Hong Kong Physics Olympiad 2016
2016 香港物理奧林匹克

Organisers 合辦機構

Education Bureau
教育局
The Hong Kong Academy for Gifted Education
香港資優教育學苑
The Hong Kong University of Science and Technology
香港科技大學

Co-organiser 協辦機構

Hong Kong Baptist University
香港浸會大學

Advisory Organisations 顧問機構

The Physical Society of Hong Kong
香港物理學會
Hong Kong Physics Olympiad Committee
香港物理奧林匹克委員會

8 May, 2016
2016 年 5 月 8 日

Rules and Regulations 競賽規則

1. All questions are in bilingual versions. You can answer in either Chinese or English, but only ONE language should be used throughout the whole paper.

   所有題目均爲中英對照。你可選擇以中文或英文作答，惟全卷必須以單一語言作答。

2. The multiple-choice answer sheet will be collected 1.5 hours after the start of the contest. You can start answering the open-ended questions any time after you have completed the multiple-choice questions without waiting for announcements.

   選擇題的答題紙將於比賽開始後一小時三十分收回。若你在這之前已完成了選擇題，你即可開始作答開放式題目，而無須等候任何宣佈。

3. On the cover of the answer book and the multiple-choice answer sheet, please write your 8-digit Contestant Number, English Name, and Seat Number.

   在答題簿封面及選擇題答題紙上，請填上你的 8 位數字參賽者號碼，英文姓名，及座位號碼。

4. After you have made the choice in answering a multiple choice question, fill the corresponding circle on the multiple-choice answer sheet **fully** using a HB pencil.

   選定選擇題的答案後，請將選擇題答題紙上相應的圓圈用 HB 鉛筆完全塗黑。

5. The open problems are long. Please read the whole problem first before attempting to solve them. If there are parts that you cannot solve, you are allowed to treat the answer as a known answer to solve the following parts.

   開放式問答題較長，請將整題閱讀完後再著手解題。若某些部分不會做，也可把它們的答案當作已知來解答其他部分。

The following symbols and constants are used throughout the exam paper unless otherwise specified:
除非特別註明，否則本卷將使用下列符號和常數：

