

**Q1: (4 points)**

In the process energy and momentum are conserved, so  
碰撞前后动量和能量守恒, 因此

$$m_A v_0 = m_B v_{Bx} \quad 0 = m_B v_{By} + m_A v_A \quad (1 \text{ point})$$

$$m_A v_0^2 = m_A v_A^2 + m_B v_{Bx}^2 + m_B v_{By}^2 + 2E \quad (1 \text{ point})$$

Where,  $v_A$  is in the y-direction, we can get 其中  $v_A$  是沿 Y-方向的速度。由此得

$$v_A = \sqrt{\frac{m_A(m_B - m_A)v_0^2 - 2Em_B}{m_A(m_B + m_A)}}, \quad (1 \text{ point})$$

$$v_{Bx} = \frac{m_A}{m_B} v_0, \quad v_{By} = -\frac{m_A}{m_B} v_A. \quad (1 \text{ point})$$

**Q2: (6 points)**

From Gauss theorem, the area density of the free charge at the surfaces of the conductor plates is  $\sigma_f = D$ . (0.5 points) 根据高斯定理, 导电板上自由电荷面密度与板间电位移的关系为  $\sigma_f = D$ 。

In the air gap  $E_1 = E_3 = D / \varepsilon_0$ , while in the dielectric  $E_2 = D / \varepsilon\varepsilon_0$ . (0.5 points)

在空气中  $E_1 = E_3 = D / \varepsilon_0$ , 在介质内  $E_2 = D / \varepsilon\varepsilon_0$ , 电压为

$$V = \frac{d}{3} E_1 + \frac{d}{3} E_2 + \frac{d}{3} E_3 = \frac{dD}{3\varepsilon_0} \left(2 + \frac{1}{\varepsilon}\right) = \frac{d\sigma_f}{3\varepsilon_0} \left(2 + \frac{1}{\varepsilon}\right),$$

由此得

$$\sigma_f = \frac{V}{d} \frac{3\varepsilon_0\varepsilon}{2\varepsilon+1}. \quad (1 \text{ point})$$

The bound charge is  $\sigma_b = \mp \varepsilon_0(E_1 - E_2) = \mp \frac{3V}{d} \frac{\varepsilon_0(\varepsilon-1)}{2\varepsilon+1}$ . (1 point)

介质上、下界面的束缚电荷面密度为  $\sigma_b = \mp \varepsilon_0(E_1 - E_2) = \mp \frac{3V}{d} \frac{\varepsilon_0(\varepsilon-1)}{2\varepsilon+1}$ 。

When the slab is moving with speed  $v$ , the electric currents at two sides are  $K = \pm \sigma_b v$ . From Ampere law,  $B_1 = B_3 = 0$  (1 point)

介质板运动时, 上、下界面的束缚电流面密度为  $K = \pm \sigma_b v$ 。由安培定理得在空隙间的磁场  $B_1 = B_3 = 0$

Inside the dielectric slab the magnetic field is 在介质板里的磁场

$$B_2 = \mu_0 K = \mu_0 \sigma_b v = \mu_0 v \frac{3V}{d} \frac{\varepsilon_0(\varepsilon-1)}{2\varepsilon+1}. \quad (1 \text{ point})$$

When the parallel conductor plates are moving with speed  $-v$ , the electric currents at two plates are  $K' = \mp \sigma_f v = \mp v \frac{V}{d} \frac{3\varepsilon_0\varepsilon}{2\varepsilon+1}$ . From the Ampere law,  $B_1 = B_2 = B_3 = -\mu_0 \sigma_f v$ . (1 point)

当导电板运动时, 上、下板的面电流密度为  $K' = \mp \sigma_f v = \mp v \frac{V}{d} \frac{3\varepsilon_0\varepsilon}{2\varepsilon+1}$ 。

由安培定理得  $B_1 = B_2 = B_3 = -\mu_0 \sigma_f v$

**Q3: (7 points)**

The electron spin (angular momentum) is  $I$ , and the associated magnetic dipole moment is

$$\vec{M} = -\frac{ge}{2m} \vec{I}.$$

In the B-field, the torque on the spin is  $\vec{M} \times \vec{B}$  and perpendicular to  $\vec{M}$ . (1 point)

