. $2\pi\sqrt{\dfrac{L}{g\cos\theta}}$ B. $2\pi\sqrt{\dfrac{L}{g\sqrt{2(1+\sin\theta)}}}$ C. $2\pi\sqrt{\dfrac{L\sin\theta}{g\cos\theta}}$

D. $2\pi\sqrt{\dfrac{L}{g\sqrt{1+3\sin^2\theta}}}$ E. $2\pi\sqrt{\dfrac{L}{g(1-\sin\theta)}}$

Solution:

The cart accelerates down the plane with a constant acceleration of $g\sin\theta$. This happens because the cart is much more massive than the bob, so the motion of the cart is unaffected by the motion of the bob oscillating back and forth. The path of the bob is quite complex in the reference frame of the inclined plane, but in the reference frame moving with the cart the path of the bob is much simpler—in this frame the bob moves back and forth along a circular arc.



To find the magnitude of effective gravity acting on the bob, g_{eff}, first we draw the vector addition diagram so as to enable us to have a clear picture of the motion of the bob with respect to the cart.

Using the cosine rule, g_{eff} is given by

$$g_{\text{eff}}^2 = g^2 + g^2\sin^2\theta - 2(g)(g\sin\theta)\cos\left(\frac{\pi}{2}-\theta\right)$$

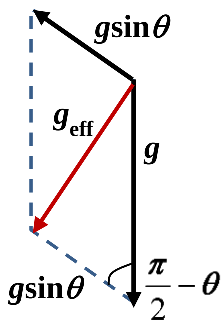
$$= g^2 + g^2\sin^2\theta - 2g^2\sin^2\theta = g^2\cos^2\theta$$

Thus, $g_{\text{eff}} = g\cos\theta$

The period of this motion is $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g_{\text{eff}}}}$

Substituting $g\cos\theta$ for g_{eff} in the equation, we have

$$T = 2\pi\sqrt{\frac{L}{g\cos\theta}}$$



Answer: A.

4. A stone of mass M is tied to a string. It is whirled in a vertical circle of radius L. At the highest point of the circle, the speed of the stone is v. Find the tension of the string.
一塊質量為 M 的石子繫於一根繩子上。石子在空中以半徑為 L 的垂直圓圈旋轉。在圓圈的最高點，石子的速率是 v。求繩子的張力。

A. $M\left(g+\frac{v^2}{L}\right)$ B. $M\left(g+\frac{v^2}{2L}\right)$ C. $M\left(g-\frac{v^2}{L}\right)$ D. $M\frac{v^2}{L}$ E. $M\left(\frac{v^2}{L}-g\right)$

Solution:
Consider forces acting on the stone. Using Newton's second law,

$$T + Mg = M\frac{v^2}{L} \Rightarrow T = M\left(\frac{v^2}{L}-g\right)$$

Answer: E.

5. A ring with mass m is hung vertically at the lower end of a uniform chain of total mass m and length L. Its upper end A is fixed, as shown in figure (a). The lower end B is raised until it is at the same position as A, and the ring slides to the midpoint of the string, as shown in figure (b). What is the minimum work required in this process?
有質量為 m 的小環，繫於一質量為 m、長度為 L 的均勻繩索的下端。繩索的上端固定，如圖(a)所示。繩索的下端被提到上端同樣高度，而小環滑到繩索的中點，如圖(b)所示。這過程所需的功，最小是多少？

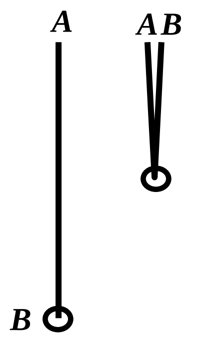


Fig. (a) Fig. (b)

A. mgL B. $\frac{3}{4}mgL$ C. $\frac{1}{2}mgL$ D. $\frac{1}{4}mgL$ E. $\frac{3}{2}mgL$

Solution:
Consider the center of mass of the chain. Its height increases by $L/2 - L/4$.
Consider the center of mass of the bob. Its height increases by $L - L/2$.
Total work done = increase in potential energy

$$= mg\left(\frac{L}{2}-\frac{L}{4}\right) + mg\left(L-\frac{L}{2}\right) = \frac{3}{4}mgL$$

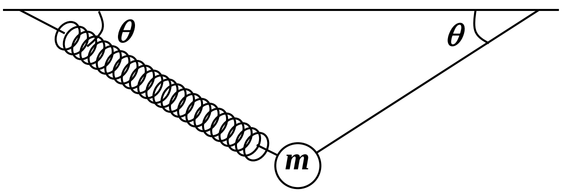
Answer: B.

6. In a spacecraft orbiting around the Earth, an astronaut has a feeling of weightlessness. In Earth's reference frame, the explanation is
   A. the weight of the astronaut becomes zero.
   B. the gravitational field inside the spacecraft becomes zero.
   C. the net force acting on the astronaut becomes zero.

D. the astronaut is falling freely.
E. there is no change in the momentum of the astronaut.
太空船環繞地球運動，其中的太空人感到沒有重量。在地球的參照系內，這現象的解釋是
A. 太空人的重量變為零。
B. 太空船內的引力場變為零。
C. 作用於太空人身上的力，總和為零。
D. 太空人在自由下墜。
E. 太空人的動量沒有改變。

Answer: D.

7. In the figure, a mass m is hung by a light spring and a light string at the ceiling. Both the spring and the string make an angle θ with the horizontal at equilibrium. If the string is suddenly cut, what is the instantaneous acceleration of the mass m?



如圖所示，有質量為 m 的物體，被輕量的彈簧和輕量的繩索繫於天花板上。在平衡狀態下，彈簧和繩索與水平方向都成角度 θ。若繩索突然斷了，物體的瞬時加速度是多少？

A. $\dfrac{2g}{\sin\theta}$ B. $\dfrac{g}{\sin\theta}$ C. $\dfrac{g}{2\sin\theta}$ D. $\dfrac{g}{\cos\theta}$ E. $\dfrac{g}{2\cos\theta}$

Solution:
Consider the balance of the vertical forces.

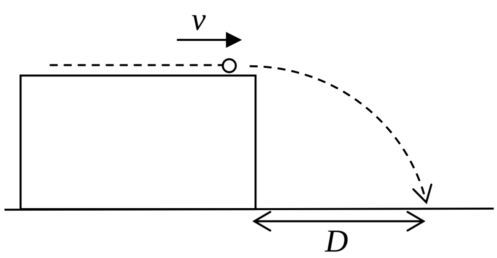
$$2T\sin\theta = mg \quad \Rightarrow \quad T = \frac{mg}{2\sin\theta}$$

When the string is suddenly cut, the mass accelerates due to the gravitational force and the tension in the spring. When these two forces are added, the result is minus the tension of the string. Using Newton's second law,

$$\frac{mg}{2\sin\theta} = ma \quad \Rightarrow \quad a = \frac{g}{2\sin\theta}$$

Answer: C.