Useful Constants（**常用常數**）：

Astronomical Unit（**天文單位**）：1 AU = 1.496×10^{8} km

Earth-Moon Distance（**地球-月球距離**）：d_M = 384,400 km

Mass of the Sun（**太陽質量**）：$M_S = 1.99\times10^{30}$ kg

Mass of the Earth（**地球質量**）：$M_E = 5.97\times10^{24}$ kg

Mass of the Moon（**月球質量**）：$M_M = 7.35\times10^{22}$ kg

Acceleration due to Gravity（**重力加速度**）：$g = 9.8$ ms^{-2}

Radius of the Sun（**太陽半徑**）：R_S = 696300 km

Radius of the Earth（**地球半徑**）：R_E = 6370 km

Radius of the Moon（**月球半徑**）：R_M = 1738 km

Gravitational Constant（**萬有引力常數**）：$G = 6.67\times10^{-11}$ m^3 kg^{-1} s^{-2}

Trigonometric Identities（**三角學恒等式**）：

$$\sin(x+y)=\sin(x)\cos(y)+\cos(x)\sin(y)$$

$$\cos(x+y)=\cos(x)\cos(y)-\sin(x)\sin(y)$$

$$\sin(2x)=2\sin(x)\cos(x)$$

$$\cos(2x)=\cos^2(x)-\sin^2(x)$$

$$\sin(x)\cos(y)=\frac{1}{2}[\sin(x+y)+\sin(x-y)]$$

$$\cos(x)\cos(y)=\frac{1}{2}[\cos(x+y)+\cos(x-y)]$$

$$\sin(x)\sin(y)=\frac{1}{2}[\cos(x-y)-\cos(x+y)]$$

Tayler Series（**泰勒級數**）：

$$\sin(x)\approx x-\frac{x^3}{6}+\frac{x^5}{120}-\cdots$$

$$\cos(x)\approx 1-\frac{x^2}{2}+\frac{x^4}{24}-\cdots$$

$$\tan(x)\approx x+\frac{x^3}{3}+\frac{2x^5}{15}+\cdots$$

$$\exp(x)\approx 1+x+\frac{x^2}{2}+\frac{x^3}{6}+\cdots$$

Series Summation（**級數總和**）：

$$\sum_{k=1}^{m}k=\frac{m(m+1)}{2}$$

$$\sum_{k=1}^{m}k^2=\frac{m(m+1)(2m+1)}{6}$$

$$\sum_{k=1}^{m}k^3=\left[\frac{m(m+1)}{2}\right]^2$$

$$\sum_{k=1}^{m}k^4=\frac{m(2m+1)(m+1)(3m^2+3m-1)}{30}$$

Hyperbolic Trigonometric Identities（**雙曲三角恒等式**）：

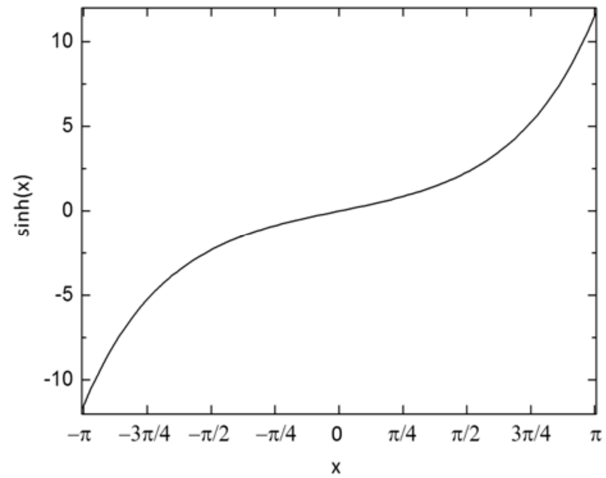
$$\sinh(x)=(e^x-e^{-x})/2$$

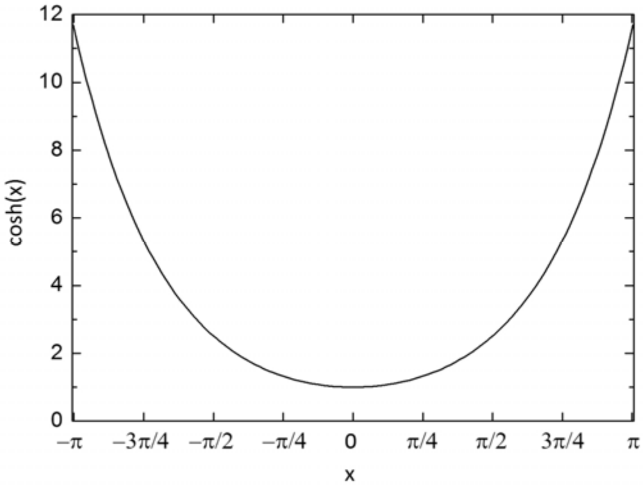
$$\cosh(x)=(e^x+e^{-x})/2$$

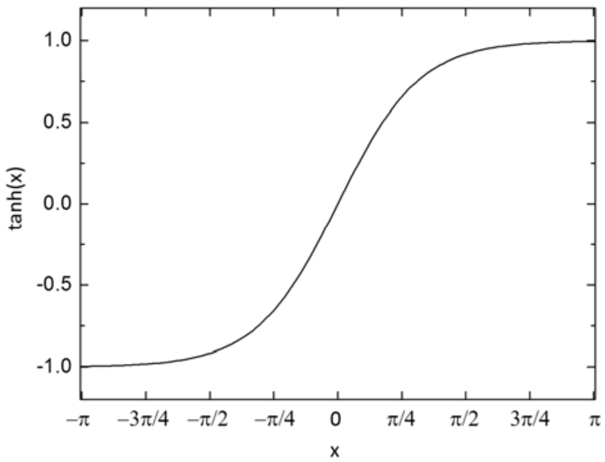
$$\tanh(x)=\sinh(x)/\cosh(x)$$

$$\cosh^2(x)-\sinh^2(x)=1$$

Sketch of Hyperbolic Functions（**雙曲函數示意圖**）：







Multiple Choice Questions

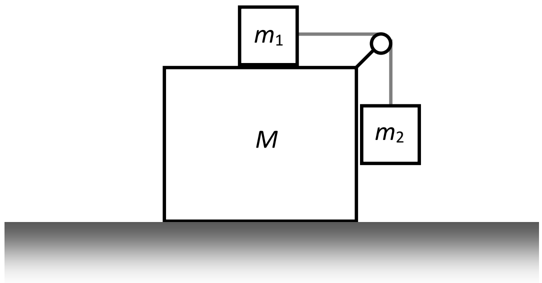
Select one answer in each question. For each question, 2 marks for correct answer, 0 mark for no answer, minus 0.25 mark for wrong answer, but the lowest mark of the multiple choice section is 0 mark.

選擇題

每題選擇一個答案，每題答對 2 分，不答 0 分，答錯扣 0.25 分，但全部選擇題最低為 0 分。

1. The following figure illustrates a pulley system with masses m_1 and m_2, and is supported by a block (mass M) on a horizontal ground. All frictions and inertia of the pulley are negligible, and the rope in the pulley system is massless. Find the horizontal force on block M such that there is no relative motion between m_1, m_2, and M.

   下圖示出了一個由質量 m_1 和 m_2 方塊所組成的滑輪系統。而這個滑輪系統是由另一個放在水平地面的方塊支撐。所有摩擦力和滑輪的轉動慣量可忽略不計，並在滑輪系統中的繩子是無質量。求作用在方塊 M 的水平力，使得 m_1，m_2，和 M 不存在相對運動。



   A. $\dfrac{m_1(M+m_1+m_2)g}{m_2}$

   B. $\dfrac{m_2(M+m_1+m_2)g}{m_1}$

   C. $\dfrac{m_1(M-m_1+m_2)g}{m_2}$

   D. $\dfrac{m_2(M-m_1+m_2)g}{m_1}$

   E. None of the above, because relative motion exists between m_1, m_2, and/or M.
   以上皆不是，因為相對運動存在於 m_1，m_2，與及/或 M 之間。

2. A man can generate a maximum propulsive power of 500 W during cycling. The air resistance has the force of $F = bv$ when the velocity of the bicycle is v, where b is a constant given by 5 N-s/m. Without wind, what is the maximum speed of the bicycle when cycling is conducted on a horizontal ground?

   一個人騎自行車時可以產生最大的推進力為 500 W。當自行車的速度是 v 時，空氣阻力是 $F = bv$，其中，常數 b 是 5 N-s/m。在沒有風的情況下，自行車在水平路面可達到的最高速度是甚麼？

   A. 1 ms^{-1}.
   B. 2 ms^{-1}.
   C. 5 ms^{-1}.
   D. 10 ms^{-1}.
   E. 20 ms^{-1}.

Questions 3 to 4. A spacecraft is orbiting in an elliptical orbit about the Sun with ~~perihelion~~ aphelion at the Earth's orbit, and ~~aphelion~~ perihelion at the Venus' orbit. The orbit equation with the Sun as the origin is given by $r = \lambda(1+\varepsilon)/(1+\varepsilon\cos\theta)$, where r is the distance in AU (Astronomical Unit) from the Sun, ε and λ are the orbital parameters. The mean distances of Venus and the Earth from the Sun are 0.73 AU and 1.0 AU, respectively.

選擇題 3 至 4. 一艘太空船正以橢圓形的軌道環繞太陽。該橢圓軌道之 ~~近日點~~ 遠日點 位於地球之軌道上，而 ~~遠日點~~ 近日點 位於金星之軌道上。以太陽為原點時該軌道的方程是 $r = \lambda(1+\varepsilon)/(1+\varepsilon\cos\theta)$，其中，$r$ 是從太陽起計之天文單位（AU）距離，ε 和 λ 則是軌道參數。金星和地球對太陽的平均距離分別是 0.73 AU 和 1.0 AU。

3. The orbital parameters ε and λ, respectively, are:

   所述的軌道參數 ε 和 λ 分別是：

   A. $\varepsilon = 0.156$, $\lambda = 0.73$ AU.
   B. $\varepsilon = 0.156$, $\lambda = 1.0$ AU.
   C. $\varepsilon = 0.312$, $\lambda = 0.73$ AU.
   D. $\varepsilon = 0.312$, $\lambda = 1.0$ AU.
   E. $\varepsilon = 0.312$, $\lambda = 1.73$ AU

4. What is the period of the spacecraft as mentioned above?

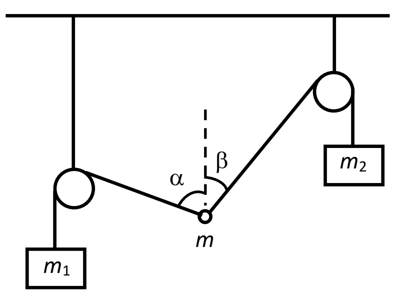
   太空船在上述軌道的週期是多少？

   A. 0.4 year (年) .
   B. 0.8 year (年) .
   C. 1.2 years (年) .
   D. 1.6 years (年) .
   E. 2.0 years (年) .

5. A point mass is hanged by 2 massless ropes, which make angles α and β to the vertical. The ropes are further connected to two masses m_1 and m_2 through frictionless and massless pulleys. Find the ratio m_1/m_2 such that the point mass hangs in equilibrium.

