

MC Key

1. B

2. D

3. A

4. B

5. E

6. C

~~7. D~~ (Cancelled)

8. E

9. D

10. B

~~11. C~~ (Cancelled)

12. A

13. C

14. A

15. E

16. A

17. B

18. C

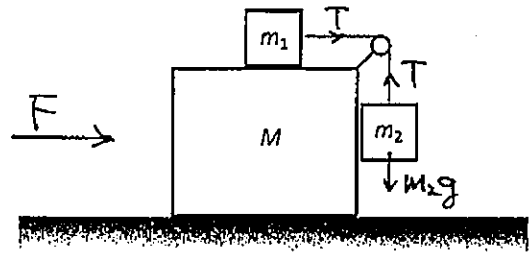
19. D

20. E

1.

Without relative motion,

$\Rightarrow m_1, m_2,$  and  $M$  have the same acceleration



Along  $x$ -axis:

$$(M + m_1 + m_2) \ddot{x} = F \quad \text{--- (1)}$$

$$m_1 \ddot{x} = T \quad \text{--- (2)}$$

Along  $y$ -axis:  $T = m_2g \quad \text{--- (3)}$

$$\text{(1), (2) and (3)} \Rightarrow (M + m_1 + m_2) \frac{m_2g}{m_1} = F$$

2.

$$\begin{cases} F = bv \\ Fv = 500 \text{ W} \end{cases}$$

$$\begin{cases} F = (5 \text{ N s/m}) (v) & \text{--- (1)} \\ Fv = 500 \text{ W} & \text{--- (2)} \end{cases}$$

$$\frac{\textcircled{2}}{\textcircled{1}} \Rightarrow v = \frac{500 \text{ W}}{5 \text{ N s/m} (v)}$$

$$v^2 = \frac{500 \text{ N m s}^{-1}}{5 \text{ N s/m}}$$

$$v = 10 \text{ m s}^{-1}$$



5.

$$\begin{cases} T_1 \cos \alpha + T_2 \cos \beta - mg = 0 \\ T_1 \sin \alpha - T_2 \sin \beta = 0 \end{cases} \Rightarrow T_1 = T_2 \frac{\sin \beta}{\sin \alpha}$$

$$T_2 \frac{\sin \beta \cos \alpha}{\sin \alpha} + T_2 \cos \beta - mg = 0$$

$$T_2 (\sin \beta \cos \alpha + \sin \alpha \cos \beta) = mg \sin \alpha$$

$$T_2 \sin(\alpha + \beta) = mg \sin \alpha$$

$$T_2 = \frac{mg \sin \alpha}{\sin(\alpha + \beta)}$$

$$\begin{aligned} T_1 &= \frac{mg \sin \alpha}{\sin(\alpha + \beta)} \frac{\sin \beta}{\sin \alpha} \\ &= \frac{mg \sin \beta}{\sin(\alpha + \beta)} \end{aligned}$$

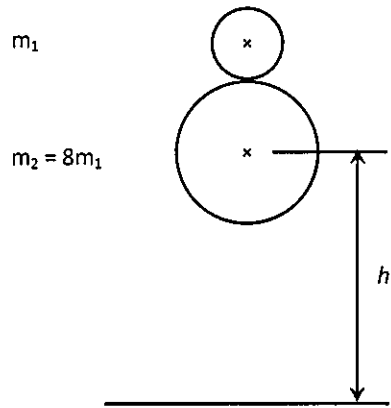
$$\begin{cases} T_1 = m_1 g \\ T_2 = m_2 g \end{cases}$$

$$\Rightarrow \frac{m_1}{m_2} = \frac{T_1}{T_2} = \frac{mg \sin \beta / \sin(\alpha + \beta)}{mg \sin \alpha / \sin(\alpha + \beta)}$$

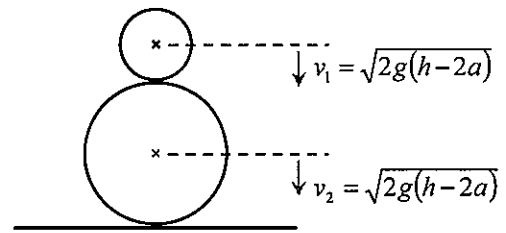
$$= \frac{\sin \beta}{\sin \alpha}$$

7.

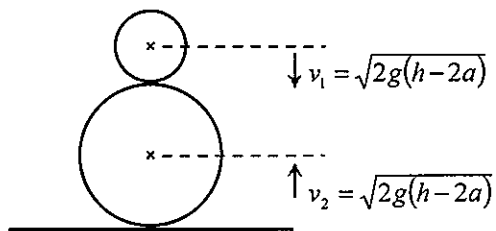
(i) Initial condition



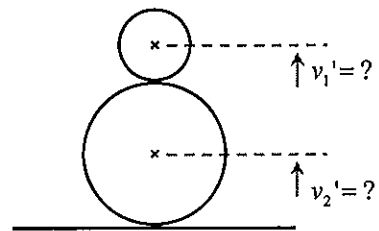
(ii) Just before collision with the ground



(iii) After lower sphere collision from the ground



(iv) Just after elastic collision between spheres



7.

Momentum:  $m_2 v_2 - m_1 v_1 = m_2 v_2' + m_1 v_1'$

$$8 m_1 v_2 - m_1 v_1 = 8 m_1 v_2' + m_1 v_1'$$

$$8 v_2 - v_1 = 8 v_2' + v_1'$$

$$v_1' = 8 v_2 - v_1 - 8 v_2'$$

$$= 7 v_1 - 8 v_2'$$

↑ +ve  
↓ -ve

Energy:  $\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_2 v_2'^2 + \frac{1}{2} m_1 v_1'^2$

$$m_1 v_1^2 + 8 m_1 v_2^2 = 8 m_1 v_2'^2 + m_1 v_1'^2$$

$$v_1^2 + 8 v_2^2 = 8 v_2'^2 + v_1'^2$$

$$9 v_1^2 = 8 v_2'^2 + (7 v_1 - 8 v_2')^2$$

$$72 v_2'^2 - 112 v_1 v_2' + 40 v_1^2 = 0$$

$$\Rightarrow v_2' = \frac{5 v_1}{9} = \frac{5 \sqrt{2g(h-2a)}}{9} \text{ or } v_2' = v_1 \text{ (rejected)}$$

