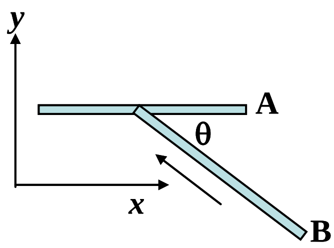
Pan Pearl River Delta Physics Olympiad 2005

Jan. 29, 2005
Morning Session (9 am – 12 pm)

Q1 (5 points)

Two identical worms of length L are lying on a smooth and horizontal surface. The mass of the worms is evenly distributed along their body length. The starting positions of the two worms are shown in the figure. The coordinate of the center of worm-A is (0, 0). Worm-B then starts to climb slowly over worm-A with their bodies always form an angle θ. After Worm-B has completely climbed over worm-A, what are the center positions of the two worms?



Q2 (13 points)

An air bubble of size 0.001m^3 and a rigid tank of the same volume and mass as the bubble are released at a depth of 2.0 km below the sea surface. Ignore friction. The temperature of the air bubble remains the same at any depth. Air density at sea level is 1.21 kg/m^3, and the atmosphere pressure is 1.0 x 10^5 N/m^2. (hint: $\int_a^b \frac{dx}{\alpha+\beta x} = \frac{1}{\beta}\ln\left(\frac{\alpha+\beta b}{\alpha+\beta a}\right)$)

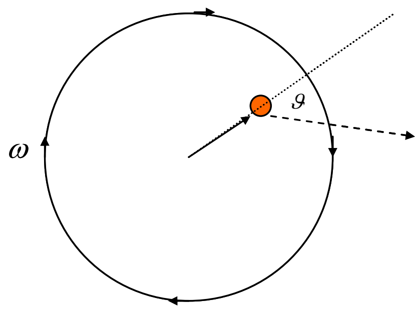
(a) What is the size of the bubble when it rises to the sea level? **(**3 points**)**

(b) Derive an expression for the net energy gained by the bubble and the tank at height h. **(**7 points**)**

(c) Find the final velocities of the bubble and the tank when they reach the sea level. **(**3 points**)**

Q3 (12 points)

A man with mass 0.5M is standing on a round table (disk shaped, uniform thickness) rotating at angular speed ω. The mass of the table is 0.5M and the friction between the table and the ground is negligible. The man carries with him 10 stones each with mass 0.01M. The radius of the table is R and the man is standing at a distance r (< R) from the center of the table.



(a) Find the total angular momentum of the system. (4 points)

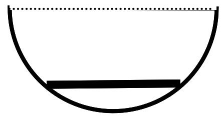
To slow down the rotation of the table, the man decides to throw the stones outward from the table, each at speed v (relative to him) and with angle ϑ relative to the radial direction (see figure).

(b) Determine the angular speed of the table after the man has thrown his first stone as a function of angle ϑ, and find the optimum angle ϑ to slow down the table. **(**4 points**)**

(c) What is the angular speed of the table after the man has thrown all his stones, each time at the optimum angle? (Leave your answer as the sum of multiple terms.) **(**4 points**)**

Q4 (8 points)

A uniform rod of length L and mass M is resting in a smooth hemisphere of radius R ($>0.5L$), as shown.



(a) Find the vibration frequency of the rod about its equilibrium position. **(**4 points**)**

(b) In the vibration motion, the maximum deviation angle of the rod from its equilibrium position is $\theta_{\max}$. Let the amplitude of the contact force from the hemisphere to the rod at each end be N. The difference between N when the rod is at $\theta_{\max}$ and when the rod is at its equilibrium position can be written as $\Delta N = \alpha Mg\theta_{\max}^2$. Find α. **(**4 points**)**

Q5 (12 points)

The electric field of an electromagnetic (EM) wave is $\vec{E} = E_0 \vec{x}_0 e^{i(kZ-\omega t)}$, where E_0 is a real constant, and $\omega = \frac{c}{\tilde{n}} k$. Here ω is real, c is the speed of light in vacuum, and $\tilde{n}$ is the complex dielectric constant of the medium.