The precession frequency is then determined by  $MB\Delta t = \Delta I$ , (1 point)

$$MB\Delta t = I\Delta\theta \quad (1 \text{ point})$$

$$\Rightarrow MB = I \frac{\Delta\theta}{\Delta t} = I\omega \quad (1 \text{ point})$$

$$\text{which leads to } \omega = \frac{ge}{2m} B. \quad (1 \text{ point})$$

The negative sign in  $\vec{M} = -\frac{ge}{2m} \vec{I}$  ensures that the spin turns in the same direction as the

electron trajectory under Lorentz force.  $\frac{mv^2}{R} = eBv \Rightarrow \omega' = \frac{eB}{m_e}$  (2 points)

电子的自旋（角动量）为  $I$ ，与其相关的磁偶极子为  $\vec{M} = -\frac{ge}{2m} \vec{I}$ 。

在磁场里磁偶极子受的力矩为  $\vec{M} \times \vec{B}$  与  $\vec{M}$  垂直。 (1 point)

$\vec{M}$  进动的频率由下式可求：

$$MB\Delta t = \Delta I, \quad (1 \text{ point})$$

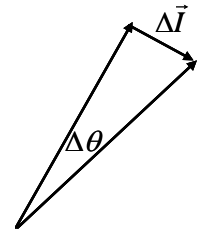
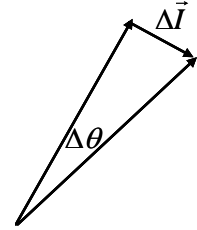
$$MB\Delta t = I\Delta\theta \quad (1 \text{ point})$$

$$\Rightarrow MB = I \frac{\Delta\theta}{\Delta t} = I\omega \quad (1 \text{ point})$$

$$\text{得 } \omega = \frac{ge}{2m} B. \quad (1 \text{ point})$$

空间的圆轨迹为：  $\frac{mv^2}{R} = eBv \Rightarrow \omega' = \frac{eB}{m_e}$  (2 points)

$\vec{M} = -\frac{ge}{2m} \vec{I}$  中的负号保证了自旋的旋转方向与它在空间的圆轨迹一致。

**Q4 (12 points)**

(a) Consider a thin layer of air at rest, the pressure difference balances the gravity,

取一薄层空气，上、下面上的压强差刚好与重力平衡，

$$p(h + \Delta h) - p(h) = -m\rho g \Delta h$$

$$\Rightarrow \frac{dp}{dh} = -m\rho g \quad (1 \text{ point})$$

(b) Put in the ideal gas law  $p = \rho RT$ , the differential equation is then

代入理想气体方程  $p = \rho RT$ ，得微分方程

$$\frac{dp}{dh} = -m\rho g = -\frac{mg}{RT} p. \quad (1 \text{ point})$$

(c) From (b), 由(b)解得

$$\frac{dp}{p} = -\frac{mg}{RT} dh \Rightarrow \ln p - \ln p_0 = -\frac{mg}{RT} h \Rightarrow p(h) = p_0 e^{-\frac{mg}{RT} h}. \quad (3 \text{ points})$$

Put in the numbers, 代入数值,

$$\frac{mg}{RT} = \frac{0.029 \times 9.8}{8.31 \times 300} = \frac{1}{8.8} \text{ km}^{-1}.$$

So the height is 得高度为

$$8.8 \times \ln 2 = 8.8 \times 0.693 = 6.1 \text{ km}. \quad (1 \text{ point})$$

(d) With a constant wind with velocity  $v$  we replace the pressure equation by the Bernoulli's equation

有风时, 微分方程为

$$\frac{dp}{dh} + \frac{mv^2}{2} \frac{d\rho}{dh} = -mg\rho.$$

Using the ideal gas law, we obtain 代入  $p = \rho RT$  理想气体方程, 得

$$\frac{dp}{dh} \left(1 + \frac{mv^2}{2RT}\right) = -\frac{mg}{RT} p. \quad (1 \text{ point})$$

Therefore 解得  $p(h) = p_0 e^{-\frac{mg}{\left(RT + \frac{mv^2}{2}\right)} h}$ . (2 point)

(e)  $\vec{\Omega} \times \vec{v} \approx \Omega \times v = 2\pi \times 500000 / (24 \times 60 \times 60 \times 60 \times 60) \approx 0.01 \text{ m/s}^2 \ll g$ . (1 point)

(f) The decay length in this case is 代入数值

$$h_0 = \ln 2 \left( \frac{RT + \frac{mv^2}{2}}{mg} \right) = 0.693 \times \frac{8.31 \times 300 + 0.5 \times 0.029 \times 2.5 \times 10^4 / 3.6}{0.029 \times 9.8} = 6.1 + 0.24 = 6.3 \text{ km}$$

(2 points)

### Q5 (11 points):

(a) When the ball arrives at point A, it begins to drop down. In this process the potential energy transforms into kinetic energy.