$$v_1' = \frac{23 v_1}{9} = \frac{23}{9} \sqrt{2g(h-2a)}$$

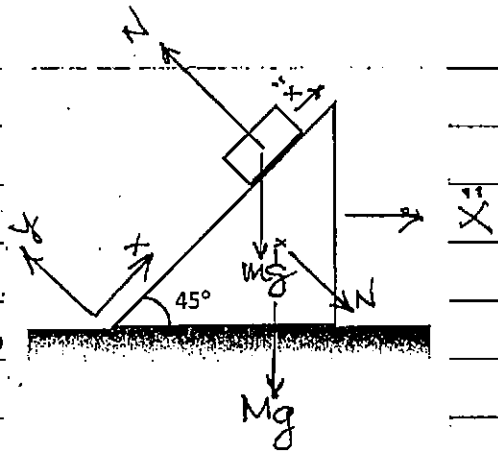
$$\text{Maximum Height} = 5a + \frac{v_1'^2}{2g} = \frac{529}{81} (h-2a)$$

8.

$$M\ddot{x} = N \sin\theta \quad \text{--- (1)}$$

$$M(\ddot{x} + \ddot{x} \cos\theta) = -mg \sin\theta \quad \text{--- (2)}$$

$$-M\ddot{x} \sin\theta = N - mg \cos\theta \quad \text{--- (3)}$$



$$\textcircled{3} \Rightarrow -M\ddot{x} \frac{\sqrt{2}}{2} = N - mg \frac{\sqrt{2}}{2}$$

$$N = mg \frac{\sqrt{2}}{2} - M\ddot{x} \frac{\sqrt{2}}{2}$$

Sub into  $\textcircled{1}$ ,

$$M\ddot{x} = \frac{N\sqrt{2}}{2}$$

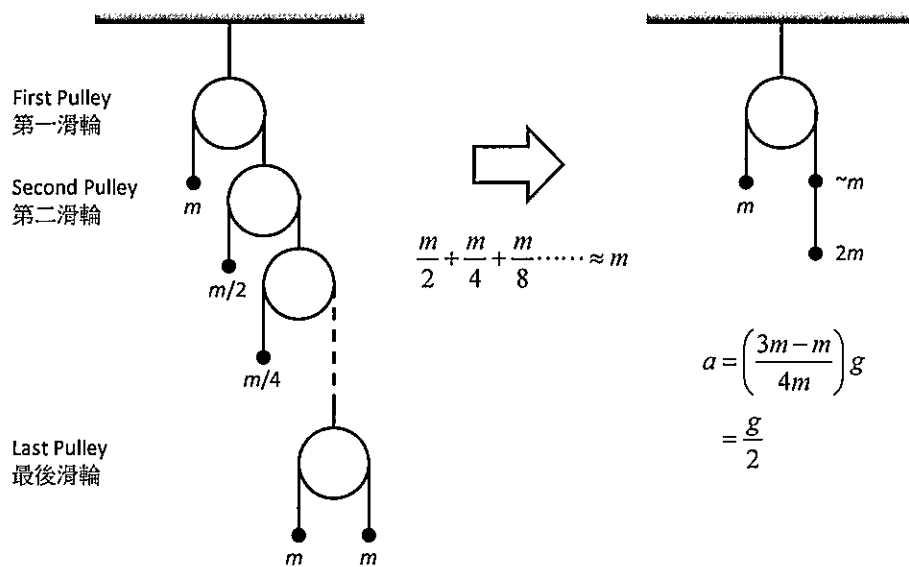
$$= \left( mg \frac{\sqrt{2}}{2} - M\ddot{x} \frac{\sqrt{2}}{2} \right) \frac{\sqrt{2}}{2}$$

$$= \frac{mg}{2} - \frac{M\ddot{x}}{2}$$

$$\Rightarrow \ddot{x} = \frac{mg}{M + 2M}$$



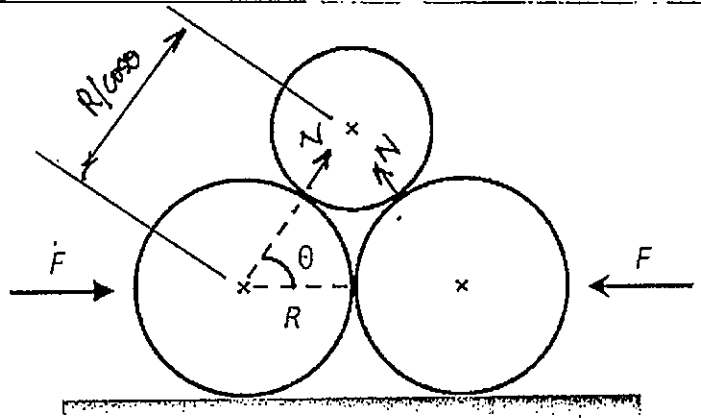
9.



10.

$$\begin{aligned} \text{Radius of the upper} \\ \text{circle} &= R - R \\ &= R \left( \frac{1}{\cos \theta} - 1 \right) \end{aligned}$$

$$\begin{aligned} \text{Mass of the upper circle} \\ &= 5 \pi R^2 \left( \frac{1}{\cos \theta} - 1 \right)^2 \end{aligned}$$



Keeping stationary :

$$2N \sin \theta = 5 \pi R^2 \left( \frac{1}{\cos \theta} - 1 \right)^2 g$$

$$N = \frac{5 \pi R^2}{2 \sin \theta} \left( \frac{1}{\cos \theta} - 1 \right)^2 g$$

~~$$F = N \sin \theta$$~~

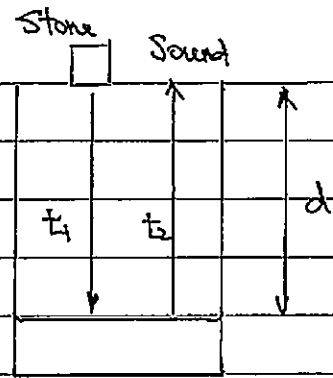
$$F = N \cos \theta = \frac{5 \pi R^2}{2 \sin \theta} \frac{(1 - \cos \theta)^2}{\cos(\theta)} g$$

11.