8. A particle is projected horizontally from the edge of a smooth table with initial speed v. The particle hits the ground at a horizontal distance D from the table. Different values of v are used and the corresponding values of D are recorded. Which of the following pair of quantities will give a straight-line curve?
有粒子從平滑桌子的邊緣以初速 v 拋射向地面，粒子在水平距離 D 處著地。不同的 v 值得到不同的 D 值。下面哪一對數量具有線性關係？



A. v and D B. v^2 and D C. v and D^2

D. v and $\dfrac{1}{D}$ E. v and $\dfrac{1}{\sqrt{D}}$

Solution:

The time of flight of the projectile is given by $h = \frac{1}{2}gt^2 \quad \Rightarrow \quad t = \sqrt{\frac{2h}{g}}$

Hence $D = vt = v\sqrt{\frac{2h}{g}}$

Answer: A.

9. Which of the following facts is/are direct evidence(s) supporting Newton's first law of motion?
   (1) A feather and a coin spend equal time to reach the ground when dropped from the same height on the Moon surface.
   (2) A satellite orbits around the Earth with uniform speed without supply of fuel.
   (3) A man is thrown forward on a bus which stops suddenly.
   下面哪些事實可以作為牛頓第一定律的直接證據？
   (1) 一根羽毛和一枚硬幣從相同高度掉到月球表面，所需的時間相同。
   (2) 衛星以均速環繞地球運動，不用消耗燃料。
   (3) 當巴士突然停下來，乘客被拋向前。
   A. (3) only B. (1) and (2) only C. (1) and (3) only D. (2) and (3) only E. (1), (2) and (3)

Answer: A.

10. Two masses, m_A and m_B ($m_B > m_A$) are put on a smooth horizontal table as shown. The maximum static friction between the two masses is f. A gradually increasing horizontal force acts on m_A and the two masses accelerate together. The masses start to slip over each other when the force attains F_1. (figure (a)) If initially, the force acts on m_B instead, the masses start to slip over each other when the force attains F_2. (figure (b)) Compare F_1 and F_2.
如圖所示，兩物體，m_A 和 m_B ($m_B > m_A$)，被置於平滑的水平桌子上。兩物體之間的最大摩擦力為 f。有一沿水平方向作用於 m_A 的外力逐漸增加，使兩物體一起加速。當外力達到 F_1 時，兩物體之間產生滑動（圖(a)）。若從起首考慮，外力作用於 m_B 而非 m_A，則當外力達到 F_2 時，兩物體之間產生滑動（圖(b)）。試比較 F_1 和 F_2。
   A. $F_1 > 2F_2$ B. $F_1 > F_2$ C. $F_1 = F_2$ D. $F_1 < F_2$ E. $F_1 < F_2/2$

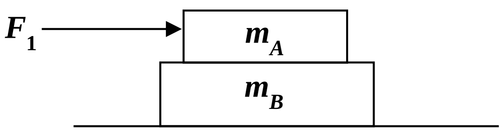


figure (a)

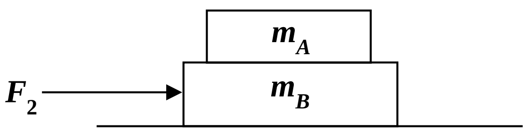


figure (b)

Solution:

In figure (a), $F_1 = (m_A + m_B)a$ and $f = m_B a$ $\Rightarrow$ $f = \frac{m_B}{m_A + m_B} F_1$ $\Rightarrow$ $F_1 = \mu m_A g \left(\frac{m_A + m_B}{m_B} \right)$.

In figure (b), $F_2 = (m_A + m_B)a$ and $f = m_A a$ $\Rightarrow$ $f = \frac{m_A}{m_A + m_B} F_2$ $\Rightarrow$ $F_2 = \mu m_A g \left(\frac{m_A + m_B}{m_A} \right)$.

$\Rightarrow \frac{F_1}{F_2} = \frac{m_A}{m_B} < 1$

Answer: D.

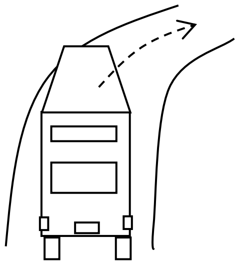
11. Planet P is moving in a circular orbit around a star X, while in another stellar system, planet Q is moving in a circular orbit around a star Y. The orbital radius of P is twice that of Q and the orbital period of P is also twice that of Q. Find the ratio of mass of X to that of Y.
行星 P 以圓形軌道環繞恆星 X，而在另一恆星系，行星 Q 以圓形軌道環繞恆星 Y。P 的軌道半徑為 Q 的兩倍，P 的軌道周期也為 Q 的兩倍。求 X 與 Y 的質量比。
   A. 8:1 B. 4:1 C. 2:1 D. 1:1 E. 1:2

Solution:
Using Newton's second law ad Newton's law of univerasal gravitation, we obtain

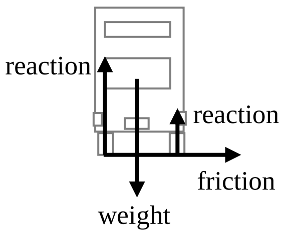
$$T^2 = \frac{4\pi^2}{GM}R^3 \Rightarrow M = \frac{4\pi^2}{G}\frac{R^3}{T^2} \Rightarrow \frac{M_X}{M_Y} = \left(\frac{R_P^3}{R_Q^3}\right)\left(\frac{T_Q^2}{T_P^2}\right) = \left(\frac{2^3}{1^3}\right)\left(\frac{1}{2^2}\right) = 2$$

Answer: C.

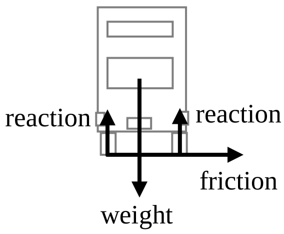
12. A bus is turning around a corner on a horizontal road. The diagram shows the rear view of the bus, which is turning to its right. Which of the following diagram best shows the force diagram of the bus?
有巴士在平路上拐彎，圖中所示為巴士右轉時，從後看的示意圖。下面哪張作用力圖最為正確？



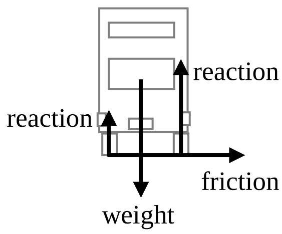
A.



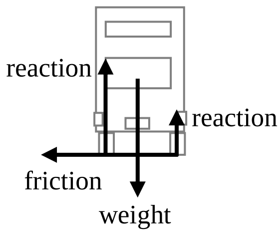
B.