球到 A 点后下落, 到 B 点时, 势能变化为

$$\Delta P = MgR(1 - \cos \theta) \quad (1 \text{ point})$$

The moment of inertia about the edge is 绕球边的转动惯量为

$$\tilde{I} = I + MR^2 = \frac{7}{5} MR^2.$$

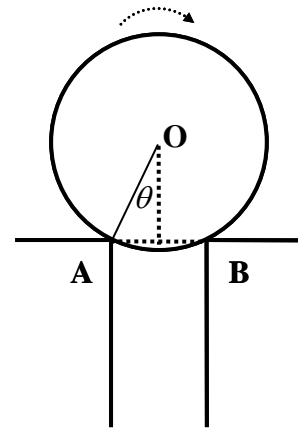
Kinetic energy is 动能为

$$K_1 = \frac{1}{2} \tilde{I} \omega^2 = \frac{1}{2} \times \frac{7}{5} MR^2 \omega^2 = \frac{7}{10} Mv^2 \quad (0.5 \text{ points})$$

$$\text{So 因此 } K_2 = \frac{7}{10} Mv'^2 = MgR(1 - \cos \theta) + \frac{7}{10} Mv^2 \quad (0.5$$

points)

$$\Rightarrow v'^2 = \frac{10}{7} gR(1 - \cos \theta) + v^2$$



$$\Rightarrow \omega'^2 = \frac{10}{7R} g(1 - \cos \theta) + \left(\frac{v}{R}\right)^2 \quad (1 \text{ point})$$

(b) The critical condition for the ball to keep contact with point A before it touches point B is that: At the moment it touches point B, the centrifugal force equals the gravity component.

要保持与 A 点接触，即球以 A 点作圆周运动，其向心力全由重力提供。

$$\frac{Mv'^2}{R} = Mg \cos \theta \quad (1 \text{ point})$$

$$\Rightarrow \frac{10}{7} gR(1 - \cos \theta) + v_{\max}^2 = gR \cos \theta \quad \Rightarrow v_{\max}^2 = \frac{gR}{7} (17 \cos \theta - 10) \quad (1 \text{ point})$$

where  $\cos \theta = \frac{\sqrt{R^2 - \frac{d^2}{4}}}{R} = \sqrt{1 - \frac{d^2}{4R^2}}$

(c) In the process of the ball collide with point B, the angular momentum of the ball around point B is unchanged. Before the collision, the angular momentum of the ball around point B is  $I\omega' + MR\tilde{v}' = I\omega' + MR(R\omega' \cos 2\theta) = MR^2\omega' \left(\frac{2}{5} + \cos 2\theta\right)$ .

After the collision, it is  $\tilde{I}\omega'' = \frac{7}{5} MR^2\omega''$ .

在与 B 点碰撞过程中，球相对于该点的角动量守恒。碰撞前的角动量为

$I\omega' + MR\tilde{v}' = I\omega' + MR(R\omega' \cos 2\theta) = MR^2\omega' \left(\frac{2}{5} + \cos 2\theta\right)$ ，碰撞后的角动量为

$$\tilde{I}\omega'' = \frac{7}{5} MR^2\omega''.$$

Hence 因此

$$\frac{7}{5} MR^2\omega'' = MR^2\omega' \left(\frac{2}{5} + \cos 2\theta\right) \Rightarrow \omega'' = \frac{2 + 5 \cos 2\theta}{7} \omega'. \quad (2 \text{ points})$$

So the total energy after collision is 碰撞后的动能为

$$K_3 = \frac{7}{10} MR^2\omega''^2 = \frac{1}{70} (2 + 5 \cos 2\theta)^2 Mv'^2 \quad (1 \text{ point})$$

The requirement for the ball to get over the ditch:

要翻上沟边，动能要克服的势能为

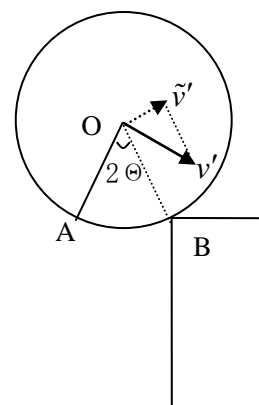
$$K_3 = \frac{1}{70} (2 + 5 \cos 2\theta)^2 Mv'^2 > MgR(1 - \cos \theta) \quad (1 \text{ point})$$

$$\Rightarrow v^2 > 10gR \left[ \frac{7}{(2 + 5 \cos 2\theta)^2} - \frac{1}{7} \right] (1 - \cos \theta)$$

$$\Rightarrow v_{\min}^2 = 10gR \left[ \frac{7}{(2 + 5 \cos 2\theta)^2} - \frac{1}{7} \right] (1 - \cos \theta) \quad (2 \text{ points})$$

(d) We must have 根据题意， $v_{\min}^2 < v_{\max}^2$

$$\text{既 } \frac{1}{10} \cos \theta - \frac{7(1 - \cos \theta)}{(2 + 5 \cos 2\theta)^2} > 0.$$



(Numerical result of Max  $\theta$ : 数值计算得最大角为 0.597797 弧度, 或  $34^\circ$ .)

**Q6 (10 points)**

Suppose the spring is extended, and choose the natural length as the origin of the coordinate of the small block  $X_2 = 0$ . The Dynamic equation of this system is

设弹簧是拉长的, 选弹簧在自然长度时小物块的坐标  $X_2 = 0$ 。系统的运动方程为

$$F + KX_2 = M_1 \ddot{X}_1 \quad (1 \text{ point}) \quad -KX_2 - M_2 \ddot{X}_1 = M_2 \ddot{X}_2 \quad (2) \quad (1 \text{ point})$$

Note that 由于  $\ddot{X}_{1,2} = -\omega^2 X_{1,2}$ , (1 point)

From (2) we get 由(2)得  $X_2 = \frac{\omega^2 M_2}{K - \omega^2 M_2} X_1$ . (1 point)

Then 因此  $F = -M_1 \omega^2 X_1 - K \frac{\omega^2 M_2}{K - \omega^2 M_2} X_1 = -\omega^2 X_1 (M_1 + \frac{KM_2}{K - \omega^2 M_2})$  (2 points)

(a) Finally 最后得  $M_{eff} = \frac{F}{-\omega^2 X_1} = M_1 + \frac{KM_2}{K - \omega^2 M_2} = M_1 - \frac{KM_2}{\omega^2 M_2 - K}$  (1 point)

(b) Negative effective mass 负有效质量:

For negative  $M_{eff}$ , we get, after some algebra,  $\omega^2 < K(\frac{1}{M_1} + \frac{1}{M_2})$ . However, the term

$\frac{KM_2}{\omega^2 M_2 - K}$  must be positive. So the final answer is

$$\frac{K}{M_2} < \omega^2 < K(\frac{1}{M_1} + \frac{1}{M_2}) \quad (3 \text{ points}) \quad \text{Missing } \frac{K}{M_2} < \omega^2, (-1 \text{ point})$$

要使  $M_{eff} < 0$ , 经过简单代数运算, 得  $\omega^2 < K(\frac{1}{M_1} + \frac{1}{M_2})$ 。但是  $\frac{KM_2}{\omega^2 M_2 - K}$  必须是正的。

因此得

$$\frac{K}{M_2} < \omega^2 < K(\frac{1}{M_1} + \frac{1}{M_2}) \quad (3 \text{ points}) \quad \text{漏掉 } \frac{K}{M_2} < \omega^2, (-1 \text{ point})$$

Part-II

**Q1 (16 points):**

(a) By Lorentz force law,  $\mathbf{F} = -q\mathbf{v} \times \mathbf{B}$ ,  $F$  has no  $z$ -component when  $\mathbf{B}$  is along the  $z$ -direction. Hence if  $v_z = 0$  initially, it always remains zero. (1 point)

磁场的力为  $\mathbf{F} = -q\mathbf{v} \times \mathbf{B}$ , 与  $XY$  面平行。由于初速度的  $Z$ -分量  $v_z = 0$ , 所以粒子保持在  $XY$  面上运动。

(b) The  $B$ -field at  $r = a$  is just right to keep the particle on a circular orbit of radius  $a$ .

在  $r = a$  处的磁场刚好可以维持粒子以  $a$  为半径的圆周运动。

$$\frac{mv_0^2}{a} = qv_0 B_0 \Rightarrow v_0 = \frac{qB_0 a}{m}. \quad (2 \text{ points})$$

(c) Let the angular momentum of the charge about the origin be  $L$ . Then  $L(t=0) = mav_0$ .