$$d = \frac{1}{2} (9.8) t_1^2 \Rightarrow d = 4.9 t_1^2 \quad \text{--- (1)}$$

$$t_1 + t_2 = 3 \text{ s} \Rightarrow t_2 = 3 - t_1 \quad \text{--- (2)}$$

$$d = 340 t_2 \quad \text{--- (3)}$$



Sub. (2) into (3),

$$d = 340 (3 - t_1)$$

$$= 1020 - 340 t_1$$

$$= 1020 - 340 \sqrt{\frac{d}{4.9}} \quad \text{(From (1))}$$

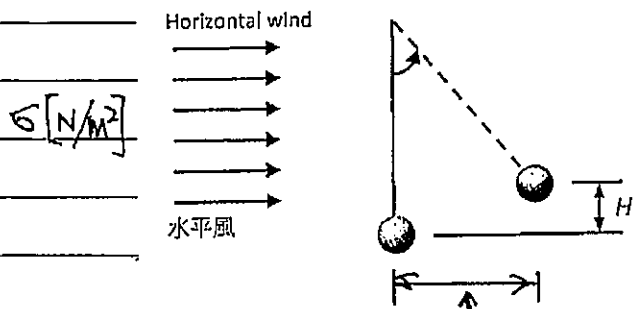
Rearranging:

~~$$d^2 - 25632d + 1020^2 = 0$$~~

~~$$d = -40.5 \text{ m} \quad \text{or} \quad d = 25.7 \text{ m}$$~~

~~$$\Rightarrow d = 25.7 \text{ m}$$~~

$$d = 40.65 \text{ m}$$



$$\sqrt{L^2 - (L-H)^2} = \sqrt{2LH - H^2}$$

$$F = \sigma \times \pi r^2$$

$$\text{Work Done by Wind} = \int F \cdot ds$$

$$= F \times \sqrt{2LH - H^2}$$

$$= \sigma \pi r^2 \sqrt{2LH - H^2}$$

Energy Conservation:

$$\sigma \pi r^2 \sqrt{2LH - H^2} = m g H$$

$$\sigma^2 \pi^2 r^4 (2LH - H^2) = m^2 g^2 H^2$$

$$(m^2 g^2 + \sigma^2 \pi^2 r^4) H = 2L \sigma^2 \pi^2 r^4$$

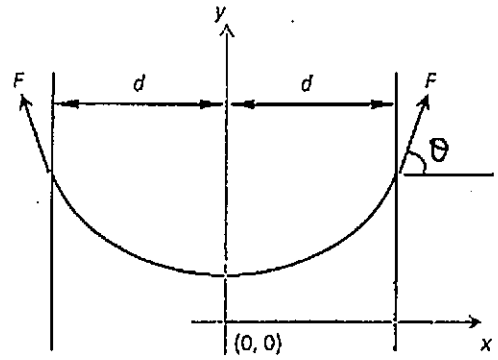
$$H = \frac{2L \sigma^2 \pi^2 r^4}{m^2 g^2 + \sigma^2 \pi^2 r^4}$$

13.

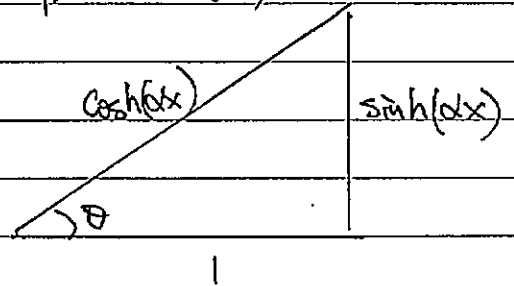
$$2F \sin \theta = mg$$

$$2F \tanh(\alpha d) = mg$$

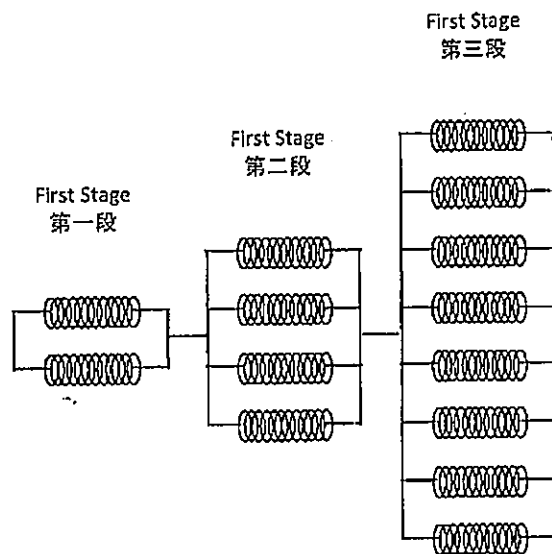
$$F = \frac{mg}{2 \tanh(\alpha d)}$$



$$\text{slope} = \sinh(\alpha x)$$



$$\begin{aligned} \Rightarrow \sin \theta &= \frac{\sinh(\alpha x)}{\cosh(\alpha x)} \\ &= \tanh(\alpha x) \end{aligned}$$



Spring Constant	$2k$	$4k$	$8k$	.....
Elongation	$x$	$\frac{x}{2}$	$\frac{x}{4}$	.....

14.

$$\frac{1}{k_{eq}} = \frac{1}{2k} + \frac{1}{4k} + \frac{1}{8k} + \dots$$

$$= \frac{1}{k} \left( \frac{1}{2} \right)$$

$$k_{eq} = k$$

15.

$$x + \frac{x}{2} + \frac{x}{4} + \dots = 10 \text{ cm}$$

$$x \left( \frac{1}{1 - \frac{1}{2}} \right) = 10 \text{ cm}$$

$$\Rightarrow x = 5 \text{ cm}$$

Fourth Stage Elongation =  $\frac{5 \text{ cm}}{8} = 0.625 \text{ cm}$

Trajectory Equation:

$$y = (\tan \theta_1) (x) - \frac{g}{2V_1^2 \cos^2 \theta_1} (x^2)$$

$$\begin{cases} x = d \cos \phi \\ y = d \sin \phi \end{cases}$$

$$\Rightarrow d \sin \phi = (\tan \theta_1) (d \cos \phi) - \frac{g}{2V_1^2 \cos^2 \theta_1} (d^2 \cos^2 \phi)$$

$$\sin \phi \cos^2 \theta_1 = \sin \theta_1 \cos \theta_1 \cos \phi - \frac{gd \cos^2 \phi}{2V_1^2}$$

$$\begin{aligned} d_1 &= \frac{2V_1^2 \cos \theta_1 (\sin \theta_1 \cos \phi - \sin \phi \cos \theta_1)}{g \cos^2 \phi} \\ &= \frac{2V_1^2 \cos \theta_1 \sin(\theta_1 - \phi)}{g \cos^2 \phi} \end{aligned}$$