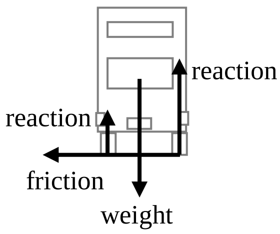
C.



D.



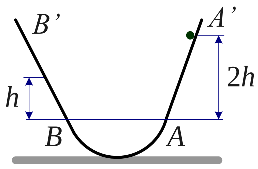
E.



Answer: A.

13. A cylindrical trough is placed on a horizontal plane. The two edges of the trough are connected with smooth inclined planes *AA*' and *BB*' at locations *A* and *B* respectively. The trough edges and the inclined planes are tangential at the connections, and *A*, *B* are located at the same horizontal level, as shown in the figure. A small mass slides freely down the slope *A*'*A* from a height of 2*h* above the horizontal level *AB*. It enters the trough at position *A*, reaches *B* and then slides upward along slope *BB*' until it reaches a height of *h* above the horizontal level *AB*. It then slides downwards along slope *B*'*B* and enters the trough again. Neglecting air resistance, the small mass
一個圓弧形槽的底部放在水平地面上，槽的兩側與光滑斜坡 *AA*'、*BB*'相切，相切處 *A*、*B* 位於同一水平面內，槽與斜坡在豎直平面內的截面如圖所示。一小物塊從斜坡 *A*'*A* 上距水平面 *AB* 的高度為 2*h* 處沿斜坡自由滑下，並自 *A* 處入槽內，到達 *B* 後沿斜坡 *BB*'向上滑行，到達距水平面 *AB* 高度為 *h* 的位置；接著小物塊沿斜坡 *B*'*B* 滑下並從 *B* 處進入槽內。若不考慮空氣阻力，則小物塊

A. reaches a height lower than *A* — A. 到達低於 *A* 的位置
B. just reaches the location *A* — B. 恰好到達 *A* 處
C. passes *A* and reaches a height less than *h* — C. 過 *A* 處後上升高度小於 *h*
D. passes *A* and reaches the height *h* — D. 過 *A* 處後上升高度等於 *h*
E. passes *A* and reaches the height 2*h* — E. 過 *A* 處後上升高度等於 2*h*



Solution:
When the mass slides down from AA’, it loses potential energy mgh and reaches a height of h above AB. If the frictional force remains the same when it slides backwards, then it should lose the same amount of potential energy and hence should reach a height of 0 above AB. However, during the slides, the frictional force is given by

$$f = \mu N = \mu\left(mg + m\frac{v^2}{R}\right)$$

Since the velocity during the second slide is reduced compared with the first slide, the frictional force is reduced, and the potential energy loss is reduced. This enables the small mass to reach a height above A, but the height cannot exceed h.
Answer: C.

14. Consider a satellite of mass m orbiting around the Earth in a circular orbit of radius R_S. Given the radius of the Earth is R_E, and the gravitational acceleration on Earth’s surface is g, and atmospheric resistance can be neglected, then the energy required to launch the satellite from Earth’s surface is
一顆質量為 m 的人造衛星以圓形軌道環繞地球，軌道半徑為 R_S。設地球半徑為 R_E，地面重力加速度為 g，大氣層對衛星的阻力忽略不計，則從地面發射該人造衛星所需能量為

A. $mgR_E\left(1 - \frac{R_E}{2R_S}\right)$
B. $mgR_E\left(1 + \frac{R_E}{2R_S}\right)$
C. $mgR_E\left(\frac{1}{2} - \frac{R_E}{2R_S}\right)$
D. $mgR_E\left(\frac{1}{2} + \frac{R_E}{2R_S}\right)$
E. $mgR_E\left(\frac{R_S}{R_E} - 1\right)$

Solution:
Orbital velocity of the satellite $u = \sqrt{\frac{GM}{R_S}}$

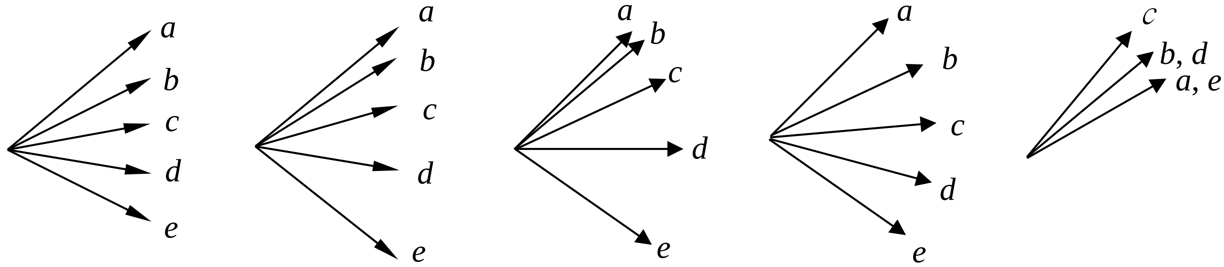
Using the conservation of energy,

$$E - G\frac{Mm}{R_E} = \frac{1}{2}mu^2 - G\frac{Mm}{R_S} \Rightarrow E = G\frac{Mm}{R_E} - G\frac{Mm}{2R_S} = G\frac{Mm}{R_E}\left(1 - \frac{R_E}{2R_S}\right)$$

Since $G\frac{Mm}{R_E^2} = mg$, we have $E = mgR_E\left(1 - \frac{R_E}{2R_S}\right)$
Answer: A.

15. The velocity vectors of an object performing projectile motion are drawn from time instants a to e at fixed time intervals. Which of the following gives a possible drawing?
把一物體在拋射過程中不同時間的速度矢量畫出來，其中時刻 a 到 e 的間隔相同。下面哪張圖是可能的？

A. B. C. D. E.

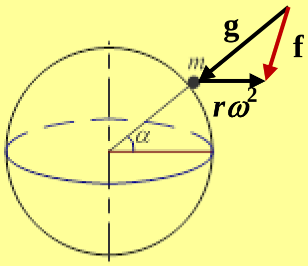


Answer: A.