令粒子绕原点的角动量为  $L$ . 则  $L(t=0) = mav_0$ 。

$$\frac{dL}{dt} = -rF \sin \theta \quad (1 \text{ point})$$

$$= -qrvB \cos \phi = -qBrv_r \quad (1 \text{ point}). \quad v_r = \frac{dr}{dt} \text{ is the radial velocity 径向速度分量 } v_r = \frac{dr}{dt}.$$

$$= qBr \frac{dr}{dt} \quad (1 \text{ point})$$

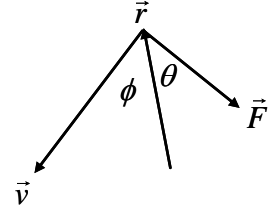
$$\Rightarrow \frac{dL}{dr} = qBr \quad (1 \text{ point})$$

One can also obtain the same differential equation by  
也可用下列公式

$$\frac{dL}{dt} = \mathbf{r} \times \mathbf{F} = -q\mathbf{r} \times (\mathbf{v} \times \mathbf{B}) = -q[(\mathbf{r} \cdot \mathbf{B})\mathbf{v} - (\mathbf{r} \cdot \mathbf{v})\mathbf{B}] = qrv_r \mathbf{B} = qr \frac{dr}{dt} \mathbf{B} \quad (2 \text{ points})$$

$$\frac{dL}{dt} = qBr \frac{dr}{dt} \quad (1 \text{ point})$$

$$\Rightarrow \frac{dL}{dr} = qBr \quad (1 \text{ point})$$



$$L(r) - L(a) = \int_a^r qBr dr = qB_0 a (r - a). \quad (1 \text{ point})$$

(d) Because  $B$  field does not do work, the speed of the charge is always  $v_0$ . Note that the angular momentum can be  $L = \pm mrv_0$

由于磁场不做功, 粒子的速率一直为  $v_0$ 。注意角动量可以是  $L = \pm mrv_0$ 。

When  $L = mrv_0$ ,  $r = a$ , no tangential motion occurs afterwards. (1 point)

当  $L = mrv_0$ ,  $r = a$ 。这只有在初始时有, 之后就不再出现了。

When 当  $L = -mrv_0$ ,  $r = a \frac{qB_0 a - mv_0}{qB_0 a + mv_0}$ . (1 point)

When  $qB_0 a < mv_0$ , the only solution is  $r = a$ , which corresponds to the initial condition. No tangential motion occurs afterwards. (1 point)

当  $qB_0 a < mv_0$ , 只有初始的  $r = a$ , 之后就不再出现了。

$$\text{当 } qB_0a > mv_0, (1 \text{ point}) \quad r = a \frac{qB_0a - mv_0}{qB_0a + mv_0} < a. (1 \text{ point})$$

It is possible. 这是个可以出现的情形。

(e) When the charge is moving in the radial direction,  $L = 0$ . (1 point)

粒子沿径向运动时,  $L = 0$ 。

Therefore, 因此

$$r = a \left( 1 - \frac{mv_0}{qB_0a} \right). \quad (1 \text{ point})$$

When  $qB_0a > mv_0$ ,  $r > 0$ , the motion can be radial. (0.5 points)

当  $qB_0a > mv_0$ ,  $r > 0$ , 粒子可沿径向运动

When  $qB_0a < mv_0$ ,  $r < 0$ , the motion never becomes radial. (0.5 points)

当  $qB_0a < mv_0$ ,  $r < 0$ , 粒子不可沿径向运动。

## Q2 (16 points)

(A.1) The differential equation 微分方程为:  $-I\dot{\omega} = \tau + \gamma\omega$  (2 points)

(A.2)

$$\frac{d\omega}{\omega + \frac{\tau}{\gamma}} = -\frac{\gamma}{I} dt \Rightarrow \int_{\omega_0}^{\omega} \frac{d\omega}{\omega + \frac{\tau}{\gamma}} = -\frac{\gamma}{I} \int_0^{t_s} dt \Rightarrow \ln \frac{\omega + \frac{\tau}{\gamma}}{\omega_0 + \frac{\tau}{\gamma}} = -\frac{\gamma}{I} t_s \Rightarrow t_s = \frac{I}{\gamma} \ln(1 + \frac{\gamma\omega_0}{\tau}) \quad (2 \text{ points})$$

$$\text{So } A = \frac{I}{\gamma}, (1 \text{ point}); \quad B = \frac{\gamma}{\tau}, (1 \text{ point})$$