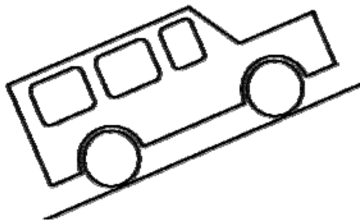
   一質點被 2 根無質量的繩子懸掛，並形成了以垂直計 α 和 β 的角度。繩子更進一步通過無摩擦且無質量的滑輪，並連接到 2 個質量為 m_1 和 m_2 的物體。在平衡狀態時，求 m_1/m_2 之比值。

   A. $\sin(\alpha + \beta)$.
   B. $\cos(\alpha + \beta)$.
   C. $\sin(\alpha) \times \cos(\beta)$.
   D. $\sin(\alpha) / \cos(\beta)$.
   E. $\sin(\beta) / \sin(\alpha)$.



6. A 4-wheel car is moving uphill without slipping, as shown in the figure. If the car is driven by the rear wheels, in which direction(s) does friction(s) act on the rear and the front wheels?

如圖所示，一輛 4 輪汽車在不滑倒的情況下正向上行駛。假設汽車是由後輪驅動，問摩擦力作用在後輪和前輪是何方向？



A. Rear wheels: uphill, front wheels: uphill.
B. Rear wheels: downhill, front wheels: uphill.
C. Rear wheels: uphill, front wheels: downhill.
D. Rear wheels: downhill, front wheels: downhill.
E. Frictions do not act on the wheels because the car is moving at constant speed.

A. 後輪：向上斜坡，前輪：向上斜坡。
B. 後輪：向下斜坡，前輪：向上斜坡。
C. 後輪：向上斜坡，前輪：向下斜坡。
D. 後輪：向下斜坡，前輪：向下斜坡。
E. 汽車以恆定速度移動，因此摩擦力不會存在。

(Remark: Question 7 is cancelled)

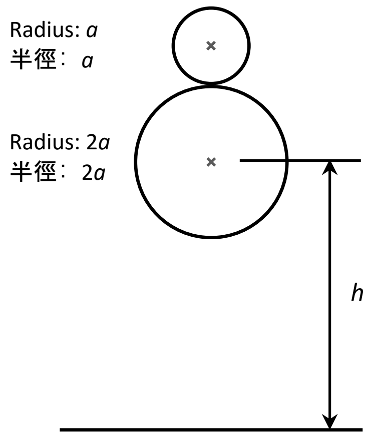
7. Two rigid spheres (uniform masses, zero initial speed, radii a and $2a$) are dropped from a height h (measured from ground to center of the lower sphere) under the influence of gravity. What is the maximum height (measured from ground to sphere center) that the upper sphere will reach? Assume all the collisions are instantaneous.

Hints: i) Air resistance is negligible; ii) Collisions are elastic; iii) Centers of the spheres (marked as "x") always lie on the same vertical line; iv) Lower sphere collides with the ground before it collides with the upper sphere.

在重力作用的影響下，兩個剛體球（均勻質量，起始速度為零，半徑 a 和 $2a$）從高處 h（由地面至較低球中心計）掉下。從地面至球中心，較上球可達到的最高高度為多少？假設所有碰撞是瞬間完成。

提示：i）空氣阻力可忽略不計；ii）碰撞是彈性的；iii）球體的中心處（以"x"為標記）總是在同一垂直線上；iv）較低球先碰撞地面，然後碰撞較上球。

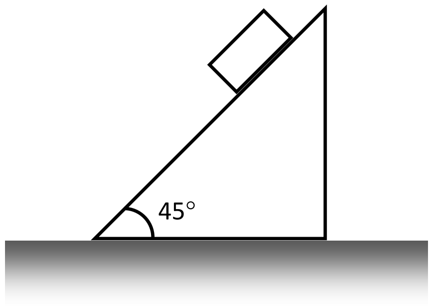
A. $\frac{286}{81}(h-a)$
B. $\frac{286}{81}(h-2a)$
C. $\frac{529}{81}(h-a)$
D. $\frac{529}{81}(h-2a)$
E. $\frac{610}{81}(h-3a)$



8. A block (mass *m*) slides on an inclined plane (mass *M*, angle 45° to the horizontal). Ignoring all the frictions between all the surfaces, find the acceleration of the inclined plane when the block is sliding.

一方塊（質量 *m*）在一斜面（質量 *M*，從水平計角度為 45°）上滑行。在所有表面之間都沒有摩擦力的情況下，求方塊滑行時斜面的加速度。

A. $\left(\frac{m/2}{m+M}\right)g$.
B. $\left(\frac{m}{m+M}\right)g$.
C. $\left(\frac{2m}{m+M}\right)g$.
D. $\left(\frac{m}{2m+M}\right)g$.
E. $\left(\frac{m}{m+2M}\right)g$.

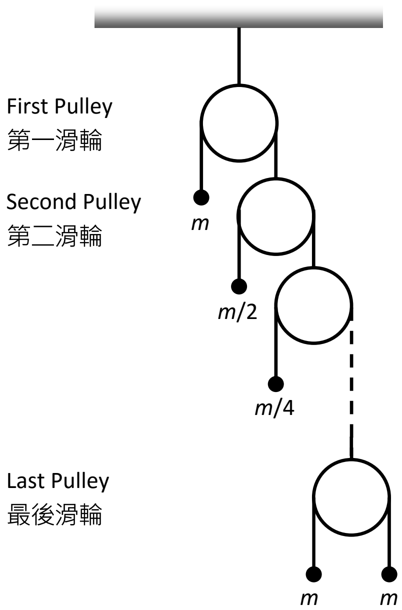


9. An Atwood machine consists of *n* masses (masses *m*, *m*/2, *m*/4, ……, respectively attached on the first, second, third, ……, pulley) and (*n* – 1) pulleys (massless, frictionless, and negligible inertia), where *n* is an integer with $n \geq 100$. At the last pulley, it is attached 2 equal masses (mass *m*). What is the acceleration of the mass attached to the first pulley?