16. It is known that the gravitational acceleration is $g = GM_E / R_S^2$ and is directed vertically towards Earth's centre. Due to Earth's rotation, the acceleration **f** of a freely falling object in Hong Kong has a magnitude *f* different from *g*, and **f** is no longer pointing vertically downwards. In fact,
已知重力加速度為 $g = GM_E / R_S^2$ 並指向地球中心。因著地球轉動，在香港的自由落體加速度則為 **f**。它的數值 *f* 與 *g* 不同，而 **f** 的方向也不是完全垂直。其實，

A. $f < g$ and **f** has a Northward component　　A. $f < g$ 而 **f** 偏向北
B. $f < g$ and **f** has a Southward component　　B. $f < g$ 而 **f** 偏向南
C. $f > g$ and **f** has a Northward component　　C. $f > g$ 而 **f** 偏向北
D. $f > g$ and **f** has a Southward component　　D. $f > g$ 而 **f** 偏向南
E. $f > g$ and **f** has a Eastward component　　E. $f > g$ 而 **f** 偏向東

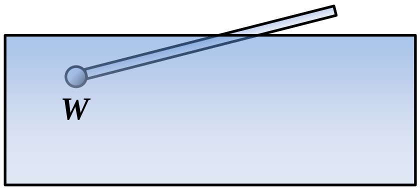
Solution:



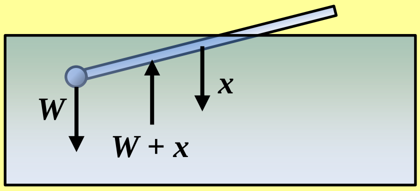
In the reference frame of the rotationg Earth, the centripetal acceleration has to be subtracted from the gravitational acceleration to obtain the free-fall acceleration. From the diagram, we see that $f < g$ and **f** has a Southward component.
Answer: B.

17. A uniform rod floats in water. A ball with weight *W* is attached to one end of the rod, and the volume of the ball is negligible. This structure causes the rod to float at an inclined position, with the other end remaining above the water surface, as shown in the figure. If the part of the rod above the water surface is $1/n$ of the total length, calculate the weight of the rod.
一根均勻桿浮在水中，一端附著重量為 *W* 而體積可忽略不計的小球。這結構令桿子傾斜地浮著，致另一端保持在水面上。若水面上的桿長為全長的 $1/n$，求桿的重量。

A. $W(n + 1)$　　B. Wn　　C. $W(n - 1)$　　D. $W/(n + 1)$　　E. $W/(n - 1)$



Solution:

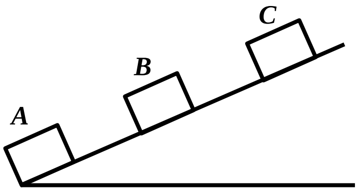


Let *x* be the weight of the rod, and *L* the length of the rod.
Since there are only 3 forces acting on the rod, the buoyancy is $W + x$.
The centre of mass is $L/2$ from the lower end of the rod.
The center of buoyancy is halfway of submegered rod. Hence its distance from the lower end of the rod is $L(1 - 1/n)/2$.
As shown in the figure, considering the torques about the lower end of the rod,

$$(W+x)\frac{L}{2}\left(1-\frac{1}{n}\right)=x\frac{L}{2} \quad \Rightarrow \quad x=W(n-1).$$

Answer: C.

18. A block has an initial kinetic energy of 128 J. It slides up from point *A* at the bottom of the inclined plane with uniform deceleration. When it passes point *B*, its kinetic energy is reduced by 80 J, and its mechanical energy is reduced by 35 J. Calculate the work done against friction when the block moves from *A* to the highest point *C* on the inclined plane.
一滑塊以 128 J 的初動能，從斜面底端的 *A* 點沿斜面向上作勻減速直線運動，它經過 *B* 點時，動能減少了 80 J，機械能減少了 35 J，求滑塊從 *A* 到最高點 *C* 對摩擦力所作的功。
A. 42 J B. 48 J C. 56 J D. 72 J E. 128 J



Solution:
Let $AB = b$. Then the kinetic energy is losing at a rate of $80/b$ Joules per unit length.
At the highest point, the kinetic energy is 0. Hence the distance is $128/(80/b) = 8b/5$.
At the same time, the mechanical energy is losing at a rate of $35/b$ Joules per unit length.
Hence at the highest point, the total mechanical energy loss is $(35/b)(8b/5) = 56$ J.
Answer: C.

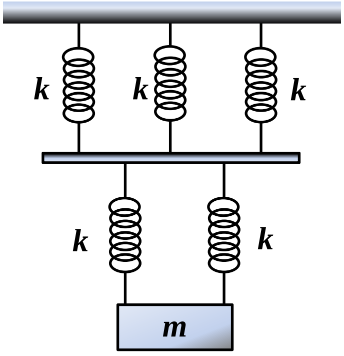
19. A block of mass *m* is placed on a smooth horizontal surface and is attached to a spring of force constant *k*. When the block is pulled sideways and released, it undergoes simple harmonic motion. At the equilibrium position, its velocity is 3 m/s. Calculate its velocity when it moves to the position at two-third of the amplitude from the equilibrium position.
一質量為 *m* 的方塊置於平滑水平面，並繫著彈性係數為 *k* 的彈簧。當方塊被拉向一旁再放鬆時，方塊即進行簡諧運動。方塊在平衡位置時的速度為 3 m/s，求在振幅三分之二處的速度。
A. $\sqrt{2}$ m/s B. $\sqrt{3}$ m/s C. 2 m/s D. $\sqrt{5}$ m/s E. $\sqrt{6}$ m/s

Solution:
At the equilibrium position, the kinetic energy is $K=\frac{1}{2}(m)(3^2)=\frac{9m}{2}$.
Using the conservation of energy, the potential energy at the maximum displaced position is $E=\frac{9m}{2}=\frac{1}{2}kA^2$.
At two-third of the amplitude from the equilibrium position, the potential energy is $U=\frac{1}{2}k\left(\frac{2A}{3}\right)^2=\left(\frac{4}{9}\right)\left(\frac{1}{2}kA^2\right)=\left(\frac{4}{9}\right)\left(\frac{9}{2}m\right)=2m$.
Hence the kinetic energy is $K=E-U=\frac{9m}{2}-2m=\frac{5m}{2} \quad \Rightarrow \quad \frac{1}{2}mv^2=\frac{5m}{2} \quad \Rightarrow \quad v=\sqrt{5}$.
Answer: D.

20. As shown in the figure, a block of mass *m* is hung from the ceiling by the system of springs consisting of two layers. The upper layer consists of 3 springs in parallel, and the lower layer consists of 2 springs in parallel. The force constants of all springs are *k*. Calculate the frequency of the vertical oscillations of the block.

如圖所示，天花板下一組兩層的彈簧懸吊著質量為 m 的方塊。上層有 3 個並排的彈簧，下層則有 2 個並排的彈簧。所有彈簧的彈力常數都是 k。求方塊上下振動時的頻率。

A. $\frac{1}{2\pi}\sqrt{\frac{k}{5m}}$ B. $\frac{1}{2\pi}\sqrt{\frac{4k}{5m}}$ C. $\frac{1}{2\pi}\sqrt{\frac{5k}{6m}}$ D. $\frac{1}{2\pi}\sqrt{\frac{6k}{5m}}$ E. $\frac{1}{2\pi}\sqrt{\frac{5k}{2m}}$



Solution:
Let x_1 = extensions of the springs in the upper layer
x_2 = extensions of the springs in the lower layer
x = displacement of the block
Consider the forces acting on the interface between the upper and lower layers: $3x_1 = 2x_2$

Hence $x_1 = \frac{2}{3}x_2$, $x = x_1 + x_2 = \frac{5}{3}x_2$, $x_2 = \frac{3}{5}x$.