(B)

- First, add each pair of blocks at equal distance to the center in order to avoid warbling, keep  $\omega_0$  fixed and measure stop time  $t_{sn}$

$$t_{sn} = \frac{I + n\Delta I}{\gamma} \ln(1 + \frac{\gamma\omega_0}{\tau}) = \frac{n\Delta I}{\gamma} \ln(1 + \frac{\gamma\omega_0}{\tau}) + \frac{I}{\gamma} \ln(1 + \frac{\gamma\omega_0}{\tau}). \text{ Here } n = 1, 2, \dots, \text{ and}$$

$\Delta I = mr^2$ , where  $m$  is the mass of the small block, and  $r$  is the distance to the center of the fan measured by the ruler. Plot  $t_{sn} \sim n\Delta I$ , one gets the slope  $K = \frac{1}{\gamma} \ln(1 + \frac{\gamma\omega_0}{\tau})$  and

the interception  $b = \frac{I}{\gamma} \ln(1 + \frac{\gamma\omega_0}{\tau})$ . Then  $I = b / K$ . (4 points)

- 首先, 保持  $\omega_0$  为常数, 将每对小重物放在离轴等距离的两边的叶片上, 测量停止时间  $t_{sn}$ 。

$$t_{sn} = \frac{I + n\Delta I}{\gamma} \ln(1 + \frac{\gamma\omega_0}{\tau}) = \frac{n\Delta I}{\gamma} \ln(1 + \frac{\gamma\omega_0}{\tau}) + \frac{I}{\gamma} \ln(1 + \frac{\gamma\omega_0}{\tau}), \text{ 其中 } n = 1, 2, \dots, \Delta I = mr^2,$$

$m$  是每块小重物的质量,  $r$  是小重物离轴的距离。将  $t_{sn} \sim n\Delta I$  作图, 得直线的斜率

$K = \frac{1}{\gamma} \ln(1 + \frac{\gamma\omega_0}{\tau})$ , 与 Y 轴的交点  $b = \frac{I}{\gamma} \ln(1 + \frac{\gamma\omega_0}{\tau})$ 。求得转动惯量  $I = b / K$ 。

- Secondly, let the initial angular  $\omega_0$  be very slow such that

$$t_s = A \ln(1 + B\omega_0) \approx AB\omega_0 = \frac{I}{\tau} \omega_0.$$

From the slope of the  $t_s \sim \omega_0$  line we can get the value of  $\tau$ . (3 points)

第二，将初始转速调小，使  $t_s = A \ln(1 + B\omega_0) \approx AB\omega_0 = \frac{I}{\tau} \omega_0$  成立。测  $t_s \sim \omega_0$  并作图。

直线的斜率为  $\frac{I}{\tau}$ 。由此得  $\tau$ 。

- Finally, let the initial angular  $\omega_0$  be very fast so that

$$t_s = A \ln(1 + B\omega_0) \approx A \ln(B\omega_0) = \frac{I}{\gamma} \left[ \ln\left(\frac{\gamma}{\tau}\right) + \ln(\omega_0) \right].$$

Change  $\omega_0$  and plot  $t_s \sim \ln(\omega_0)$  which forms a straight line. The slope is  $I/\gamma$ . Since  $I$  is already known,  $\gamma$  can be readily obtained. (3 points)

最后，将初始转速调大，使  $t_s = A \ln(1 + B\omega_0) \approx A \ln(B\omega_0) = \frac{I}{\gamma} \left[ \ln\left(\frac{\gamma}{\tau}\right) + \ln(\omega_0) \right]$  成立。测  $t_s \sim \ln(\omega_0)$  并作图。直线的斜率为  $I/\gamma$ 。由此得  $\gamma$ 。

### Q3 (18 points)

Ans:

- a) (i) For  $R < R_0$  there must be a constant current  $i$  is flowing in the system to increase  $R_{NR}$  so the total resistance of the system will not be negative.