一阿特伍德機由 *n* 個質量物（質量為 *m*，*m/2*，*m/4*，. . . . . . ，分別安裝在第一，第二，第三，. . . . . . ，滑輪上）和（*n* – *1*）滑輪（無質量，無摩擦，及轉動慣量可忽略不計）所組成，其中 *n* 是整數且 $n \geq 100$。在最後的滑輪上，是與 2 個相等質量物（質量 *m*）連接在一起。附在第一滑輪的質量物之加速度為何值？

A. Zero acceleration.
B. g / 4.
C. g / 3.
D. g / 2.
E. Greater than or equal to 2^{100} g.

A. 零加速。
B. g / 4。
C. g / 3。
D. g / 2。
E. 大於或等於 2^{100} g。



10. Consider the following 3 planar circles with identical mass density per unit area of σ. One of the circles is put between and above the other 2 identical circles (radius R). The radii of contact points between the upper circle and the lower circles make an angle θ with the horizontal. Ignoring all the frictions, what is the horizontal force, F, on each side of the lower circle in order to keep them stationary?

以下是 3 個相同質量密度的平面圓，其每單位面積的質量密度是 σ。其中一個圓圈被置於另外 2 個相同的圓圈之上面（半徑 R）。上圓和下圓的半徑接觸點與水平線之間形成一個 θ 的角度。在沒有摩擦力的影響下，為了保持平面圓靜止不動，兩邊在下圓的水平力，F，應該是多少？

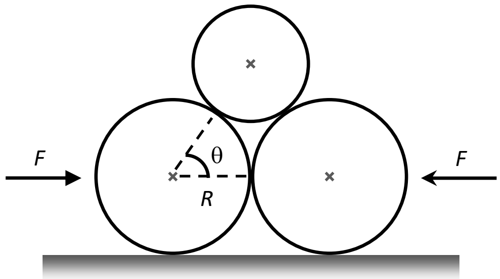
A. $\frac{\sigma\pi R^2}{2\sin\theta}\frac{(1-\sin\theta)^2}{\cos\theta}g$.

B. $\frac{\sigma\pi R^2}{2\sin\theta}\frac{(1-\cos\theta)^2}{\cos\theta}g$.

C. $\frac{\sigma\pi R^2}{2\cos\theta}\frac{(1-\sin\theta)^2}{\sin\theta}g$.

D. Revised to: $\frac{\sigma\pi R^2}{2\cos\theta}\frac{(1-\cos\theta)^2}{\tan\theta}g$

E. $\frac{\sigma\pi R^2}{2(\sin\theta+\cos\theta)}(1-\tan\theta)^2 g$.

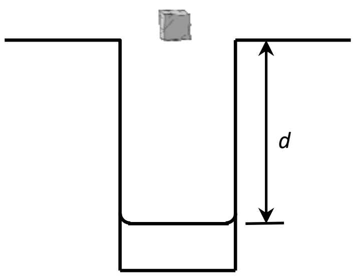


(Remark: Question 11 is cancelled)

11. A stone is dropped from rest into a deep well. After three seconds, a sound is heard when the stone hits the well water. Assume that the speed of sound in air is 340 m/s, and air resistance is negligible; what is the approximate depth (from the top of the well to water surface), d, of the well?

一塊石頭從靜止狀態中被掉入一個深井。三秒之後，當石擊中井水時便聽到聲音。假定聲音在空氣中的傳播速度是 340 m/s，及空氣阻力可忽略不計；井的深度（從井的頂部至水的表面），d，大概是多少？

A. 15 m.
B. 20 m.
C. 25 m.
D. 30 m.
E. 35 m.



12. A sphere (mass m and radius r) is hanged vertically with a massless rope (length L). A wind is exerting constant horizontal force density σ (force per unit area), on the sphere. Without energy loss, what is the maximum height H that the sphere can rise?

一球體（質量為 m，半徑為 r）被一根無質量的繩子（長度 L）懸掛。一陣風在球體上施加了恒定且水平的力密度 σ（即每單位面積之力）。在沒有能量損耗的情況下，該球體可以上升的最高高度 H 是何值？

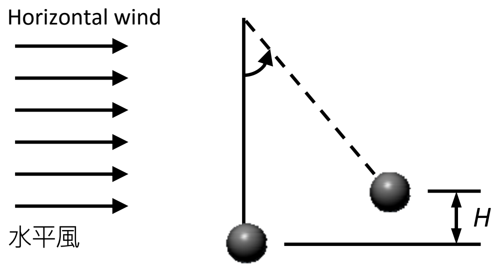
A. $\dfrac{2L\sigma^2\pi^2 r^4}{m^2 g^2 + \sigma^2\pi^2 r^4}$.

B. $\dfrac{32L\sigma^2\pi^2 r^4}{9m^2 g^2 + 16\sigma^2\pi^2 r^4}$.

C. $\dfrac{2L\sigma\pi r^2}{9m^2 g^2 + 16\sigma\pi r^2}$.

D. $\dfrac{32L\sigma\pi r^2}{9m^2 g^2 + 16\sigma\pi r^2}$.

E. $\dfrac{2L^2\sigma^2\pi^2 r^4}{m^2 g^2 + \sigma^2\pi^2 r^4}$



13. A rope (mass m) is hanged between two supports (separation $2d$) at the same level. The shape of the hanging rope can be approximated to the function $y(x) = (1/\alpha)\cosh(\alpha x)$ in an x-y coordinate system, where α is a constant. The slope of the function is $\sinh(\alpha x)$. What is the magnitude of force F at each of the support?

一根繩子（質量 m）在兩個同一水平的支架點（距離 $2d$）之間懸掛。該懸掛繩子的形狀可以被近似地描述為以 x-y 坐標系統中的函數 $y(x) = (1/\alpha)\cosh(\alpha x)$，其中，$\alpha$ 是一個常數。該函數的斜率是 $\sinh(\alpha x)$。問在每一個支架點的力 F 是何值？

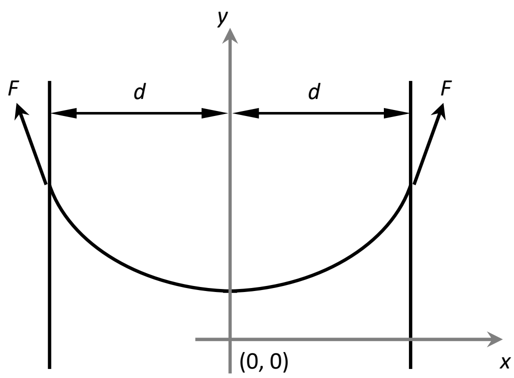
A. $\dfrac{mg}{2\sinh(\alpha d)}$.