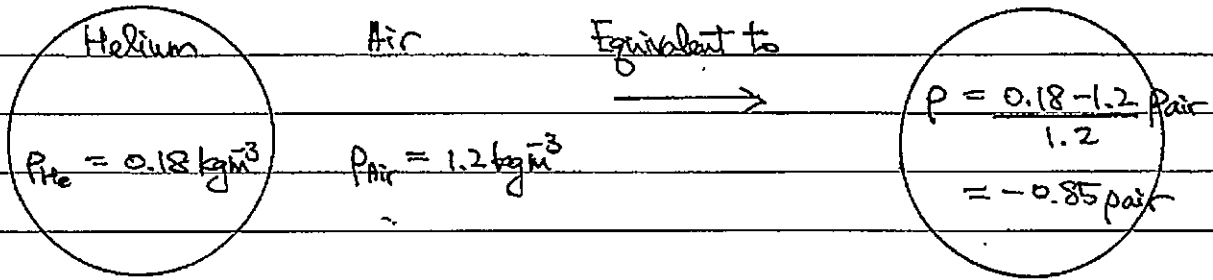
Similarly,  $d_2 = \frac{2V_2^2 \cos \theta_2 \sin(\theta_2 - \phi)}{g \cos^2 \phi}$

$$d_1 = d_2, \Rightarrow 2V_1^2 \cos \theta_1 \sin(\theta_1 - \phi) = 2V_2^2 \cos \theta_2 \sin(\theta_2 - \phi)$$

$$\left(\frac{V_1}{V_2}\right)^2 = \frac{\cos \theta_2 \sin(\theta_2 - \phi)}{\cos \theta_1 \sin(\theta_1 - \phi)}$$

17.

Consider an equivalent case that the density of the balloon is  $-0.85 \rho_{\text{air}}$  (Density of air), and neglect the effect of air.



$$\text{Effective gravitational acceleration} = -0.85 g$$

$$\text{Effective centrifugal acceleration} = -0.85 a$$

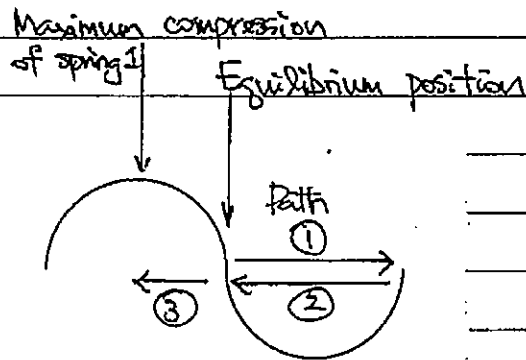
$$a = \frac{v^2}{R} = \frac{(105 \text{ km/hour})^2}{1000 \text{ m}} = 0.85 \text{ m s}^{-2}$$

$$\begin{aligned} \Rightarrow \tan \theta &= \frac{-0.85 \left( \frac{a}{g} \right)}{-0.85 \left( \frac{g}{g} \right)} \\ &= \frac{0.85}{9.8} \end{aligned}$$

$$\begin{aligned} \Rightarrow \theta &= \tan^{-1} \left( \frac{0.85}{9.8} \right) \\ &= 5^\circ \end{aligned}$$



Effective spring constant,  $k = k_1 + k_2 = 9 \text{ N/m}$ .



Maximum compression of Spring 1 occurs after  $\frac{3}{4}$  period,

$\Rightarrow$  The time that the block will take

$$= \frac{3T}{4}$$

$$= \frac{3}{4} (2\pi) \sqrt{\frac{M}{k}}$$

$$= \frac{3}{4} (2\pi) \sqrt{\frac{1}{9}}$$

$$= \frac{\pi}{2} \text{ seconds}$$

$$\text{Center of Mass} = \frac{\sum m_i x_i}{M}$$

$$\begin{aligned} \sum m_i x_i &= \pi r^2 (r) + \pi \left(\frac{r}{2}\right)^2 \left(2r + \frac{r}{2}\right) \\ &+ \pi \left(\frac{r}{4}\right)^2 \left[2r + 2\left(\frac{r}{2}\right) + \frac{r}{4}\right] \\ &+ \pi \left(\frac{r}{8}\right)^2 \left[2r + 2\left(\frac{r}{2}\right) + 2\left(\frac{r}{4}\right) + \frac{r}{8}\right] \\ &+ \pi \left(\frac{r}{16}\right)^2 \left[2r + 2\left(\frac{r}{2}\right) + 2\left(\frac{r}{4}\right) + 2\left(\frac{r}{8}\right) + \frac{r}{16}\right] \\ &+ \dots \end{aligned}$$

$$= \pi r^3 \left[ 1 + \frac{5}{8} + \frac{13}{64} + \frac{29}{512} + \frac{61}{4096} + \dots \right]$$

$$= \pi r^3 \sum_{n=1}^{\infty} 8^{1-n} (2^{n+1} - 3)$$

$$= \frac{460}{21} \pi r^3$$

$$M = \pi r^2 + \pi \left(\frac{r}{2}\right)^2 + \pi \left(\frac{r}{4}\right)^2 + \pi \left(\frac{r}{8}\right)^2 + \dots$$

$$= \pi r^2 \left( 1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \dots \right)$$

$$= \pi r^2 \left( 1 + \sum_{n=1}^{\infty} 4^{-n} \right)$$

$$= \pi r^2 \left( 1 + \frac{1}{3} \right)$$

$$= \pi r^2 \left( \frac{4}{3} \right)$$

Note: The mass proportional constant is omitted.  
It will be cancelled finally.

$$\Rightarrow \text{Center of Mass} = \frac{\frac{460}{21} \pi r^3}{\frac{4}{3} \pi r^2} = \frac{10}{7} r$$

$$\left\{ \begin{array}{l} p \quad mV = mV_{1P} + 2mV_{45P} \end{array} \right.$$

$$\left\{ \begin{array}{l} \frac{1}{2} mV^2 = \frac{1}{2} mV_{1P}^2 + \frac{1}{2} (2m) V_{45P}^2 \end{array} \right.$$