当  $R < R_0$ ，需有电流使  $R_{NR}$  增加，使系统的总电阻不为负。

$$\left( R - R_0 \left( 1 - \left( \frac{i}{i_0} \right)^2 \right) \right) i = 0 \quad (\text{Kirchhoff's law}) \quad (0.5 \text{ points})$$

$$\Rightarrow i = \pm \sqrt{\frac{R_0 - R}{R_0}} i_0 \quad (0.5 \text{ points})$$

$$\text{Voltage drop across } R_{NR} = -R_0 \left( 1 - \left( \frac{i}{i_0} \right)^2 \right) i = -Ri = - \text{voltage drop across } R. \quad (0.5$$

points)

$$\text{电压为 } i R_{NR} = -R_0 \left( 1 - \left( \frac{i}{i_0} \right)^2 \right) i = -Ri$$

For  $R > R_0$ , the total resistance  $> 0$ , so the solution is  $i = 0$ . There is no voltage drop. (0.5 points)

若  $R > R_0$ ，则系统的总电阻为正，电流为零。

(ii)  $L$  has no resistance. So a constant current  $i = \pm i_0$  flows through the system and  $R_{NR} = 0$ . There is no voltage drop anywhere. (1 point)

电感无直流电阻，所以  $R_{NR} = 0$ 。电流为  $i = \pm i_0$ ，但无电压。



(iii) In this case  $i = 0$  and charge  $q = \pm q_o$  is needed to maintain the circuit in equilibrium ( $R_{NR} = 0$ ). (1 point)

Therefore a minimum voltage  $V_o = \pm q_o / C$  is needed to maintain the system at equilibrium.

There is no voltage drop across  $R_{NR}$ . (1 point)

直流电流  $i = 0$ 。电容上的电荷为  $q = \pm q_o$ ， $R_{NR} = 0$ 。所需最小电压为  $V_o = \pm q_o / C$ 。 $R_{NR}$  上无电压。

b) (i) For  $R < R_0$  the Kirchhoff's Law becomes 当  $R < R_0$ ，有

$$\left( R - R_0 \left( 1 - \left( \frac{i}{i_o} \right)^2 \right) \right) i = V_o$$

Writing  $i = \sqrt{\frac{R_0 - R}{R_0}} i_o + j = i' + j$  where  $j$  is small,

代入  $i = \sqrt{\frac{R_0 - R}{R_0}} i_o + j = i' + j$ ，其中  $j$  为一级小量，

we obtain to linear order in  $j$  保持  $j$  的一级小量，得

$$V_o = \left[ R - R_0 \left( 1 - \left( \frac{i' + j}{i_o} \right)^2 \right) \right] (i' + j) \approx 2R_0 \sqrt{\frac{R_0 - R}{R_0}} \frac{j}{i_o} (i' + j) \approx 2(R_0 - R)j, \text{ (1 point)}$$

so  $j = \frac{V_o}{2(R_0 - R)}$  for both AC and DC. (1 point)

最后的（无论是 AC 或 DC） $j = \frac{V_o}{2(R_0 - R)}$ 。

For  $R > R_0$ , the original current is zero.  $R_0 \left( \left( \frac{j}{i_o} \right)^2 - 1 \right) \approx -R_0$ .

So we obtain  $j = \frac{V_o}{(R - R_0)}$ . (1 point)

当  $R > R_0$ ，原来的电流为零。因此  $R_0 \left( \left( \frac{j}{i_o} \right)^2 - 1 \right) \approx -R_0$ ，得  $j = \frac{V_o}{(R - R_0)}$ 。

<<Notice that for  $R = R_0$ , we obtain 当  $R = R_0$  时，得

$$R_0 \left( \frac{j}{i_o} \right)^2 j = V \Rightarrow j = \left( \frac{V}{R_0} i_o^2 \right)^{\frac{1}{3}} \gg$$

(ii) In this case we obtain 在此情形，我们有

$$-R_0 \left( 1 - \left( \frac{i_o + j}{i_o} \right)^2 \right) (i_o + j) + L \frac{d(i_o + j)}{dt} = V, \quad (1 \text{ point})$$

Note that 但是  $\frac{di_o}{dt} = 0$ .

$$-R_0 \left( 1 - \left( \frac{i_o + j}{i_o} \right)^2 \right) (i_o + j) = R_0 \left( 1 + \frac{2j}{i_o} - 1 \right) (i_o + j) = 2R_0 j$$

We then obtain 由此得微分方程  $2R_0 j + L \frac{dj}{dt} = V$ . (1 point)

For AC voltage  $V(t) = V_0 \sin \omega t$ , the solution of this equation is  $j = j_o \sin(\omega t + \delta)$  where

$$\tan \delta = -\frac{L\omega}{2R_0} \quad \text{and} \quad j_o = \frac{V_0}{2R_0 \cos \delta - L\omega \sin \delta}. \quad (1 \text{ point})$$