B. $\dfrac{mg}{2\cosh(\alpha d)}$.

C. $\dfrac{mg}{2\tanh(\alpha d)}$.

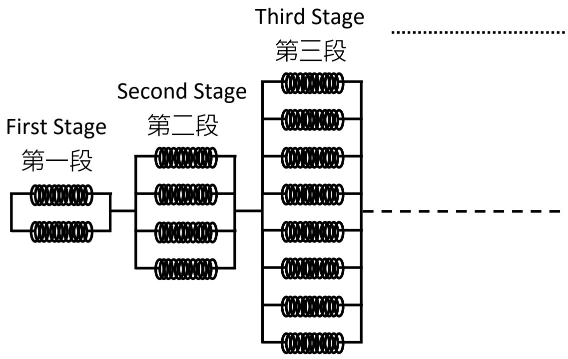
D. $\dfrac{mg\sinh(\alpha d)}{2}$.

E. $\dfrac{mg\cosh(\alpha d)}{2}$.



Questions 14 to 15. A spring system consists of *n* stages, where *n* tends to infinity. The first stage contains 2 springs, and the number of springs in the next stages are a geometric sequence, that is, the second stage contains 4 springs, the third stage contains 8 springs, and so on. All the springs are massless and identical having the same spring constant of *k*.

選擇題 14 至 15. 一個彈簧系統由 *n* 段彈簧所組成，其中 *n* 趨於無窮。第一段包含 2 個彈簧，而在下一段的彈簧數量是以幾何序列，亦即第二段有 4 個彈簧，第三段有 8 個彈簧，等等。所有彈簧均是無質量的，並且具有相同的彈簧常數 *k*。



14. The equivalent spring constant of this spring system is:

    所述的彈簧系統之當量彈簧常數是：

    A. *k*.
    B. 2*k*.
    C. 4*k*.
    D. 8*k*.
    E. 16*k*.

15. If the total elongation of the spring system is 10 cm, what is the elongation in the fourth stage, that is, the stage that contains 16 springs?

    如果彈簧系統的總伸長為 10 cm，問第四段，亦即該段有 16 個彈簧的伸長是何值？

    A. 7.5 cm.
    B. 5.0 cm.
    C. 2.5 cm.
    D. 1.25 cm.
    E. 0.625 cm.

16. Two identical balls are projected up an incline (angle ϕ) with initial speed v_1 at angle θ_1 and v_2 at angle θ_2 with respect to the horizontal, where θ_1 and $\theta_2 > \phi$. The two balls eventually reach the same distance on the incline. The relationship between θ_1, θ_2, v_1, and v_2 is:

兩個相同的球在一個斜坡（角度 ϕ）上被投射。它們被投射的初始條件是當角度為 θ_1 時，速度是 v_1，而當角度為 θ_2 時，速度是 v_2，其中，θ_1 和 $\theta_2 > \phi$。角度都是以水平線起計。這兩個球最終被投射到在斜坡上的相同距離。θ_1，θ_2，v_1，和 v_2 之間的關係式是：

A. $\left(\frac{v_1}{v_2}\right)^2 = \frac{\cos\theta_2 \sin(\theta_2 - \phi)}{\cos\theta_1 \sin(\theta_1 - \phi)}$.

B. $\left(\frac{v_1}{v_2}\right)^2 = \frac{\cos\theta_1 \sin(\theta_1 - \phi)}{\cos\theta_2 \sin(\theta_2 - \phi)}$.

C. $\frac{v_1}{v_2} = \frac{\cos\theta_2 \sin(\theta_2 - \phi)}{\cos\theta_1 \sin(\theta_1 - \phi)}$.

D. $\frac{v_1}{v_2} = \frac{\cos\theta_1 \sin(\theta_1 - \phi)}{\cos\theta_2 \sin(\theta_2 - \phi)}$.

E. $\frac{v_1}{v_2} = \cos\theta_1 \cos\theta_2 \sin(\theta_1 - \phi)\sin(\theta_2 - \phi)$.

17. A helium-filled balloon is held on a horizontal floor of a MTR train by a massless string (length 0.5 m). The train is travelling at constant speed of 105 km/hour and turning a circular path of radius 1000 m. The densities of air and helium gases are 1.2 kg/m^3 and 0.18 kg/m^3, respectively. The angle of the balloon string making with the vertical is:

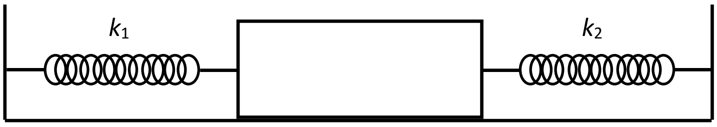
一個氦氣球被一根無質量的繩子（長度 0.5 m）綁緊在地鐵列車的水平地板上。列車以每小時 105 公里的恆定速度行駛在一條半徑為 1000 m 的圓形軌道。空氣和氦氣的密度分別是 1.2 kg/m^3 和 0.18 kg/m^3。氣球繩子與垂直線的角度是：

A. 1°.
B. 5°.
C. 10°.
D. 15°.
E. 18°.

18. A wooden block (mass 1 kg) is connected to 2 massless springs (Spring 1 with spring constant k_1 = 5 N/m; and Spring 2 with spring constant k_2 = 4 N/m), and is oscillating on a frictionless horizontal table with small amplitude. At the equilibrium position (time t = 0 second), the block is moving at 2 m/s towards the Spring 2. The time that the block will take when the Spring 1 has maximum compression is:

一木塊（質量 1 kg）被連接到 2 條無質量的彈簧（彈簧 1 具有彈簧常數 k_1 = 5 N/m; 而彈簧 2 具有彈簧常數 k_2 = 4 N/m），並以小幅度的振幅在一水平及無摩擦力的桌子上擺動。處於平衡位置時（時間 t = 0 秒），該木塊以 2 m/s 移向彈簧 2。當彈簧 1 處於最大壓縮時，木塊所需要的時間是：

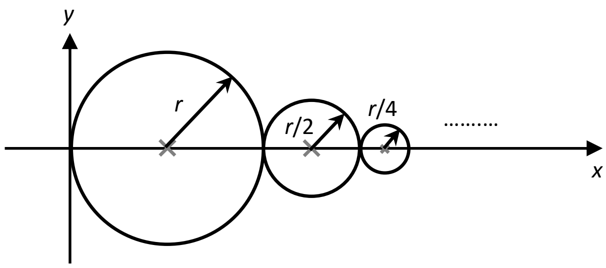
A. π seconds（秒）.
B. 3π/4 seconds（秒）.
C. π/2 seconds（秒）.
D. π/4 second（秒）.
E. π/8 second（秒）.