Solutions give :

$$\left\{ \begin{array}{l} V_{1P} = -\frac{1}{3}V \end{array} \right.$$

$$V_{45P} = \frac{2}{3}V$$

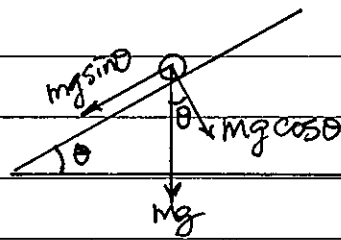
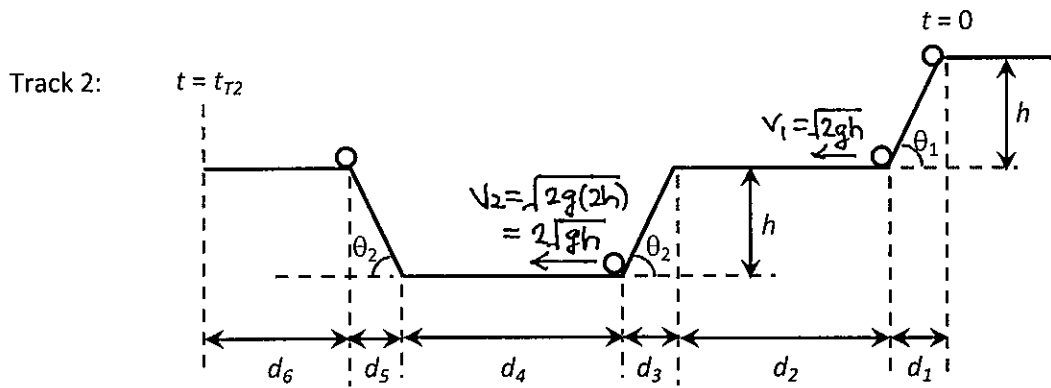
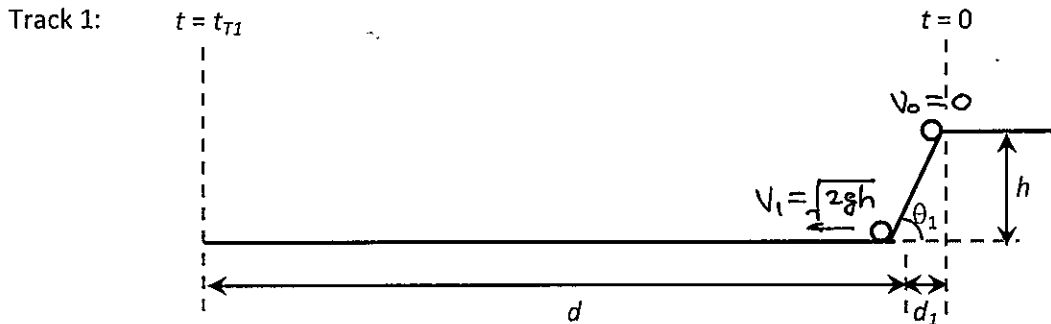
1.

$$(a) \quad d = d_2 + d_3 + d_4 + d_5 + d_6$$

$$= d_2 + \frac{h}{\sin \theta_2} + d_4 + \frac{h}{\sin \theta_2} + d_6$$

$$= d_2 + d_4 + d_6 + \frac{2h}{\sin \theta_2}$$

(b)



$$t_{d1} = \frac{\sqrt{2gh}}{g \sin \theta_1}$$

$$t_{d3} = \frac{\sqrt{gh}(2 - \sqrt{2})}{g \sin \theta_2}$$

$$t_{d4} = \frac{d_4}{2\sqrt{gh}}$$

$$t_d = \frac{d}{\sqrt{2gh}}$$

$$t_{d5} = t_{d3} = \frac{\sqrt{gh}(2 - \sqrt{2})}{g \sin \theta_2}$$

$$t_{d2} = \frac{d_2}{\sqrt{2gh}}$$

$$t_{d6} = \frac{d_6}{\sqrt{2gh}}$$

1.

$$(b) t_{T1} = t_{d1} + t_d$$

$$= \frac{\sqrt{2gh}}{g \sin \theta_1} + \frac{d}{\sqrt{2gh}}$$

$$t_{T2} = t_{d1} + t_{d2} + \dots + t_{d6}$$

$$= \frac{\sqrt{2gh}}{g \sin \theta_1} + \frac{d_2}{\sqrt{2gh}} + \frac{\sqrt{gh}(2-\sqrt{2})}{g \sin \theta_2} + \frac{d_4}{2\sqrt{gh}}$$

$$+ \frac{\sqrt{gh}(2-\sqrt{2})}{g \sin \theta_2} + \frac{d_6}{\sqrt{2gh}}$$

$$= \frac{\sqrt{2gh}}{g \sin \theta_1} + \frac{d_2 + d_6}{\sqrt{2gh}} + \frac{d_4}{2\sqrt{gh}} + \frac{2\sqrt{gh}(2-\sqrt{2})}{g \sin \theta_2}$$

$$(c) \Delta t = \frac{d}{\sqrt{2gh}} - \frac{d_2 + d_6}{\sqrt{2gh}} - \frac{d_4}{2\sqrt{gh}} - \frac{2\sqrt{gh}(2-\sqrt{2})}{g \sin \theta_2}$$

$$= \frac{1}{\sqrt{2gh}} \left( d - d_2 - d_6 + d_4 - \frac{d_4}{\sqrt{2}} - d_6 \right) - \frac{2\sqrt{gh}(2-\sqrt{2})}{g \sin \theta_2}$$