Consider forces acting on the block: $ma = -2kx_2 = -\frac{6}{5}kx \Rightarrow a = -\frac{6k}{5m}x$

Hence $f = \frac{1}{2\pi}\sqrt{\frac{6k}{5m}}$.
Answer: D.

《END OF MC's 選擇題完》

Open Problems 開放題

Total 5 problems 共 5 題

The Open Problem(s) with the '*' sign may require information on page 2.
帶 * 的開放題可能需要用到第二頁上的資料。

1*. James Bond Ski Chased by a Killer 殺手追殺占士邦 (10 marks)

In James Bond movie "The Spy Who Loved Me" (鐵金鋼勇破海底城), James skiied down a snowy slope in an attempt to escape from the killer. Unfortunately, the killer had a higher skiing speed than James's; they are 45 m/s and 40 m/s respectively. Now consider an alternative version of the movie.
James noted that he and the killer carried the same kind of rifle and estimated that their masses M (including body mass, skis, weapon, backpack, etc) were about the same. Recalling high school physics knowledge, James realised that each time he fired his weapon back at the killer, his momentum would change; whereas when the killer fired, the killer's momentum would also change. Every time James fired a bullet, the killer would fire back accordingly.

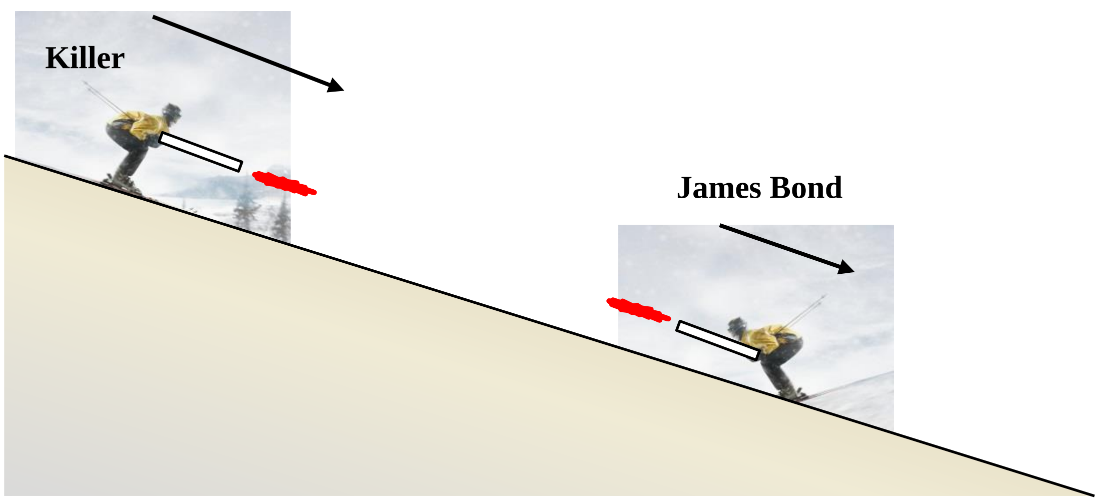
(a) How many bullets James had to fire in order to assure that the killer couldn't catch up with him? Assume that all bullets missed their targets (otherwise this exercise would terminate). Given M = 100 kg, mass and the muzzle velocity of a bullet are m = 0.02 kg and v = 500 m/s respectively.

(b) Traveling with the final velocity obtained in part (a), James Bond escaped by sliding down a cliff at an inclination of 20^o. After 20 seconds, he opened his parachute and landed safely. The average air drag during his fall is 600 N in both the vertical and horizontal directions, as long as the velocity components are nonzero. Calculate the landing position of James Bond and the height of the cliff. (Neglect the distance he traveled with a parachute.)

在占士邦電影"鐵金剛勇破海底城"裡，占士邦滑下雪坡，企圖逃離殺手的追殺。不幸殺手的滑速比占士邦高，分別為 45 m/s 和 40 m/s。現考慮電影的另一版本。
占士邦留意到殺手和他採用同樣的手槍，又推測兩人的質量 M（包括身體、滑雪裝備、武器、背包等）相若。他想起中學學到的物理學，理解到每當他向對方開火，他的動量就會改變；同樣當對方開火，對方的動量也會改變。每次占士邦發射一顆子彈，對方也同樣回射一顆。

(a) 占士邦需要發射多少顆子彈，才能保證殺手不會追上他？設 M = 100 kg，子彈的質量為 m = 0.02 kg，手槍射擊速度為 v = 500 m/s。

(b) 占士邦以(a)部的最終速度，滑下傾角為 20^o 的懸崖，成功逃脫。20 秒後，他打開降傘，安全著陸。設他在下墜過程中，垂直方向和水平方向的空氣阻力平均值均為 600 N(只要該速度的分量為非零)，計算占士邦著陸的位置和懸崖的高度。（可忽略降傘滑翔的距離。）



Solution:

(a) Using the conservation of linear momentum, the change in velocity after shooting a bullet in the backward direction by Jame Bond is given by

$$Mv_i = (M - m)v_f + m(-v + v_i)$$

$$v_f - v_i = \frac{mv}{M - m} \approx \frac{mv}{M} = \frac{(0.02)(500)}{100} = 0.1 \text{ m/s}$$

Similarly, using the conservation of linear momentum, the change in velocity after shooting a bullet in the forward direction by the killer is given by

$$Mv_i = (M - m)v_f + m(v + v_i)$$

$$v_f - v_i = -\frac{mv}{M - m} \approx -\frac{mv}{M} = -\frac{(0.02)(500)}{100} = -0.1 \text{ m/s}$$

Hence after both James Bond and the killer both shoot a bullet, the relative velocity reduces by 0.2 m/s. Since the relative velocity is 45 – 40 = 5 m/s, the number of bullets shot by James Bond is 5/0.2 = 25.

(b) Final velocity of James Bond $= 40 + (0.1)(25) = 42.5 \text{ m/s}$.

Horizontal direction: Acceleration during James Bond's fall: $a = -\frac{600}{100} = -6 \text{ m/s}^2$

Horizontal distance from the cliff: $x = 42.5\cos 20^\circ - (6)(20) = -80.1 \text{ m} < 0$

Hence instead, we have to calculate the horizontal distance up to the point of zero velocity.

$$0^2 - (42.5\cos 20^\circ)^2 = 2(-6)x \quad \Rightarrow \quad x = -\frac{(42.5\cos 20^\circ)^2}{2(-6)} = 133 \text{ m}$$

Vertical direction: Acceleration during James Bond's free fall: $a = \frac{(100)(9.8) - 600}{100} = 3.8 \text{ m/s}^2$

Distance of free fall: $y = 42.5\sin 20^\circ(20) + \frac{1}{2}(3.8)(20^2) = 1{,}051 \text{ m}$

James Bond is 133 m horizontally from the edge of the cliff, and 1,051 m below the cliff.