19. Find the center of gravity of the integrated shape in the *x*-*y* coordinate system. The integrated shape is an infinite series of circular discs with radii of *r*, *r*/2, *r*/4, *r*/8, ………., and their centers are on the *x*-axis. The masses of the circular discs are proportional to their areas.

尋找集成形狀在 *x*-*y* 坐標系統中的重心。該集成形狀是由無窮級數的圓盤所組成，半徑為 *r*， *r*/2， *r*/4， *r*/8， ……….，並且它們的中心位於在 *x* 軸上。圓盤的質量與其面積成正比。

Hints 提示：$\sum_{n=1}^{\infty} 4^{-n} = 1/3$；$\sum_{n=1}^{\infty} 8^{1-n}\left(2^{n+1}-3\right) = 40/21$·

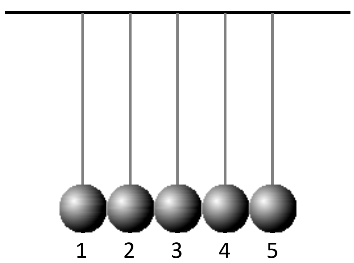


A. $x = (40/7)\,r$, $y = 0$.
B. $x = (80/21)\,r$, $y = 0$.
C. $x = (40/21)\,r$, $y = 0$.
D. $x = (10/7)\,r$, $y = 0$.
E. $x = (40/63)\,r$, $y = 0$.

20. A Newton kinetic ball stress reliever gadget contains 5 identical hard balls supported by identical strings (negligible mass) of equal lengths. From left to right, the hard balls are labelled as ball 1, ball 2, ball 3, ball 4 and ball 5. Balls 4 and 5 are glued together (glue is massless). When Ball 1 is slightly pulled out and released, velocity of Ball 1 just before collision with Ball 2 is +*v* (positive velocity represents moving to the right). Assuming all the collisions are elastic, the velocities of Ball 1 (v_{1f}), and Balls 4, 5 (v_{45f}) after collision are:

牛頓動力球應力消除小擺設由 5 個相同的硬球組成，硬球被 5 根相同且長度相等的繩子（質量可忽略不計）懸掛。從左到右，硬球被標記為球 1，球 2，球 3，球 4，和球 5。球 4 和 5 被緊緊地膠粘在一起（膠是無質量的）。當球 1 被稍微拉出並釋放，球 1 剛碰撞到球 2 前的速度是 +*v*（正速度表示向右移動）。假設所有碰撞都是彈性碰撞，球 1（v_{1f}），和球 4，5（v_{45f}）在碰撞後的速度是：

A. $v_{1f} = 0$；$v_{45f} = +(1/2)v$.
B. $v_{1f} = -v$；$v_{45f} = 0$.
C. $v_{1f} = +v$；$v_{45f} = 0$.
D. $v_{1f} = +(1/3)v$；$v_{45f} = -(2/3)v$.
E. $v_{1f} = -(1/3)v$；$v_{45f} = +(2/3)v$.



--- END OF MULTIPLE CHOICE SESSION 選擇題完 ---

Open Problems 開放題

Total 5 Problems 共 5 題

1. Faster track (9 Marks)

A school student designed a frictionless track to allow a ball (negligible radius) to move on a track, which consisted of a downward slope and a horizontal path (Fig. 1, Track 1). At time t = 0 second, a ball is released from rest to move down the slope (angle θ_1, height h). After the slope, it is a horizontal path (length d). The time duration to complete this track (Track 1) is t_{T1}. However, the student did not satisfy the time duration. He/she would like to design a faster track with shorter completion time.

Another faster track (Fig. 1, Track 2) is designed, which includes 2 downward slopes (angles θ_1 and θ_2, height h), 1 upward slope (angle θ_2, height h), and 3 horizontal paths. The length d_1, d_2, d_3, d_4, d_5, and d_6 are the horizontal lengths.

You may assume that the ball speeds on the 2 tracks are low enough such that the balls are always moving on the tracks without projectile motion.

(a) Express d in terms of h, θ_2, d_2, d_4, and d_6.

(b) Find the time durations to complete Track 1 (t_{T1}) and Track 2 (t_{T2}). Express your answers in terms of θ_1, θ_2, g (acceleration due to gravity), d, h, d_2, d_4, and d_6.

(c) Prove that Track 2 is always faster than Track 1, that is, $t_{T1} > t_{T2}$. Find the time difference (Δt) between the 2 tracks. Express your answer in terms of g, h, d_4, and θ_2.

更快的賽道（9 分）

一名學生設計了一條無摩擦力的軌道好讓一珠子（半徑很小可忽略不計）在軌道上移動。此軌道是由一塊向下斜面和一個水平路徑所組成（圖 1，軌道 1）。在時間 t = 0 秒時，該珠子從靜止位置釋放，並沿斜面（角度 θ_1，高度 h）向下。在斜面之後是一條水平路徑（長度 d）。完成此軌道（軌道 1）所需的時間是 t_{T1}。然而，該學生並不滿足此時間。他/她想設計另一條更快且時間更短的軌道。

另一條更快的軌道（圖 1，軌道 2）已經設計好了。新的軌道包括 2 塊向下斜面（角度 θ_1 和 θ_2，高度 h），1 塊向上斜面（角度 θ_2，高度 h），與及 3 條水平路徑。長度 d_1，d_2，d_3，d_4，d_5 和 d_6 則是水平之長度。