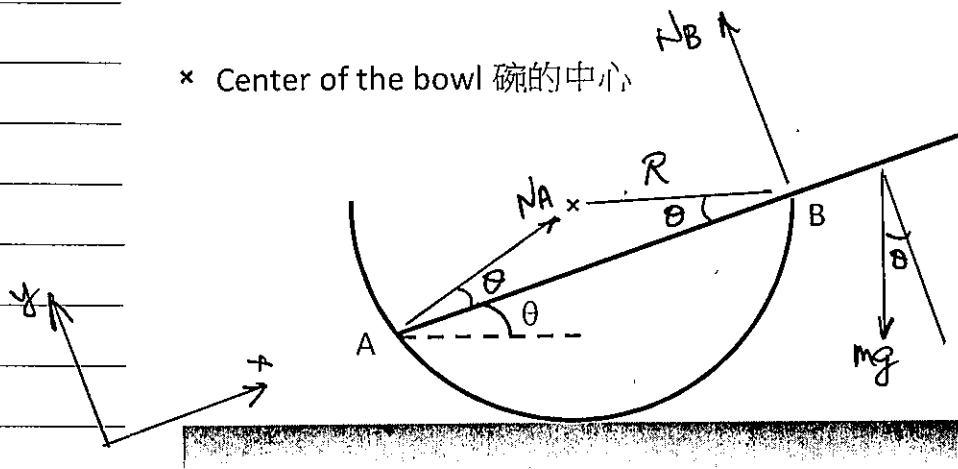
$$= \frac{1}{\sqrt{2gh}} \left( \frac{2h}{\sin \theta_2} + \frac{\sqrt{2}-1}{\sqrt{2}} d_4 \right) - \frac{2\sqrt{gh}(2-\sqrt{2})}{g \sin \theta_2}$$

$$= \frac{\sqrt{2h}}{\sqrt{g \sin \theta_2}} - \frac{2\sqrt{h}(2-\sqrt{2})}{\sqrt{g \sin \theta_2}} + \left( \frac{\sqrt{2}-1}{\sqrt{2}} \right) \frac{1}{\sqrt{2gh}} d_4$$

$$= \frac{\sqrt{h}(3\sqrt{2}-4)}{\sqrt{g \sin \theta_2}} + \left( \frac{\sqrt{2}-1}{\sqrt{2}} \right) \frac{d_4}{2gh}$$

2.

(a)



$$\Sigma F_x = 0, \quad N_A \cos \theta = mg \sin \theta \quad \text{--- (1)}$$

$$\Sigma F_y = 0, \quad N_A \sin \theta + N_B = mg \cos \theta \quad \text{--- (2)}$$

$$\Sigma M_A = 0, \quad \text{Moment about A:}$$

← Length of AB

$$N_B (2R \cos \theta) - \frac{L}{2} mg \cos \theta = 0$$

$$\therefore N_B = \frac{L mg}{4R}$$

Sub. into (2),

$$N_A \sin \theta + \frac{L mg}{4R} = mg \cos \theta$$

$$N_A = \left( mg \cos \theta - \frac{L mg}{4R} \right) \frac{1}{\sin \theta} \quad \text{--- (3)}$$

Sub. into (1),

$$\left( mg \cos \theta - \frac{L mg}{4R} \right) \frac{1}{\sin \theta} \cos \theta = mg \sin \theta$$

$$\frac{4R \cos^2 \theta - L \cos \theta}{4R} = \sin^2 \theta$$

2.

$$(a) \quad 4R \cos^2 \theta - L \cos \theta = 4R (1 - \cos^2 \theta)$$

$$8R \cos^2 \theta - L \cos \theta - 4R = 0$$

$$\cos \theta = \frac{L \pm \sqrt{L^2 + 128R^2}}{16R}$$

$$\Rightarrow \cos \theta = \frac{L + \sqrt{L^2 + 128R^2}}{16R}$$

$$AB = 2R \left( \frac{L + \sqrt{L^2 + 128R^2}}{16R} \right)$$

$$(b) \quad \text{When } L = 4R, \quad \cos \theta = 1$$

$$\Rightarrow \theta = 0^\circ$$

3.

(a) Consider the upper sphere, energy conservation gives

$$\frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\dot{\beta}^2 + mg(R+r)\cos\alpha = mg(R+r)$$

$$v = \dot{\beta}r = (R+r)\dot{\alpha}$$

$$\dot{\alpha}^2 = \dot{\beta}^2 \frac{r^2}{(R+r)^2}$$

$$\frac{v^2}{2} + \frac{1}{5}r^2\dot{\beta}^2 + g(R+r)\cos\alpha = g(R+r)$$

$$\frac{\dot{\beta}^2 r^2}{2} + \frac{r^2 \dot{\beta}^2}{5} + g(R+r)\cos\alpha = g(R+r)$$

$$\frac{7r^2 \dot{\beta}^2}{10} = g(R+r)(1 - \cos\alpha)$$

$$r^2 \dot{\beta}^2 = \frac{10}{7}g(R+r)(1 - \cos\alpha)$$

$$\dot{\beta}^2 = \frac{10}{7r^2}g(R+r)(1 - \cos\alpha)$$

$$\dot{\alpha}^2 = \frac{10}{7}g \frac{(1 - \cos\alpha)}{R+r}$$

$$v = (R+r)\dot{\alpha}$$

$$= (R+r) \sqrt{\frac{10}{7}g \frac{(1 - \cos\alpha)}{R+r}}$$

$$= \sqrt{\frac{10}{7}g(R+r)(1 - \cos\alpha)}$$



3.

$$(b) \quad mg \cos \alpha - N = \frac{mv^2}{R+r}$$

Upper sphere will detach from the lower sphere when  $N=0$ .

$$g \cos \alpha = \frac{v^2}{R+r}$$

$$g \cos \alpha = \frac{10}{7} g (R+r) (1 - \cos \alpha) \times \frac{1}{R+r}$$

$$\cos \alpha = \frac{10}{7} (1 - \cos \alpha)$$

$$\Rightarrow \cos \alpha = \frac{10}{17}$$

$$\alpha \approx 54^\circ$$

$$(c) \quad \left\{ \begin{array}{l} mg \sin \alpha - f = m\dot{v} \quad \Rightarrow \quad \dot{v} = g \sin \alpha - \frac{f}{m} \\ \end{array} \right.$$

$$f r = \left( \frac{2}{5} m r^2 \right) \ddot{\beta}$$

$$v = \dot{\beta} r = (R+r)\dot{\alpha} \quad \Rightarrow \quad \dot{v} = \ddot{\beta} r = (R+r)\ddot{\alpha}$$

$$f r = \left( \frac{2}{5} m r^2 \right) \ddot{\beta}$$

$$= \left( \frac{2}{5} m r^2 \right) \frac{\dot{v}}{r}$$

$$= \left( \frac{2}{5} m r^2 \right) \frac{1}{r} \left( g \sin \alpha - \frac{f}{m} \right)$$

$$f = \left( \frac{2}{5} m \right) \left( g \sin \alpha - \frac{f}{m} \right)$$

$$= \frac{2}{5} m g \sin \alpha - \frac{2}{5} f$$

3.