(a) Briefly discuss what will happen to the EM wave amplitude as it propagates in the medium if $\tilde{n}$ is real, imaginary, or complex. **(**4 points**)**

(b) Find the magnetic field $\vec{B}$, and the time-averaged (over one period) Poynting's vector $<\vec{S} = \frac{1}{\mu_0}(\vec{E} \times \vec{B})>$. **(**4 points**)**

(c) The quantity $q = \frac{d<\vec{S}>}{dz}$ describes the loss of EM wave energy to the medium. Calculate q and briefly discuss the physical meanings of the results if $\tilde{n}$ is real, imaginary, or complex. **(**3 points**)**

(d) With reference to the results above, does an EM wave that decreases in amplitude while propagating always loose energy to the medium? **(**1 points**)**

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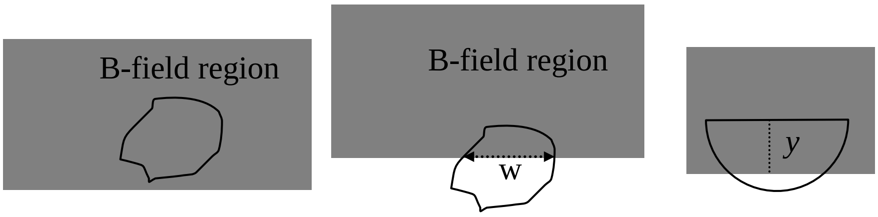
Afternoon Session (2 pm – 5 pm)

Q6 (12 points)

Consider a uniform magnetic field B within the shaded region and pointing out of the paper plane, as shown below.



(a) (b) (c)

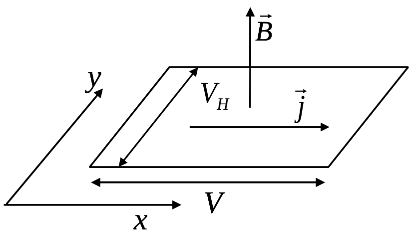
(a) Find the total force of the magnetic field on a closed thin wire coil carrying a steady electric current I, all of which is inside the field region. The coil plane is within the paper. (3 points)

(b) The total force of the magnetic field on the coil when part of it is outside the field region can be expressed as $F = \alpha wBI$, where w is the distance between the two points where the coil intersects the bottom edge of the field region, and its direction is either upward or downward depending on the direction of the current. Find the value of α. (3 points)

(c) A semicircle thin wire coil of radius r, resistance R, and mass m is falling down and out of the field region. The plane of the coil remains in the paper plane, and its straight edge remains parallel to the horizontal bottom edge of the field region. Ignore self-inductance of the coil. Derive the differential equation for the distance between the straight edge of the coil and the edge of the field region y (< R). If you have not found α in (b) you may take it as a known constant in solving this part of the problem. (6 points)

Q7 (15 points)

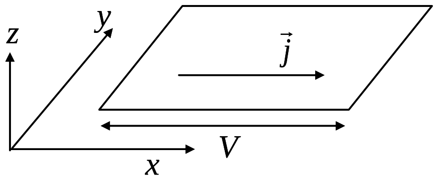
It is well known that when crossed electric and magnetic fields are applied to a piece of semiconductor, a voltage V_H perpendicular to the direction of charge motion will be induced (see figure). The phenomenon is called the *Hall Effect*.



(a) Assume that the semiconductor is a square sheet of size $W \times W$. The electric current is due to the motion of positive charge carriers each carrying charge e. The surface density of the carriers is n, and the conductivity of the semiconductor is σ. There is also a negative charge background so there is charge neutrality everywhere except at the side edges. The electric field is uniform in the semiconductor. A magnetic field B is applied in the direction perpendicular to the sheet. When a voltage V is applied a voltage V_H across the two edges parallel to the current along the x-direction will be induced, in addition to the electric current $\vec{j}$. When the steady state is reached, find the Hall Coefficient $R_H \equiv V_H / V$. (Note that $\vec{j}$ is a unknown quantity) (6 points)

Nowadays it is also known that for certain semiconductor structures, a *spin-Hall* effect will also occur. The effect is associated with the magnetic moment $\vec{m}$ of the charge carriers. For two dimensional structures it is known that an additional force $\vec{F}_R = \eta_R(\vec{m}\times\vec{v})$ (called Rashba force) will act on the carriers, where $\vec{v}$ is the velocity of the carriers on the 2-dimensional (X-Y) plane, and η_R is a constant. The magnetic moment $\vec{m}$ is restricted to point perpendicular to the plane, i.e. $\vec{m} = \pm m\hat{z}$. The external magnetic field is absence. Ignore the magnetic dipole interactions between the carriers.