2. The Bicycle 自行車 (10 marks)

A student rides a bicycle on a slope of inclination θ. Due to air drag, he found that the bicycle can barely move down the slope without his pedaling. He would like to estimate the power he needs to drive the bicycle up the same slope at a uniform velocity.

To achieve this, he measured that during the up-slope drive, one of his feet pedaled N cycles in a time interval T (assuming that the pedaling is continuous and at a uniform rate). He also obtained the following data: the total mass of the bicycle and the rider m, length of pedal crank L, radius of gear 1 R_1, radius of gear 2 R_2, radius of rear wheel R_3, as shown in the figure.

It is given that the air drags during the up-slope and down-slope drives have the same magnitude, and there are no slippings between the wheels and the slope during both the up-slope and down-slope drives. The energy loss due to the relative motion of the bicycle components is negligible.

(a) Derive an expression for the force needed to drive the bicycle up-slope at uniform velocity.

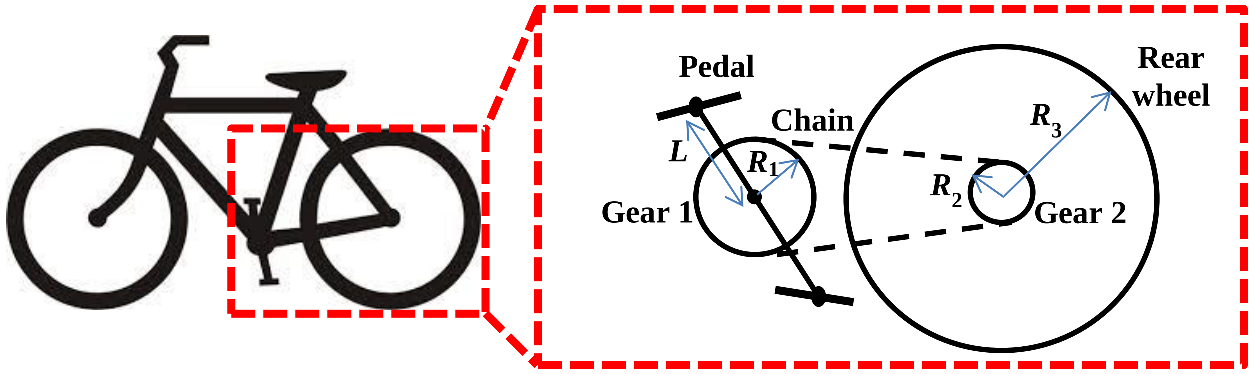
(b) Derive an expression for the power needed to drive the bicycle up-slope at uniform velocity.

某同學選了一個傾角為 θ 的斜坡，因有風阻，他騎在自行車上剛好能在不踩踏板的情況下讓自行車沿斜坡勻速向下行駛。現在他想估測沿此斜坡向上勻速行駛時的功率。

為此，他數出在上坡過程中某一隻腳蹬踩踏板的圈數 N(設不間斷的勻速蹬踏)，並測得所用的時間 T，再測得下列相關資料：自行車和人的總質量 m、踏板桿的長度 L、輪盤半徑 R_1、飛輪半徑 R_2、車後輪半徑 R_3，如圖所示。

已知上、下坡過程中的風阻大小相等，不論是在上坡還是下坡過程中，車輪與坡面接觸處都無滑動。不計自行車內部各部件之間因相對運動而消耗的能量。

(a) 試導出駕駛自行車勻速上坡所需作用力的表達式。
(b) 試導出估測功率的表達式。



Solution:

Down-slope: $f = mg\sin\theta$.

Up-slope: $F = f + mg\sin\theta = 2mg\sin\theta$.

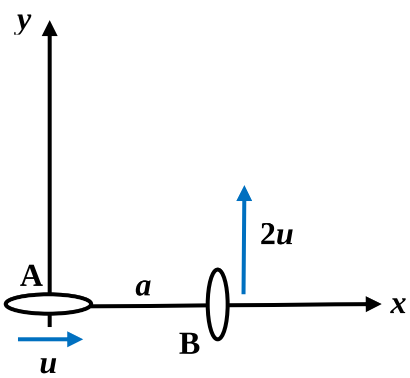
$$\omega = \frac{2\pi N}{t},\; v_1 = R_1\omega = v_2,\; \frac{v_3}{R_3} = \frac{v_2}{R_2},\; v_3 = \frac{v_2 R_3}{R_2} = \omega\frac{R_1R_3}{R_2} = \frac{2\pi N}{t}\frac{R_1R_3}{R_2}$$

$$\therefore\quad P = Fv_3 = \frac{2\pi N}{T}\frac{R_1R_3}{R_2}\cdot 2mg\sin\theta = \frac{4\pi NR_1R_3mg\sin\theta}{TR_2}.$$

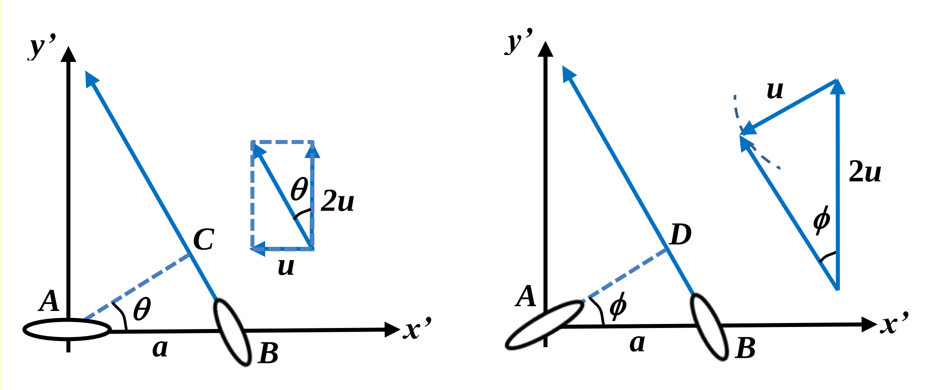
3. The Ships 輪船 (10 marks)

Consider two ships on the sea as shown in the figure. Ship A moves with velocity u directed to East. Ship B moves with velocity $2u$ directed to North. At time $t = 0$, ship B crosses the path of ship A at a distance a in front of ship A.
(a) Find the shortest distance between the ships, and the time they reaches this position.
(b) Suppose at $t = 0$, the speed of ship A remains at u, but its direction can be adjusted. What should be the direction of ship A such that the shortest distance between the two ships is minimum, and how much is this minimum shorter than the result obtained in part (a)?