(Or  $\frac{dj}{dt} = -i\omega j$ , and  $j_o = \frac{V_0}{2R_0 - iL\omega}$ )

令  $V(t) = V_0 \sin \omega t$ , 解为  $j = j_o \sin(\omega t + \delta)$ , 代入微分方程得  $\tan \delta = -\frac{L\omega}{2R_0}$ ,

$$j_o = \frac{V_0}{2R_0 \cos \delta - L\omega \sin \delta}.$$

(或用  $\frac{dj}{dt} = -i\omega j$ , 代入微分方程得  $j_o = \frac{V_0}{2R_0 - iL\omega}$ 。两种解等价。)

(iii) For small additional voltage source there is additional small amount of charge  $q'$ . The equation becomes

有小电源时, 原来的电荷会增加一小量  $q'$ , 原来的方程变为

$$-R_0 \left( 1 - \left( \frac{q_o + q'}{q_o} \right)^2 \right) \left( \frac{d(q_o + q')}{dt} \right) + \frac{q_o + q'}{C} = V + V' \quad (1 \text{ point})$$

Note that  $\frac{dq_o}{dt} = 0$ , and  $\frac{q_o}{C} = V$ . The above equation then becomes

$$R_0 \left( 1 + \frac{2q'}{q_o} - 1 \right) \frac{dq'}{dt} + \frac{q'}{C} = V' \Rightarrow \frac{q'}{C} = V'.$$

It is true for both AC and DC. (1 point)

由于  $\frac{dq_o}{dt} = 0$ ,  $\frac{q_o}{C} = V$ , 上述方程简化成  $R_0 \left( 1 + \frac{2q'}{q_o} - 1 \right) \frac{dq'}{dt} + \frac{q'}{C} = V' \Rightarrow \frac{q'}{C} = V'$ 。

AC 或 DC 都适用。

(iv) For DC voltage at equilibrium  $i = 0$  and the situation is same as (b(ii)). A minimum voltage  $V_o = \pm q_o / C$  is needed to maintain the system at equilibrium. (2 points)

和(b(ii))相同。  $V_o = \pm q_o / C$

(v) In the presence of an additional AC voltage, Kirchhoff's Law becomes  
多一个 AC 电源, 方程为

$$R_0 \left( \left( \frac{q_o + q'}{q_o} \right)^2 + i_o^{-2} \left( \frac{d(q_o + q')}{dt} \right)^2 - 1 \right) \left( \frac{d(q_o + q')}{dt} \right) + \frac{q_o + q'}{C} + L \frac{d^2(q_o + q')}{dt^2} \quad (1 \text{ point})$$

$$= V + V'$$

Note  $\frac{q_o}{C} = V$ , and  $\frac{dq_o}{dt} = 0$ , the first term becomes

由于  $\frac{q_o}{C} = V$ ,  $\frac{dq_o}{dt} = 0$ , 上述方程第一项简化成

$$R_0 \left( \left( \frac{q_o + q'}{q_o} \right)^2 + i_o^{-2} \left( \frac{d(q_o + q')}{dt} \right)^2 - 1 \right) \left( \frac{d(q_o + q')}{dt} \right)$$

$$= R_0 \left( \frac{2q'}{q_o} + i_o^{-2} \left( \frac{dq'}{dt} \right)^2 \right) \left( \frac{dq'}{dt} \right) = 0$$

So we obtain 最终得  $\frac{q'}{C} + L \frac{d^2 q'}{dt^2} = V'$ , (1 point)

Therefore, for  $V'(t) = V_o \sin \omega t$ . We obtain  $q'(t) = \frac{V_o}{(C^{-1} - L\omega^2)} \sin \omega t$ . (0.5 points)

And  $j(t) = \frac{\omega V_o}{(C^{-1} - L\omega^2)} \cos \omega t$  (0.5 points)

用交流形式解代入, 得  $q'(t) = \frac{V_o}{(C^{-1} - L\omega^2)} \sin \omega t$ ,  $j(t) = \frac{\omega V_o}{(C^{-1} - L\omega^2)} \cos \omega t$ 。

Or 或

$$\lll q' = \frac{V_o}{(C^{-1} - L\omega^2)}, j = \frac{dq'}{dt} = \frac{-i\omega V_o}{(C^{-1} - L\omega^2)} \ggg$$