$$(c) \quad \Rightarrow \quad f = \frac{2}{7} mg \sin \alpha \quad (\text{Given})$$

$$\text{Condition to slide:} \quad f = \mu N$$

$$\Rightarrow \quad \frac{2}{7} mg \sin \alpha = \mu N$$

$$\frac{2}{7} mg \sin \alpha = \mu \left( mg \cos \alpha - \frac{mv^2}{R+r} \right)$$

Substitute  $v$  using the results from (a),

$$\frac{2}{7} g \sin \alpha = \mu \left[ g \cos \alpha - \frac{10g}{7} (1 - \cos \alpha) \right]$$

$$\frac{2}{7} \sin \alpha = \mu \cos \alpha - \frac{10}{7} \mu + \frac{10}{7} \mu \cos \alpha$$

$$2 \sin \alpha = 17 \mu \cos \alpha - 10 \mu$$

$$2(\sqrt{1 - \cos^2 \alpha}) = 17 \mu \cos \alpha - 10 \mu$$

$$4(1 - \cos^2 \alpha) = 289 \mu^2 \cos^2 \alpha + 100 \mu^2 - 340 \mu^2 \cos \alpha$$

$$\cos^2 \alpha (289 \mu^2 + 4) - \cos \alpha (340 \mu^2) + 100 \mu^2 - 4 = 0$$

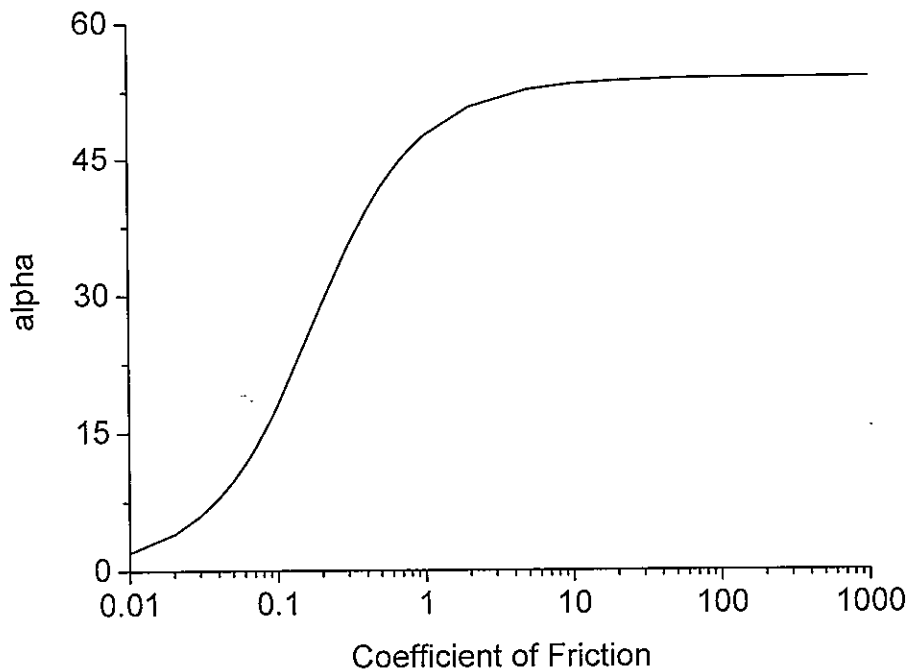
$$\begin{aligned} \cos \alpha &= \frac{340 \mu^2 \pm \sqrt{340^2 \mu^4 - 4(289 \mu^2 + 4)(100 \mu^2 - 4)}}{2(289 \mu^2 + 4)} \\ &= \frac{340 \mu^2 \pm \sqrt{16(189 \mu^2 + 4)}}{2(289 \mu^2 + 4)} \\ &= \frac{170 \mu^2 + 2\sqrt{189 \mu^2 + 4}}{289 \mu^2 + 4} \end{aligned}$$

Since  $0^\circ \leq \alpha \leq 90^\circ$ ,

$$\Rightarrow \quad \cos \alpha = \frac{170 \mu^2 + 2\sqrt{189 \mu^2 + 4}}{289 \mu^2 + 4}$$

3.

(c) For reference :



(d) When  $\mu = 0$ ,  $\cos \alpha = 1$  and  $\alpha = 0^\circ$

Remark: Without friction, the upper sphere slides immediately.

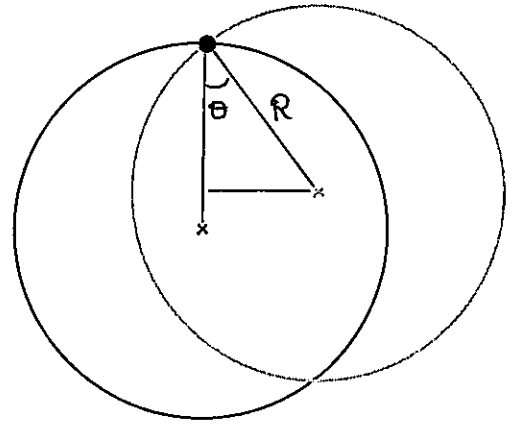
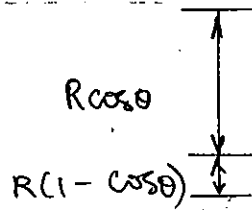
$$\begin{aligned} \text{When } \mu \rightarrow \infty, \quad \cos \alpha &= \frac{170 + 2\sqrt{189/\mu^2 + 4/\mu^4}}{289 + 4/\mu^4} \\ &= \frac{170}{289} \\ &= \frac{10}{17} \end{aligned}$$

$$\Rightarrow \alpha \approx 54^\circ$$

4.

$$KE = \frac{1}{2} (2M_1 R_1^2) \left( \frac{d\theta}{dt} \right)^2 + \frac{1}{2} (2M_2 R_2^2) \left( \frac{d\theta}{dt} \right)^2$$

$$= (M_1 R_1^2 + M_2 R_2^2) \left( \frac{d\theta}{dt} \right)^2$$



(a)

$$PE = -m_1 g R_1 (1 - \cos \theta) + m_2 g R_2 (1 - \cos \theta)$$

(b)  $KE + PE = \text{constant}$ 

$$(M_1 R_1^2 + M_2 R_2^2) \left( \frac{d\theta}{dt} \right)^2 - m_1 g R_1 (1 - \cos \theta) + m_2 g R_2 (1 - \cos \theta) = \text{constant}$$

$$(M_1 R_1^2 + M_2 R_2^2) \left( \frac{d\theta}{dt} \right)^2 - m_1 g R_1 \left( 2 \frac{\sin^2 \theta}{2} \right) + m_2 g R_2 \left( 2 \frac{\sin^2 \theta}{2} \right) = \text{constant}$$