如圖所示，海面上有兩艘輪船，船 A 以速度 u 向正東方向航行，船 B 以速度 $2u$ 向正北方向航行。在時刻 $t = 0$ 時，船 B 恰好同時經過船 A 的航線並位於船 A 的前方，船 B 到船 A 的距離為 a。
(a) 求兩船最接近的距離，和到達這距離的時刻。
(b) 假如在時刻 $t = 0$，船 A 的速率仍是 u，但方向可以自由調節，那麼船 A 應走什麼方向，兩船最接近的距離才是最短，這最短距離比(a)部得出的距離短多少?



Solution:



(a) As shown in the left figure, the path of ship B is given by the line BC in the reference frame of ship A. The shortest distance is given by the length AC.

Shortest distance $= a\cos\theta = \dfrac{2a}{\sqrt{5}}$

Velocity of ship B relative to ship A $= \sqrt{(2u)^2 + u^2} = \sqrt{5}u$

Time to reach the shortest distance $= \dfrac{a\sin\theta}{\sqrt{5}u} = \dfrac{a}{\sqrt{5}u}\left(\dfrac{1}{\sqrt{5}}\right) = \dfrac{a}{5u}$

(b) As shown in the right figure, to minimize the shortest distance, the velocity of ship B relative to ship A should make an angle with AB as small as possible. Considering variable directions of the velocity of ship A, the tip of this velocity vector generates a circle as shown in the inset of the figure. The best angle of the relative velocity is given by the tangent to the circle. Hence $\phi = \sin^{-1}(1/2) = 30^\circ$.

Shortest distance $= a\cos\phi = \dfrac{\sqrt{3}}{2}a$

Hence to minimize the shortest distance, ship A should move at an angle of 30° North of the East direction. Compared with the result obtained in part (a), the minimum shortest distance is shorter by

$$\frac{2a}{\sqrt{5}} - \frac{\sqrt{3}}{2}a = \frac{4-\sqrt{15}}{2\sqrt{5}} = 0.028\ a$$

4*. STEP (15 marks)

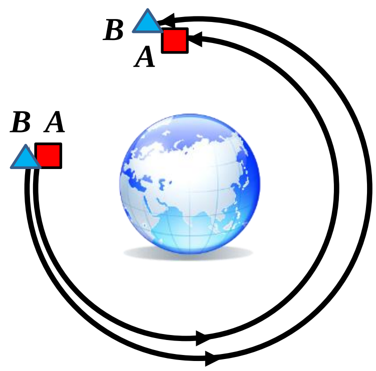
According to Newton's second law of motion, $F = m_I a$, where m_I is the *inertial mass*. According to Newton's law of universal gravitation, the gravitational force between Earth and an object is $F = GM_E m_G / R^2$, where m_G is the *gravitational mass* of the object (here R is the distance between Earth's center and the object). Presently, it is widely accepted that $m_G = m_I$, but some physicists would like to test the validity of this assumption. If there is a difference between m_I and m_G, even as small as one part in 10^{18}, our present understanding about gravity has to be revised. Hence they proposed a satellite experiment called STEP to measure the mass ratio $r = m_G/m_I$. (STEP represents Satellite Test of the Equivalence Principle.)

In the proposed experiment, several test bodies are enclosed in a vacuum box in a satellite that orbits around the Earth. The box protects the test bodies from outside disturbances and all forces from the satellite acting on the test bodies have been carefully eliminated, so that each test body can be considered as a mini-satellite orbiting around the Earth.

(a) The proposed satellite has a circular orbit with a period of 24 hours. Calculate its orbital radius R. Express your answer in multiples of R_E, the radius of Earth.

(b) Consider test bodies A and B with mass ratios r_A and r_B respectively, as shown in the figure. Suppose the two bodies have the same position at a point on the orbit. When body A completes one orbit, what is the displacement of body B relative to body A?
(c) Simplify your result in part (b) using the approximation $(1 + x)^n \approx 1 + nx$ when $|x| << 1$. Suppose the position sensors in the satellite can detect position changes as little as 10^{-15} m. What is the duration of the satellite flight before differences in the mass ratio of the order 10^{-18} can be detected? Express your answer in hours.

根據牛頓運動第二定律，$F = m_I a$，其中 m_I是*慣性質量*。根據牛頓萬有引力定律，地球作用於物體的引力是 $F = GM_E m_G / R^2$，其中 m_G 是*引力質量*（這裡 R 是地球中心與物體的距離）。現時普遍接納 $m_G = m_I$，但有物理學家希望驗證這假設的真確性。如果 m_I與 m_G 有差別，即使是 10^{18} 分之一那麼微小，我們對萬有引力的理解也要改寫。所以他們提出名為 STEP 的衛星實驗以量度質量比 $r = m_G/m_I$。（STEP 代表 Satellite Test of the Equivalence Principle。）
在提出的實驗中，環繞地球的衛星中有一真空箱，數個實驗物封在其中。箱子保護實驗物免受外力干繞，衛星作用在實驗物上的所有作用力都被仔細隔絕，使每一實驗物都可考慮成環繞地球的袖珍衛星。
(a) 提出的衛星軌道為圓形，周期為 24 小時。試計算其軌道半徑 R。答案應以地球半徑 R_E的倍數為單位。
(b) 考慮實驗物 A 和 B，質量比分別為 r_A和 r_B，如圖所示。設兩物在軌道某點位置相同。當物體 A 完成一周，物體 B 相對於物體 A 的位移是多少?
(c) 試用$|x| << 1$時的近似$(1 + x)^n \approx 1 + nx$ 簡化(b)部的結果。設衛星上的位置感應器可檢測到小至 10^{-15} m 的位移。衛星航行多久，才能檢測到 10^{-18} 量階的質量比？答案應以小時為單位。



Solution:
(a) Using Newton's second law of motion and Newton's law of universal gravitation,

$$M\frac{v^2}{R} = \frac{GM_E M}{R^2} \Rightarrow v = \sqrt{\frac{GM_E}{R}} \Rightarrow T = \frac{2\pi R}{v} = 2\pi\sqrt{\frac{R^3}{GM_E}} \Rightarrow T^2 = \frac{4\pi^2}{GM_E}R^3 \Rightarrow$$

$$R = \sqrt[3]{\frac{GM_E T^2}{4\pi^2}} = \sqrt[3]{\frac{(6.67\times 10^{-11})(5.98\times 10^{24})(24\times 3600)^2}{4\pi^2}} = 4.2250\times 10^7 \text{ m} = 6.62 R_E .$$

(b) Using Newton's second law of motion and Newton's law of universal gravitation,

$$m_I\frac{v^2}{R} = \frac{GM_E m_G}{R^2} \Rightarrow v = \sqrt{\frac{GM_E r}{R}} \Rightarrow T_A = \frac{2\pi R}{v_A} = 2\pi\sqrt{\frac{R^3}{GM_E r_A}} .$$

Similarly, $v_B = \sqrt{\frac{GM_E r_B}{R}}$.