你可假設珠子在軌道上移動時的速度比較慢，使得珠子總是在軌道上移動，並無抛物運動。

（a） 以 h，θ_2，d_2，d_4，和 d_6 作表達，計算 d。

（b） 以 θ_1，θ_2，g（重力加速度），d，h，d_2，d_4，和 d_6 作表達，計算完成軌道 1（t_{T1}）和軌道 2（t_{T2}）所需的時間。

（c） 證明軌道 2 總是比軌道 1 快。以 g，h，d_4，和 θ_2 作表達，計算 2 條軌道的時間差異。

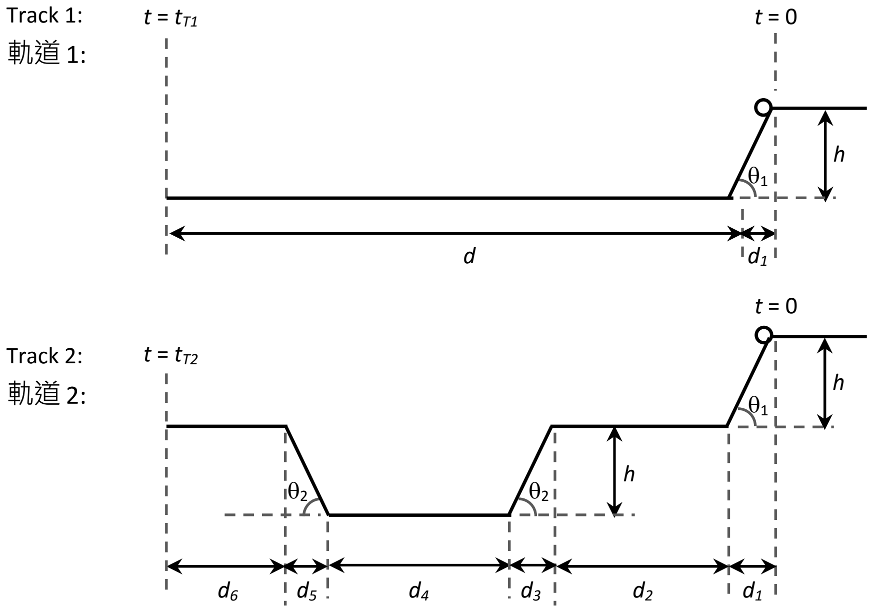


Fig 1. Faster track.
圖 1. 更快的賽道.

2. Rod in semispherical bowl (9 Marks)

A uniform rod (mass m, length L) is placed partially within and partially outside a fixed hemispherical bowl (radius R, where $L > R$). The bowl rests on a horizontal table. When the rod is in equilibrium, the rod makes an angle θ with the horizontal. AB is the length of the rod partially within the bowl. Ignore all the frictions.

(a) Derive the angle θ and the length AB. Express your answers in terms of L and R.

(b) Give the numerical value θ for the special case of $L = 4R$.

半球形碗上的棒（9 分）

一均匀質量的棒（質量 m，長度 L），部分被放置於半球形碗的內部，而部分則處於碗（半徑 R，其中 $L > R$）的外部。該碗被放置在水平的桌子上。當棒子處於平衡狀態時，棒子形成了以水平計 θ 的角度。AB 是棒子在碗內之長度。所有摩擦力可忽略不計。

（a）以 L 和 R 作表達，推導角度 θ 和 AB 的長度。

（b）對於 $L = 4R$ 的特殊情況下，計算 θ 的數值。

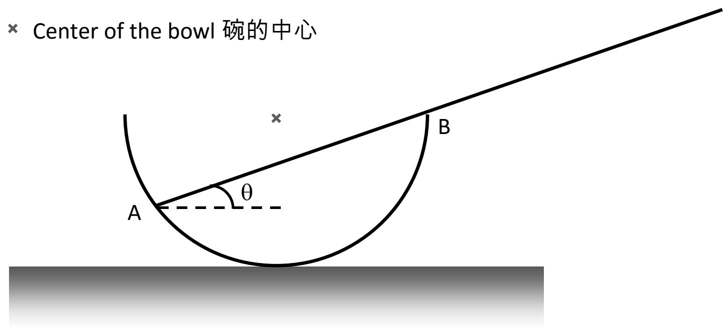


Fig 2. Rod in semispherical bowl.
圖 2. 半球形碗上的棒.

3. Stacked spheres (24 Marks)

Two spheres are stacked such that an upper sphere (radius *r*, mass *m*) is put on the top of a stationary lower sphere (radius *R*; where $R > r$). Eventually, under the influence of gravity, the upper sphere rolls without sliding on the surface of the lower stationary sphere. α ($0° \le \alpha \le 90°$) is the angle between the centers of the upper sphere and the lower sphere, whereas β is the angle that the upper sphere rotates. Initially, $\alpha = 0°$ and $\beta = 0°$.

(a) Find the angular velocities of α and β. Also obtain the velocity of the upper sphere as a function of α. Express your answers in terms of *r*, *R*, g (acceleration due to gravity), and α.

(b) Find the angle at which the upper sphere will detach from the lower sphere.

(c) Suppose that coefficient of static friction between the two spheres is μ. Find the minimum value of α that the upper sphere starts to slide. Express your answer in terms of μ.

(d) By considering the coefficient of static friction, find the minimum and maximum values of α.

Hints: i) Kinetic energy of the upper sphere at an angular velocity $\dot{\beta}$ is given by [Revised to: $\frac{1}{2}\left(\frac{2}{5}mr^2\right)(\dot{\beta})^2$].

ii) In case of existence of friction, the frictional force, *f*, at an angle α is: $f = \frac{2}{7}mg\sin\alpha$.

疊球（24 分）

兩個球體堆疊在一起。上層球體（半徑 *r*, 質量 *m*）被放置在靜止的下層球體（半徑 *R*, 其中 $R > r$）的頂部。最終, 在重力的影響下, 上層球體在不滑動的情況下, 沿著靜止的下層球體表面滾下。α（$0° \le \alpha \le 90°$）是上層球體和下層球體的中心角度, 而 β 是上層球體旋轉角度。最初, $\alpha = 0°$ 和 $\beta = 0°$。

（a）計算 α 和 β 的角速度。以 α 作為函數, 計算上層球體的速度。答案請以 *r*, *R*, g （重力加速度）, 和 α 作表達。

（b）計算上層球體與下層球體分離時的角度。

（c）假設兩個球之間的靜摩擦係數為 μ, 計算上層球體開始滑動時的最小 α 值。

（d）通過考慮靜摩擦係數, 計算 α 的最小值和最大值。

提示： i）上層球體速度為 $\dot{\beta}$ 時, 其動能是 [Revised to: $\frac{1}{2}\left(\frac{2}{5}mr^2\right)(\dot{\beta})^2$]。

ii）在摩擦存在的情況下, 其摩擦力, *f*, 於角度 α 時是 $f = \frac{2}{7}mg\sin\alpha$。

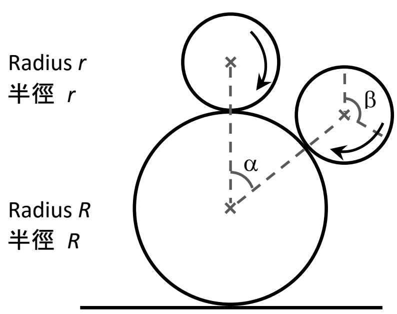


Fig 3. Stacked sphere.
圖 3. 疊球.