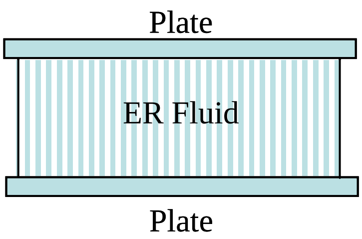
(b) Assume again that the electric field is uniform and its force along the *x*-axis is much stronger than $\vec{F}_R$, find the currents flowing in the *y*-direction in terms of the voltage *V* and other parameters given in (a). How are the currents related to $\vec{m}$? (6 points)



(c) Due to collision with the boundary, the carriers with particular magnetic moment will loose their sense of direction within a 'life time' τ after they reach the edge. In other words, each second there are n_m/τ of carriers loosing their moment direction within a unit length of edge, where n_m is the surface density of the carriers still maintaining their moment direction ($\pm\hat{z}$). Find the magnetization $\vec{M}$ near the edges. (3 points)

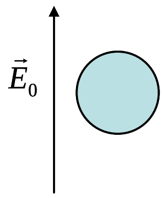
Q8 (23 points)

Electrorheological (ER) fluids, which are composed of small dielectric spheres suspended in an insulating liquid, such as silicone oil, are materials that can transform from liquid-like form to solid-like under an external electric field. A typical test setting of ER fluids is shown in the figure, where ER fluid is filled between two parallel conducting plates of area *A* separated by a distance *D*. When no voltage is applied between the plates the ER fluid is liquid-like so the plates can moved horizontally almost without friction.



When a voltage *V* is applied, the small spheres are polarized and aligned into vertical columns, and to move a conducting plate relative to the other by a small displacement δx requires a small force δf. The shear modulus η is defined as $\eta = \frac{D}{A}\frac{\delta f}{\delta x}$. The radius of the spheres is *R* (<< *D*), their dielectric constant is ε, and the volume fraction of spheres to fluid is *m*. The dielectric constant of the liquid without the spheres is 1. Ignore gravity. You are to find η in terms of the physical qualities given above.

(a) The first step is to find the polarization $\vec{P}$ of an isolated sphere in a uniform external electric field $\vec{E}_0$. This can be done by solving (a1) – (a3) below, and <u>utilizing the known fact</u> that under such circumstance the polarization is <u>uniform</u> in the sphere and parallel to $\vec{E}_0$.



(a1) Find the electric field <u>due to the polarization</u> $\vec{P}$ at the center of the sphere. (3 points)
(a2) Find the total electric field inside the sphere. (3 points)

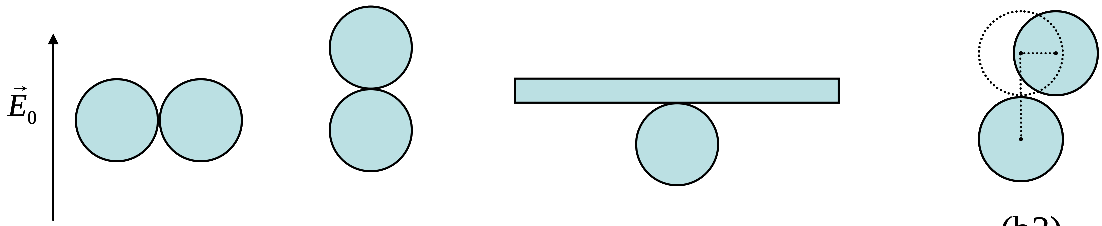
(a3) The total induced electric dipole moment of a sphere can be expressed as $\vec{p}_0 = \alpha \vec{E}_0$. Find the constant α. (3 points)

(b) Treat each sphere as an ideal electric dipole located at the center of the sphere, and assume that the dipole moment depends only on $\vec{E}_0$. If you have not found α in (a3) you may take it as a known constant in solving the following problems. (Hint: Keep the expansion terms up to d^2, where d is the length of the dipole.)

(b1) Find the electrostatic energies of two spheres in contact in the side-by-side and the top-bottom configurations, as shown in the figures below. (4 points)

(b2) Find the electrostatic force of the conductor plate on the sphere that is in contact with the plate. (3 points)

(b3) Find the restoring horizontal force between two spheres when the upper one in the top-bottom configuration is displaced horizontally by a small distance δa, as shown. (3 points)



(b1) (b2) (b3)

(c) Assume that under the applied electric field, <u>all</u> spheres form continuous, straight, and single file thin columns between the plates. According to your answers in (b1), do the columns like to bunch together? Consider only the force between adjacent spheres within a column, when the top plate is displaced by a small distance δx, the top sphere of each column remains stick to the plate and is moved by the same distance. As shown in the figure, each sphere in the column below is then displaced uniformly relative to the one just above. The bottom spheres remain fixed to the bottom plate. Find the shear modulus η. (4 points)