Hence the distance moved by body B is $v_B T_A = \left(\sqrt{\frac{GM_E r_B}{R}}\right)\left(2\pi\sqrt{\frac{R^3}{GM_E r_A}}\right) = 2\pi R\sqrt{\frac{r_B}{r_A}}$.

Displacement of body B from body A $= 2\pi R\left(\sqrt{\frac{r_B}{r_A}} - 1\right)$.

Since $r_A \approx r_B \approx 1$, displacement $= \frac{2\pi R}{\sqrt{r_A}}\left(\sqrt{r_B} - \sqrt{r_A}\right) \approx 2\pi R\left\{[1 + (r_B - 1)]^{\frac{1}{2}} - [1 + (r_A - 1)]^{\frac{1}{2}}\right\}$

$\approx 2\pi R\left\{\left[1 + \frac{1}{2}(r_B - 1)\right] - \left[1 + \frac{1}{2}(r_A - 1)\right]\right\} = \pi R(r_B - r_A)$

(c) For differences in the mass ratio of the order 10^{-18}, displacement in one orbit

$= \pi R(r_B - r_A) = \pi(4.2250 \times 10^7)(10^{-18}) = 1.3273 \times 10^{-10}$ m

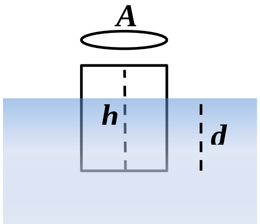
Duration of satellite flight $= 24\left(\frac{10^{-15}}{1.3273 \times 10^{-10}}\right) = 0.000181$ h $= 0.65$ s

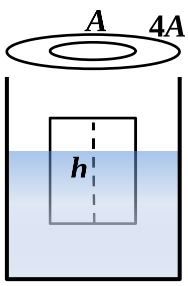
Remark: Hence in principle, the sensor should be sensitive enough to detect this effect. In fact, the main challenge of the experiment is that the sensor is also sensitive to other disturbances, and a lot of effort has to be made to screen out the other disturbances.

5. The Floating Ice 浮冰 (15 marks)

As shown in the left figure, a cylindrical piece of ice floats in water. Its cross sectional area is A and its height is h. The density of ice and water are ρ_I and ρ_W respectively.
(a) Find d, the depth of ice immersed in water.
(b) Suppose the ice is pushed slightly in the vertical direction. Find the frequency of oscillations. You may assume that the motion of water is negligible.
(c) Suppose the ice floats in water in a container of cross-section area $4A$, as shown in the right figure. The ice is displaced from equilibrium by z in the vertical direction. (i) Calculate the change in the total potential energy of the system, up to order z^2. (ii) Calculate the kinetic energy of the system during the push when the ice moves at velocity v. You may assume that the water below the bottom level of the ice does not move, and the water above the bottom level of the ice moves with the same velocity. (iii) Hence find the frequency of oscillations.

如左圖所示，一圓柱形的冰塊浮在水里。它的横切面積是 A，高度是 h。冰和水的密度分別為 ρ_I和ρ_W。
(a) 求冰塊浸在水里的深度 d。
(b) 設冰塊在垂直方向被輕推，求它的振動頻率。你可假設水的流動可以忽略。
(c) 如右圖所示，設冰塊浮在容器的水裡，容器的横切面積為 $4A$。冰塊在垂直方向被推，相對於平衡態的位移為 z。(i) 試計算系統總位能的改變，算至 z^2 階。(ii) 試計算當在冰塊被推過程中，冰塊速率為 v 時的系統總動能。你可假設低於冰塊底部水平的水不動，而高於冰塊底部水平的水以同一速度流動。(iii) 由此找出系統的振動頻率。





Solution:
(a) Using Archimedes' principle, weight of ice = buoyancy = weight of water displaced

$$\rho_I Ahg = \rho_W Adg \quad \Rightarrow \quad d = \frac{\rho_I h}{\rho_W}$$

(b) Since the motion of water is negligible, we only have to consider weight and buoyancy acting on the ice. Using Newton's second law,

$$ma = mg - \rho_W A(d+z)g = -\rho_W Agz \quad \Rightarrow \quad a = -\frac{\rho_W Ag}{m}z = -\frac{\rho_W Ag}{\rho_I Ah}z = -\frac{\rho_W g}{\rho_I h}z = -\frac{g}{d}z$$

Frequency of oscillation: $f = \frac{1}{2\pi}\sqrt{\frac{g}{d}}$

(c) (i) The potential energy change of ice $= -mgz = -\rho_I Ahgz$

Rise in water level $= \frac{Az}{3A} = \frac{z}{3}$

The potential energy change of water is the work done in moving the water from the bottom of the ice to the water surface. The initial center of mass is $d + z/2$ below the equilibrium water surface. The final center of mass is $z/6$ above the water surface. Hence the potential energy change of water

$$= (\rho_W Az)g\left(d + \frac{z}{2} + \frac{z}{6}\right) = \rho_W Ag\left(dz + \frac{2}{3}z^2\right)$$

Total potential energy change of the system $= -\rho_I Aghz + \rho_W Ag\left(dz + \frac{2}{3}z^2\right) = \frac{2}{3}\rho_W Agz^2$

(ii) Kinetic energy of ice $= \frac{1}{2}mv^2 = \frac{1}{2}\rho_I Ahv^2$

The velocity of water $= \frac{Av}{3A} = \frac{v}{3}$

Kinetic energy of water $= \frac{1}{2}(\rho_W 3Ad)\left(\frac{v}{3}\right)^2 = \frac{1}{6}\rho_W Adv^2$

Total kinetic energy of the system $= \frac{1}{2}\rho_I Ahv^2 + \frac{1}{6}\rho_W Adv^2 = \frac{2}{3}\rho_W Adv^2$

(iii) Total energy change of the system $= \frac{2}{3}\rho_W Adv^2 + \frac{2}{3}\rho_W Agz^2$

This is equivalent to the energy of a harmonic oscillator with an effective mass of $m_{\text{effective}} = \frac{4}{3}\rho_W Ad$ and a spring constant of $k_{\text{effective}} = \frac{4}{3}\rho_W Ag$.

Frequency of oscillation: $f = \frac{1}{2\pi}\sqrt{\frac{k_{\text{effective}}}{m_{\text{effective}}}} = \frac{1}{2\pi}\sqrt{\frac{g}{d}}$.

The result is the same as part (b). Hence in this approximation, the frequency of oscillation is independent of the cross-sction area of the container.

《END 完》