4. A ‘8’ shape oscillator (11 Marks)

Two uniform, thin, and hollow cylinders (radii R_1 and R_2, masses m_1 and m_2, respectively) are stacked to form an “8” shape such that their diameters are aligned in a straight line. The stacked cylinders is free to oscillate about its axis of oscillation (Fig. 4). The kinetic energy of the stacked cylinders is $KE = \frac{1}{2}\left(2m_1 R_1^{\,2} + 2m_2 R_2^{\,2}\right)\omega^2$, where ω is the instantaneous angular velocity about the oscillation axis. A small angle θ is made to displace the stacked cylinders from the equilibrium position. Figure 4 shows the equilibrium position of the stacked cylinders. At equilibrium position, $\theta = 0°$.

(a) Find the potential energy of the stacked cylinders in terms of m_1, m_2, R_1, R_2, g (acceleration due to gravity) and θ. Take zero potential energy at the level of oscillation axis.

(b) Find the total energy of the oscillator of the stacked cylinders. Express your answer in the form of energy transformation in a simple harmonic mass-spring system, that is, Total Energy $E = \frac{1}{2} m v_x^{\,2} + \frac{1}{2} k x^2$, where you may consider x as an angular displacement.

(c) Determine the effective mass (m_{eff}), effective spring constant (k_{eff}), and period of the oscillation.

(d) According to the above results, estimate the oscillation period of a single hollow cylinder about the same oscillation axis, that is, without the upper hollow cylinder.

一個 “8” 字形的振盪（11 分）

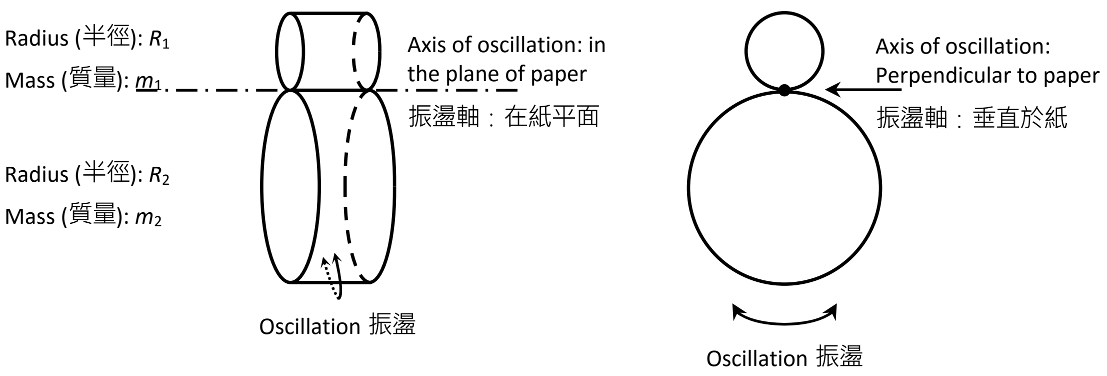
兩個均勻、薄、及中空的圓柱體（半徑 R_1 和 R_2，質量分別為 m_1 和 m_2）被堆疊成一個 “8” 字的形狀，而它們的直徑都在同一條直線上對齊。該堆疊圓柱體可以在其軸線上自由擺動（圖 4）。該堆疊圓柱體的動能是 $KE = \frac{1}{2}\left(2m_1 R_1^{\,2} + 2m_2 R_2^{\,2}\right)\omega^2$，而 ω 是軸線上擺動的瞬時角速度。該堆疊圓柱體被弄至偏離其平衡點計的細小角度。圖 4 示出了堆疊圓柱的平衡位置。在平衡位置時，$\theta = 0°$。

（a） 計算堆疊圓柱體的勢能。答案請以 m_1，m_2，R_1，R_2，g（重力加速度）和 θ 作表達。設勢能在振盪軸的水平位置為零。

（b） 計算堆疊圓柱體擺動系統的總能量。答案請以質量與彈簧的簡諧運動能量轉換形式表達，即是總能量 $E = \frac{1}{2} m v_x^{\,2} + \frac{1}{2} k x^2$，其中你可考慮 x 為角位移。

（c） 計算該系統的有效質量（m_{eff}），有效彈簧常數（k_{eff}），與及擺動週期。

（d） 根據以上的結果，計算單一中空圓柱體在同一擺動軸的擺動週期，亦即沒有上部中空圓柱體時的擺動週期。



Three dimensional view
三維視圖

Two dimensional view
二維視圖

Fig 4. A ‘8’ shape oscillator.
圖 4. 一個 “8” 字形的振盪.

5. Energy to break up the Sun completely (7 Marks)

By considering the gravitational potential energy to separate all the solar mass to infinity, estimate the energy required to break up the Sun completely. Express your answer in terms of M_S (mass of the Sun), R_S (radius of the Sun), and G (Gravitational Constant). You may assume i) the Sun is spherical, and ii) the Sun density is uniform.

Hint: Other than solving integral, the following summation formula may help:

$$\sum_{k=1}^{m} k^4 = \frac{m(2m+1)(m+1)(3m^2+3m-1)}{30}$$, where *k* and *m* are integers.

完全打散太陽所需的能量（7 分）

通過考慮引力勢能來分隔所有太陽的質量至無窮遠，估計要完全打破太陽所需的能量。答案請以 M_S（太陽質量），R_S（太陽半徑），和 G（萬有引力常數）作表達。你可假設 i）太陽是球形，和 ii）太陽密度是均勻。

提示：求積分以外，下列總和公式或可幫助計算：

$$\sum_{k=1}^{m} k^4 = \frac{m(2m+1)(m+1)(3m^2+3m-1)}{30}$$，其中，*k* 和 *m* 為整數。

--- END OF EXAM PAPER 全卷完 ---