For small  $\theta$ ,  $\sin^2 \left( \frac{\theta}{2} \right) \approx \frac{\theta^2}{4}$ .

$$(M_1 R_1^2 + M_2 R_2^2) \left( \frac{d\theta}{dt} \right)^2 + \frac{\theta^2}{2} g (M_2 R_2 - M_1 R_1) = \text{constant}$$

$$\frac{1}{2} (2) (M_1 R_1^2 + M_2 R_2^2) \left( \frac{d\theta}{dt} \right)^2 + \frac{1}{2} g (M_2 R_2 - M_1 R_1) \theta^2 = \text{constant}$$

$$(c) \quad k_{\text{eff}} = g (M_2 R_2 - M_1 R_1)$$

$$T = \frac{2\pi}{\omega}$$

$$m_{\text{eff}} = 2 (M_1 R_1^2 + M_2 R_2^2)$$

$$= 2\pi \sqrt{\frac{2 (M_1 R_1^2 + M_2 R_2^2)}{g (M_2 R_2 - M_1 R_1)}}$$

$$\omega = \sqrt{\frac{k_{\text{eff}}}{m_{\text{eff}}}} = \sqrt{\frac{g (M_2 R_2 - M_1 R_1)}{2 (M_1 R_1^2 + M_2 R_2^2)}}$$

4.

Alternative Method to (b) and (c):

$$(m_1 R_1^2 + m_2 R_2^2) \left( \frac{d\theta}{dt} \right)^2 - m_1 g R_1 (1 - \cos\theta) + m_2 g R_2 (1 - \cos\theta) = \text{constant}$$

$$(m_1 R_1^2 + m_2 R_2^2) (2) \left( \frac{d\theta}{dt} \right) \left( \frac{d^2\theta}{dt^2} \right) - m_1 g R_1 \sin\theta \frac{d\theta}{dt} + m_2 g R_2 \sin\theta \frac{d\theta}{dt} = 0$$

$$2(m_1 R_1^2 + m_2 R_2^2) \left( \frac{d^2\theta}{dt^2} \right) + g(m_2 R_2 - m_1 R_1) \sin\theta = 0$$

$$\frac{d^2\theta}{dt^2} \approx - \frac{g(m_2 R_2 - m_1 R_1)}{2(m_1 R_1^2 + m_2 R_2^2)} \theta$$

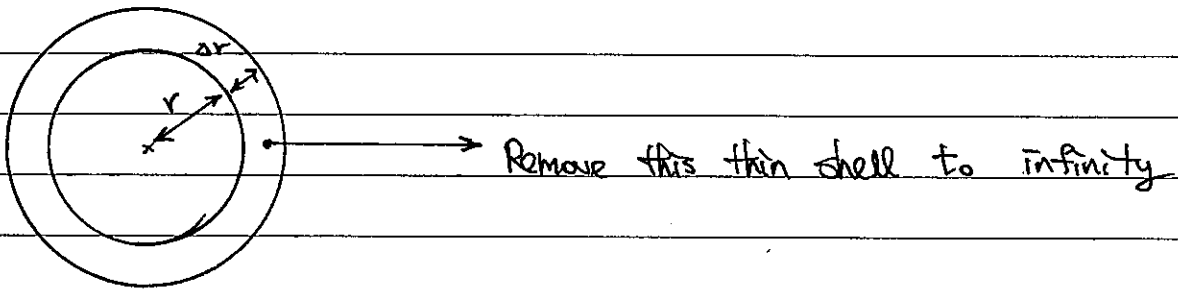
$$\Rightarrow \omega = \frac{g(m_2 R_2 - m_1 R_1)}{2(m_1 R_1^2 + m_2 R_2^2)}$$

$$T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{2(m_1 R_1^2 + m_2 R_2^2)}{g(m_2 R_2 - m_1 R_1)}}$$

(d) When  $m_1 R_1 \rightarrow 0$ ,

$$T = 2\pi \sqrt{\frac{2m_2 R_2^2}{g m_2 R_2}} = 2\pi \sqrt{\frac{2R_2}{g}}$$

5.



Energy to remove a thin shell at radius  $r$ :

$$E = - \frac{G M_r m_r}{r}$$

where  $M_r$  = mass within the radius  $r = \frac{4\pi r^3 \rho}{3}$

$$m_r = \text{mass of thin shell at radius } r \text{ and thickness } \Delta r. \\ = -4\pi r^2 \rho \Delta r \quad (\rho = \text{Density of the Sun})$$

Energy to remove the  $i$ th thin shell:

$$E_i = - \frac{G}{r} \left( \frac{4\pi r^3 \rho}{3} \right) (-4\pi r^2 \rho \Delta r) \\ = \frac{G (4\pi)^2 \rho^2 r^4 \Delta r}{3}$$

$$E_{\text{total}} = \sum_i E_i \\ = \frac{(4\pi)^2 G \rho^2}{3} \sum_i \left( \frac{R_s}{N} i \right)^4 \left( \frac{R_s}{N} \right)$$

↑

[ Suppose the Sun is divided into  $N$  thin shells, where  $N \rightarrow \infty$  ]

5.

$$\begin{aligned}
 E_{\text{total}} &= \frac{(4\pi)^2}{3} \frac{G \rho^2}{N^5} R_S^5 \sum_i i^4 \\
 &= \frac{(4\pi)^2}{3} \frac{G \rho^2}{N^5} R_S^5 \left( \frac{6N^5}{30} \right) \\
 &= \frac{(4\pi)^2}{3} G \rho^2 \frac{R_S^5}{5} \\
 &= \frac{(4\pi)^2}{3} G \left( \frac{M_S}{\frac{4}{3}\pi R_S^3} \right)^2 \frac{R_S^5}{5} \\
 &= \frac{3}{5} G \frac{M_S^2}{R_S}
 \end{aligned}$$